Discretization Correction Particle Strength Exchange (DC PSE) method for Linear Elasticity

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Linear Elasticity

$$(\lambda+\mu)\frac{\partial^2 u_j}{\partial x_i \partial x_j}+\mu\frac{\partial^2 u_i}{\partial x_j^2}=-\rho b_i$$

$$u_i=f_i$$

$$\lambda n_i\frac{\partial u_k}{\partial x_k}+\mu n_j\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)=\tau_i$$

Matrix notation

$$\begin{bmatrix} L_{xx} & L_{xy} & L_{xz} \\ B_{xx} & B_{xy} & B_{xz} \\ \Phi & \mathbf{0} & \mathbf{0} \\ L_{yx} & L_{yy} & L_{yz} \\ B_{yx} & B_{yy} & B_{yz} \\ \mathbf{0} & \Phi & \mathbf{0} \\ L_{zx} & L_{zy} & L_{zz} \\ B_{zx} & B_{zy} & B_{zz} \\ \mathbf{0} & \mathbf{0} & \Phi \end{bmatrix}\begin{bmatrix} U_i \\ V_i \\ W_i \end{bmatrix}=\begin{bmatrix} -\rho b_x \\ \tau_x \\ f_x \\ -\rho b_y \\ \tau_y \\ f_y \\ -\rho b_z \\ \tau_z \\ f_z \end{bmatrix}$$

$$L_{ij}=\mu\delta_{ij}\frac{\partial^2}{\partial x_k^2}+(\lambda+\mu)\frac{\partial^2}{\partial x_i \partial x_j}, \qquad i,j=x,y,z$$

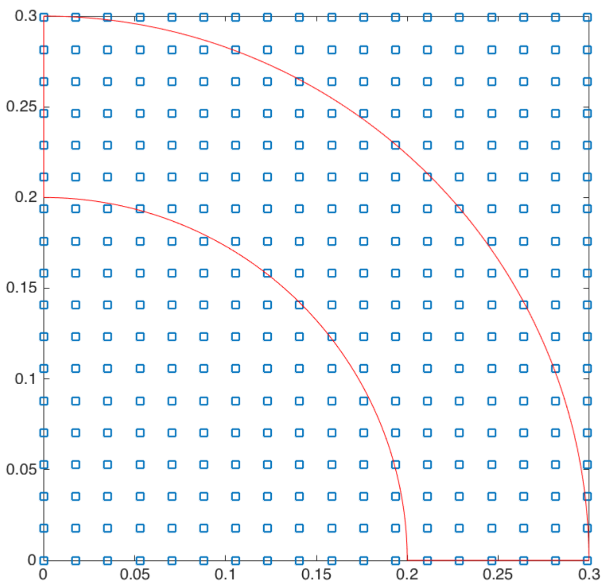
$$B_{ij}=\lambda n_i\frac{\partial}{\partial x_j}+\mu n_j\frac{\partial}{\partial x_i}+\mu\delta_{ij}n_k\frac{\partial}{\partial x_k}, \qquad i,j=x,y,z$$

