NEW STRUCTURAL EVOLVING ALGORITHMS

FOR FUZZY SYSTEMS

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UNIVERSITI SAINS MALAYSIA

2018

NEW STRUCTURAL EVOLVING ALGORITHMS

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by

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Thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

September 2018

ACKNOWLEDGEMENT

All Praises be to Allah the Almighty, for delivering me the patience, the potency and the guidance to conclude this thesis fruitfully.

I owe the greatest debt to my parents, brothers, sisters and all my relatives for not only their never-ending affection, love and patience, but also their encouragement and unconditioned support to me in all my decisions. Their guidance helped me to keep my motivations high throughout my studies. I would like to share this moment of happiness and express the appreciation and love to my father (**Haider Yusef**) and mother (**Mrs. Huda Abdallah**) who encouraged me in every step in my life.

I would like to express my utmost gratitude and appreciation to my main supervisor, **Professor Dr. Nor Ashidi Mat Isa** and my co-supervisor **Dr. Md. Manjur Ahmed** for their valuable and constructive suggestions that enabled this thesis to run smoothly. They provided excellent guidance through these years with a mix of trust in my ideas and instant feedback on my questions. I am very grateful to **Assc. Prof. Dr. Junita Bt. Mohamad** for the supporting materials she has provided me during PhD.

I am greatly indebted to my brothers **Dr. Adnan** and **Dr. Abdulmalik Haider**, for their guidance and great source of motivation. I thank all of my lab friends **Tariq Adnan**, **Abduallah Amer**, **Mohammed Alswaitti**, **Sami Salah** and all staffs of University Sains Malaysia for their supports. Special thanks to **Mrs. Nor Hidayah Saad** for translating the abstract to Bahasa Melayu.

I would also like to thank University Sains Malaysia (USM) for their financial support by awarding me the USM Fellowship. This work is also partially supported by

the Universiti Sains Malaysia (USM) through Research University Grants (RU) (Grant no. 1001/PELECT/814205) entitled —Development of an Intelligent Auto-Immune Diseases Diagnostic System by Classification of Hep-2 Immunofluorescence Patterns.

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LIST OF ABBREVIATIONS

AI	Artificial Intelligence
ANFIS	Adaptive Neuro-Fuzzy Inference System
CL	Closure
Clas	Classification
CONS	Consistency
CPU	Relative Central Processing Unit
DENFIS	Dynamic Evolving Neural-Fuzzy Inference System
D-FNN	Dynamic Fuzzy Neural Networks
DPFNN	Dynamic Parsimonious Fuzzy Neural Network
ECSFS	Evolving Construction Scheme for Fuzzy Systems
EER	Error-Evolving Rate
EFS	Evolving Fuzzy Systems
EFTI	Enhancement of Fuzzy Term Identification
EFuNNs	Evolving Fuzzy Neural Networks
EGE	Evolving Granule Error
ELM	Extreme Learning Machine
eTS	Evolving Takagi-Sugeno
FCM	Fuzzy C-Means
FGP	Fixed Grid Partitioning
FRBSs	Fuzzy Rule Based Systems
GA	Genetic Algorithm
GAE	Global Average Error
GD-FNNs	Generalized Dynamic Fuzzy Neural Networks
GS-EFS	Generalized smart evolving fuzzy systems

GSETSK	Generic Self-Evolving TSK
KNN	K-Nearest Neighbors
LEOA	Local Error Optimization Approach
LFM	Linguistic Fuzzy Modelling
LOLIMOT	Local Linear Model Trees
LSM	Least Square Method
MAD	Mean Absolute Deviation
MF	Membership Function
MISO	Multi Input Single Output
MSE	Mean Square Error
NA	Not Applied
NC	Number of Conditions
NFEAT	Number of Features
NFIRED	Number of Fired Rules
NFS	Neuro Fuzzy Systems
NMF	Number of Membership Functions
NR	Number of Rules
OSAMNN	Online Self-Adaptive Modular Neural Network
PANFIS	Parsimonious Network Based On Fuzzy Inference System
PFM	Precise Fuzzy Modelling
POPFNN	Pseudo Outer-Product Based Fuzzy Neural Network
PRP	Published Relative Performance
RANFIS	Randomized Adaptive Neuro-Fuzzy Inference System
Reg	Regression
R-ELANFIS	Regularized Extreme Learning Adaptive Neuro-Fuzzy Inference System

RMSE	Root Mean Square Error
SIM	Structure Identification Method
RSPOP	Rough Set–Based Pseudo Outer-Product
SAFIS	Sequential Adaptive Fuzzy Inference System
SE	Square Error
SEA	Structural Evolving Approach
SELM	Structure Evolving Learning Method
SISO	Single Input Single Output
SOFMLS	Self-Organizing Fuzzy Modified Least-Squares Network
SOFNN	Self-Organizing Fuzzy Neural Networks
SP	Split
SSE	Sum Square Error
SSEM	Simplified Structure Evolving Method

