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## ***Numerical modelling options for cracked masonry buildings***

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MORADABADI, EHSAN<sup>1</sup>, LAEFER, DEBRA F.<sup>2</sup>

**ABSTRACT:** In most numerical modelling of buildings, there is an assumption that the structure is undamaged. However, with historic buildings, defects often exist. Failing to incorporate such damage may cause an unconservative estimation of a building's response. Nowhere is this more critical than in the case of urban tunnelling where hundreds of unreinforced masonry structures may be impacted by ground movements. This paper examines the effectiveness and limitations of four numerical approaches in the modelling of existing discontinuities, in the form of masonry cracking when compared to traditional finite element methods. The comparative methods include a micro-poly method, a distinct element method, a discontinuity deformation method, and a combined continuum-interface method. Particular attention is paid to the ease of model implementation, the availability of input data, applicability of crack modelling, and the ability to define the initial state of the structure as part of the model. The methods are compared to each other and finite element modelling. Relative qualitative assessments are provided as to their applicability for modelling damaged masonry.

*Keywords:* Pre-existing damage, micro-poly methods, distinct element methods, discontinuity deformation method, and mesoscopic modelling

### **1 INTRODUCTION**

Dramatic rises in global population and urbanization have led to unprecedented levels of tunnelling. Such underground activities pose risks to unreinforced masonry buildings (UMBs), especially since many have existing defects in the form of cracks and deformations. As part of preventive measures, numerical models are not infrequently used to predict whether the subsurface construction will imperil the aboveground structure. If the actual condition of the building in the form of pre-existing damage is not incorporated into the numerical model, then the computational output may not be sufficiently conservative.

Cracking in masonry structures may be induced by deformation in bending/shear or volumetric changes of the component bricks, blocks, or mortar arising from natural expansion or shrinkage, temperature change, corrosion, or associated reactions [1]. In 1975 and 1976 much of Europe was subjected to severe droughts. As a consequence, many buildings on clay soils experienced damage. Based on that experience, Burland et al. (1977) [2] classified masonry building damage according to ease of repair into three broad categories: aesthetic, serviceability, and stability. Having considered the tensile strain as a serviceability parameter, limiting tensile strain was proposed as the crucial factor instead of critical strain, as is likely to progress at points where cracks have begun[3].

Notably, there are no current analytical methods that explicitly address the damaged state of a building, as part of its future vulnerability, damage should the building be subjected to further ground movements. Since masonry is an anisotropic structural material composed of natural or artificial blocks and mortar bonding layers, the constitutive model for the material and/or the details of component arrangements may be highly complex. Another difficulty in the analysis of historical structures is the

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1 PhD student, University College Dublin (UCD), Urban Modelling Group (UMG), School of Civil, Structural, and Environmental Engineering (SCSEE), [ehsan.moradabadi@ucdconnect.ie](mailto:ehsan.moradabadi@ucdconnect.ie)

2 Associate Professor, UCD, UMG, SCSEE, [debra.laefer@ucd.ie](mailto:debra.laefer@ucd.ie)

generation of the current loading level and distribution [4]. A direct application of self-weight and other expected loads may not reflect correctly the load paths and distribution that has resulted from a building's long history. These factors often demand parametric studies, in addition to the usual uncertainties about material and interface properties. Arguably, such cases are better handled with numerical modelling than experimental studies.

To propose a framework for addressing pre-existing damage in masonry modelling, this paper briefly describes examples of each of the following four discrete methods and considers their benefits and drawbacks when compared to traditional FEM.

1. Micro-poly methods (MPM)
2. Distinct element methods (DEM)
3. Discontinuity deformation analysis (DDA)
4. Combined continuum-interface methods (CIM)

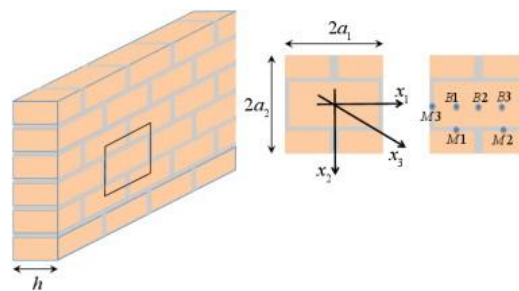
## 2 MODELLING WITH CONTINUUM MICRO-POLY METHODS

In the Cosserat/micro-poly method, the microscopic and macroscopic scales are intrinsically coupled. The main idea of that method is to analyse at a micromechanical level a Representative Volume Element (RVE) that contains all relevant information about the micro-structure and then to solve two different Boundary Value Problem (BVPs): one at the micro-scale level and the other at the macro-scale. When a finite element method (FEM) has been applied, different homogenization procedures are used to overcome the computational cost resulting from solving a non-linear, micro-mechanical FEM scheme at each Gauss point in the structural mesh [5].