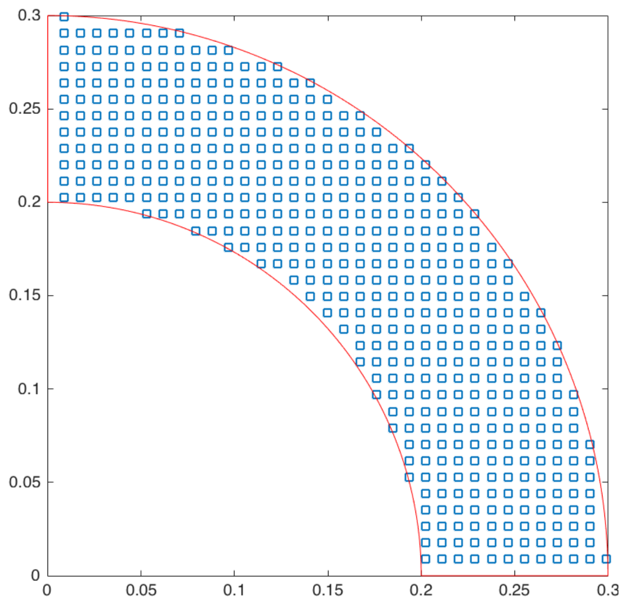
DC PSE

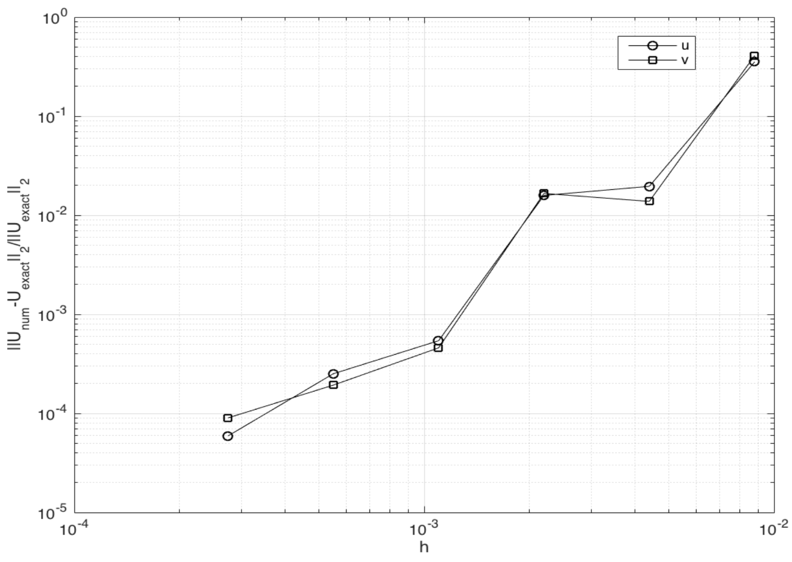
DC PSE was originally formulated as a correction of the Particle Strength Exchange method (PSE) on irregular nodal distributions. PSE is used for the evaluation of spatial derivatives of any order of a sufficiently smooth function discretized over scattered colocation points.

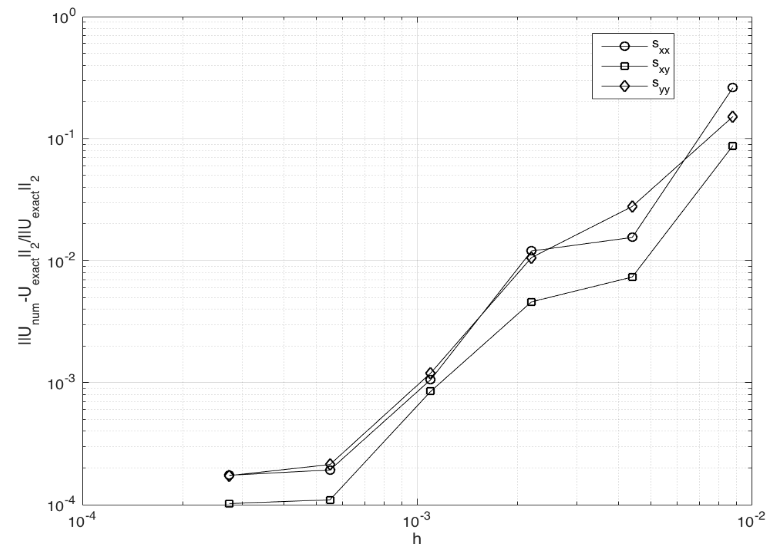
$$\boldsymbol{Q}^{m,n} f(\boldsymbol{x}_p) = \frac{1}{\epsilon(\boldsymbol{x}_p)^{m+n}} \sum_{\boldsymbol{x}_q \in \mathcal{N}(\boldsymbol{x})} \Big(f(\boldsymbol{x}_q) - f(\boldsymbol{x}_p) \Big) \boldsymbol{p}\left(\frac{\boldsymbol{x}_p - \boldsymbol{x}_q}{\epsilon(\boldsymbol{x}_p)} \right) \boldsymbol{a}^T(\boldsymbol{x}_p) e^{-\frac{(x_p - x_q)^2 + (y_p - y_q)^2}{\left(\epsilon(x_p)\right)^2}}$$