LIST OF SYMBOLS

В	The number of possible rules to be excluded without
	having a significant loss of the system accuracy
C_i	The number of membership functions in the attribute i
е	The sample error
Ε	The average error of the subregion
g	The index of the current evolving stage
LAE	Local average error
n	The number of input attributes
n _i	The number of attributes of subregion <i>i</i>
Ν	The number of training samples
Q	The consequent parameters
$Q_r^{q_1q_2\dots q_n}$	Consequent part of the $subregion_r$
r	The index of the selected subregion or subrange
R	The number of rules of the subregion
$R_{cs}(g)$	The number of rules for the current evolving stage with g
	index
R_{mx}	The maximum number of rules
RMSE(g)	The root mean square error of the fuzzy system in the
	evolving stage g
$RMSE_{cmb}(L,H)$	The RMSE of the combination of rules of (L, H) after
	training
$RMSE_{ref}(L,H)$	The reference weight of each combination
<i>RMSE</i> _{thr}	The rule weight threshold
Ř,	The number of subregion

Я	The desired number of subregion
$subregion_r$	The selected subregion
S	The accumulated local error of the subregion
t	The target of sample <i>i</i>
Т	The antecedent part
$T_r^{q_1q_2\dots q_n}(\boldsymbol{x})$	Antecedent part of the <i>subregion</i> _r
υ	The system output of sample <i>i</i>
$\boldsymbol{x}\left(x_{1} \ x_{2} \ \dots x_{n}\right)$	A training input sample
x_s	S represents the selected attribute
x_{sp}	The splitting point
χ_{spl}^{int}	The initial splitting point of EFTI
x_{spl}^{opt}	The optimal splitting point of EFTI
μ	Membership functions
β	Predefined threshold
η	Number of subrange
ή	Desired number of subrange
α_i	The number of conditions of subregion <i>i</i>
ς	The influence of the accuracy part
б	The influence of the interpretability part
σ, φ	The minimal and maximal value of the input variable x
${\Phi}$	The total number of rules of $subregion_r$ after training

ALGORITMA PERKEMBANGAN STRUKTUR BAHARU UNTUK SISTEM KABUR

ABSTRAK

Pada masa kini, isu kompromi antara kejituan dan penafsiran semakin mendapat perhatian dalam merekabentuk sistem kabur yang baharu. Dalam tesis ini, tiga model kabur berkembang iaitu peningkatan pengenalan istilah kabur (EFTI), kaedah pengenalan struktur (SIM) dan pendekatan perkembangan struktur (SEA) dicadangkan untuk menangani isu kompromi antara kejituan dan penafsiran. EFTI, SIM dan SEA direkabentuk berdasarkan kaedah-kaedah pengurangan ralat. EFTI dibangunkan untuk disesuaikan dengan masalah-masalah masukan tunggal keluaran tunggal (SISO) (iaitu satu dimensi), manakala SIM dan SEA dibangunkan untuk disesuaikan dengan masukan berbilang keluaran tunggal (MISO) (iaitu dimensi sederhana dan tinggi). EFTI bermula dengan struktur kabur mudah yang terdiri daripada dua istilah kabur dalam ruang masukan. Kemudian, EFTI terus berkembang dengan mengenal pasti titik-titik pemisahan pada ruang masukan yang serasi dengan parameter-parameter yang dihasilkan. Sebaliknya, SIM dan SEA bermula dengan satu peraturan kabur yang tidak mempunyai istilah-istilah kabur dalam ruang masukan tanpa mengira tahap dimensi masukan. Kemudian kedua-dua kaedah berkembang berdasarkan proses penutupan atau pemisahan untuk sifat masukan yang terpilih pada subkawasan yang dipilih. Sekiranya sifat yang terpilih tidak mempunyai istilah kabur, penutupan dilakukan, tetapi jika berlaku sebaliknya, pemisahan dilaksanakan. Proses perkembangan berlanjutan sehingga kejituan yang memuaskan dipenuhi atau bilangan subkawasan maksimum dicapai. Teknik pemetakan berdasarkan ciri persamaan dan teknik pemilihan-pemetakan statik dibangunkan untuk SIM. Manakala, teknik pemetakan berdasarkan pemisahan subkawasan yang terpilih kepada dua subkawasan dengan ralat purata maksimum dan minimum dan teknik pemilihan pemetakan yang dinamik dibangunkan untuk SEA. Selain itu, teknik pemangkasan berdasarkan tahap kepentingan peraturan kabur dicadangkan untuk mengecilkan asas peraturan SEA. Berbanding dengan model SISO dan dengan menggunakan tiga set data, EFTI menghasilkan RMSE terendah dengan bilangan peraturan yang paling rendah. Bagi model MISO pula dan dengan menggunakan sembilan set data penanda aras, SIM mencapai RMSE terendah dengan saiz terkecil sistem asas-peraturan. Demikian juga untuk model-model terkini MISO dan dengan menggunakan enam set data penanda aras, SEA juga menghasilkan RMSE terendah dengan saiz terkecil sistem asas-peraturan. Sebagai kesimpulan, keputusan yang diperolehi membuktikan bahawa EFTI, SIM dan SEA dapat menghasilkan kompromi yang ketara antara kejituan dan penafsiran.

