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(Revised)

SUBROUTINE CTM1 (ITYPE, SQUARE,N,AF, INFIX,
A,B,INFDAT,UØ,U1,C,CMOD,
HEIGHT, IFAIL)

A conformal transformation method for the solution of a class of Laplacian problems

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$t3 = (1/m^2, 0)$ and $t_4 = (1,0)$, where $m = \left\{ \left(\frac{a_4 - a_1}{a_4 - a_2} \right) \left(\frac{a_3 - a_2}{a_3 - a_1} \right) \right\}^{\frac{1}{2}}$.

A Schwarz-Christoffel transformation is then used to map the upper half t-plane onto the rectangle

$$\overline{\Omega}' = \left\{ (\xi\, \eta) : 0 \le \xi \le 1, 0 \quad \le \eta \le \frac{K\left\{ (1 - m^{2})^{\frac{1}{2}} \right\}}{K(m)} = H \right\}$$

in the w'-plane (w'=•+iη), so that the four points z_1, z_2, z_3 and z_4 are mapped respectively onto the four vertices (0,0), (0,H), (1,H) and (1,0) of the rectangle $\overline{\Omega}$. In the above K(m) denotes the complete elliptic integral of the first kind with modulus m,0<m<l.

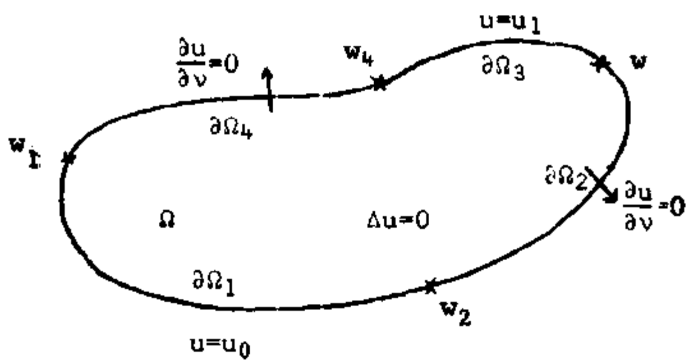
CTM1 can also be used to map conformally the upper half z-plane onto the unit square

$\overline{\Omega}$'- {(•,n):0••,n•1}.

In this case only two points z_1 and z2, on the real z-axis, can be mapped onto two vertices of $\overline{\Omega}'$. These two vertices are always the corners (0,0) and (0,1) of $\overline{\Omega}'$.

Given any point (a,b) in the upper half z-plane CTM1 computes its image (•,η) in the rectangle $\overline{\Omega}' \in w'$ -plane. The computation requires the calculation of two complete elliptic integrals of the first kind, and for each co-ordinate of a transformed point, the calculation of an incomplete elliptic integral of the first kind. For further details see reference [3].

<u>Problem of Type 2</u> (Solution of harmonic boundary value problems). CTM1 can also be used for the solution of a class of harmonic boundary value problems defined in two-dimensional simply-connected domains. Let •, with boundary ∂•, be the domain of definition of such a problem and let consist of four successive segments $\partial•_1$, $\partial•_2$, $\partial•_3$ and $\partial•_4$. Then the routine can be used provided that the boundary conditions of the problem are constant Dirichlet of the form $u = u_0$ and $u = u_1$ on the segments $\partial•_1$ and $\partial•_3$ and homogeneous Neumann $\partial u/\partial v=0$ on $\partial•_2$ and $\partial•_4$ see fig. 1.



w=x+iy

<u>Figure 1</u>

This application of CTM1 requires that the conformal transformation of • = •U∂• onto the upper half z-plane is known. The user must supply the images, under this transformation, of the four end points w_i; i = 1(1)4, of the boundary segments

$\partial\Omega_i$; i = 1(1)4, and also the images of the points in Ω at which the solution of the boundary value problem is required. The solution of the boundary value problem is obtained by using the Conformal Transformation Method described in references [1,2]. This method gives the solution u at any point (x,y) Ω by means of the formula

$$u = (u_1 - u_0)\left(\xi + \frac{u_0}{u_1 - u_0}\right)$$

where is the real co-ordinate of the image of (x,y) in the rectangle $\overline{\Omega}'$.

In problems of this type the height H of the rectangle $\overline{\Omega}'$ is the so called "conformal module" of the "quadrilateral" defined by the four points w_i; i- 1,2,3,4. From this conformal module, the "capacitance" between the arcs w_1w_2 and w_3w_4 can be easily computed; see references [6,7].

4. References

[1] J.R. Whiteman and N. Papamichael: Treatment of harmonic boundary value problems containing singularities by conformal transformation methods, Z.angew.Math.Phys. 23 (1972) 655-664.

[2] N. Papamichael and J.R. Whiteman: A numerical conformal transformation method for harmonic mixed boundary value problems in polygonal domains, Z.angew.Math.Phys. *24* (1973) 304-316.

[3] N. Papamichael and G.T. Symm: Numerical techniques for two-dimensional Laplacian problems, Comp.Meths Appl. Mech. Eng. *6* (1975) 175 -194.

[4] D.J. Hofsommer and R.P. Van De Riet: On the numerical calculations of elliptic integrals of the first and second kind and the elliptic functions of Jacobi, Numer.Math. 5 (1963) 291-302.

[5] N. Papamichael and A. Sideridis: Formulae for the approximate conformal mapping of some simply-connected domains, Technical Report TR/72, Dept, of Maths, Brunei University 1977.

[6] D. Gaier: Ermittlung des konformen Moduls von Vierecken mit Differenzenmethoden, Numer.Math, 19 (1972) 179-194.

[7] D. Gaier: Capacitance and the conformal module of quadrilaterals, J.Math.Anal.Applies., 70 (1979) 236-239.

5. Parameters

ITYPE - INTEGER.
On entry, ITYPE is assigned the value 1 if CTM1 is to be used for the solution of a problem of type 1, or the value 2 if CTM1 is to be used for the solution of a problem of type 2. Unchanged on exit.

SQUARE - LOGICAL.
A variable which on entry specifies whether or not the transformed domain is required to be a square. If SQUARE is .TRUE, then will be the unit square

$$\overline{\Omega}' = \{(\xi,\eta); 0 \le \xi,\eta \le 1\}.$$

Otherwise it will be the rectangle

$$\overline{\Omega}' = \{(\xi,\eta): 0 \le \varsigma \le 1, 0 \le \eta \le H\}.$$

For a problem of type 2 SQUARE must always be false so that is a rectangle. Unchanged on exit.

N - INTEGER.
For a problem of type 1, on entry N contains the number of points (a,b) in the upper half z-plane, whose images (ξ,η) in the rectangle $\overline{\Omega}'$ are to be computed. For a problem of type 2, N contains the number of points in $\overline{\Omega} \in w$ -plane at which the solution of the boundary value problem is required.
Unchanged on exit.

AF - *real* array of DIMENSION (4).
On entry, if SQUARE is .FALSE., AF contains the real co-ordinates of the four "fixed" points $z_i = (a_i,0)$; i = 1(1)4, in the z-plane, which are to be mapped onto the vertices of the rectangle $\overline{\Omega}'$. For a problem of type 2 these four fixed points are the images, in the upper half z-plane, of the end points w_i; i = 1(1)4, of the boundary segments $\partial\Omega_i$; i - 1(1)4^1. It is essential that the four fixed points z_i ; i = 1(1)4, are correctly ordered. To obtain a correct ordering, the boundary of the upper half z-plane is considered to consist of the real axis and a semi-circular arc of infinite radius. Then, starting with the point z_1 the other three points may be arranged in either clockwise or anti-clockwise direction so that z_1, z_2, z_3 , z_4 form a cyclic set along the boundary . If one of the four fixed points is the point at infinity then this point must be taken to correspond to z_4, and AF(4) can then be assigned any value.

If, for a problem of type 1, $\overline{\Omega}'$ is required to be the unit square then AF(1) and AF(2) contain the real co-ordinates of the two fixed points z_1 and z_2, which are to be mapped onto the vertices (0,0) and (0,1) of $\overline{\Omega}'$, and AF(3), AF(4) are ignored. In this case, if one of the two fixed points is the point at infinity then this point must be taken to correspond to z_2 and AF(2) can be assigned any value.
Unchanged on exit.

INFIX - INTEGER.
On entry, INFIX indicates whether or not one of the fixed points z_2 or z_4 is the point at infinity.
Thus, if

INFIX = 0 none of the fixed points z_i; i = 1(1)4, is the point at infinity,

INFIX = 2 the point Z_2 is the point at infinity,

INFIX = 4 the point z_4 is the point at infinity.

