An Improved H_∞ Approach for Networked Control Systems with Transmission Delays and Packet Dropout

Milad Farsi[1], Arash Farnam[2] and Reza Mahboobi Esfanjani[1]
[1]Electrical Engineering Department, Sahand University of Technology, Tabriz, Iran, m_farsi/mahboobi@sut.ac.ir
[2]SYSTeMS Research Group, Ghent University, Gent, Belgium, arash.farnam@ugent.be

Abstract— **This paper is concerned with the H_∞ controller design for robust stabilization of networked control systems with the network-induced delay, data packet dropout and norm-bounded parameter uncertainties. In order to obtain less conservative results, a new augmented Lyapunov-Krasovskii functional is used and novel free-weighting matrices are employed to make some extra degree of freedom in the H_∞ design conditions. The feedback gain of a memoryless controller, maximum allowable delay bound and minimum disturbance attenuation level can be derived by solving a set of linear matrix inequalities (LMIs). The advantages of the proposed method are demonstrated by numerical example.**

Keywords-Networked Control Systems; Robust Control; Stabilization; Lyapunov-Krasovskii Theorem; Linear Matrix Inequality (LMI); Minimum attenuation level; Parameter uncertainties.

I. INTRODUCTION

A networked control system (NCS) is a feedback control configuration wherein the sensors, controllers and actuators exchange data via a communication network. In NCS the communication network is included in control loops to achieve low cost, simple installation, easy maintenance and high flexibility. However, the presence of communication link brings hard to solve problems compared with traditional point-to-point control approaches. The Data packet dropout and latency in the communication channels are the main issues in the analysis and design of NCSs.

Robust H_∞ stabilization for uncertain linear systems with the assumption that the controller is continuous time has been investigated already in the literature [1], [2]. However, in NCSs, a continuous-time system often is controlled by a discrete-time controller. This issue motivated a lot of researches in the stabilization [3-4] and H_∞ stabilization [5-6] of NCSs during the recent years.

The common method to investigate stability analysis and controller gain synthesis is based utilizing different Lyapunov-Krasovskii functional including double-integral terms [3-6]. [3] surveyed the problem of stability and controller design according to using Lyapunov-Krasovskii functional, and the results of [3] were improved in [4] by utilizing new Lyapunov-Krasovskii functional. For the first time, augmented Lyapunov-Krasovskii functional to obtain sufficient conditions for designing robust H_∞ controller gain to satisfy robust stability for NCSs was introduced in [6] and this paper [6] also improved the results of the proposed approach in [3]. Further improvement for investigating robust stability for NCSs was achieved in [7] by introducing new weighting matrices to enhance the degree of freedom considerably. [8] and [9] investigated the robust stability problem for NCSs with considering the closed-loop system as discrete time model with binary random delay and Markovian jumping parameters, respectively.

In this paper, an approach is proposed to design H_∞ static state feedback controller for NCSs based on a new augmented Lyapunov-Krasovskii functional, including tripe-integral terms. The continuous-time plant is controlled by discrete-time controller; hence the closed loop system has the sample and hold devices.

This paper is organized as follows: In section II, a continuous time model for NCSs is described. Sufficient conditions for the H_∞ stability analysis and state feedback control design of NCSs are introduced in section III. In section IV, numerical benchmark example is presented to illustrate the efficiency of the proposed approach. Section V concludes the paper.

Notation: In this paper, * denotes block in the symmetric matrix. I is identity matrix of appropriate dimension. The notation $P > 0$ (respectively$P \geq 0$) means that P is real symmetric and positive define (respectively, positive semi definite). The superscript T stands for matrix transposition.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

The controlled system is described as follows:

$$\begin{gathered} \dot{x}(t) = Ax(t) + Bu(t) + E\omega(t), \\ Z(t) = Cx(t) + Du(t) \\ A = A_0 + \Delta A(t), \quad B = B_0 + \Delta B(t) \end{gathered} \tag{1}$$

where $x(t)\epsilon R^n$, $u(t)\epsilon R^m$, $\omega(t)\epsilon R^n$ and $Z(t)\epsilon R^q$ are the state vector, control input vector, disturbance vector and controlled output, respectively; A_0, B_0, E, C and D are known system matrices with appropriate dimensions. It is assumed that the pair *(A,B)* is completely controllable. $\Delta A(t)$ and $\Delta B(t)$ denotes the norm-bounded parameter uncertainties in plant satisfying :

$$[\Delta A(t) \quad \Delta B(t)] = J\Delta(t)[H_1 \quad H_2] \tag{2}$$

where J, H_1 and H_2 are known constant matrices with appropriate dimensions and $\Delta(t)$ is unknown time-varying matrix satisfying $\Delta^T(t)\Delta(t) \leq I$. Without loss of generality, we mention the following assumption:

Assumption: we suppose A and E are matrices with same dimensions. The system (1) is controlled through the network. The considered NCS structure is shown in Fig.1, where the controller and actuator are event-driven and sampler is clock-driven. The sampling period is assumed to be h, where h is a positive constant. The transmission delay may not be necessarily integer multiples of the sampling period, so zero order hold (Z.O.H) device's information may be updated between sampling instants.

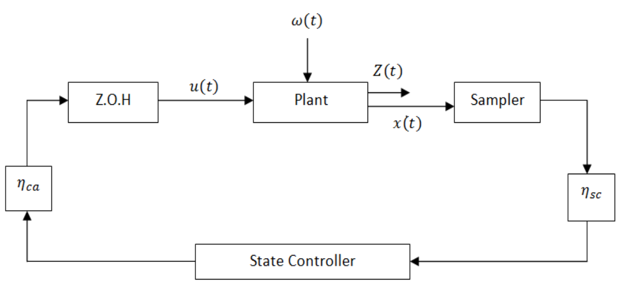

Figure 1. Networked Control System

Since the controller is a constant gain, the feedback and forward delays are combined together at each sampling time. The updating instant of Z.O.H are t_k and the successfully transmitted signals from the sampler to the controller and from the controller to Z.O.H at the instant t_k experience signal transmission delay η_k, where $\eta_k = \eta_{sc_k} + \eta_{ca_k}$ (η_{sc_k} and η_{ca_k} are delays from the sampler to the controller and from the controller to the Z.O.H at the updating instant t_k, respectively). Therefore, the state feedback with considering the behavior of the Z.O.H takes the following form:

$$u(t_k) = Kx(t_k - \eta_k) \qquad t_k \leq t < t_{k+1} \tag{3}$$

in which t_{k+1} is next updating state after t_k . The network-induced delay η_k is bounded as the following inequality:

$$\eta_m \leq \eta_k \leq \eta_M \tag{4}$$

where η_M and η_m are the lower and upper bounds of the network-induced delay, respectively. Then, the closed-loop system in Fig. 1 is described by:

$$\dot{x}(t) = Ax(t) + BKx(t_k - \eta_k) + E\omega(t)$$

$$Z(t) = Cx(t) + DKx(t_k - \eta_k) \qquad t_k \leq t < t_{k+1} \tag{5}$$

which is the form of sampled-data system. Moreover, at the updating instant t_k, the number of accumulated data packet dropout since the last updating instant t_{k-1} is denoted by τ_k, where $0 \leq \tau_k \leq \tau_M$. Combining the above-mentioned facts yields to:

$$t_{k+1} - t_k = \eta_{k+1} - \eta_k + (\tau_{k+1} + 1)h \tag{6}$$

Now, let $\eta(t) = t - t_k + \eta_k$ is replaced in (5), then the following continuous time model is obtained for the closed-loop NCS in Fig. 1:

$$\begin{gathered}\dot{x}(t) = Ax(t) + BKx\big(t - \eta(t)\big) + E\omega(t) \\ Z(t) = Cx(t) + DKx(t - \eta(t))\end{gathered} \tag{7}$$

in which,

$$\eta_m \leq \eta(t) \leq \eta \tag{8}$$

with $\eta = \eta_M + (\tau_M + 1)h$. It's evident that η is related to the maximum number of accumulated data packet dropouts τ_M, the upper bound of network-induced delay η_M and the sampling period h of the sampler device.

H_∞ *Control Problem:* System (7) is said robustly asymptotically stable with H_∞ norm bound $\gamma > 0$ if the following conditions are satisfied:

1) The closed-loop system (7) is asymptotically stable when $\omega(t) = 0$ for all uncertainties $\Delta A(t)$ and $\Delta B(t)$.

2) Under the zero conditions, the controlled output $Z(t)$ satisfies $\|Z(t)\|_2 \leq \gamma\|\omega(t)\|_2$ for all nonzero $\omega(t)\epsilon L_2[0, \infty)$.

Before proceeding further, the following lemma is introduced to handle the norm-bounded parameter uncertainties:

Lemma: Given real matrices Σ, Σ_1 and Σ_2 with appropriate dimensions, with $\Sigma^T = \Sigma$, then

$$\Sigma + \Sigma_2\Delta(t)\Sigma_1 + \Sigma_1^T\Delta^T(t)\Sigma_2^T < 0 \tag{9}$$

holds if and only if for all $\Delta^T(t)\Delta(t) \leq I$ and some $\epsilon > 0$ the following inequality holds $\Sigma + \epsilon\Sigma_2\Sigma_2^T + \epsilon^{-1}\Sigma_1^T\Sigma_1 < 0$ which can be modified by Schur complement to the following matrix inequality:

$$\begin{bmatrix} \Sigma & \Sigma_1^T & \epsilon\Sigma_2 \\ \Sigma_1 & -\epsilon I & 0 \\ \epsilon\Sigma_2^T & 0 & -\epsilon I \end{bmatrix} < 0 \tag{10}$$

III. MAIN RESULTS

In this section, a new delay-dependent H_∞ stability condition is proposed in Theorem 1 to ensure robust stability of the closed-loop system (7) for all delays satisfying (8). Then, controller synthesis condition is derived in Theorem 2.

Theorem 1: For given η_m, η, J, H_1, H_2 and K, the closed-loop system (7) is robustly asymptotically stable with the H_∞ norm bound γ if there exist matrices N_z, $L_z (z = 0,1,2)$, M, R, S, F, symmetric matrices $P = [P_{ij}]_{5\times5}$, $Q_1 = [Q_{1_{ij}}]_{2\times2} > 0$, $Q_2 =$

$\left[Q_{2_{ij}}\right]_{2\times 2} > 0$, $T_1 = \left[T_{1_{ij}}\right]_{2\times 2} > 0$, $T_2 = \left[T_{2_{ij}}\right] > 0$, $Z_1 > 0$, $Z_2 > 0$, U_1, U_2, $V_z = \left[V_{z_{ij}}\right]_{8\times 8} (z = 0,1)$, X_0, X_1, X_2, $W_z = \left[W_{z_{ij}}\right]_{8\times 8}$, $(z = 0,1)$ with appropriate dimensions and scalar ϵ, satisfying (11-17).

$$\begin{bmatrix} P & R & S \\ * & U_1 & T \\ * & * & U_2 \end{bmatrix} > 0 \tag{11}$$

$$\begin{bmatrix} \Sigma & \Sigma_1^T & \epsilon\Sigma_2 \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} V_0 & L_0 + \psi_0 & N_0 \\ * & T_{1_{11}} & T_{1_{12}} + X_0 \\ * & * & T_{1_{22}} \end{bmatrix} \geq 0 \tag{13}$$

$$\begin{bmatrix} V_1 & L_1 + \psi_1 & N_1 \\ * & T_{2_{11}} & T_{2_{12}} + X_1 \\ * & * & T_{2_{22}} \end{bmatrix} \geq 0 \tag{14}$$

$$\begin{bmatrix} V_1 & L_1 + \psi_1 & N_2 \\ * & T_{2_{11}} & T_{2_{12}} + X_2 \\ * & * & T_{2_{22}} \end{bmatrix} \geq 0 \tag{15}$$

$$\begin{bmatrix} W_0 & L_0 + \varphi_0 \\ * & Z_1 \end{bmatrix} \geq 0 \tag{16}$$

$$\begin{bmatrix} W_1 & L_1 + \varphi_1 \\ * & Z_2 \end{bmatrix} \geq 0 \tag{17}$$

where $\hat{\eta} = \eta - \eta_m, \bar{\eta} = \frac{1}{2}(\eta^2 - \eta_m^2)$,

$\Sigma = \pi_1 + \pi_{2i} + \pi_{2i}^T + \pi_{3_0} + \pi_{3_0}^T + \eta_m V_0 + \hat{\eta}\, V_1 + \frac{\eta_m^2}{2} W_0 + \bar{\eta}\, W_1$,