NEW STRUCTURAL EVOLVING ALGORITHMS FOR FUZZY SYSTEMS

ABSTRACT

Recently, the issue of accuracy and interpretability trade-off has been getting more attention when designing new fuzzy systems. In this thesis, three evolving fuzzy models, namely enhancement of fuzzy term identification (EFTI), structure identification method (SIM) and structural evolving approach (SEA) are proposed to spot the best trade-off between accuracy and interpretability. EFTI, SIM and SEA are designed based on error reducing methods. EFTI is developed to fit with single input single output (SISO) problems (i.e. one dimension), while SIM and SEA are developed to fit with multi input single output (MISO) (medium and high dimension). EFTI begins with a simple fuzzy structure that is composed of two fuzzy terms in the input space. Then EFTI continues evolving by identifying splitting points of the input space that are compatible with the consequent parameters. On the other hand, SIM and SEA start with one fuzzy rule that has no fuzzy term in the input space regardless of the degree level of input dimension. Then they evolve on the basis of either closure or split processes for the selected input attribute of the selected subregion. If the selected attribute has no fuzzy terms, closure is performed, otherwise split is done. The evolving continues until a satisfactory accuracy is fulfilled or maximum number of subregion is reached. A partitioning technique based on the similarity feature and a static partition-selection technique are developed for SIM. While, a partitioning technique based on splitting the selected subregion into two subregions with maximum and minimum average error and a dynamic partition-selection technique are developed for SEA. Furthermore, a pruning technique based on the importance level of the fuzzy rules is proposed to shrink the rule-base of SEA. Compared with SISO models and using three datasets, EFTI produces the lowest RMSE with lowest number of rules. For MISO models and using nine benchmark datasets, SIM achieves the lowest RMSE with the smallest size of rule-base systems. Similarly, for MISO state-of-the-art models and using six benchmark datasets, SEA also produces the lowest RMSE with the smallest size of rule-base systems. In conclusion, the results proved that EFTI, SIM and SEA are able to produce a significant trade-off between accuracy and interpretability.

CHAPTER ONE

INTRODUCTION

1.1 Introduction

Fuzzy modelling is considered one of the main techniques used in computational intelligence. It is widely known that it can represent systems with semantic description. Fuzzy systems are designed to produce a rule-base composed of many fuzzy rules (i.e. IF THEN). Natural language is used to express the terms involved in these rules. In fact, the reasoning form of fuzzy rules expressed by human language offers a significant feature that helps users, who are in charge to make crucial decisions, understand how the systems' outputs are concluded. From users view, this feature (i.e. interpretability) which is provided by fuzzy set theory, grants any created systems with more reliability.

Fuzzy rules of any fuzzy system are mainly generated from two different types, namely designed by human expert or by data. Fuzzy systems designed by human experts were popular in the early approaches. These approaches exploit the knowledge and experience of the human experts to form the IF-THEN fuzzy rules. The lack and difficulty of knowledge acquisition has led the researchers to move to design fuzzy systems using input-output (I/O) data by applying some machine learning techniques. The fuzzy rules generated from numerical data result a better performance than the ones generated by human experts (Alonso et al., 2015). However, these data-driven models suffer from lack of interpretability that the models, built by human expert, have. Subsequently, an issue of accuracy and interpretability trade-off has emerged. As a result, fuzzy systems have been divided on the basis of accuracy and interpretability trade-off into two different tracks as follow:

Linguistic fuzzy modelling (LFM) (Ahmed and Isa, 2017): the main goal of this type is to build fuzzy rule based systems (FRBSs) with high interpretability using linguistic fuzzy rules. Mamdani structure is usually utilized to build LFM models due to the use of linguistic variables in both the antecedent and consequent parameters (Gacto et al., 2011).

Precise fuzzy modelling (PFM) (Ahmed and Isa, 2017): the main goal of this type is to build fuzzy rule based systems (FRBSs) with high accuracy. PFM models are mainly designed based on Takagi–Sugeno structure which build approximate FRBSs that use non-linguistic fuzzy sets for the antecedent and consequent parameters (Gacto et al., 2011).

In this work, the main focus is to find the best trade-off between accuracy and interpretability. These two characteristics have a conflict relation. They are contradicting each other. Therefore, identifying the best trade-off between accuracy and interpretability, which means to produce systems that have low RMSE with small and high interpretable rule-base, is considered a real challenge.

Many fuzzy systems have been proposed regarding the issue of accuracy and interpretability trade-off. These fuzzy systems can be categorized into three types namely, fuzzy clustering, nonlinear parameters optimization and grid and tree partitioning methods based on error reduction mechanism (i.e. error reducing methods).