Continuum micro-poly methods employ first order homogenization (using the classical Cauchy continuum). From a mechanical point of view, the Cosserat model is able to describe typical micromechanical mechanisms not covered by the Cauchy medium, like block rotation, rotational deformation, and two other double curvature [6]. For masonry, there are also several disadvantages [5, 7]. Firstly, as the distribution and morphology of constituents do not incorporate the absolute size of the microstructure, it is impossible to address geometrical size effects. Secondly, there is a difficulty in the intrinsic assumption of uniformity of the macroscopic fields attributed to each micro-structural RVE. Finally, there are localization and mesh dependencies problems.

To overcome these drawbacks, researchers have inserted an embedded fracture band [8] or employed a generalized continua with a higher order [9] to describe the behaviour of either the microstructural constituents or the homogenized macrostructure. Notably, developing a systematic methodology for assigning equivalent continuum properties in this approach is not easily achieved. Consequently, attempting to solve practical engineering problems using these methodologies is rare [10], even when using Cosserat homogenization [11, 12].

However, a recent development in the application of the Cosserat model in a masonry structure used an enriched plane state formulation [13]. In that, (1) the RVE of a masonry panel is governed by a 3D displacement vector ( $u(x)$ , where  $x$  denotes the position vector), and (2) the equilibrium equations are solved based on the Gauss-Green theorem and an approximate numerical solution (i.e. FEM). The RVE are characterized by dimensions  $2a_1 \times 2a_2 \times h$  ( $a_1$ ,  $a_2$  and  $h$  are the length, depth, and thickness of bricks, respectively), and the principal strain is evaluated for six points of RVE, as denoted by M1, M2, M3, B1, B2, and B3 (as shown in Fig. 1). This multi-scale model is able to perform in-plane macro-mechanical analysis of masonry walls accounting for the micro-mechanical transversal effects. However, when a crack pattern or crack width needs to be considered, this method is not applicable.



**Figure 1.** Masonry wall, RVE and reference system used by Addessi et.al [13]

### 3 DISTINCT ELEMENT METHODS

The distinct element method (DEM) is applied to problems in which a substantial part of the deformation occurs at the joints or contact points. The method treats the structure as being comprised of multiple blocks interacting with one another through contacts. This assumption removes two main difficulties intrinsically connected with FEMs: (1) the generation of compatible meshes among blocks and joints, and (2) the inability to provide a remeshing methodology to update the size of contacts or make new ones, when large relative displacements are encountered [14-16].

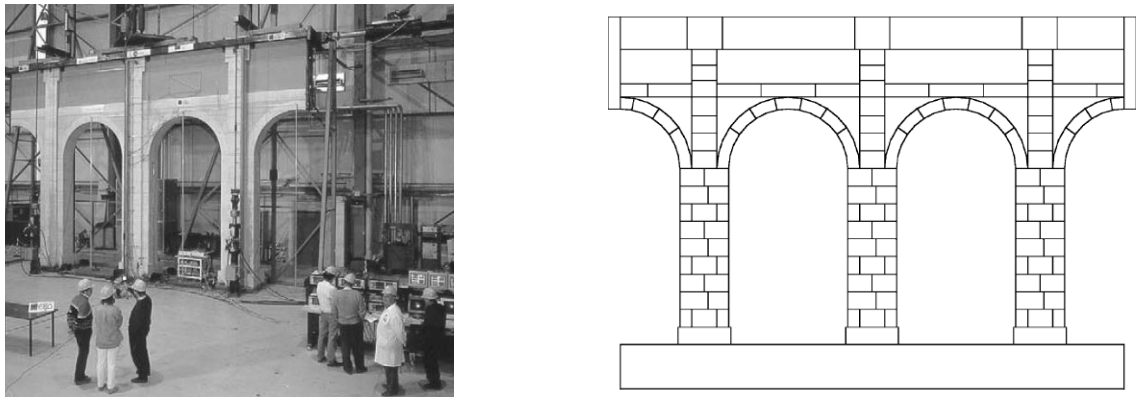
In commercial DEM software (e.g. UDEC and 3DEC), individual blocks behave as either rigid or deformable materials. Deformable blocks are subdivided into a mesh of finite-difference elements, with each element responding according to a prescribed linear or nonlinear stress-strain law. The relative motion of the discontinuities is also governed by linear or nonlinear force-displacement relations for movement in both the normal and shear directions. Several built-in material behaviour models are provided, for both the intact blocks and their discontinuities, which permit the simulation of representative responses in discontinuous geological materials (including mortar). DEM software programmes are based on a “Lagrangian” calculation scheme that is well-suited to model the large movements and deformations of a blocky system [17].