If INFIX is assigned any other value, the routine assumes the value to be zero.

Unchanged on exit.

A - *real* array of DIMENSION (N).
On entry, A contains the real co-ordinates of the N "data" points in the upper half z-plane whose images in the rectangle are required. For a problem of type 2, these data points are the images, in the upper half z-plane, of the points in at which the solution of the boundary value problem is required.

If the point at infinity is one of the data points then this point must be taken to correspond to the point (A(1),0), and A(l) can be assigned any value.

For a problem of type 1, on exit A contains the real co-ordinates ξ of the transformed points in the rectangle Ω'. For a problem of type 2, on exit A contains the solution of the harmonic problem at the required points.

B - *real* array of DIMENSION (N).
On entry, B contains the imaginary co-ordinates of the N data points in the z-plane whose images in the rectangle are required. For a problem of type 2, these data points are the images in the upper half z-plane, of the points in at which the solution of the boundary value problem is required. For a problem of type 1, on exit B contains the imaginary co-ordinates n of the transformed points in the rectangle . For a problem of type 2, on exit B remains unchanged.

INFDAT - LOGICAL.
A variable which on entry specifies whether or not the data point (A(1),B(1)), B(l)-0, is the point at infinity. If INFDAT is TRUE, then (A(1),B(1)) corresponds to the point at infinity. Otherwise the point at infinity is not a data point.

Unchanged on exit.

Uϕ - *real.*
For a problem of type 1, Uϕ is ignored on entry. For a problem of type 2, Uϕ specifies the boundary value along the boundary segment.

Unchanged on exit.

U1 - *real.*
For a problem of type 1, U1 is ignored on entry.
For a problem of type 2, U1 specifies the boundary value along the boundary segment 3-
Unchanged on exit.

C - *real.*
On entry, C is ignored. On exit it contains the value of the constant k of the bilinear transformation which maps the upper half z-plane onto the upper half t-platie; see [3], formula (32).

CMOD - *real.*
On entry, CMOD is ignored. On exit it contains the value of the modulus m of the complete elliptic integral of the first kind involved in the Schwarz-Christoffel transformation*

HEIGHT -*real.*
On entry, HEIGHT is ignored- On exit it contains the value of the height H of the rectangle $\overline{\Omega'}$.

IFAIL - INTEGER.
Before entry IFAIL must be assigned a value.
For users not familiar with this parameter (described in Chapter P01 of NAG Library) the recommended value is ϕ. Unless the routine detects an error (see Section 6), IFAIL contains 0 on exit.

6. Error indicators

Errors detected by the routine:

IFAIL = 1 On entry, ITYPE < 1 or ITYFE > 2.

IFAIL = 2 On entry, ITYPE = 2 but SQUARE is .TRUE. which is incompatible.

IFAIL =3 On entry, SQUARE is .FALSE, but INFIX =2 which is incompatible.

IFAIL =4 On entry, SQUARE is .TRUE, but INFIX = 4 which is incompatible.

IFAIL = 5 The modulus m of the complete elliptic integral of the first kind, which is involved in the Schwarz Christoffel transformation, does not satisfy the condition 0< m< I. This indicates that the fixed points (AF(1),0), (AF(2),0), (AF(3),0) and (AF(4),0) are incorrectly ordered.

7. Auxiliary Routines

CTM1 calls the NAG Library routines AØ2ACF, PØ1AAF, XØ1AAF and XØ2AAF It also uses real functions CTM1X and CTM1Y to compute respectively the complete and incomplete elliptic integrals of the first kind which are needed for the determination of the transformed points in the rectangle. These auxiliary routines CTMIX and CTMIY call the NAG Library routines XØ1AAF and XØ2AAF.

CTMIX and CTMIY are FORTRAN versions of the ALGOL routines given by Hofsommer and Van de Riet in reference [4]. These FORTRAN versions are given in full in the accompanying example program.

8. Timing

The time taken is approximately proportional to N.

9. Storage

There are no internally declared arrays.

10. Accuracy

In principle, for a problem of type 1, the routine is capable of producing full machine precision. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error.

For a problem of type 2, the accuracy depends on the accuracy of the first transformation employed by the user to perform the mapping of the domain of definition onto the upper half z-plane.

11. Further Comments

The auxiliary routines CTM1X and CTMlY, used respectively for the computation of the complete and incomplete elliptic integrals of the first kind, can be replaced by any other routines available for this purpose.

12. Keywords

Harmonic mixed boundary value problems.
Conformal transformations.
Bilinear and Schwarz-Christoffel transformations.
Dirichlet, Neumann boundary conditions.
Complete and incomplete elliptic integrals of the first kind.

13. Examples

(1) Problem of Type 1

To obtain the conformal mapping of the upper half z-plane onto a rectangle $\bar{\Omega}$ so that the points (-1,0), (0,0), (1,0), (2,0) are mapped respectively onto the four vertices (0,0), (0,H), (1,H), (1,0) of the rectangle $\bar{\Omega}'$.

(2) Problem of Type 2

(i) To solve Laplace's equation in the rectangle

$$\bar{\Omega}' = \{(x,y): -1\leq x\leq 1,\ 0\leq y\leq 1\}$$

subject to the boundary conditions u_0 = 500 along the straight line joining the vertex (-1,0) to the origin (0,0), u_1 = 1000 along the side joining the vertex (1,0) to (1,1) and homogeneous Neumann conditions $\partial \upsilon/\partial v=0$ on the remainder of the boundary; see Fig. 2. This is the well-known problem of Motz; see reference [1].

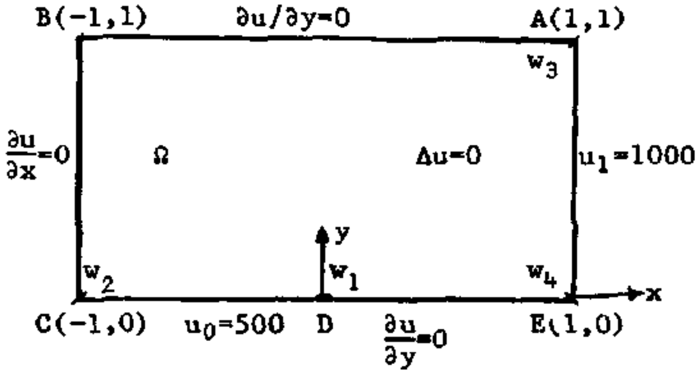


Figure 2

The end points of the four boundary segments are w_1 = (0,0), w_2 - (-1,0), w_3 = (1,1) and w_4 = (1,0). The conformal transformation of $\bar{\Omega}\in w$-plane onto the upper half z-plane is

$$z = T(w)$$
$$= sn(Kw,k)$$

where sn denotes the Jacobian elliptic sine and K = K(k) is the complete elliptic integral of the first kind with modulus $k = 1/\sqrt{2}$. Under this transformation $z_1 = T(w_1) = (0,0)$, $z_2 = T(w_2) = (-1,0)$, $z_3 = T(w_3) - (\sqrt{2},0)$ and $z_4 - T(w_4) = (1,0)$; see ref [1]. The above transformation is used in a preliminary routine C0MAP1 to determine the images z_1, z_2, z_3, z_4 and also the images, in the upper half z-plane, of the points in Ω at which the solution of the boundary value problem is to be computed. The elliptic sine in the transformation is found from a series expansion. See reference [1].

(ii) To solve Laplace's equation in the L-shaped region $\bar{\Omega}$ illustrated in Fig. 3 subject to the boundary conditions u_0 = 0 along AB, u_1 = 1 along EF and $\partial u/\partial v=0$ along BC, CD, DE and AF.