$\pi_2 = [N_0 + \eta_m L_0 + \hat{\eta}\, L_1 \quad -N_0 + N_1 \quad -N_2 \;\; 0 \;\; 0 \;\; 0 \;\; -N_1 + N_2 \;\; 0]$,

$$\pi_1 = \begin{bmatrix} (1,1) & (1,2) \\ * & (2,2) \end{bmatrix}, \quad (1,1) = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ * & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ * & * & \Lambda_{33} & \Lambda_{34} \\ * & * & * & \Lambda_{44} \end{bmatrix},$$

$\Lambda_{11} = P_{14} - R_1 + P_{14}^T - R_1^T + Q_{1_{11}} + \eta_m T_{1_{11}} + \hat{\eta}\, T_{2_{11}} + X_0 + C^T C$,

$\Lambda_{12} = -P_{14} + P_{15} + R_1 - S_1 + P_{24}^T - R_2^T, \Lambda_{13} = -P_{15} + S_1 + P_{34}^T - R_3^T$, $\Lambda_{14} = P_{11} + \eta_m R_1 + \hat{\eta}\, S_1 + Q_{1_{12}} + \eta_m T_{1_{12}} + \hat{\eta}\, T_{2_{12}}$,

$\Lambda_{22} = -P_{24} + P_{25} + R_2 - S_2 - P_{24}^T + P_{25}^T + R_2^T - S_2^T - Q_{1_{11}} + Q_{2_{11}} - X_0 + X_1$, $\Lambda_{23} = -P_{25} + S_2 - P_{34}^T + P_{35}^T + R_3^T - S_3^T$

$\Lambda_{24} = P_{12}^T + \eta_m R_2 + \hat{\eta}\, S_2$, $\Lambda_{34} = P_{13}^T + \eta_m R_3 + \hat{\eta}\, S_3$

$\Lambda_{33} = -P_{35} + S_3 - P_{35}^T + S_3^T - Q_{2_{11}} - X_2$,

$\Lambda_{44} = Q_{1_{22}} + \eta_m T_{1_{22}} + \hat{\eta}\, T_{2_{22}} + \frac{\eta_m^2}{2} Z_1 + \bar{\eta} Z_2$,

$$(1,2) = \begin{bmatrix} P_{12} & P_{13} & C^T DK & 0 \\ P_{22} - Q_{1_{12}} + Q_{2_{12}} & P_{23} & 0 & 0 \\ P_{23}^T & P_{33} - Q_{2_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$(2,2) = \begin{bmatrix} -Q_{1_{22}} + Q_{2_{22}} & 0 & 0 & 0 \\ * & -Q_{2_{22}} & 0 & 0 \\ * & * & -X_1 + X_2 + K^T D^T DK & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix},$$

$\Sigma_2 = [-J^T M_1^T, -J^T M_2^T, -J^T M_3^T, -J^T M_4^T, -J^T M_5^T, -J^T M_6^T, -J^T M_7^T, -J^T M_8^T]^T$, $\Sigma_1 = [H_1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad H_2 K \quad 0]$,

$\pi_{3_0} = [-M A_0 \quad 0 \quad 0 \quad M \quad 0 \quad 0 \quad -M B_0 K \quad -ME]$,

$$\psi_0 = \begin{bmatrix} -P_{44} + R_4^T \\ P_{44} - P_{45}^T - R_4^T + S_4^T \\ P_{45}^T - S_4^T \\ -P_{14} - \eta_m R_4^T - \hat{\eta} S_4^T \\ -P_{24} \\ -P_{34} \\ 0 \\ 0 \end{bmatrix}, \quad \psi_1 = \begin{bmatrix} -P_{45} + R_5^T \\ P_{45} - P_{55} - R_5^T + S_5^T \\ P_{55} - S_5^T \\ -P_{15} - \eta_m R_5^T - \hat{\eta} S_5^T \\ -P_{25} \\ -P_{35} \\ 0 \\ 0 \end{bmatrix},$$

$$\varphi_0 = \begin{bmatrix} -R_4 + U_1 \\ -R_4 - R_5 - U_1 + F^T \\ R_5 - F^T \\ -R_1 - \eta_m U_1 - \hat{\eta} F^T \\ -R_2 \\ -R_3 \\ 0 \\ 0 \end{bmatrix} \text{ and } \varphi_1 = \begin{bmatrix} -S_4 + F \\ S_4 - S_5 - F + U_2 \\ S_5 - U_2 \\ -S_1 - \eta_m F - \hat{\eta} U_2 \\ -S_2 \\ -S_3 \\ 0 \\ 0 \end{bmatrix}.$$

Proof : Define a Lyapunov-Krasovskii functional as follows:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t) \tag{18}$$

$$V_1(x_t) = \xi^T(t) \begin{bmatrix} P & R & S \\ * & U_1 & F \\ * & * & U_2 \end{bmatrix} \xi(t) \tag{19}$$

$$V_2(x_t) = \int_{t-\eta_m}^{t} \tau^T(\alpha) Q_1 \tau(\alpha) d\alpha + \int_{t-\eta}^{t-\eta_m} \tau^T(\alpha) Q_2 \tau(\alpha) d\alpha \tag{20}$$

$$V_3(x_t) = \int_{-\eta_m}^{0} \int_{t+\beta}^{t} \tau^T(\alpha) T_1 \tau(\alpha) d\alpha d\beta + \int_{-\eta}^{-\eta_m} \int_{t+\beta}^{t} \tau^T(\alpha) T_2 \tau(\alpha) d\alpha d\beta \tag{21}$$