Although the DEM approach was originally designed for problems in rock mechanics, it is also applicable to block-mortar systems due to its intrinsic ability for modelling blocky structures. Having compared the modelling of a historical cloister façade under pseudo-dynamic and cyclic, quasi-static loading using different approaches (i.e., DEM, FEM, and CIM), Giordano et al. [15] noted many advantages of DEM for masonry structures [16]. These included fully universality (i.e., the majority of constitutive laws are developed for DEM) and reliability for non-linear materials under large displacements. Additionally, the DEM approach is typified by low storage needs (hard drive and RAM), simplicity in coding, cross-applicability of one algorithm for both static and dynamics problems, and suitability for parallel processing. However, as the actual blocks and joints distribution must be modelled, this approach is not well-suited for rendered surfaces where that masonry block unit distribution is not visible. A sample model is shown in Fig. 2.

Experimental failure mechanisms of a masonry shear wall (stone-mortar type) were assessed by a numerical model generated in UDEC [18]. Results correspond well with the experimental tests of [19]. This research confirmed the effect of stone size in the failure mechanisms of a shear wall. Large stones lead to a blocking of the stones, a stiffening of the masonry wall, and an increase in stone failure, whereas small stones reduced the load bearing capacity due to larger rotations. The simulation showed good compatibility for predicting failure mechanisms and crack propagation.

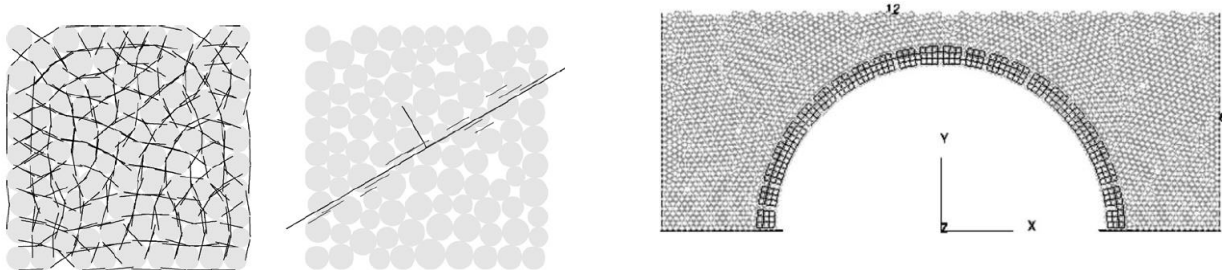
Thavalingam et al. [20] used a particle flow code (PFC) type of DEM to model a masonry arch bridge. The blocks were modelled as discs (Fig. 3), thereby allowing the particles to displace independently and interacting only at contacts or interfaces between the adjacent particles. Although no attempt was made to conduct a comprehensive sensitivity study of all parameters, a displacement-controlled, loading procedure was used to compare the different approaches. The results showed no convergence

problems. However, the disc-shaped elements could not model appropriately the bricks and mortar for cracking.



**Figure 2.** Test model and relevant UDEC model used by Giordano et al. [16]

In the other research, Schlegel and Rautenstrauch [21] compared a continuum model (i.e. an FEM approach) for the load carrying capacity calculated by a discontinuum model (i.e. 3DEC) for a masonry bridge. In that study, the DEM was better able to model the blocking effect of the masonry assembly resulting in a prediction 5% closer to the experimental results than the FEM.