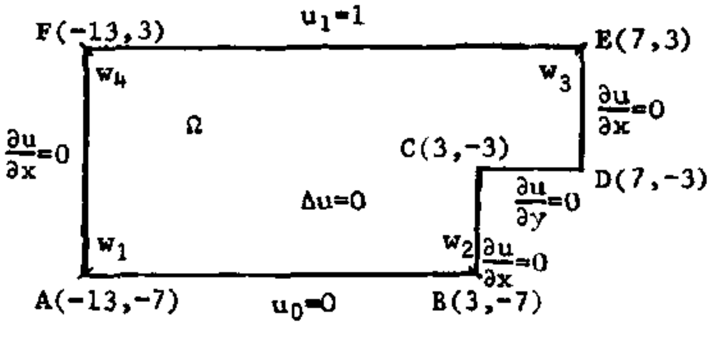


Figure 3

The end points of the four boundary segments are $W1 \equiv A$, $W2 \equiv B$, $w_3 \equiv E$ and $W4 \equiv F$. For the conformal mapping of $\overline{\Omega}$ onto the upper half z-plane, the preliminary routine C0MAP2 uses the approximate conformal transformation

$$z = \tilde{T}(w) = i\left\{\frac{1+\tilde{F}(w)}{1-\tilde{F}(W)}\right\},$$

where $\tilde{F}(w)$ is the approximation to the mapping function which maps conformally onto the unit disc, given in reference [5,Ex.2.4],

AKNOWLEDGEMENT

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```
C
C
C                              TEST DRIVER FOR CTM1
C
      INTEGER NIN, NOUT, L,ITYPE, N, INFIX, I, IFAIL
      LOGICAL SQUARE, INFDAT
      REAL AF(4),A(62),B(62),A1(62),Bl(62),XF(4),YF<4), X(62), Y(62)
      REAL UO.Ul,C,CMOD, HEIGHT
      DATA NIN /5/, NOUT /6/
C
C                                        FOR L=l
C          MAP CONFORMALLY THE UPPER HALF Z-PLANE ONTO A RECTANGLE
C
C                                        FOR L=2
C                        SOLVE PROBLEM 2, REFERENCE /l/, PAGE 660
C
C                                        FOR L=3
C                     SOLVE PROBLEM 2(IV), REFERENCE /2/, PAGE 310
C
      L=l
   10 READ (NIN,8000) ITYPE
      READ (NIN,8000) N
      READ (NIN,8000) INFIX
      SQUARE= . FALSE.
      INFDAT= . FALSE.
      IF (ITYPE.EQ. 2) GO TO 40
      DO 20 1=1, 4
      READ (NIN,8010) AF(I)
   20 CONTINUE
      WRITE (NOUT,9000)
      WRITE (NOUT, 9010)
      WRITE (NOUT,9020)
      DO 30 1=1, N
      READ (NIN,8020) A(I),B(I)
      A1(I)=A(I)
      B1(I)=B(I)
   30 CONTINUE
      GO TO 80
   40 DO 50 1=1,4
      READ (NIN, 8020) XF(I),YF(I)
   50 CONTINUE
      DO 60 1=1,N
      READ (NIN,8020) X(I),Y(I)
   60 CONTINUE
      READ (NIN,8020) UO, Ul
      IF (L. EQ. 3) GO TO 70
C
C                          CALL USERS ROUTINE COMAP1
C COMAP1 MAPS THE DOMAIN OMEGA OF PROBLEM 2 ONTO THE UPPER HALF Z-PLANE
C THE MAPPING FUNCTION IS GIVEN IN TERMS OF AN ELLIPTIC SINE.
C                  SEE REFERENCE /l/, FORMULAE (17) AND (18).
C
      CALL COMAP1(N, XF, YF, AF, X, Y, A, B, INFIX, INFDAT)
      WRITE (NOUT,9030)
      GO TO 80
C
C                          CALL USERS ROUTINE C0MAP2
C COMAP2 MAPS THE DOMAIN OMEGA OF PROBLEM 2(IV) ONTO THE UPPER HALF
C Z-PLANE. THE MAPPING FUNCTION IS NOT KNOWN ANALYTICALLY AND THE
C TRANSFORMATION IS PERFORMED USING AN APPROXIMATE MAPPING FUNCTION.
C                          SEE REFERENCE /5/, EXAMPLE 2. 4.
C
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```
   70 CALL COMAP2(N. XF, YF, AF, X, Y, A, B)
      WRITE (N0UT,9040)
   80 IFAIL-1
C
C                                CALL MAIN ROUTINE CTM1
C                               -------------------------
C
      CALL CTM1 ( ITYPE, SQUARE, N, AF, INFIX, A, B, INFDAT, UO, Ul, C, CMOD,
     *HEIGHT, IFAIL)
      IF (IFAIL. NE. 0) G0 TO 120
      IF (ITYPE. EG. 2) GO TO 100
      DO 90 1=i,N
      WRITE (N0UT,9050) Al (I ), B1 (I). A( I). B( I )
   90 CONTINUE
      GO TO 130
  100 WRITE (NOUT,9060)
      DO 110 I-i.N
      WRITE (NOUT,9070) X(I),Y(I),A(I)
  110 CONTINUE
      GO TO 130
  120 WRITE (NOUT,9080) L,IFAIL
  130 L=L+1
      IF (L. LE. 3) G0 TO 10
      STOP
 8000 FORMAT (13)
 8010 FORMAT (E16. 8)
 8020 FORMAT (2E16. 8)
 9000 FORMAT (1H1////11X. 54HC0NFORMAL MAP OF THE UPPER HALF Z-PLANE TO A
     *RECTANGLE)
 9010 FORMAT (//2IX.7HZ-PLANE, 22X,8HW'-PLANE)
 9020 FORMAT (//16X,1HA, 14X,1HB, 13X, 3HKSI, 12X, 3HETA)
 9030 FORMAT (1H1/////7X. 43HHARM0NIC PROBLEM 2, REFERENCE /l/, PAGE 660)
 9040 FORMAT (1H1/////7X, 47HHARM0NIC PROBLEM 2(IV), REFERENCE /2/, PAGE
     *310)
 9050 FORMAT (/7X.4E15. 6)
 9060 FORMAT (//7X, 9HP0INT: (,4X, 1HX,4X, 1H,,4X, 1HY, 4X, 1H), 4X. 8HSOLUTION
      *)
 9070 FORMAT ( /15X, 1H(, F8. 4, 2H , , F8. 4, 2H ),2X,F10. 4)
 9080 FORMAT (//5X,47HCTM1 FAILS FOR THE PROBLEM CORRESPONDING TO L= ,
      *        12, 1H, , 8H IFAIL=, 12)
       END
C
C
C
      SUBROUTINE CTM1 (ITYPE, SQUARE, N, AF, INFIX, A, B, INFDAT, UO, Ul, C,
     *                  CMOD, HEIGHT, IFAIL)
      INTEGER ITYPE, N. INFIX, IFAIL
      LOGICAL SQUARE, INFDAT
      REAL AF(4), A(N),B(N),U0,U1.C, CMOD, HEIGHT
C                                                                       C
C***********************************************************************C
C                                                                       C
C                                                                       C
C                 N.PAPAMICHAEL AND A. B. SIDERIDIS 30/8/1979           C
C                                                                       C
C                                                                       C
C      THIS SUBROUTINE CAN BE USED FOR TWO TYPES OF PROBLEMS            C
C (1)  TO MAP CONFORMALLY THE COMPLEX UPPER HALF Z-PLANE ONTO A         C
C      RECTANGLE(SQUARE) SO THAT FOUR(TWO) SPECIFIED POINTS ON THE      C
C      REAL AXIS OF THE Z-PLANE ARE MAPPED RESPECTIVELY ONTO THE        C
C      FOUR(TWO OF THE FOUR) VERTICES OF THE RECTANGLE(SQUARE).         C
C (2)  TO SOLVE LAPLACES EQUATION IN A SIMPLY-CONNECTED DOMAIN,         C
C      SUBJECT TO CONSTANT DIRICHLET AND HOMOGENEOUS NEUMANN BOUNDARY   C
C      CONDITIONS RESPECTIVELY ON FOUR SUCCESSIVE BOUNDARY SEGMENTS.    C
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C      THIS APPLICATION OF CTM1 REQUIRES THAT THE CONFORMAL TRANS-
C      FORMATION OF THE DOMAIN OF DEFINITION OF THE BOUNDARY VALUE
C      PROBLEM ONTO THE UPPER HALF PLANE IS KNOWN. THE USER MUST
C      SUPPLY THE IMAGES, UNDER THIS TRANSFORMATION, OF THE POINTS AT
C      WHICH THE SOLUTION OF THE PROBLEM IS REQUIRED.
C
C INPUTS:
C --------
C
C ITYPE    SPECIFIES THE TYPE OF PROBLEM TO BE CONSIDERED. I.E..
C          ITYPE=1 FOR A PROBLEM OF TYPE (1),
C          ITYPE=2 FOR A PROBLEM OF TYPE (2).
C SQUARE   SPECIFIES WHETHER OR NOT THE TRANSFORMED DOMAIN IS REQUIRED
C          TO BE A SQUARE. I. E. ,
C          SQUARE= . TRUE. THE TRANSFORMED DOMAIN IS THE UNIT SQUARE,
C          SQUARE= . FALSE. THE TRANSFORMED DOMAIN IS A RECTANGLE.
C N        NUMBER OF POINTS TO BE MAPPED ONTO A RECTANGLE FOR A PROBLEM
C          OF TYPE (1), OR NUMBER OF POINTS AT WHICH THE SOLUTION IS
C          REQUIRED, FOR A PROBLEM OF TYPE (2).
C AF       ARRAY OF DIMENSION (4) STORING THE REAL CO-ORDINATES OF THE
C          FOUR FIXED POINTS ON THE REAL Z-AXIS.
C INFIX    INDICATES WHETHER OR NOT THE POINT AT INFINITY IS A POINT
C          OF THE SET (AF(I),0); I=1(1)4. I.E.
C          INFIX=0 NONE OF THE FIXED POINTS CORRESPONDS TO THE POINT AT
C                  INFINITY,
C          INFIX=2 THE POINT (AF(2),0) CORRESPONDS TO THE POINT AT
C                  INFINITY,
C          INFIX=4 THE POINT (AF(4),0) CORRESPONDS TO THE POINT AT
C                  INFINITY.
C A, B     ARRAYS OF DIMENSION (N) CONTAINING THE REAL AND IMAGINARY
C          CO-ORDINATES OF THE N DATA POINTS.
C INFDAT   SPECIFIES WHETHER OR NOT THE DATA POINT (A(1),B(1))
C          CORRESPONDS TO THE POINT AT INFINITY. I. E. ,
C          INFDAT= . TRUE. THE POINT AT INFINITY IS A DATA POINT,
C          INFDAT= . FALSE. THE POINT AT INFINITY IS NOT A DATA POINT.
C UO, U1   CONSTANTS SPECIFYING THE DIRICHLET BOUNDARY CONDITIONS
C          FOR A PROBLEM OF TYPE (2).
C IFAIL    ERROR PARAMETER INDICATOR SET TO INDICATE THE TYPE OF
C          FAILURE IN CTM1.
C
C ALL ABOVE PASSED AS ARGUMENTS
C
C
C OUTPUTS.
C---------
C
C A        ARRAY OF DIMENSION (N) STORING THE REAL CO-ORDINATES OF
C          THE TRANSFORMED POINTS IN THE RECTANGLE(SQUARE) FOR A
C          PROBLEM OF TYPE (1), OR THE SOLUTION VECTOR OF THE HARMONIC
C          PROBLEM FOR A PROBLEM OF TYPE (2).
C B        ARRAY OF DIMENSION (N) STORING THE IMAGINARY CO-ORDINATES
C          OF THE TRANSFORMED POINTS IN THE RECTANGLE(SQUARE) FOR
C          A PROBLEM OF TYPE (1).
C C        CONSTANT OF THE BILINEAR TRANSFORMATION
C                            T-C*(Z-AF(1))/(Z-AF(2))
C          MAPPING THE UPPER HALF Z-PLANE ONTO THE UPPER HALF T-PLANE.
C          SEE REFERENCE /1/, FORMULA (3)
C CMOD     MODULUS OF THE COMPLETE ELLIPTIC INTEGRAL OF THE FIRST
C          KIND INVOLVED IN THE SCHWARZ-CHRISTOFFEL TRANSFORMATION.
C HEIGHT   HEIGHT OF THE RECTANGLE.
C IFAIL    ERROR INDICATOR,
C          IFAIL=0 CORRECT RETURN
C          IFAIL=1 ITYPE INVALID
C          IFAIL=2 SQUARE: TRUE INCOMPATIBLE WITH ITYPE=2
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```
C           IFAIL=3 SQUARE FALSE  INCOMPATIBLE WITH  INFIX=2  C
C           IFAIL=4  SQUARE  TRUE  INCOMPATIBLE  WITH  INFIX-4                      C
C           IFAIL=5 MODULUS OF THE ELLIPTIC  INTEGRAL GREATER  THAN  1.             C
C                                                                                   C
C  ALL  PASSED  AS  ARGUMENTS                                                       C
C                                                                                   C
C                                                                                   C
C  ROUTINES USED:                                                                   C
C------------------                                                                 C
C                                                                                   C
C  XOIAAF(NAG)          COMPUTES THE CONSTANT PI.                                   C
C  XO2AAF(NAG)          COMPUTES THE MACHINE-DEPENDENT CONSTANT EPS                 C
C                       FOR  WHICH  ((1.+EPS). GT. 1.)  WITHIN THE MACHINE.         C
C  PO1AAF(NAG)          ERROR HANDLING ROUTINE.                                     C
C  AO2ACF(NAG)          DIVIDES  A  COMPLEX  NUMBER  BY  ANOTHER  COMPLEX           C
C                       NUMBER.                                                     C
C  CTM1X(AUXILIARY)     COMPUTES THE COMPLETE ELLIPTIC  INTEGRAL OF THE             C
C                       FIRST KIND OF MODULUS CMOD.                                 C
C  CTM1Y(AUXILIARY)     COMPUTES THE  INCOMPLETE ELLIPTIC  INTEGRAL OF              C
C                       THE FIRST  KIND OF AMPLITUDE PHI  AND MODULAR               C
C                       ANGLE ALFA.                                                 C
C                                                                                   C
C********************************************************************************** C
C                                                                                   C
      INTEGER  IERR,PO1AAF. K, M
      REAL PI. HALFPI, EPS. Al, A2. A3. A4. 01. CMODP. ALFA. ONEC, CM0DP2,
     *      ALFAP. ONECP. CM0D2, CEIP. CTM1X,TWOCEP. SRNAME, CEI, TWOCEI,
     *      U10. AK2. AK1. BK1, AK3, AK4. AK. BK. TR. TI. G. H. G2. H2. S. SI, S2.
     *      V. W. T, R, DONE. TA, D, PHI, CON1, CON2, CON3. F. AB, CTM1Y, KSI, ETA,
     *      XO1AAF,XO2AAF, EPS1
      DATA SRNAME  /5H CTM1/
C
C  CHECK DATA FOR  OBVIOUS ERRORS
C
      IF  (ITYPE. EQ. 1  .OR.  ITYPE. EQ. 2)  GO TO 2
C
C  ERROR  1
C
      IERR=1
      GO TO 570
    2 IF  (ITYPE. EQ. 1  .OR.  . NOT. SQUARE)  GO TO  4
C
C  ERROR  2
C
      IERR=2
      GO  TO  570
    4 IF  (SQUARE  .OR.   INFIX. NE. 2)  GO  TO  6
C
C  ERROR  3
C
      IERR=3
      GO TO  570
    6 IF  (.NOT. SQUARE   OR.  INFIX. NE. 4)  GO  TO 8
C
C  ERROR  4
C
      IERR=4
      GO TO  570
C
C  USE NAG  LIBRARY  ROUTINES  TO  COMPUTE  THE  CONSTANTS  PI  AND  EPS.
C
    8 PI=XO1AAF(PI)
      HALFPI=0. 5*PI
```

