Bond University
Research Repository

# Using Simulations to Better Appreciate Game Outcomes: Implications for Bet Minimums and Maximums 

Spence, Mark T.; Kale, Sudhir; Sugden, Stephen

Unpublished: 28/05/2013

Link to publication in Bond University research repository.

Recommended citation(APA):
Spence, M. T., Kale, S., \& Sugden, S. (2013). Using Simulations to Better Appreciate Game Outcomes: Implications for Bet Minimums and Maximums. The 15th International Conferenceon Gambling \& Risk Taking, Las Vegas, United States.

[^0]For more information, or if you believe that this document breaches copyright, please contact the Bond University research repository coordinator.

# Using game simulations to shed insight into common gaming behaviours 

Mark T. Spence
Sudhir Kale
Stephen Sugden
Bond University, Gold Coast, Australia and GamePlan Consultants
www.gameplanconsultants.net

November 2012

## Using game simulations to shed insight into common gaming behaviours

Fair games of chance, such as Baccarat, are exact: the relevant elements of randomness are known, hence probability theory can be applied to deduce outcomes, notably the house advantage.

However, applied probability theory can prove unwieldy for computing the outcome of everyday gambling behaviours, such as betting systems. For example: how exposed is the house if a winning streak causes a highly skewed playerbanker differential?
Simulation is a proven way to assess possible outcomes in such situations.

## Using game simulations to shed insight into common gaming behaviours

## To start: Define the simulation parameters

| Symbol | Meaning | Value | Domain |
| :---: | :--- | :--- | :--- |
| $n_{\max }$ | maximum number of games | 50 |  |
| $n$ | game number |  | $1 \leq \mathrm{n} \leq \mathrm{n}_{\max }$ |
| $p$ | probability of success for player in 1 game | 0.44625 |  |
| $b$ | probability of success for banker in 1 game | 0.45860 |  |
| $t$ | probability of tie in 1 game | 0.09515 |  |
| $z_{n}$ | outcome on game n |  | $\{P, B, T\}$ |
| $L_{n}$ | current run length, game n |  |  |
| $F_{n}$ | final run length, game n |  |  |
| $R_{n}$ | bet of run follower on game n |  | $\{P, B\}$ |
| $x_{n}$ | betting index |  | $1 \leq \mathrm{x}_{n}$ |

## Using game simulations to shed insight into common gaming behaviours

## Runs and their lengths

A run may be a Player run or a Banker run.

## Example of Player run

We define a player run to be any contiguous sequence of outcomes beginning with a player win and containing no banker win. Ties may appear anywhere in a player run, but may not begin it. A run may be terminated by either a win for the Banker or by the last game (set at 50, the approximate number of plays in a shoe). We define the length of a player run to be simply the number of player wins in the run.

## Using game simulations to shed insight into common gaming behaviours

Examples of various run lengths

| Game \#, n | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | Run <br> Length |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Example 1 | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{B}$ |  |  |  |  | $\mathbf{5}$ |
| Example 2 | $\mathbf{P}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{B}$ |  |  |  | $\mathbf{4}$ |
| Example 3 | $\mathbf{P}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{B}$ |  |  |  |  |  | $\mathbf{1}$ |
| Example 4 | $\mathbf{B}$ | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{T}$ | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{P}$ | $\mathbf{T}$ | $\mathbf{P}$ | $\mathbf{7}$ |

## Using game simulations to shed insight into common gaming behaviours

## The run follower

We propose the existence of a player who follows runs, and call him the run follower (RF). Management in Macau suggest this is the case as evidenced by a widening differential. On the first game he bets on Player or Banker with probability 0.5 each. Thereafter, he bets on the same outcome ( P or B ) as he did on the previous game, unless there is a run of length two on the "other side", in which cases he switches to that side.
RF will stay on the side of a run even if the run breaks. He will only switch allegiance if a run of length 2 becomes evident "on the other side".

## Using game simulations to shed insight into common gaming behaviours

Example of following runs. Here, a follower run length of 5 is achieved - and the player would have lost that hand.

| 18 | 0.65 | B | -1 | 0 | B | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | 0.97 | T | -1 | -1 | B | 2 |
| 20 | 0.24 | P | 1 | 0 | B | 2 |
| 21 | 0.13 | P | 2 | 0 | B | 1 |
| 22 | 0.08 | P | 3 | 0 | P | 1 |
| 23 | 0.30 | P | 4 | 4 | P | 2 |
| 24 | 0.61 | B | -1 | -1 | P | 3 |
| 25 | 0.24 | P | 1 | 0 | P | 1 |
| 26 | 0.32 | P | 2 | 0 | P | 2 |
| 27 | 0.09 | P | 3 | 0 | P | 3 |
| 28 | 0.95 | T | 3 | 0 | P | 4 |
| 29 | 0.37 | P | 4 | 4 | P | 4 |
| 30 | 0.79 | B | -1 | -1 | P | 5 |
| 31 | 0.40 | P | 1 | 0 | P | 1 |
| 32 | 0.14 | P | 2 | 2 | P | 2 |
| 33 | 0.63 | B | -1 | 0 | P | 3 |
| 34 | 0.69 | B | -2 | 0 | P | 1 |
| 35 | 0.91 | T | -2 | -2 | B | 1 |

## Using game simulations to shed insight into common gaming behaviours

## Betting patterns

The amount wagered for a hand is a function of the length of "winning runs". We consider the following betting patterns: constant, linear, an actual observed pattern provided by a local establishment, and two hypothetical bet patterns (we created 2, but any patterns can be inserted).
Cumulative winnings/losses
We may now compute cumulative house profits (losses) for each of the betting patterns. We could determine insights for wins/losses for individual hands.