a) Original specimen of PFC model with parallel bonds

b) Specimen after insertion of smooth-joint contacts

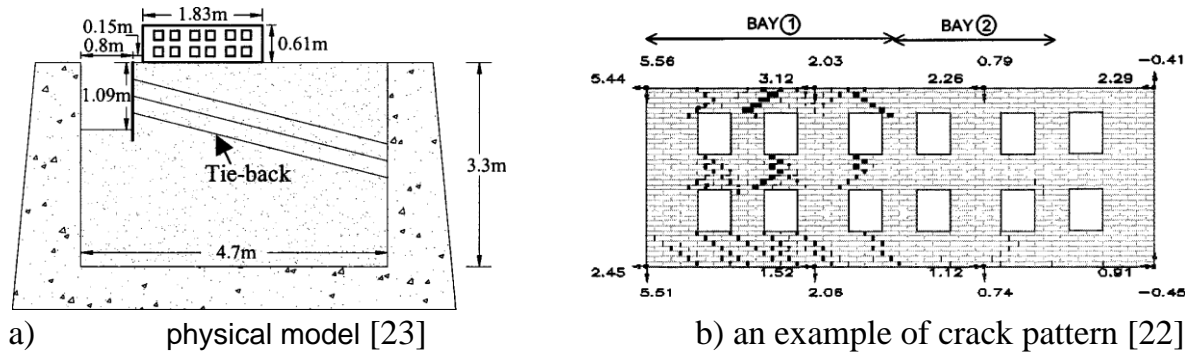
c) Masonry bridge modelled by PFC disks

**Figure 3.** Example of PFC model [20]

Work by Son et. al [22] based on a physical model introduced by Laefer et. al [23] (Fig. 4) evaluated building response from an adjacent excavation. The comparison verified that DEM is suitable for validation of masonry experiments in the case of angular distortion and horizontal strains including simulating large cracks and the relative rotation and translations between the masonry units. These last features cannot be done in FEM. The appropriateness of a DEM model for the cyclic behaviour of stone masonry has also been validated against experimental and field data for seismic loading [4]. That experimental data on shear behaviour of masonry joints established that a high level of uncertainty in dynamic parameters and the representation of stress dissipation during shear behaviour remain unsolved obstacles in the DEM modelling [4].

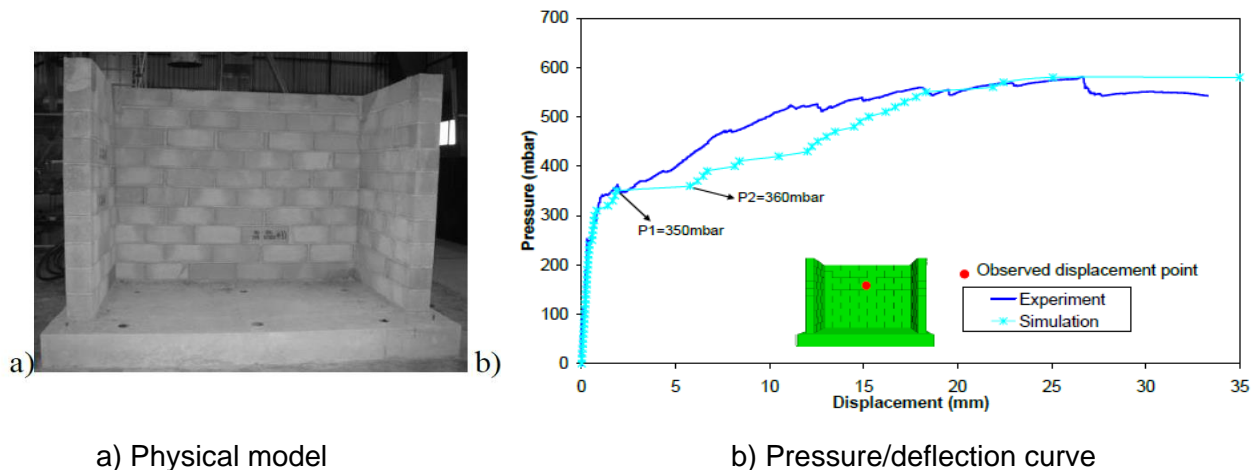
Based on a discrete element approach, Caliò et. al [24, 25] and Churilov and Dumova-Jovanoska [26] showed that while DEM is able to reduce computational costs with respect to a non-linear finite element simulation for both in-plane and the out-of-plane behaviour of masonry buildings, all material properties of all components must be carefully selected to achieve this. Given the nature of existing

structures, this is not always possible; notably the FEM homogenization approach does not have this limitation.