```
      EPS=XO2AAF(EPS)
      EPS1=10. *EPS
      A1=AF(1)
      IF (INFIX. EQ. 2) GO TO 30
      A2=AF(2)
      IF (SQUARE) GO TO 30
      A3=AF(3)
      IF (INFIX. EQ. 4) GO TO 10
      A4=AF(4)
C
C COMPUTE THE CONSTANT C OF THE BILINEAR TRANSFORMATION WHICH MAPS
C THE UPPER HALF Z-PLANE ONTO THE UPPER HALF T-PLANE.
C
      C=(A4-A2)/(A4-A1)
      GO TO 20
   10 C=i. 0
   20 G1=C*(A3-Al)/(A3-A2)
      IF (G1. LT. 1.0) GO TO 60
C
C COMPUTE THE CONSTANTS OF THE SCHWARZ-CHRISTOFFEL TRANSFORMATION,
C I.E., CMOD=MODULUS OF THE COMPLETE ELLIPTIC INTEGRAL OF THE FIRST
C               KIND, AND
C        ALFA=MODULAR ANGLE OF THE INCOMPLETE ELLIPTIC INTEGRAL OF
C               THE FIRST KIND.
C
      CMOD=1. 0/SQRT(Gl)
      GO TO 40
   30 CMOD=0. 5*SQRT(2. 0)
      CM0D2=0. 5
      C=l.0
      CMODP=CMOD
      CMODP2-CMOD2
      ALFA=ATAN(1. O)
      ONEC=0. 5
      CMQDP2-0. 5
      ALFAP=ALFA
      ONECP=ONEC
      GO TO 50
   40 CMQD2=CM0D*CM0D
      CMODP=SQRT(1. 0-CM0D2)
      ALFA=ATAN(CMOD/CMODP)
      ONEC=1. 0-CMOD2
      IF (ITYPE. EQ. 2) GO TO 80
      CMODP2=CMODP*CMODP
      ALFAP=ATAN(CMODP/CMOD)
      ONECP=1 O-CMODP2
C
C COMPUTE THE COMPLETE ELLIPTIC INTEGRALS CEI AND CEIP OF MODULUS
C CMOD AND COMPLEMENTARY MODULUS CMODP RESPECTIVELY USING CTM1X.
C
   50 CEIP=CTM1X(CMODP)
      TWOCEP=CEIP+CEIP
      GO TO 70
C
C ERROR 5
C
   60 IERR=5
      GO TO 570
   70 IF (SQUARE) CO TO 90
   80 CEI=CTM1X(CM0D)
      TWOCEI=CEI+CEI
      GO TO 100
   90 CEI=CEIP
      TWOCEI=TWOCEP
```