$$V_4(x_t) = \int_{-\eta_m}^{0} \int_{\beta}^{0} \int_{t+\theta}^{t} \dot{x}^T(\alpha) Z_1 \dot{x}(\alpha) d\alpha d\theta d\beta + \int_{-\eta}^{-\eta_m} \int_{\beta}^{0} \int_{t+\theta}^{t} \dot{x}^T(\alpha) Z_2 \dot{x}(\alpha) d\alpha d\theta d\beta \tag{22}$$

wherein, $\xi(t) = col[x(t), \quad x(t-\eta_m), \quad x(t-\eta), \quad \int_{t-\eta_m}^{t} x(\alpha)d\alpha, \quad \int_{t-\eta}^{t-\eta_m} x(\alpha)d\alpha, \quad \eta_m x(t) - \int_{t-\eta}^{t-\eta_m} x(\alpha)d\alpha, \quad (\eta - \eta_m)x(t) - \int_{t-\eta}^{t-\eta_m} x(\alpha)d\alpha, \quad \tau(\alpha) = col[x(\alpha) \quad \dot{x}(\alpha)]$. Now consider the following equation:

$$j_{z\omega} = \int_0^\infty [z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t)] dt \tag{23}$$

under zero-initial conditions, we have $V(x_0) = 0$ and $V(x_\infty) \geq 0$, so (22) can be rewritten to the following inequality;

$$j_{z\omega} = \int_0^\infty \left[z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V}(x_t)\right] dt - V(x_\infty) \leq \int_0^\infty [z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V}(x_t)] \mathrm{dt} \tag{24}$$

So the closed-loop system (7) is robustly asymptotically stable with disturbance attenuation level γ if and only if satisfying (24):

$$z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V}(x_t) < 0 \tag{25}$$

The time derivative of $V(x_t)$ along the trajectories of (7) is obtained as follows:

$$\dot{V}_1(x_t)=2\xi^T(t)\begin{bmatrix}P & R & S\\ * & U_1 & F\\ * & * & U_2\end{bmatrix}\dot{\xi}(t) \tag{26}$$

$$\begin{aligned}\dot{V}_2(x_t)&=\tau^T(t)Q_1\tau(t)-\tau^T(t-\eta_m)Q_1\tau(t-\eta_m)\\&+\tau^T(t-\eta_m)Q_2\tau^T(t-\eta_m)-\tau^T(t-\eta)Q_2\tau(t-\eta)\end{aligned} \tag{27}$$

$$\begin{aligned}\dot{V}_3(x_t)&=\tau^T(t)(\eta_m T_1+\hat{\eta}T_2)\tau(t)-\int_{t-\eta_m}^{t}\tau^T(\alpha)T_1\tau(\alpha)d\alpha-\\&\int_{t-\eta(t)}^{t-\eta_m}\tau^T(\alpha)T_2\tau(\alpha)d\alpha-\int_{t-\eta}^{t-\eta(t)}\tau^T(\alpha)T_2\tau(\alpha)d\alpha\end{aligned} \tag{28}$$

$$\begin{aligned}\dot{V}_4(x_t)&=\dot{x}^T(t)\left(\frac{\eta_m^2}{2}Z_1+\bar{\eta}Z_2\right)\dot{x}(t)-\int_{-\eta_m}^{0}\int_{t+\beta}^{t}\dot{x}^T(\alpha)Z_1\dot{x}(\alpha)\,d\alpha d\beta\\&-\int_{-\eta(t)}^{-\eta_m}\int_{t+\beta}^{t}\dot{x}^T(\alpha)Z_2\dot{x}(\alpha)d\alpha d\beta-\int_{-\eta}^{-\eta(t)}\int_{t+\beta}^{t}\dot{x}^T(\alpha)Z_2\dot{x}(\alpha)d\alpha d\beta\end{aligned} \tag{29}$$

For any matrices N_0, N_1, N_2, M, L_0, L_1 and L_2 and symmetric matrices V_0, V_1,, W_0, W_1, X_0, X_1, and X_2 with appropriate dimensions, the following equalities hold:

$$\varepsilon_1(t)=2\zeta^T(t)N_0(x(t)-x(t-\eta_m)-\int_{t-\eta_m}^{t}\dot{x}(\alpha)d\alpha)=0 \tag{30}$$

$$\varepsilon_2(t)=2\zeta^T(t)N_1\left(x(t-\eta_m)-x(t-\eta(t))-\int_{t-\eta(t)}^{t-\eta_m}\dot{x}(\alpha)d\alpha\right)=0 \tag{31}$$

$$\varepsilon_3(t)=2\zeta^T(t)N_2\left(x\big(t-\eta(t)\big)-x(t-\eta)-\int_{t-\eta}^{t-\eta(t)}\dot{x}(\alpha)d\alpha\right)=0 \tag{32}$$

$$\varepsilon_4(t)=2\zeta^T(t)M\left(\dot{x}(t)-Ax(t)-BKx\big(t-\eta(t)\big)-E\omega(t)\right)=0 \tag{33}$$

$$\begin{aligned}\varepsilon_5(t)&=2\zeta^T(t)L_0(\eta_m x(t)-\int_{t-\eta_m}^{t}x(\alpha)d\alpha\\&-\int_{-\eta_m}^{0}\int_{t+\beta}^{t}\dot{x}(\alpha)d\alpha d\beta=0\end{aligned} \tag{34}$$

$$\begin{aligned}\varepsilon_6(t)&=2\zeta^T(t)L_1[(\eta-\eta_m)x(t)-\int_{t-\eta(t)}^{t-\eta_m}x(\alpha)d\alpha-\int_{t-\eta}^{t-\eta(t)}x(\alpha)d\alpha\\&-\int_{-\eta}^{-\eta_m}\int_{t+\beta}^{t}\dot{x}(\alpha)d\alpha d\beta]=0\end{aligned} \tag{35}$$

$$\varepsilon_7(t)=\eta_m\zeta^T(t)V_0\zeta(t)-\int_{t-\eta_m}^{t}\zeta^T(t)V_0\zeta(t)d\alpha=0 \tag{36}$$