**Figure 4.** Building response from an adjacent excavation

As shown in Figure 5, when reproduced in a DEM (i.e. 3DEC), the non-linear behaviour of a masonry wall under bending loads was observed in the experimental load-deflection curves as globally correct from the initiation through final failure [27]. Crack appearances and propagation appeared correctly, with respect to stress distribution. This allowed a good estimation of the load capacity and the collapse mode, in part because the discrete element method allowed for the simulation of rupturing.



a) Physical model

b) Pressure/deflection curve

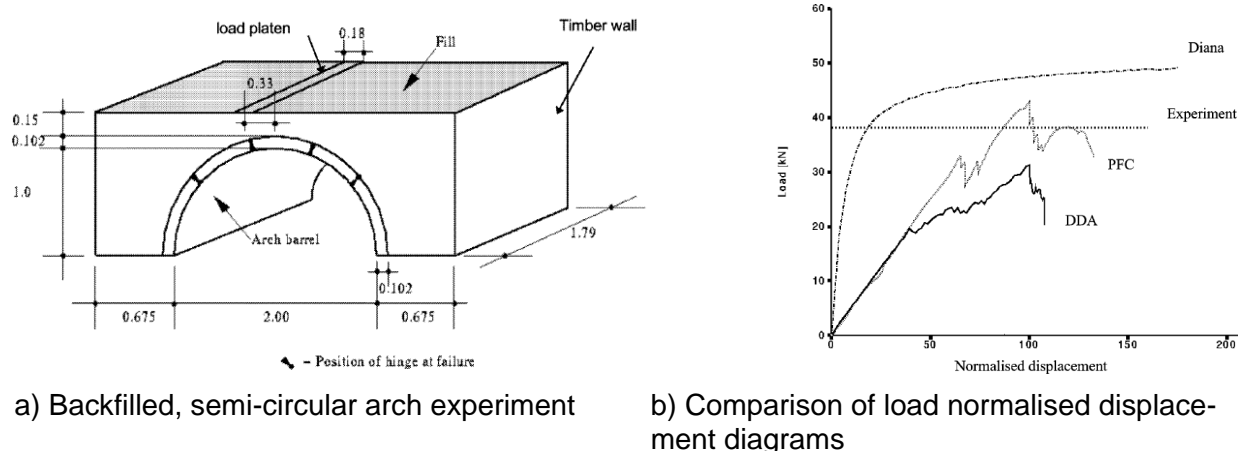
**Figure 5.** Behaviour of a masonry wall under bending loading[27]

#### 4 DISCONTINUITY DEFORMATION ANALYSIS (DDA)

DDA was first introduced for rock joint analysis in 1992 [28]. The approach is based on an assumed deformation field within distinct domains of arbitrary shapes and on a rigorous imposition of contact constraints. Unlike most discrete element techniques, which employ an explicit, time marching scheme to solve the equations of motion directly, the DDA method is usually implemented in an implicit formulation with opening-closing iterations within each time step. This enables equilibrium of the blocks under contact constraints. Incremental equilibrium equations are based on the minimization of the potential energy, which stems from the block deformability and applied loads, as well as from the contact between the blocks (normal contact, shear sliding, and stick and slip conditions). Similar to DEM, DDA provides a systematic approach to modelling discretized blocks in an efficient mesh. Thus, the model can be divided into simple, deformable blocks of any shape, which is useful in searching for critical failure mechanisms.

Thavalingam et al. [20] in a comparison of DDA with PFC to FEM and DEM for a brick, arch bridge, demonstrated DDA's usefulness in modelling the fill of randomly distributed particles of a definable

average size. The DDA model predicted a lower collapse load compared to those using DEM and FEM, and with fewer convergence problems than with the FEM.



a) Backfilled, semi-circular arch experiment      b) Comparison of load normalised displacement diagrams