## Using game simulations to shed insight into common gaming behaviours

Betting patterns examined. All patterns start with a bet of 100 'units'; the actuals follow an S-shaped betting pattern, levelling off at a run of length 10..

| run length | ls1* | actuals2 | actuals3 | linear | constant |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 100 | 100 | 100 | 100 | 100 |
|  | 100 | 100 | 100 | 200 | 100 |
|  | 133 | 443 | 1200 | 300 | 100 |
|  | 315 | 1050 | 10500 | 400 | 100 |
|  | 728 | 2427 | 24267 | 500 | 100 |
|  | 1064 | 3547 | 35467 | 600 | 100 |
|  | 1596 | 5320 | 53200 | 700 | 100 |
|  | 2183 | 7277 | 72767 | 800 | 100 |
|  | 2911 | 9703 | 97033 | 900 | 100 |
|  | 3000 | 10000 | 100000 | 1000 | 100 |

## Using game simulations to shed insight into common gaming behaviours

Insights from the simulation (30,000 shoes of 50 hands each). Performance from a single shoe that had a max run length of 5 .

| 30000 | $$ | $$ | $\begin{aligned} & \frac{\pi}{\pi} \\ & \frac{\pi}{0} \\ & \frac{U}{4} \end{aligned}$ | $\begin{aligned} & \mathbf{N} \\ & \frac{0}{工} \\ & \pm \\ & \mathbf{U} \end{aligned}$ | $\frac{m}{n}$ $\frac{1}{2}$ $\frac{U}{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CURRENT SHOE |  |  |  |  |  |
| Min bet | 100 | 100 | 100 | 100 | 100 |
| Max bet | 100 | 500 | 728 | 2427 | 24267 |
| Total bet (handle) | 5000 | 8000 | 6190 | 10600 | 54367 |
| Net player payout | -625 | -1425 | -1104 | -2689 | -16592 |
| Min player payout | -100 | -500 | -728 | -2427 | -24267 |
| Max player payout | 100 | 400 | 315 | 1050 | 10500 |
| RTP | -12.50\% | -17.81\% | -17.84\% | -25.36\% | -30.52\% |

## Using game simulations to shed insight into common gaming behaviours

Outcome after 30,000 simulations (about a month's worth of play across 20 tables).

| CUMULATIVE |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Prizes | -1587370 | -3045755 | -2901008 | -6693614 | -50996036 |
| Player Contribution | 150000000 | 293756900 | 264054128 | 614046955 | 4521359306 |
| RTP | -1.06\% | -1.04\% | -1.10\% | -1.09\% | -1.13\% |
| SUMMARY STATISTICS |  |  |  |  |  |
| Min shoe revenue | -2635 | -3535 | -2635 | -5334 | -77278 |
| Max shoe revenue | 2460 | 17010 | 36446 | 121346 | 1213072 |
| Median | -55 | -375 | -655 | -2397 | -18496 |
| Mean | -53 | -102 | -97 | -223 | -1700 |
| Std deviation | 664 | 1559 | 2120 | 6813 | 67212 |
| Skewness | 0 | 1 | 5 | 5 | 5 |
| Kurtosis | 0 | 4 | 32 | 36 | 36 |

## Using game simulations to shed insight into common gaming behaviours

This translates into 1,500,000 hands


|  | 0.44629 | 0.45860 | 0.095 | 1.000 |
| :--- | :--- | :--- | :--- | :--- |
| Sim frequencies |  |  |  |  |
| Expected frequencies | 0.44625 | 0.4586 | 0.095 | 1.000 |

## Using game simulations to shed insight into common gaming behaviours

Frequency distribution of net player payouts across the 30,000 simulations. Escalating bets means the player is more likely to lose; but if they win, they can win large amounts. Note how the mode shifts left as the differential increases.

Player NET rel freq: $\mathbf{3 0 0 0 0}$ simulations


## Using game simulations to shed insight into common gaming behaviours

Betting patterns can result in "outcome reversals" relative to constant betting. This could result in interesting behavioural ramifications.


$\frac{N}{N}$
$\frac{0}{Z}$
世

| m |  |
| :--- | :---: |
|  |  |
| 艺 |  |


| \# | 1528 | 1093 | 1525 | 1957 | 3768 | 7529 | 8818 | 9217 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\%$ | 0.051 | 0.036 | 0.051 | 0.065 | 0.126 | 0.251 | 0.294 | 0.307 |

## Using game simulations to shed insight into common gaming behaviours

## Conclusion

Regardless of differential (which can be used as a product differentiation strategy), the house advantage does not change; but the greater the differential, the greater the number of shoes that end with players losing and the more extreme are the payouts in the positive domain for players (the house paying out for a hand of play).

This simulation is therefore intended to help casino managers set betting limits that maximize total winnings while bearing in mind both the likelihood and magnitude of negative outcomes of increased differentials.

# Using game simulations to shed insight into common gaming behaviours 

## Questions?

## Thank you!

www.gameplanconsultants.net


[^0]:    General rights
    Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