$$\varepsilon_8(t)=(\eta-\eta_m)\zeta^T(t)V_1\zeta(t)-\int_{t-\eta}^{t-\eta_m}\zeta^T(t)V_1\zeta(t)d\alpha=0 \tag{37}$$

$$\varepsilon_9(t)=\frac{\eta_m^2}{2}\zeta^T(t)W_0\zeta(t)-\int_{-\eta_m}^{0}\int_{t+\beta}^{t}\zeta^T(t)W_0\zeta(t)d\alpha d\beta=0 \tag{38}$$

$$\varepsilon_{10}(t)=\frac{(\eta^2-\eta_m^2)}{2}\zeta^T(t)W_1\zeta(t)-\int_{-\eta}^{-\eta_m}\int_{t+\beta}^{t}\zeta^T(t)W_1\zeta(t)d\alpha d\beta=0 \tag{39}$$

$$\begin{aligned}\varepsilon_{11}(t)&=x^T(t)X_0x(t)-x^T(t-\eta_m)X_0x(t-\eta_m)-\\&2\int_{t-\eta_m}^{t}\dot{x}^T(\alpha)X_0x(\alpha)d\alpha=0\end{aligned} \tag{40}$$

$$\begin{aligned}\varepsilon_{12}(t)&=x^T(t-\eta_m)X_1x(t-\eta_m)-x^T\big(t-\eta(t)\big)X_1x(t-\eta(t))-\\&2\int_{t-\eta(t)}^{t-\eta_m}\dot{x}^T(\alpha)X_1x(\alpha)d\alpha=0\end{aligned} \tag{41}$$

$$\begin{aligned}\varepsilon_{13}(t)&=x^T(t-\eta(t))X_2x(t-\eta(t))-x^T(t-\eta)X_2x(t-\eta)-\\&2\int_{t-\eta}^{t-\eta(t)}\dot{x}^T(\alpha)X_2x(\alpha)d\alpha=0\end{aligned} \tag{42}$$

Where $\zeta(t)=col[\ x(t),\ \ x(t-\eta_m),\ \ x(t-\eta),\ \ \dot{x}(t),\ \ \dot{x}(t-\eta_m)$ $,\dot{x}(t-\eta),x(t-\eta(t)),\omega(t)]$. Now based on (26-29) and combining (30-42), $z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)+\dot{V}(x_t)$ can be stated as follows:

$$\begin{aligned}&z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)+\dot{V}(x_t)\\&=\dot{V}_1(x_t)+\dot{V}_2(x_t)+\dot{V}_3(x_t)+\dot{V}_4(x_t)+\textstyle\sum_{i=1}^{i=13}\varepsilon_i(t)+\\&+(Cx(t)+DKx(t-\eta(t))^T(Cx(t)+DKx\big(t-\eta(t)\big)-\gamma^2\omega^T(t)\omega(t)\end{aligned} \tag{43}$$

The $\dot{V}+Z^TZ-\gamma^2\omega^T\omega$ in (45) can be rewritten as

$$\dot{V}(x_t)+z^T(t)z(t)-\gamma^2\omega^T(t)\omega(t)=\zeta^T(t)\,\pi\zeta(t)+\textstyle\sum_{i=1}^{i=5}\Omega_i(t) \tag{44}$$

where $\pi=\pi_1+\pi_2+\pi_2^T+\pi_3+\pi_3^T+\eta_m V_0+\hat{\eta}V_1+\frac{\eta_m^2}{2}W_0+\bar{\eta}W_1$

$$\Omega_1(t)=-\int_{t-\eta_m}^{t}\begin{bmatrix}\zeta(t)\\x(\alpha)\\\dot{x}(\alpha)\end{bmatrix}^T\begin{bmatrix}V_0 & L_0+\psi_0 & N_0\\ * & T_{1_{11}} & T_{1_{12}}+X_0\\ * & * & T_{1_{22}}\end{bmatrix}\begin{bmatrix}\zeta(t)\\x(\alpha)\\\dot{x}(\alpha)\end{bmatrix}d\alpha,$$

$$\Omega_2(t)=-\int_{t-\eta(t)}^{t-\eta_m}\begin{bmatrix}\zeta(t)\\x(\alpha)\\\dot{x}(\alpha)\end{bmatrix}^T\begin{bmatrix}V_1 & L_1+\psi_1 & N_1\\ * & T_{2_{11}} & T_{2_{12}}+X_1\\ * & * & T_{2_{22}}\end{bmatrix}\begin{bmatrix}\zeta(t)\\x(\alpha)\\\dot{x}(\alpha)\end{bmatrix}d\alpha,$$

$$\Omega_3(t)=-\int_{t-\eta}^{t-\eta(t)}\begin{bmatrix}\zeta(t)\\x(\alpha)\\\dot{x}(\alpha)\end{bmatrix}^T\begin{bmatrix}V_1 & L_1+\psi_1 & N_2\\ * & T_{2_{11}} & T_{2_{12}}+X_2\\ * & * & T_{2_{22}}\end{bmatrix}\begin{bmatrix}\zeta(t)\\x(\alpha)\\\dot{x}(\alpha)\end{bmatrix}d\alpha,$$

$$\Omega_4(t)=-\int_{-\eta_m}^{0}\int_{t+\beta}^{t}\begin{bmatrix}\zeta(t)\\\dot{x}(\alpha)\end{bmatrix}^T\begin{bmatrix}W_0 & L_0+\varphi_0\\ * & Z_1\end{bmatrix}\begin{bmatrix}\zeta(t)\\\dot{x}(\alpha)\end{bmatrix}d\alpha d\beta, \text{ and}$$

$$\Omega_5(t)=-\int_{-\eta}^{-\eta_m}\int_{t+\beta}^{t}\begin{bmatrix}\zeta(t)\\\dot{x}(\alpha)\end{bmatrix}^T\begin{bmatrix}W_1 & L_1+\varphi_1\\ * & Z_2\end{bmatrix}\begin{bmatrix}\zeta(t)\\\dot{x}(\alpha)\end{bmatrix}d\alpha d\beta.$$

Provided $\pi<0$, and $\Omega_i\geq 0\ (i=1,\dots,5)$, the Lyapunov-Krasovskii theorem ensures that the system (7) is asymptotically stable. ■

The sufficient conditions derived for the H_∞ stability of the closed-loop system (7) in Theorem 1 are in the form of nonlinear matrix inequalities. For a given controller K, this Theorem can be used to determine the maximum value of allowable delay η and minimum disturbance attenuation level γ which retain the robust stability of the controlled system. In the Theorem 2, utilizing changing variable technique, the nonlinear conditions in Theorem 1 is modified to obtain equivalent linear matrix inequalities (LMIs) which are computationally more tractable to obtain controller gain.