**Figure 6.** Comparison of DDA, PFC and Diana against experimental testing (adapted from [20])

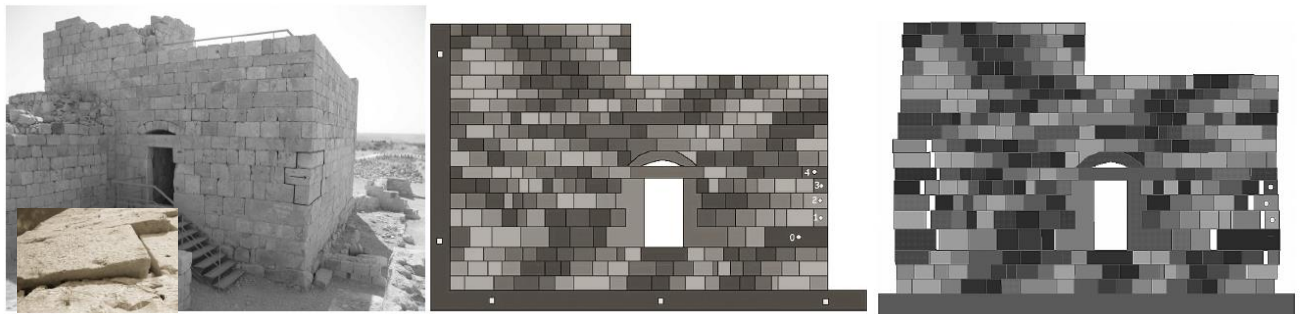
As a DDA benchmarking problem, Bićanić and Stirling [29] proposed the stability of a semi-circular masonry Couplet/Heyman arch under self-weight. The DDA results were compared to the classical Couplet/Heyman analytical solution. This research provided a critical thickness to span ratio of 0.105869 regardless of the material's unit weight, under the conditions of no sliding, no tension, and no crushing along the block boundaries. Rizzi, et al. [30] also provided further information to support the validity of the obtained solutions, with a good overall matching to the Couplet/Heyman analytical solution. To validate the DDA as an alternative modelling procedure for masonry arches, Bićanić et al. [27] considered this case, as well as the Edinburgh University Model Arch and the Bridgemill Arch Bridge, in Girvan, Scotland. The results showed that the DDA approach is viable for structural integrity assessment, ultimate load prediction, and failure mode analysis of masonry arches.

Another example of DDA experimental validation was performed by Chiou et al. [31], who modelled masonry structures, including an end-loaded cantilever beam. The proposed numerical model was shown to be capable of simulating the discontinuous behaviour of the masonry structure when subjected to in-plane monotonic loading and identifying the failure regions. Chang [32] also used the case of a cantilever beam as an example of the enhanced deformability facilitated by the incorporation of finite element meshes within DDA blocks, but those results presented were only qualitative in nature. Dynamic analysis is another concern in using DDA. Having attempted to analyse the stability of masonry retaining walls that do not require mortar or concrete joint filling, Nishiyama et al. [33] used a DDA technique and then proposed a quantitative method for evaluating static and dynamic behaviours of masonry retaining walls. They showed that the masonry retaining walls were stable for the existing and anticipated loadings.

Since the failure mode of the block system in DDA is a result of the analysis, there is no need to presume a failure mode as shown by Kamai and Hatzor [34, 35] who used DDA to study the response of ancient masonry structures under seismic ground motions (Fig. 7). Different loading functions were used to isolate the parameters that produced the unusual failure mechanisms observed in the field (namely being able to simulate an initial state of the structure that incorporated blocks that were already in displaced positions). The structural deformation patterns and localized failure of particular blocks observed in the two case studies were modelled with DDA. Both deformation patterns and displacement magnitudes were replicated accurately in two different structural geometries and at two distinct sites.

Notably both DDA and DEM are discrete element methods, but these are not the only ones. There are also momentum-exchange (ME) methods and modal methods (MM) [36]. However, they were not developed for either rock or masonry. Providing attributes of four mentioned methods (i.e. DDA, DEM, ME and MM) compared to limit equilibrium, Cundall et al. [36] illustrated that DDA is not particularly appropriate for non-linear material. Additional limitations of DDA include an inability to support fracturing

and dynamic analysis, whereas the most significant limitation of DEM relates to fracture modelling when deformable blocks are considered.



**Figure 7.** An ancient masonry structure simulated by DDA with the displaced blocks modelled by two different loading functions [after 35]

## 5 COMBINED CONTINUUM INTERFACE METHOD

The final model considered in this paper is a combined continuum interface method. Two forms developed to address discontinuity and crack propagation in masonry structures are (1) the advanced continuum method with weak discontinuity (WD) and (2) the strong discontinuity Joint Finite Element Model (JFEM). Arguably, these enable the explicit representation of discontinuities within FEM. In JFEM, there is an assumption of an interface of negligible thickness. Thus, for the joint element, kinematic (stress) and kinematic (strain) terms associated with the joint thickness can be disregarded, and the equations of virtual work of interface elements are represented by sliding and normal displacement along the joints [37]. In some masonry structures, the mortar thickness is very small compared to the blocks. Thus, it is argued that in these cases the mortar joints can be replaced with continuum elements through interface or joint elements.

Notably, the underlying physics of DEM and JFEM are identical. The key difference lies in how contacts between discrete blocks are allowed to change. In DEM, contacts are free to change and blocks can change their neighbouring units, thus breaking old contacts and creating new ones. With JFEM, contacts always remain the same; a block cannot separate from its neighbouring units nor develop new contacts with other blocks. The practical importance of this difference is that there are classes of problems that DEM can handle but that JFEM cannot [38], specifically when large displacements occur between masonry units. However, for typical masonry loading, such displacement between units does not occur. Under those conditions, the limiting point of equilibrium can be assessed with JFEM.