```
   100 IF (ITYPE. EQ. 2) GO TO 110
C
C COMPUTE THE HEIGHT OF THE RECTANGLE
C
       HEIGHT=CEIP/CEI
       GO TO 120
   110  U10=U1-U0
   120  K=l
        M=l
        IF (.NOT. INFDAT) GO TO 130
        IF (INFIX. EQ. 2) GO TO 510
        IF (INFIX. EG. 4) GO TO 530
        GO TO 180
   130  CONTINUE
        M=l
        IF (I N F I X . EG. 2) GO TO 150
        AK2=ABS(A(K)-A2)
   150  AKl=ABS(A(K-Al)
        BK1=ABS(B(K))
        IF (SQUARE) GO TO 160
        AK3=ABS(A(K)-A3)
        IF (INFIX. EQ. 4) GO TO 160
        AK4=ABS(A(K)-A4)
        IF (AK4.LE.EPS.AND.BK1.LE.EPS) GO TO 530
   160  IF (AK1.LE.EPS.AND.BK1.LE.EPS) GO TO 500
        IF (INFIX. EG. 2) GO TO 190
        IF (AK2.LE.EPS.AND.BK1.LE.EPS) GO TO 510
        IF (SQUARE) GO TO 170
        IF (AK3.LE.EPS.AND.BK1.LE.EPS) GO TO 520
   170  AK=A(K)
        BK=B(K)
C
C MAP EACH POINT (A,B) OF THE UPPER HALF Z-PLANE ONTO A POINT (G , H )  IN
C THE UPPER HALF T-PLANE USING THE BILINEAR TRANSFORMATION. SEE
C SECTION 3 OF THE DOCUMENT AT I ON.
C
        CALL A02ACF(AK-A1, BK,AK-A2, BK, TR, T I )
        G=C*TR
        H=C*TI
        GO TO 200
   180  G=C
        H=0. 0
        GO TO 200
   190  G=A(K)-A1
        H=B(K)
C
C COMPUTE THE AMPLITUDES PHI REQUIRED FOR THE CALCULATION OF THE
C INCOMPLETE ELLIPTIC INTEGRALS OF THE FIRST KIND.
C
   200  Gl = l. 0-G
        G2=G1*G1
        H2=H*H
        S=SQRT(G2+H2)
        S1=SQRT(G*G+H2)
        S2=S1*S1
        V=l. 0-S2*CMOD2
        GO TO 220
   210  V=S*S-S2*CMODP2
   220  W=ABS(V)
        IF (W-EPS) 230,230,250
   230  IF (ITYPE. EQ. 1. AND. M. GT. 1.  GO TO 240
        T=0. 5*(S-S2*0NEC/S)
        GO TO 300
   240  T=0. 5*(1. 0-S2*ONECP)/S
```

```
      GO TO 300
  250 IF (I TYPE. EQ. 1. AND. M. GT. 1 ) GO TO 260
      R=S*S*CMOD2+ONEC*V
      GO TO 270
  260 R=CM0DP2+V*ONECP
  270 IF (R) 280,290,290
  280 R=O. O
  290 T=(S-S1*SQRT(R))/V
  300 DONE=l. O-T
      IF (DONE. LT. 0. ) GO TO 310
      D0NE=1. +T
      IF (DONE. GE. O. ) GO TO 320
  310 T=SIGN(1. ,T)
  320 TA=ABS(T)
      IF (ABS(TA-1. ) .LE. EPS1) GO TO 330
      IF (TA.LE.EPS) GO TO 360
      D=ABS(1. -T*T)
      PHI=SNGL(DATAN(DBLE(SQRT(D)/T) ) )
      GO TO 340
  330 PH 1=0.
  340 IF (T) 350,360,370
  350 PHI=PHI+PI
      IF ( I TYPE. EQ. 1. AND. M. GT. 1 ) GO TO 380
      GO TO 390
  360 PHI=HALFPI
  370 IF (ITYPE. EG. 1. AND. M. GT. 1) GO TO 380
      GO TO 390
  380 C0N1=CEIP
      C0N2=TW0CEP
      C0N3=ALFAP
      GO TO 400
  390 C0N1=CEI
      C0N2=TW0CEI
      C0N3=ALFA
  400 IF (PHI) 420,410,420
  410 F=0. 0
      IF (M. EQ. 1) GO TO 480
      GO TO 490
  420 AB=ABS(PHI-PI)
      IF (AB-EPS) 430,430,440
  430 F=C0N2
      IF (M. EQ. 1) GO TO 480
      GO TO 490
  440 IF (PHI-HALFPI) 460,450,470
  450 F=CONl
      IF (M. EQ. 1) GO TO 480
      GO TO 490
C
C COMPUTE THE INCOMPLETE ELLIPTIC INTEGRALS OF THE FIRST KIND OF
C AMPLITUDES PHI AND MODULAR ANGLE C0N3 USING CTM1Y.
C
  460 F=CTM1Y(PHI.C0N3)
      IF (M. EQ. 1) GO TO 480
      GO TO 490
  470 F=C0N2-CTM1Y(PI-PHI, C0N3)
      IF (M. GT. 1) GO TO 490
C
C COMPUTE THE CO-ORDINATES OF THE POINT (K5I,ETA) IN THE WPRIME-PLANE
C CORRESPONDING TO THE POINT (G,H) IN THE LPPER HALF T-PLANE.
C
  480 KSI=F/TWOCEI
      M=M+1
      IF (ITYPE. EQ. 1) G0 TO 210
      Go TO 540
```

```
  490 ETA=F/TWOCEI
      GO TO 550
  500 KSI=0. 0
      IF (ITYPE.EQ. 2) GO TO 540
      ETA=0. 0
      GO TO 550
  510 KSI=0. 0
      IF (ITYPE. EQ. 2) GO TO 540
      ETA=HEIGHT
      GO TO 550
  520 KSI=1.0
      IF ( ITYPE. EQ. 2) GO TO 540
      ETA=HEIGHT
      GO TO 550
  530 KSI-1. 0
      IF (ITYPE. EQ. 2) GO TO 540
      ETA-O. O
      GO TO 550
C
C COMPUTE THE SOLUTION U AT EACH POINT ( x , Y )  OF THE ORIGINAL
C DOMAIN OMEGA.
C
  540 A(K)=U10*(KSI+U0/U10)
      GO TO 560
  550 A(K)=KSI
      B(K)=ETA
  560 K=K+1
      IF (K. LE. N) GO TO 130
      IFAIL=0
      RETURN
C
C FAILURE EXIT
C
  570 IFAIL=P01AAF(IFAIL, IERR, SRNAME)
      RETURN
      END
C
C
C
      REAL FUNCTION CTMIX(B)
      REAL B
C
C THIS ROUTINE IS USED FOR THE COMPUTATION OF THE COMPLETE ELLIPTIC
C INTEGRAL OF THE FIRST KIND OF MODULUS B. B MUST BE POSITIVE LT. 1.
C SEE REFERENCE /3/ OF THE DOCUMENTATION.
C
      INTEGER N
      REAL PI. A, Al. B l . A2, B2, C1 . Z, X01AAF
      PI=X01AAF(PI)
      A=ATAN(B/SQRT(1. 0-B*B))
      IF(B-O. 9539) 20, 10. 10
   10 A1=0. 5*(1. 0+B)
      B1=SQRT(B)
      A2=0. 5*(A1+B1)
      B2=C0S(A)
      B1=SQRT(A1*B1)
      C1-A2+B1
      Z=128. 0*C1*A2*A1*A1
      CTM1X=0. 5*ALOG(Z/(B2**4))/C l
      RETURN
   20 B1=ABS(C0S(A))
      Al = l. O
      D0 30 N=l.3
      A2=0. 5*(A1+B1)
```