Theorem 2: For given constants η_m, η and γ and scalars $\rho_i(i=2,\dots,8)$, the closed-loop system (7) is robustly asymptotically stable for H_∞ level γ with the control gain $K=YX^{-T}$ if there exist nonsingular matrix X, matrices $\bar{N}_z$, $\bar{L}_z$ $(z=0,1,2)$, $\bar{R}$, $\bar{S}$, $\bar{F}$, Y and symmetric matrices $\bar{P}=\left[\bar{P}_{ij}\right]_{5\times 5}$, $\bar{Q}_1=\left[\bar{Q}_{1_{ij}}\right]_{2\times 2}>0$, $\bar{Q}_2=\left[\bar{Q}_{2_{ij}}\right]_{2\times 2}>0$, $\bar{T}_1=\left[\bar{T}_{1_{ij}}\right]_{2\times 2}>0$, $\bar{T}_2=\left[\bar{T}_{2_{ij}}\right]_{2\times 2}>0$, $\bar{Z}_1>0$, $\bar{Z}_2>0$, $\bar{U}_1$, $\bar{U}_2$, $\bar{V}_z=\left[\bar{V}_{z_{ij}}\right]_{8\times 8}$, $\bar{W}_z=\left[\bar{W}_{z_{ij}}\right]_{8\times 8}$, $\bar{X}_z$ $(z=0,1,2)$ with appropriate dimensions and scalar $\epsilon>0$ such that the following LMIs hold (45-51):

$$\begin{bmatrix} \bar{P} & \bar{R} & \bar{S} \\ * & \bar{U}_1 & \bar{F} \\ * & * & \bar{U}_2 \end{bmatrix}>0 \tag{45}$$

$$\begin{bmatrix} \bar{\bar{\Sigma}} & \bar{\Sigma}_1^T & \epsilon\bar{\Sigma}_2 & \bar{\mho}_1 & \bar{\mho}_2 \\ \bar{\Sigma}_1 & -\epsilon I & 0 & 0 & 0 \\ \epsilon\bar{\Sigma}_2^T & 0 & -\epsilon I & 0 & 0 \\ \bar{\mho}_1^T & 0 & 0 & -I & 0 \\ \bar{\mho}_2^T & 0 & 0 & 0 & I \end{bmatrix}<0 \tag{46}$$

$$\begin{bmatrix} \bar{V}_0 & \bar{L}_0+\bar{\psi}_0 & \bar{N}_0 \\ * & \bar{T}_{1_{11}} & \bar{T}_{1_{12}}+\bar{X}_0 \\ * & * & \bar{T}_{1_{22}} \end{bmatrix}\geq 0 \tag{47}$$

$$\begin{bmatrix} \bar{V}_1 & \bar{L}_1+\bar{\psi}_1 & \bar{N}_1 \\ * & \bar{T}_{2_{11}} & \bar{T}_{2_{12}}+\bar{X}_1 \\ * & * & \bar{T}_{2_{22}} \end{bmatrix}\geq 0 \tag{48}$$

$$\begin{bmatrix} \bar{V}_1 & \bar{L}_1+\bar{\psi}_1 & \bar{N}_2 \\ * & \bar{T}_{2_{11}} & \bar{T}_{2_{12}}+\bar{X}_2 \\ * & * & \bar{T}_{2_{22}} \end{bmatrix}\geq 0 \tag{49}$$

$$\begin{bmatrix} \bar{W}_0 & \bar{L}_0+\bar{\varphi}_0 \\ * & \bar{Z}_1 \end{bmatrix}\geq 0 \tag{50}$$

$$\begin{bmatrix} \bar{W}_i & \bar{L}_i+\bar{\varphi}_1 \\ * & \bar{Z}_2 \end{bmatrix}\geq 0 \tag{51}$$

where

$\bar{\bar{\Sigma}}=\bar{\bar{\pi}}_1+\bar{\pi}_2+\bar{\pi}_2^T+\bar{\pi}_{3_0}+\bar{\pi}_{3_0}^T+\eta_m\bar{V}_0+\hat{\eta}\,\bar{V}_1+\frac{\eta_m^2}{2}\bar{W}_0+\bar{\eta}\,\bar{W}_1$,

$\bar{\pi}_2=$
$[\bar{N}_0+\eta_m\bar{L}_0+\hat{\eta}\,\bar{L}_1 \quad -\bar{N}_0+\bar{N}_1 \quad -\bar{N}_2 \quad 0 \quad 0 \quad 0 \quad -\bar{N}_1+\bar{N}_2 \quad 0]$,

$$\bar{\bar{\pi}}_1=\begin{bmatrix} \overline{\overline{(1,1)}} & \overline{\overline{(1,2)}} \\ * & \overline{\overline{(2,2)}} \end{bmatrix},\qquad \overline{\overline{(1,1)}}=\begin{bmatrix} \bar{\Lambda}_{11} & \bar{\Lambda}_{12} & \bar{\Lambda}_{13} & \bar{\Lambda}_{14} \\ * & \bar{\Lambda}_{22} & \bar{\Lambda}_{23} & \bar{\Lambda}_{24} \\ * & * & \bar{\Lambda}_{33} & \bar{\Lambda}_{34} \\ * & * & * & \bar{\Lambda}_{44} \end{bmatrix},$$