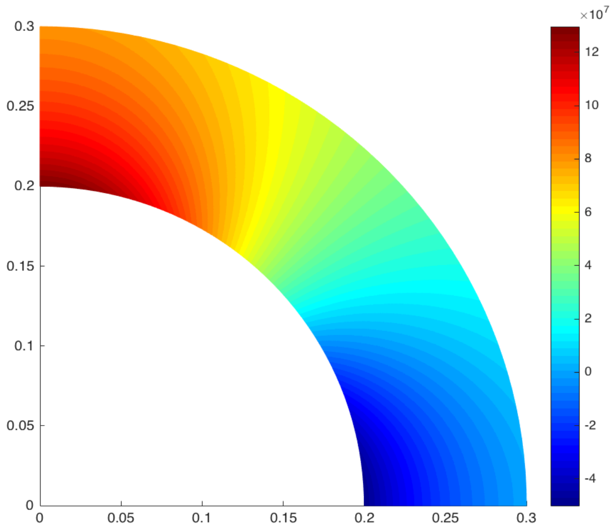
Pressurized cylinder

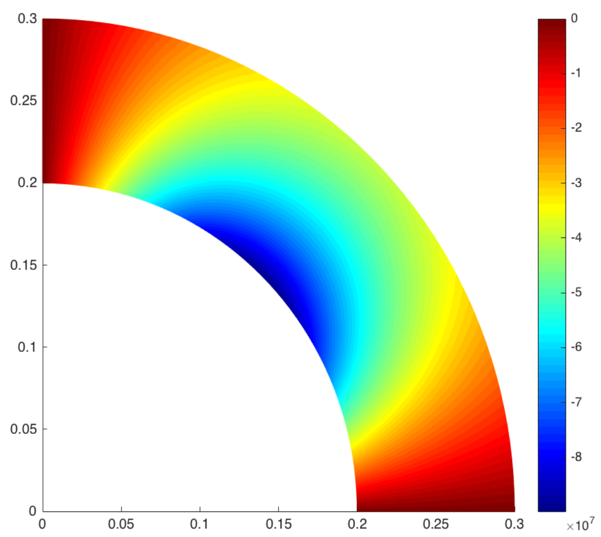





u
v
h
$||U_{num}-U_{exact}||_2/||U_{exact}||_2$


s_{xx}
s_{xy}
s_{yy}
h
$||U_{num}-U_{exact}||_2/||U_{exact}||_2$