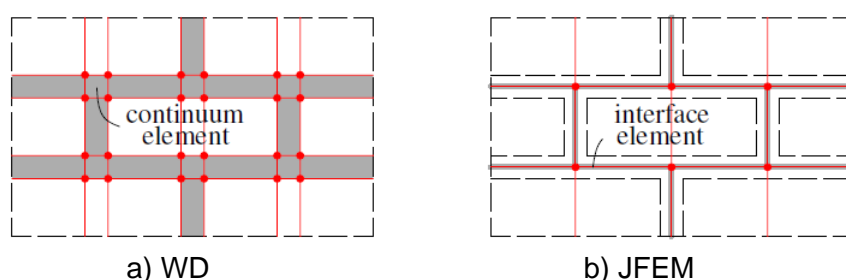
Using the joint finite element model, Giordano et al. [16] simulated a masonry façade under seismic loading. Compared to DEM, the main difficulty was remeshing the contacts during large displacements. However, under the adopted constitutive hypotheses model, the simulated response agreed well with the experimental monotonic envelope. However, like DDA and DEM, since each masonry unit should be modelled in its proper place and discontinuities are difficult to detect, the geometric modelling may lead to errors in the output.

As an example of the usefulness of this approach, Zucchini et al. [39] numerically modelled a pre-compressed shear wall test conducted by Vermeltfoort et al. [39]. Each masonry unit was subdivided into interior brick elements. Interior brick elements had boundaries either representing the mortar interfaces or internal brick interfaces using a Mohr–Coulomb failure surface with a tension cut-off. The results showed that the procedure captured the pre-peak and post-peak responses and the cracking characteristics reasonably well.

If mortar joints are considered as two parallel weak discontinuities, the arrangement can be addressed as an advanced continuum model with weak discontinuities. However, some researchers categorized JFEM as a continuum model with strong discontinuities [40]. In classical strong

discontinuity models (e.g. JFEM), mortar joints are assumed to have zero thicknesses, which is not consistent with reality, especially when the joint thickness is significant in comparison with the brick [41]. However, in weak discontinuity approaches the mortar joint thickness is incorporated and acts as a length-scale parameter of the localization band (Fig. 8).

A comparative analysis made by [42] between weak and strong discontinuity models through a three-point bending test and a shear wall test showed that compared to JFEM, continuum material models can be employed in WD to describe joint behaviour. Additionally, the failure mode of the masonry structure remains the same upon mesh refinement when WD was used. However, WD suffers from mesh dependent results when the element size is smaller than the weak bandwidth, while JFEM always results in mesh-independent results. Additionally, JFEM allows an easier implementation, since less quadrature sub-polygons are required, as there are only half the number of enhanced degrees of freedom for an equal mesh size when modelling regular bond masonry (i.e. when the joints are aligned with the finite element mesh).



**Figure 8.** Alternative Concepts (after [41])

## 6 DISCUSSION

Figure 9 provides an overview of the four methods that were investigated with respect to the ability to define the initial state, availability of input data, applicability of crack modelling, ease of model implementation, and the computing costs. The differences are shown qualitatively by the size of the marker, with the larger markers indicating beneficial attributes, and the smaller ones less beneficial characteristics. What is readily apparent is that standard FEM approaches are poorly suited to modelling damaged masonry, DEM is extremely well-suited (at least on static and pseudo-static loading) and that the other three methods are somewhere in between. The CIM is nearly as good as the DEM, except in the case of pre-deformed structures or when large deformations are expected. The DEM also has the advantage of being independent upon mesh size. The MPM and DDA methods both had a wider range of limitations. Although more applicable in explicit crack modelling than FEM or MPM, the complexity of DDA model generation, computational effort, and high input data requirements are all sources for concern. When properly applied, it does however perform well, especially for establishing a masonry structure's initial state of stress.

Although DEM works in the majority of cases, especially in avoiding convergence problems under large displacements, it is less effective when computational effort and material parameter requirements are considered. On the other hand, since discontinuities are pre-defined as joints in DEM, new joint generation is avoided, as well as joint thickness concerns. Thus, problematic mechanisms relevant to joints are circumvented. The method has also been extensively validated. While CIM is used less often than FEM and DEM in masonry structures, its performance is identical with DEM in most areas. Notably CIM can be an effective alternative to DEM, when large displacement analysis is unnecessary.