$\bar{\bar{\Lambda}}_{11}=\bar{P}_{14}-\bar{R}_1+\bar{P}_{14}^T-\bar{R}_1^T+\bar{Q}_{1_{11}}+\eta_m\bar{T}_{1_{11}}+\hat{\eta}\,\bar{T}_{2_{11}}+\bar{X}_0$,

$$\overline{\overline{(1,2)}}=\begin{bmatrix} \bar{P}_{12} & \bar{P}_{13} & 0 & 0 \\ \bar{P}_{22}-\bar{Q}_{1_{12}}+\bar{Q}_{2_{12}} & \bar{P}_{23} & 0 & 0 \\ \bar{P}_{23}^T & \bar{P}_{33}-\bar{Q}_{2_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\overline{\overline{(2,2)}}=\begin{bmatrix} -\bar{Q}_{1_{22}}+\bar{Q}_{2_{22}} & 0 & 0 & 0 \\ * & -\bar{Q}_{2_{22}} & 0 & 0 \\ * & * & -\bar{X}_1+\bar{X}_2 & 0 \\ * & * & * & 0 \end{bmatrix},$$

$\bar{\Sigma}_2=[-J^T,-\rho_2J^T,-\rho_3J^T,-\rho_4J^T,-\rho_5J^T,-\rho_6J^T,-\rho_7J^T,-\rho_8J]^T$,

$\bar{\Sigma}_1=[H_1X^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad H_2Y \quad 0]$,

$\bar{\pi}_{3_0}=$

$$\begin{bmatrix} -A_0X^T & 0 & 0 & X^T & 0 & 0 & -B_0Y & -EX^T \\ -\rho_2A_0X^T & 0 & 0 & \rho_2X^T & 0 & 0 & -\rho_2B_0Y & -\rho_2EX^T \\ -\rho_3A_0X^T & 0 & 0 & \rho_3X^T & 0 & 0 & -\rho_3B_0Y & -\rho_3EX^T \\ -\rho_4A_0X^T & 0 & 0 & \rho_4X^T & 0 & 0 & -\rho_4B_0Y & -\rho_4EX^T \\ -\rho_5A_0X^T & 0 & 0 & \rho_5X^T & 0 & 0 & -\rho_5B_0Y & -\rho_5EX^T \\ -\rho_6A_0X^T & 0 & 0 & \rho_6X^T & 0 & 0 & -\rho_6B_0Y & -\rho_6EX^T \\ -\rho_7A_0X^T & 0 & 0 & \rho_7X^T & 0 & 0 & -\rho_7B_0Y & -\rho_7EX^T \\ -\rho_8A_0X^T & 0 & 0 & \rho_8X^T & 0 & 0 & -\rho_8B_0Y & -\rho_8EX^T \end{bmatrix},$$

$\bar{\mho}_1=[CX^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad DY \quad 0]^T$, and

$\bar{\mho}_2=[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \gamma X^T]^T$

and the rest of the elements $(\bar{\Lambda}_{12},\dots,\bar{\Lambda}_{44})$ is equivalent to $\Lambda_{12},\dots,\Lambda_{44}$.

Proof : By Schur complement (12) is equivalent to

$$\begin{bmatrix} \tilde{\Sigma} & \Sigma_1^T & \epsilon\Sigma_2 & \mho_1 & \mho_2 \\ \Sigma_1 & -\epsilon I & 0 & 0 & 0 \\ \epsilon\Sigma_2^T & 0 & -\epsilon I & 0 & 0 \\ \mho_1^T & 0 & 0 & -I & 0 \\ \mho_2^T & 0 & 0 & 0 & I \end{bmatrix}<0 \tag{52}$$

where

$\tilde{\Sigma}=\tilde{\pi}_1+\pi_2+\pi_2^T+\pi_{3_0}+\pi_{3_0}^T+\eta_mV_0+\hat{\eta}\,V_1+\dfrac{\eta_m^2}{2}W_0+\bar{\eta}\,W_1$,

$$\tilde{\pi}_1=\begin{bmatrix} \widetilde{(1,1)} & \widetilde{(1,2)} \\ * & \widetilde{(2,2)} \end{bmatrix},\quad \widetilde{(1,1)}=\begin{bmatrix} \tilde{\Lambda}_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ * & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ * & * & \Lambda_{33} & \Lambda_{34} \\ * & * & * & \Lambda_{44} \end{bmatrix},$$

$\tilde{\Lambda}_{11}=P_{14}-R_1+P_{14}^T-R_1^T+Q_{1_{11}}+\eta_mT_{1_{11}}+\hat{\eta}\,T_{2_{11}}+X_0$,

$$\widetilde{(1,2)}=\begin{bmatrix} P_{12} & P_{13} & 0 & 0 \\ P_{22}-Q_{1_{12}}+Q_{2_{12}} & P_{23} & 0 & 0 \\ P_{23}^T & P_{33}-Q_{2_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{(2,2)}=\begin{bmatrix} -Q_{1_{22}}+Q_{2_{22}} & 0 & 0 & 0 \\ * & -Q_{2_{22}} & 0 & 0 \\ * & * & -X_1+X_2 & 0 \\ * & * & * & 0 \end{bmatrix},$$

$\mho_1=[C \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad DK \quad 0]^T$, and

$\mho_2=[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \gamma I]^T$.

Let $M=[M_1^T \quad M_2^T \quad M_3^T \quad M_4^T \quad M_5^T \quad M_6^T \quad M_7^T \quad M_8^T]^T$. Replace $M_1=M_0$, $M_2=\rho_2M_0$, $M_3=\rho_3M_0$, $M_4=\rho_4M_0$, $M_5=\rho_5M_0$, $M_6=\rho_6M_0$, $M_7=\rho_7M_0$, $M_8=\rho_8M_0$. Feasibility of inequality (52) implies that M_0 become nonsingular. Let $X=M_0^{-1}$ then pre and post multiply simultaneously the two side of (52) with diag $[X \ X \ X \ X \ X \ X \ X \ X \ I \ I \ I \ I]$ (13-15) with diag $[X \ X \ X \ X \ X \ X \ X \ X \ X \ X]$, (16-17) with diag $[X \ X \ X \ X \ X \ X \ X \ X \ X]$ and (11) with diag $[X \ X \ X \ X \ X \ X \ X]$ and its transpose, respectively. Therefore the inequalities (11-17) leads to inequalities (45-51) with $Y=KX^T$. ■

IV. ILLUSTRATIVE EXAMPLE

An illustrative example is presented to verify the effectiveness of the proposed approach compared to previous results in the literature. The YALMIP Toolbox is utilized to solve the LMI feasibility problems [12].