```
      B1=SQRT(A1*B1)
      A1=A2
   30 CONTINUE
      CTM1X=PI/(A1+B1)
      RETURN
      END
C
C
C
      REAL FUNCTION CTMlY(PHI,ALFA)
      REAL PHI. ALFA
C
C THIS ROUTINE IS USED FOR THE COMPUTATION OF THE INCOMPLETE ELLIPTIC
C INTEGRAL OF THE FIRST KIND OF AMPLITUDE PHI AND MODULAR ANGLE ALFA.
C SEE REFERENCE /3/ OF THE DOCUMENTATION.
C
      INTEGER N, M
      REAL ER, P l , C , B , B I - A, SI, Al, X01AAF, X02AAF
      ER=X02AAF(ER)
      PI=XO1AAF(PI)
      C=ALFA
      B=ABS(COS(C))
      B1=ABS(SIN(C))
      IF (B1-0. 9539) 10,30,30
   10 A=l. O
      SI=COS(PHD/SIN(PHI)
      N=0
      C=B
      DO 20 M=l, 3
      SI=0. 5*(SI-C/SI)
      N=2*N+IFIX(0. 5*(1. 0-SIGN(l. 0, SI))+ER)
      A=0. 5*(A+B)
      B=SQRT(C)
      C=A*B
   20 CONTINUE
      SI=0. 5*(SI-C/SI)
      N=2*N+1-IFIX(SIGN(1. 0, SI))
      A=0. 5*(A+B)
      CTM1Y=0. 03125*(2. 0*ATAN(A/SI)+FLOAT(N)*PI)/A
      RETURN
   30 A=l. O
      B=B1
      SI=COS(PHl)/SIN(PHI)
      A1=B*B
      DO 40 M=l/ 2
      C=A*B
      SI=A*SI+saRT(A*A*(SI*SI-(-l. 0)-Al)
      A=0. 5*(A+0)
      SI=0. 5*SI/A
      B=SQRT(C)
      Al=C
   40 CONTINUE
      SI=A*SI+SORT(A*A*(SI*SI*H. 0)c)
      A=0. 5*(A+B)
      SI=0. 5*SI/A
      CTM1Y=ALOG((1. 0+SQRT(1. 0+SI*SI))/SI ./A
      RETURN
      END
C
C
C
      SUBROUTINE C0MAP1 (N, XF, YF, AF, (, Y, A, E , INFIX, INFDAT)
      INTEGER N, INFIX
      LOGICAL INFDAT
```

```
      REAL XF(4),YF(4).AF(4),X(N),Y(N),A(N),B(N)
C
C THIS ROUTINE MAPS THE SET OF POINTS (X,Y) OF THE RECTANGULAR
C DOMAIN OMEGA ONTO A SET OF POINTS (A, B) ON THE COMPLEX UPPER
C HALF PLANE USING A CONFORMAL TRANSFORMATION IN TERMS OF AN ELLIPTIC
C SINE. SEE FORMULAE (17),(18) OF REFERENCE /l/ OF THE DOCUMENTATION.
C
      INTEGER NO. I, Kl, K, J
      REAL PI.HALFPI.E, QN(3),GD(3), XI, Yl, PIX, PIY, RJ, A1.B1.RJX.
     *      U, V, AR1, AC1. AR2. AC2, XOIAAF
C
C COMPUTE THE CONSTANTS OF THE TRANSFORMATION.
C
      PI=X01AAF(PI)
      HALFPI-O. 5*PI
      NO=1
      DO 10 1=1, 3
      E=FLOAT(I)-0. 5
      QN(I)=EXP (-E*E*PI)*FLOAT(NO)
      NO=-NO
      E=FLOAT(I)
      QD ( I )-EXP (-E*E*PI)*FLOAT(NO)
   10 CONTINUE
C
C MAP EACH POINT (X(I),Y(D) ONTO A POINT (A(I).B(I).
C
      Kl-N-4
      DO 70 K=4, N, Kl
      DO 60 1 = 1, K
      IF (K. EQ. 4) GO TO 20
      IF (INFDAT. AND. I. EQ. 1) GO TO 60
      X1»X(I)
      Yl=Y(I)
      GO TO 30
   20 IF (INFIX. EQ. 4 .AND. I. EQ. 4) GO TO 60
      X1=XF(I)
      Y1=YF(I)
   30 PIX=PI*X1
      PIY=PI*Y1
      AR 1=0.
      AC 1=0.
      AR 2=0.
      AC 2=0.
      DO 40 J=i, 3
      RJ=FLOAT(J)
      A1=EXP(RJ*PIY)
      Bl = l. O/Al
      RJX=RJ*PIX
      U=0.5*C0S(RJX)*(A1+B1)
      V=-0. 5*SIN(RJX)*(A1-B1)
      AR1=U*QD(J)+AR1
      AC1«V*QD(J)+AC1
      RJ=FLOAT(J)-0. 5
      A1=EXP(RJ*PIY)
      Bl = l. O/Al
      RJX=RJ*PIX
      U=0. 5*SIN(RJX)*(A1+B1)
      V=0. 5*C0S(RJX)*(A1-B1)
      AR2=U*QN(J)+AR2
      AC2*V*QN(J)+AC2
   40 CONTINUE
      AR1 = 1. +2. *AR1
      AC 1=2. *AC1
      CALL A02ACF(AR2, AC2. AR1. AC1, Al, Bl)
```

```
       IF (K. EG. 4) GO TO 50
       A(I)=A1
       B(I)=B1
       G0 TO 60
   50  AF(I)=A1
   60  CONTINUE
   70  CONTINUE
       RETURN
       END
C
C
C
       SUBROUTINE C0MAP2(N, XF, YF, AF, X, Y. A, B)
       INTEGER N
       REAL XF(4), YF(4), AF(4), X(N),Y(N),A(N), B(N)
C
C THIS ROUTINE MAPS THE SET OF POINTS ( X,Y) OF THE L-SHAPED DOMAIN
C OMEGA ONTO A SET OF POINTS (A,B) ON THE COMPLEX UPPER HALF PLANE.
C THE MAPPING FUNCTION IS NOT KNOWN ANALYTICALLY. FOR THIS REASON
C AN APPROXIMATE MAPPING FUNCTION OF OMEGA ONTO THE UNIT DISC IS
C USED, GIVEN IN EXAMPLE 2. 4 OF REFERENCE /5/ OF THE DOCUMENTATION.
C THE SET OF POINTS IN THE UNIT DISC IS THEN MAPPED ONTO THE UPPER
C HALF PLANE ANALYTICALLY.
C
       COMPLEX AN(i7),C0, CI, C2, W, SUM, D, F, WO, WI, Z
       INTEGER NIN, L, K1,K, I,M
       REAL C, PI,TWOPI,PHI, RC, XI, Yl, E, THETA, Rl, X2, Y2, R, X01AAF
       DATA NIN /5/
C
C INPUT THE PARAMETERS OF THE MAPPING FUNCTION I. E. THE DEGREE L OF
C THE POLYNOMIAL EXPRESSION, THE REAL CONSTANT C AND THE COMPLEX
C COEFFICIENTS AN.
C
       READ (NIN,8OOO)  L
       READ (NIN, 8010) C
       DO 10 1=1, L
       READ (NIN,8010) AN(I)
    10 CONTINUE
       CO=CMPLX(C, O. O)
       C1=(0. 0, 1. 0)
       C2=(l. 0,0. 0)
       PI=X01AAF(PI)
       TWOPI=PI+PI
       PHI-O. 75*PI
       RC=3. 0*SQRT(2. O)
       Kl=N-4
       DO 250 K=4, N, Kl
       DO 240 1 = 1, K
       IF (K. EQ. 4) GO TO 20
       X1(=X(I)
       Y1=Y(I)
       GO TO 30
   20  X1=XF(I)
       Y1=YF(I)
   30  W=CMPLX(X1, Y1)
       SUM=(0. O. 0. O)
C
C FORM THE SERIES EXPANSION APPROXIMATION OF THE MAPPING FUNCTION
C
      M=l
   40 CONTINUE
      GO TO (50, 60, 70, 80, 100, 170, 100, 180, 1 10, 190, 120, 200, 130), M
   50 D=W-(14. 0, 0. 0)
      GO TO 90
```