## 7 CONCLUSIONS

The paper described four different methods for numerical modelling of masonry structure with pre-existing cracks with respect to 1) ease of model implementation, 2) availability of input data, 3) applicability of crack modelling, and 4) ability to define the structure's initial state. With respect to modelling masonry structures with existing defects and large deformations, the discrete element



approaches was definitely superior. Quantification of such advantages with respect to computational costs is still needed.

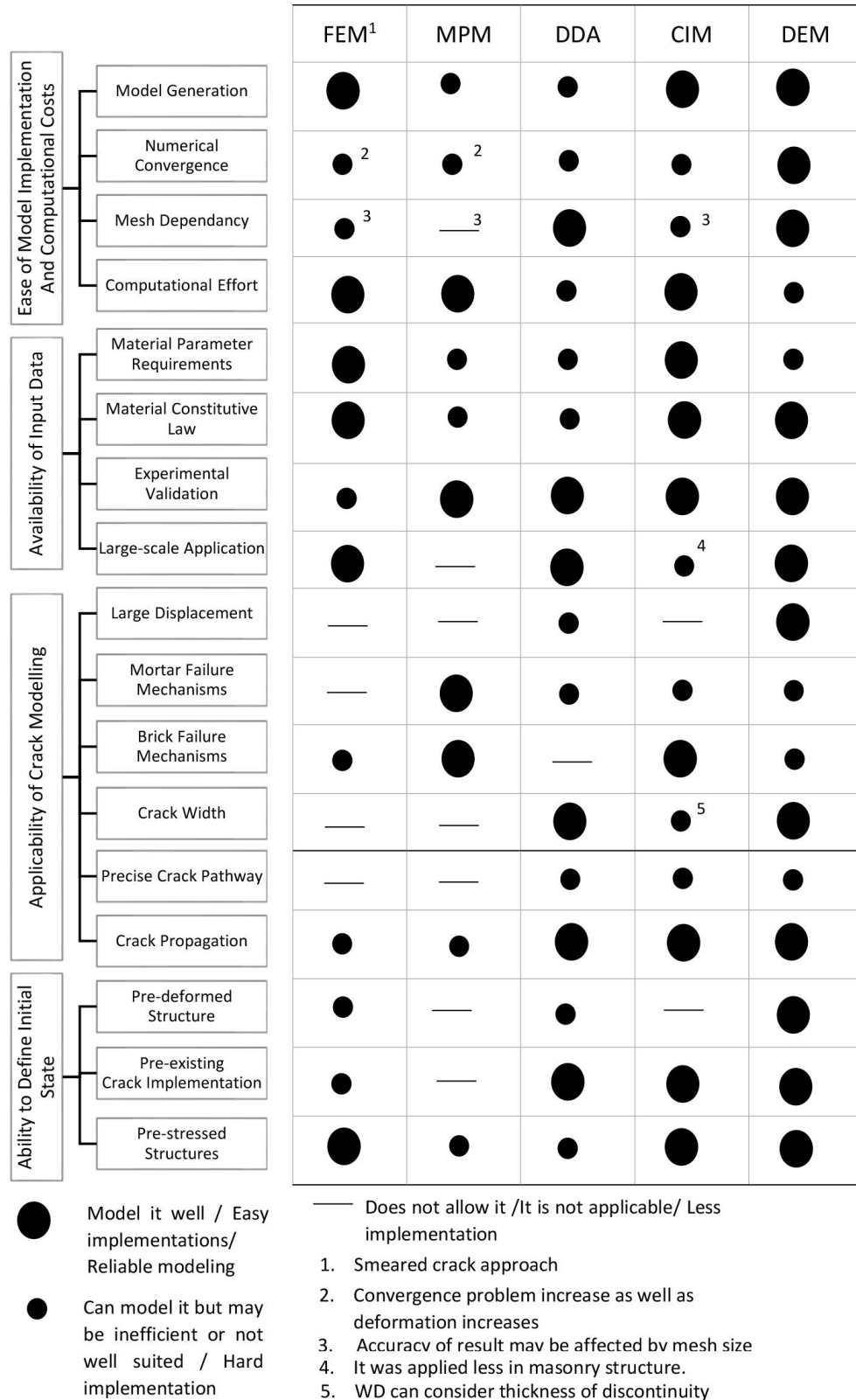


Figure. 9. Comparison of reviewed models versus FEM

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