Example: Consider the following system with norm-bounded uncertainty controlled over a network [6]:

$$\dot{x}(t) = \left(\begin{bmatrix} -1 & 0 & -0.5 \\ 1 & -0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} + \Delta A(t) \right) x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \omega(t)$$

$$Z(t) = [1 \quad 0 \quad 1] x(t) + 0.1u(t) \tag{53}$$

where $\|\Delta A(t)\| \leq 0.01$. Choose $J = 0.1I$, $H_1 = 0.1I$, $H_2 = 0$, $\rho_2 = 0.01$, $\rho_3 = 0.01$, $\rho_4 = 144$, $\rho_5 = 0.01$, $\rho_6 = 0.01$, $\rho_7 = 0.01$ and $\rho_8 = 0.1$.

In Table 1, the minimum disturbance attenuation level corresponding to the rival design methods are compared for different values of η_m and η. As seen in the similar situation, larger attenuation level is achieved with the proposed scheme.

TABLE I: Minimum disturbance attenuation level corresponding to the different design methods for different values of η_m and η.

η_m	η	γ			obtained K
		[6]	[7]	Proposed Method	
0.1	0.5	1.843	1.714	1.531	$-[0.8006 \quad 0.00204 \quad 1.7832]$
0.3	0.7	2.642	2.455	2.260	$-[0.3814 \quad 0.0010 \quad 1.2941]$
0.5	1.0	5.829	4.415	3.381	$-[0.1723 \quad 0.0 \quad 1.0162]$

Fig. 2 shows the simulation results of system (53) with state feedback controller $K = -[0.8006 \quad 0.0204 \quad 1.7832]$ and $0.1 \leq \eta(t) \leq 0.5$. The initial values of the states are $x_1(0) = 0.1$, $x_2(0) = 0.1$ and $x_3(0) = 0.8$ and the disturbance signal $\omega(t)$ is as follows:

$$\omega(t) = \begin{cases} 0.2, & 2 \leq t \leq 6 \\ 0, & otherwise \end{cases} \tag{54}$$

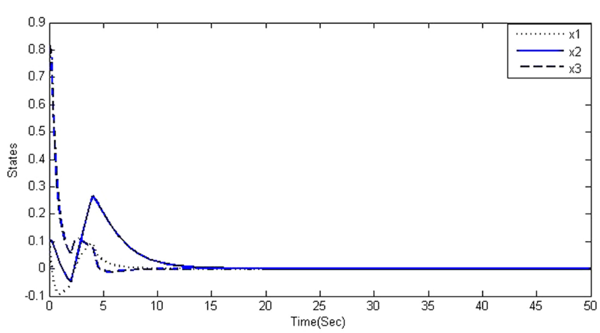

Figure 2. Simulation Results

V. CONCLUSION

This paper proposed a new approach to synthesize robust H_∞ state feedback controller for the linear time invariant system which is controlled via communication network with considering interval time delay, data packet dropout, disturbance input and parameter uncertainties effects. A set of linear matrix inequalities (LMIs) are developed to design controller gain. A novel augmented Lyapunov-Krasovskii functional and free-weighting matrices method are introduced to achieve less conservative results. An illustrative example demonstrates the superiority of the proposed method.

REFERENCES

[1] R. A. Gupta and M-Y. Chow, “Networked control system, overview and research Trends” IEEE Transactions on Industrial Electronics, vol. 57, pp. 2527-2535, 2010.

[2] J. P. Hespanha, P. Naghshtabrizi and Y. Xu, “A Survey of recent results in Networked Control Systems,” Proceedings of IEEE,vol. 95, pp . 138-162, 2007.

[3] M. Yu, L. Wang, T. Chu and F. Hao, “Stabilization of networked control systems with data packet dropout and transmission delays: continuous-time case,” European Journal of Control, vol.11, pp. 40-49, 2005.

[4] D. Yue, Q.L. Han, and C. Peng, "State feedback controller design of networked control systems,” IEEE Transactions. Circuits and Systems II : Express Briefs, vol. 51, pp. 640-644, 2004.

[5] Bin Tang, Guo-Ping Liu and Wei-Hua Gui, “Improvement of state feedback controller design for networked control systems,” IEEE Transaction. Circuits and Systems II : Express Briefs, vol. 55, pp. 464–468, 2008.

[6] D. Yue, Q.L. Han and J. Lam, “Netwrok-based robust H_∞ control of systems with uncertainty,” Automatica, vol. 41,pp. 999-1007, 2005.

[7] S.H. Kim, P. Park and C. Jeong, “Robust H_∞ stabilization of networked control systems with packet analyser,” IET Control Theory and Applications, vol. 4, pp. 1828-1837, 2010.

[8] F. Yang, Z. Wang, Y.S. Hung and M. Gani, “H_∞ Control for networked control systems with random communication delays,” IEEE Transactions . Automatic Control ,vol. 51,pp. 511-518, 2006.

[9] P. Siler and R. Sengupta, “An H_∞ approach to networked control systems,” IEEE Transaction. Automatic Control, vol. 50,pp. 356-364, 2005.

[10] H. Gao, Member IEEE and T. Chen, Fellow IEEE, “Network-based H_∞ output tracking control,” IEEE Transactions on Automatic Control, vol. 53, pp. 655-666, 2008.

[11] J. Löfberg, “Yalmip: A toolbox for modeling and optimization in MATLAB,” Proceeding of the CACSD Conference, Taipei, Taiwan, 2004.

[12] P. Gahinet, A. Nemirovskii, A.J. Laub and M. Chilali, “LMI Control Toolbox User's Guide,” Natick, MA: Math Works, 1995.