```
  60 D=W-(0.0,6.0)
     GO TO 90
  70 D=W+(0.0,14.0)
     GO TO 90
  80 D=W+(26.0,0,0)
  90 F=W/D
     GO TO 220
 100 E=(FL0AT(M)--3. 0)/3. 0
     GO TO 140
 110 E=8. 0/3.0
     GO TO 140
 120 E=10. 0/3.0
     GO TO 140
 130 E=14. 0/3.0
 140 THETA=PHI*E
     R1=RC**E
     W0-CMPLX(R1*C0S(THETA),R1*SIN(THETA))
     Wl=W-(3. 0.-3. 0)
     X2=REAL(W1)
     Y2=AIMAG(W1)
     IF (X2. EQ. 0. 0. AND. Y2. EQ. 0. 0) GO TO 150
     THETA=ATAN2(Y2, X2)
     IF (Y2. LT. 0. 0) THETA-THETA+TWOPI
     GO TO 160
 150 THETA-O.0
 160 R=CABS(W1)**E
     THETA=E*THETA
     F=CMPLX(R*COS(THETA),R*SIIM(THETA))-WO
     GO TO 220
 170 F=W
     GO TO 220
 180 F=W*W/2
     GO TO 220
 190 F=W**3/3
     GO TO 220
 200 F=W**4/4
     GO TO 220
 210 F=W**(M-9)/(M-9)
 220 SUM=SUM+AN(M)*F
     M=M+1
     IF (M. LE. 13) GO TO 40
     IF (M. LE. L) GO TO 210
     W=CO*SUM
C
C MAP THE UNIT DISC ONTO THE UPPER HALF Z-PLANE.
C
     Z=C1*(C2+W)/(C2-W)
     IF (K. EQ. 4) GO TO 230
     A(I)=REAL(Z)
     B(I)=AIMAG(Z)
     IF (B(I). LT. l.E-2) B(I)=0 0
     GO TO 240
 230 AF(I)=REAL(Z)
 240 CONTINUE
 250 CONTINUE
8000 FORMAT (I3)
8010 FORMAT (2E18. 12)
     RETURN
     END
```

```
 1
26
 0
-1.00000000E 00
 0.00000000E 00
 1.00000000E 00
 2.00000000E 00
-1.00000000E 00      0.00000000E 00
 0.00000000E 00      0.00000000E 00
 1.00000000E 00      0.00000000E 00
 2.00000000E 00      0.00000000E 00
-13.00000000E 00     1.00000000E 00
-11.00000000E 00     1.00000000E 00
-9.00000000E 00      1.00000000E 00
-7.00000000E 00      1.00000000E 00
-5.00000000E 00      1.00000000E 00
-3.00000000E 00      1.00000000E 00
-1.00000000E 00      1.00000000E 00
 1.00000000E 00      1.00000000E 00
 3.00000000E 00      1.00000000E 00
 5.00000000E 00      1.00000000E 00
 7.00000000E 00      1.00000000E 00
-13.00000000E 00     3.00000000E 00
-11.00000000E 00     3.00000000E 00
-9.00000000E 00      3.00000000E 00
-7.00000000E 00      3.00000000E 00
-5.00000000E 00      3.00000000E 00
-3.00000000E 00      3.00000000E 00
-1.00000000E 00      3.00000000E 00
 1.00000000E 00      3.00000000E 00
 3.00000000E 00      3.00000000E 00
 5.00000000E 00      3.00000000E 00
 7.00000000E 00      3.00000000E 00
 2
28
 0
 0.00000000E 00      0.00000000E 00
-1.00000000E 00      0.00000000E 00
 1.00000000E 00      1.00000000E 00
 1.00000000E 00      0.00000000E 00
-0.85714287E 00      0.85714287E 00
-0.57142858E 00      0.85714287E 00
-0.28571429E 00      0.85714287E 00
 0.00000000E 00      0.85714287E 00
 0.28571429E 00      0.85714287E 00
 0.57142858E 00      0.85714287E 00
 0.85714287E 00      0.85714287E 00
-0.85714287E 00      0.57142858E 00
-0.57142B58E 00      0.57142858E 00
-0.28571429E 00      0.57142858E 00
 0.00000000E 00      0.57142858E 00
 0.28571429E 00      0.57142858E 00
 0.57142858E 00      0.57142858E 00
 0.85714287E 00      0.57142858E 00
-0.85714287E 00      0.2B571429E 00
-0.57142858E 00      0.28571429E 00
-0.28571429E 00      0.28571429E 00
 0.00000000E 00      0.28571429E 00
 0.28571429E 00      0.28571429E 00
 0.57142858E 00      0.28571429E 00
 0.85714287E 00      0.28571429E 00
-0.85714287E 00      0.00000000E 00
-0.57142858E 00      0.00000000E 00
```

Data (Contd)