$\times 10^7$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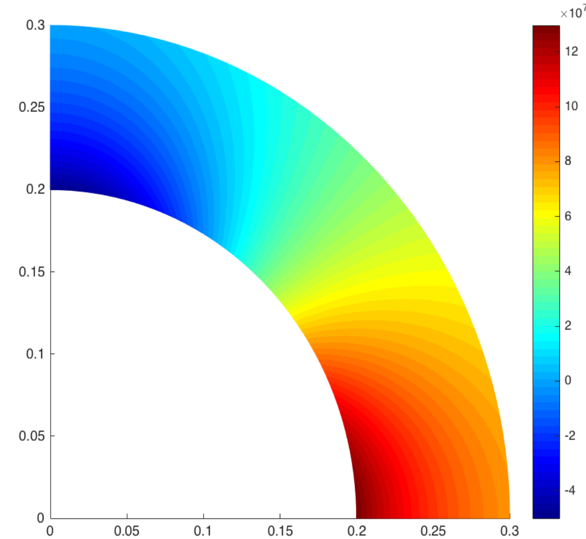
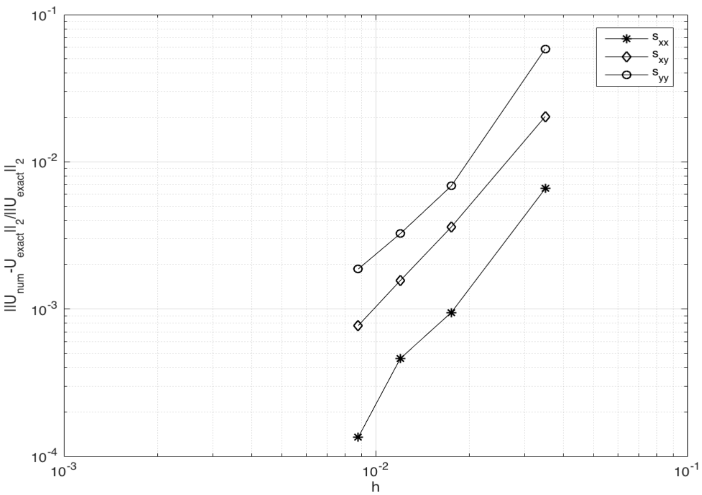
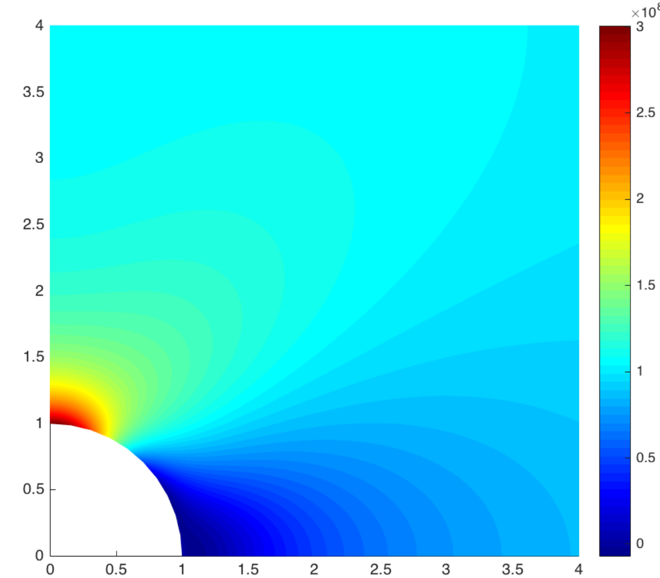
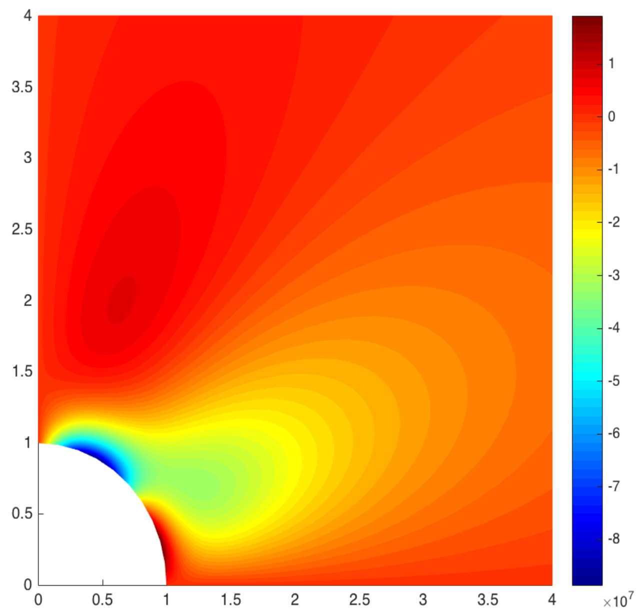
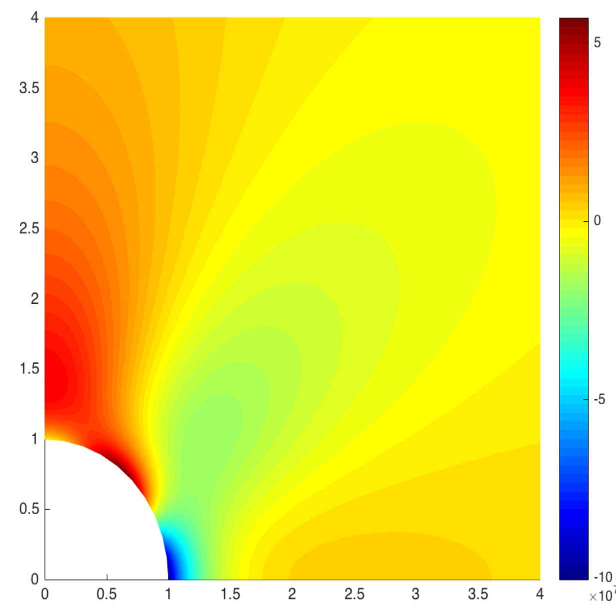