```
-0.28571429E 00      0.00000000E 00
 0.00000000E 00      0.00000000E 00
 0.28571429E 00      0.00000000E 00
 0.57142858E 00      0.00000000E 00
 0.85714287E 00      0.00000000E 00
 0.50000000E 03      1.00000000E 03
 2
62
 0
-13.00000000E 00    -7.00000000E 00
 3.00000000E 00     -7.00000000E 00
 7.00000000E 00      3.00000000E 00
-13.00000000E 00     3.00000000E 00
 7.00000000E 00      3.00000000E 00
 7.00000000E 00      1.00000000E 00
 7.00000000E 00     -1.00000000E 00
 7.00000000E 00     -3.00000000E 00
 5.00000000E 00      3.00000000E 00
 5.00000000E 00      1.00000000E 00
 5.00000000E 00     -1.00000000E 00
 5.00000000E 00     -3.00000000E 00
 3.00000000E 00      3.00000000E 00
 3.00000000E 00      1.00000000E 00
 3.00000000E 00     -1.00000000E 00
 3.00000000E 00     -3.00000000E 00
 3.00000000E 00     -5.00000000E 00
 3.00000000E 00     -7.00000000E 00
 1.00000000E 00      3.00000000E 00
 1.00000000E 00      1.00000000E 00
 1.00000000E 00     -1.00000000E 00
 1.00000000E 00     -3.00000000E 00
 1.00000000E 00     -5.00000000E 00
 1.00000000E 00     -7.00000000E 00
-1.00000000E 00      3.00000000E 00
-1.00000000E 00      1.00000000E 00
-1.00000000E 00     -1.00000000E 00
-1.00000000E 00     -3.00000000E 00
-1.00000000E 00     -5.00000000E 00
-1.00000000E 00     -7.00000000E 00
-3.00000000E 00      3.00000000E 00
-3.00000000E 00      1.00000000E 00
-3.00000000E 00     -1.00000000E 00
-3.00000000E 00     -3.00000000E 00
-3.00000000E 00     -5.00000000E 00
-3.00000000E 00     -7.00000000E 00
-5.00000000E 00      3.00000000E 00
-5.00000000E 00      1.00000000E 00
-5.00000000E 00     -1.00000000E 00
-5.00000000E 00     -3.00000000E 00
-5.00000000E 00     -5.00000000E 00
-5.00000000E 00     -7.00000000E 00
-7.00000000E 00      3.00000000E 00
-7.00000000E 00      1.00000000E 00
-7.00000000E 00     -1.00000000E 00
-7.00000000E 00     -3.00000000E 00
-7.00000000E 00     -5.00000000E 00
-7.00000000E 00     -7.00000000E 00
-9.00000000E 00      3.00000000E 00
-9.00000000E 00      1.00000000E 00
-9.00000000E 00     -1.00000000E 00
-9.00000000E 00     -3.00000000E 00
-9.00000000E 00     -5.00000000E 00
-9.00000000E 00     -7.00000000E 00
-11.00000000E 00     3.00000000E 00
```

-11.00000000E 00	1.00000000E 00
-11.00000000E 00	-1.00000000E 00
-11.00000000E 00	-3.00000000E 00
-11.00000000E 00	-5.00000000E 00
-11.00000000E 00	-7.00000000E 00
-13.00000000E 00	3.00000000E 00
-13.00000000E 00	1.00000000E 00
-13.00000000E 00	-1.00000000E 00
-13.00000000E 00	-3.00000000E 00
-13.00000000E 00	-5.00000000E 00
-13.00000000E 00	-7.00000000E 00
0.00000000E 00	1.00000000E 00
17	
. 149098601250E 02	
-.171147593328E-01	-.945691444885E-03
. 953821713728E-05	-.530104068099E-01
. 955178615453E-03	.233189952548E-01
. 362715490460E-01	-.258868194269E-02
. 216876372822E-01	.157865852619E-01
. 857275037075E-01	.855965062994E-02
-.408292565825E-02	.407932O02784E-O2
-.231704817365E-01	-.288822137143E-01
.142602603603E-02	-.133972804815E-02
-.809913758946E-03	.315437414610E-02
-.187405614369E-03	-.234230541311E-03
-.272980529148E-04	.121172385031E-O3
-.415596940524E-05	-.156407058589E-05
.549528008224E-05	-.212981873915E-05
.767447829679E-07	.379621311941E-07
.182068289580E-08	.186980825904E-08
.330153679120E-10	.318968182170E-10

CONFORMAL MAP OF THE UPPER HALF Z-PLANE TO A RECTANGLE

Z-PLANE		W'-PLANE	
A	B	KSI	ETA
-0.100000E+01	0.000000E+00	0.000000E+00	0.000000E+00
0.000000E+00	0.000000E+00	0.000000E+00	0.731701E+00
0.100000E+01	0.000000E+00	0.100000E+01	0.781701E+00
0.200000E+01	0.000000E+00	0.100000E+01	0.000000E+00
-0.130000E+02	0.100000E+01	0.465762E+00	0.254777E-02
-0.110000E+02	0.100000E+01	0.459858E+00	0.351272E-02
-0.900000E+01	0.100000E+01	0.451310E+00	0.515159E-02
-0.700000E+01	0.100000E+01	0.438834E+00	0.827738E-02
-0.500000E+01	0.100000E+01	0.417416E+00	0.154359E-01
-0.300000E+01	0.100000E+01	0.374782E+00	0.382908E-01
-0.100000E+01	0.100000E+01	0.302639E+00	0.172752E+00
0.100000E+01	0.100000E+01	O.607621E+00	0.327245E+00
0.300000E+01	0.100000E+01	0.663306E+00	0.743234E-01
0.500000E+01	0.100000E+01	O.599803E+00	0.231125E-01
0.700000E+01	0.100000E+01	0.570311E+00	0.110331E-01
-0.130000E+02	0.300000E+01	0.467209E+00	0.731880E-02
-0.110000E+02	0.300000E+01	0.462163E+00	0.992926E-02
-0.900000E+01	0.300000E+01	0.455496E+00	0.141736E-01
-0.700000E+01	0.300000E+01	0.446E49E+03	0.216587E-01
-0.500000E+01	0.300000E+01	0.434981E+00	0.362382E-01
-0.300000E+01	0.300000E+01	0.424746E+00	0.670999E-01
-0.100000E+01	0.300000E+01	0.442879E+00	0.122483E+00
0.100000E+01	0.300000E+01	0.522259E+00	0.144809E+00
0.300000E+01	0.300000E+01	0.573141E+00	0.926152E-01
0.500000E+01	0.300000E+01	0.571064E+00	0.487654E-01
0.700000E+01	0.300000E+01	0.538948E+00	0.276596E-01

HARMONIC PROBLEM 2, REFERENCE /1/. PAGE 660

POINT:

(X ,	Y)	SOLUTION
(-0.8571 ,	0.8571)	591. 3407
(-0.5714 ,	0.8571)	608. 8885
(-0.2857 ,	0.8571)	645. 4917
(0.0000 ,	0.8571)	702. 1380
(0.2857 ,	0.8571)	776. 2938
(0.5714 ,	0.8571)	862. 0200
(0.8571 ,	0.8571)	953. 4631
(-0.8571 ,	0.5714)	574. 0998
(-0.5714 ,	0.5714)	589. 8034
(-0.2857 ,	0.5714)	624. 7562
(0.0000 ,	0.5714)	683. 9083
(0.2857 ,	0.5714)	764. 8423
(0.5714 ,	0.5714)	856. 6767
(0.8571 ,	0.5714)	951. 9812
(-0.8571 ,	0.2857)	541. 7604
(-0.5714 ,	0.2857)	551. 9745
(-0.2857 ,	0.2857)	578. 5592
(0.0000 ,	0.2857)	641. 5595
(0.2857 ,	0.2857)	743. 8118
(0.5714 ,	0.2857)	848. 6436
(0.8571 ,	0.2857)	949. 9306
(-0.8571 ,	0.0000)	500. 0000
(-0.5714 ,	0.0000)	500. 0000
(-0.2857 ,	0.0000)	500. 0000
(0.0000 ,	0.0000)	500. 0000
(0.2857 ,	0.0000)	728. 4743
(0.5714 ,	0.0000)	844. 3682
(0.8571 ,	0.0000)	948. 9331

HARMONIC PROBLEM 2 (IV), REFERENCE /2/, PAGE 310

POINT	(X ,	Y)	SOLUTION
	(7. 0000 ,	3. 0000)	1. 0000
	(7. 0000 ,	1. 0000)	0. 9004
	(7. 0000 ,	-1. 0000)	0. 8220
	(7. 0000 ,	-3. 0000)	0. 7907
	(5. 0000 ,	3. 0000)	1. 0000
	(5. 0000 ,	1. 0000)	0. 8906
	(5. 0000 ,	-1. 0000)	0. 7980
	(5. 0000 ,	-3. 0000)	0. 7555
	(3. 0000 ,	3. 0000)	1. 0000
	(3. 0000 ,	1. 0000)	0. 8660
	(3. 0000 ,	1. 0000)	0. 7320
	(3. 0000 ,	-3. 0000)	0. 5676
	(3. 0000 ,	-5. 0000)	0. 2469
	(3. 0000 ,	-7. 0000)	0. 0000
	(1. 0000 ,	3. 0000)	1. 0000
	(1. 0000	1. 0000)	0. 8400
	(1. 0000	-1. 0000)	O. 6700
	(1. 0000 ,	-3. 0000)	0. 4692
	(1. 0000 ,	-5. 0000)	0. 2260
	(1. 0000 ,	-7. 0000)	0. 0000
	(-1. 0000 ,	3. 0000)	1. 0000
	(-1. 0000 ,	1. 0000)	0. 8222
	(-1. 0000 ,	1. 0000)	0. 6365
	(-1. 0000 ,	-3. 0000)	0. 4360
	(-1. 0000 ,	-5. 0000)	0. 2212
	(-1. 0000 ,	-7. 0000)	0. 0000
	(-3. 0000 ,	3. 0000)	1. 0000
	(-3. 0000 ,	1. 0000)	0. 8119

(-3. 0000 , -1.0000)	0.6193
(-3. 0000 , -3.0000)	0.4192
(-3. 0000 , -5.0000)	0.2116
(-3. 0000 , -7.0000)	0.0000
(-5. 0000 , 3.0000)	0.0000
(-5. 0000 , 1.0000)	0.8064
(-5. 0000) -1.0000)	0.6103
(-5. 0000 , -3.0000)	0.4103
(-5. 0000 , -5.0000)	0.2063
(-5. 0000 , -7.0000)	0.0000
(-7. 0000 , 3.0000)	0.0000
(-7. 0000 , 1.0000)	0.8034
(-7. 0000 , -1.0000)	0.6056
(-7. 0000 , -3.0000)	0.4056
(-7. 0000 , -5.0000)	0.2034
(-7. 0000 , -7.0000)	0.0000
(-9. 0000 , 3.0000)	0.0000
(-9. 0000 , 1.0000)	0.8019
(-9. 0000 , -1.0000)	0.6031
(-9. 0000 , -3.0000)	0.4031
(-9. 0000 , -5.0000)	0.2019
(-9. 0000 , -7.0000)	0. 0000
(-11. 0000 , 3.0000)	0. 0000
(-11. 0000 , 1.0000)	0.8011
(-11. 0000 , -1.0000)	0.6019
(-11. 0000 , -3.0000)	0.4020
(-11. 0000 , -5.0000)	0.2013
(-11. 0000 , -7.0000)	0. 0000
(-13. 0000 , 3.0000)	1. 0000
(-13. 0000 , 1.0000)	0.8006
(-13. 0000 , -1.0000)	0.6016
(-13. 0000 , -3.0000)	0.4016

(-13. 0000 , -5.0000)	0.2012
(-13. 0000 , -7.0000)	0. 0000