$\times 10^7$

Infinite plate with hole

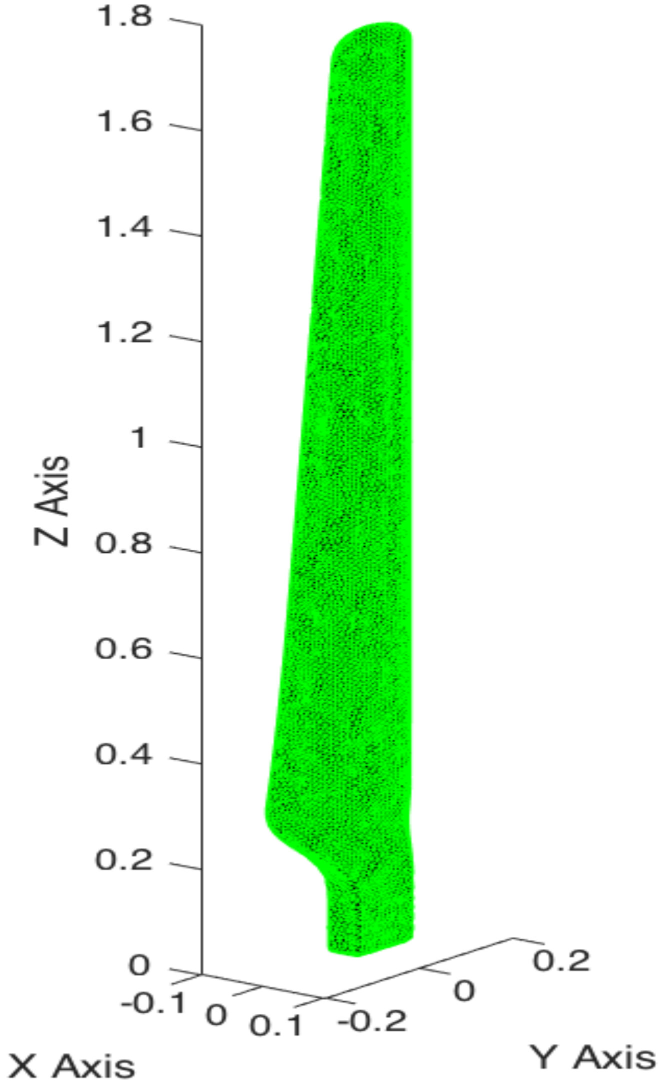




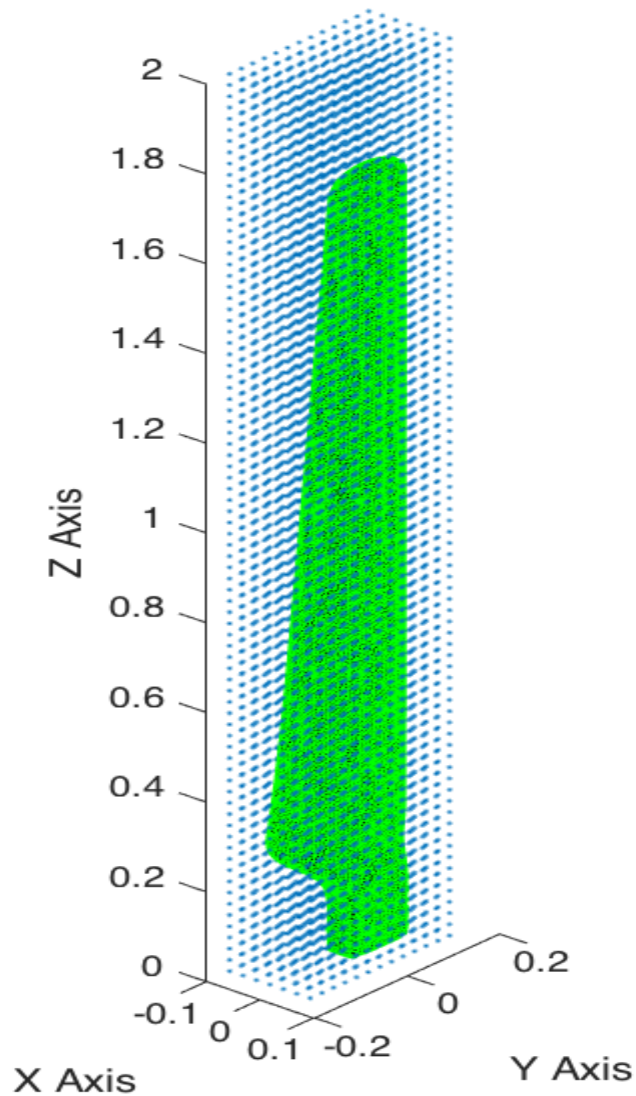




Blade

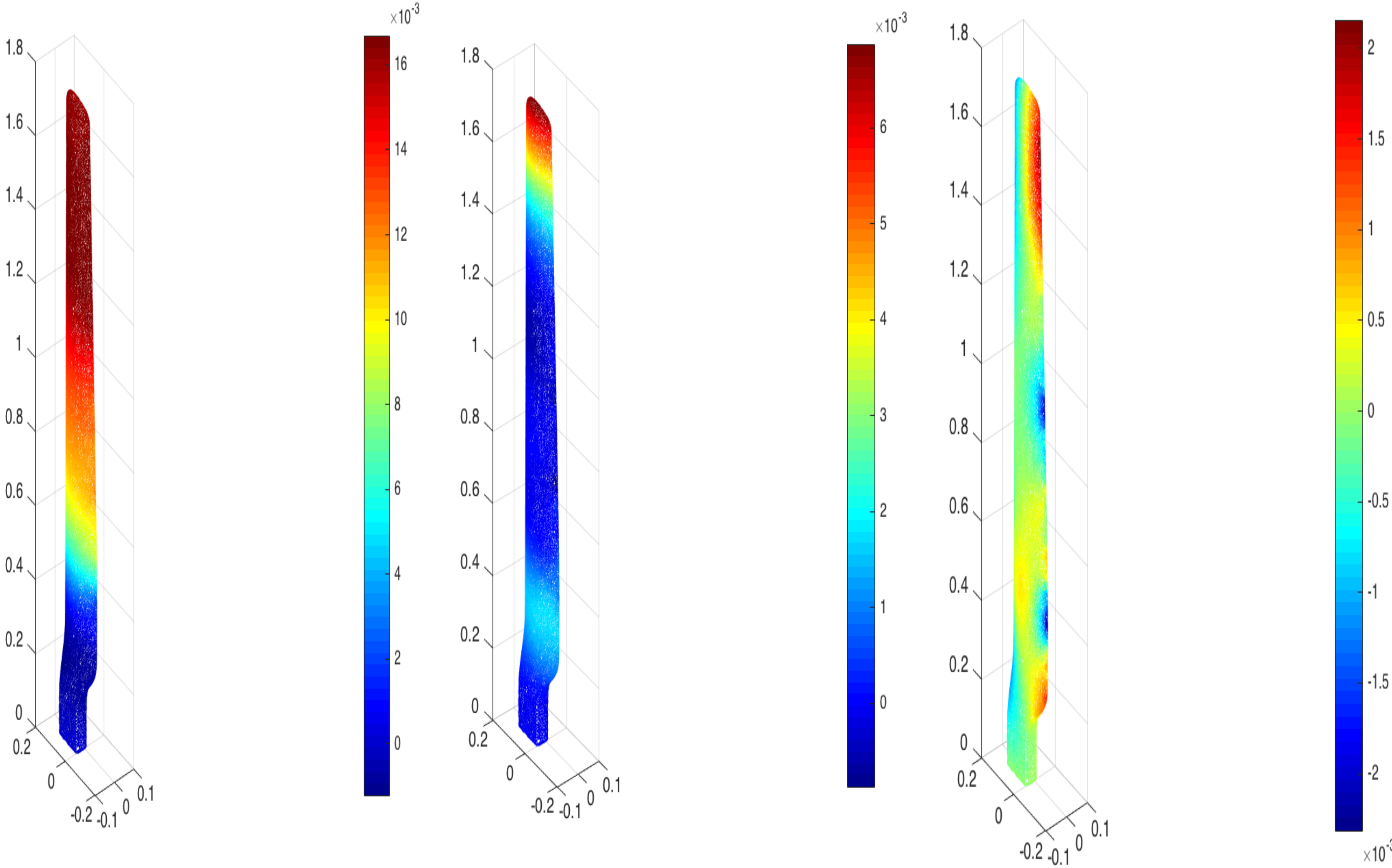


1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Z Axis
-0.1
0
0.1
X Axis
-0.2
0
0.2
Y Axis



2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Z Axis
-0.1
0
0.1
X Axis
-0.2
0
0.2
Y Axis

Blade



1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.2
0
-0.2
-0.1
0
0.1
×10^-3
16
14
12
10
8
6
4
2
0
×10^-3
6
5
4
3
2
1
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
×10^-3