

Review

# Health Monitoring of Civil Infrastructures by Subspace System Identification Method: An Overview

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**Abstract:** Structural health monitoring (SHM) is the main contributor of the future's smart city to deal with the need for safety, lower maintenance costs, and reliable condition assessment of structures. Among the algorithms used for SHM to identify the system parameters of structures, subspace system identification (SSI) is a reliable method in the time-domain that takes advantages of using extended observability matrices. Considerable numbers of studies have specifically concentrated on practical applications of SSI in recent years. To the best of author's knowledge, no study has been undertaken to review and investigate the application of SSI in the monitoring of civil engineering structures. This paper aims to review studies that have used the SSI algorithm for the damage identification and modal analysis of structures. The fundamental focus is on data-driven and covariance-driven SSI algorithms. In this review, we consider the subspace algorithm to resolve the problem of a real-world application for SHM. With regard to performance, a comparison between SSI and other methods is provided in order to investigate its advantages and disadvantages. The applied methods of SHM in civil engineering structures are categorized into three classes, from simple one-dimensional (1D) to very complex structures, and the detectability of the SSI for different damage scenarios are reported. Finally, the available software incorporating SSI as their system identification technique are investigated.

**Keywords:** structural health monitoring (SHM); vibration-based damage detection; system identification; subspace system identification (SSI)

## 1. Introduction

In the recent years, there has been a considerable interest in “Smart City” concept and the monitoring, controlling, and preservation of the health state of critical infrastructures, like roads, buildings, bridges, and tunnels. Structural health is required to be diagnosed at every moment during a structure life cycle in order to provide high quality services. Civil engineering structures are designed for the lifetime of the occupants or facilities and their failure might lead to catastrophic consequences in terms of human life and economic assets [1]. Structural health monitoring (SHM) is one of the main

stakeholders of the said Smart City concept [2]. SHM is an effective solution contributing to the need for safety, lowering the maintenance costs and reliable condition assessment of structures [3,4].

SHM is an interdisciplinary subject that incorporates knowledge and experiences from synergetic technologies in civil, mechanical, control, and computer engineering to deal with the health assessment of structures. The health monitoring of structures has been the subject of many studies for the past three decades [5,6]. The development of a reliable SHM method for civil structures is a challenging task due to ambient-induced uncertainty and the associated complexity measures. Four analysis levels of damage detection are applied in the context of SHM that include: (1) detection, (2) localization, (3) quantification, and (4) prediction of the remaining life, whereas the first three levels are more explicitly reported in literature [7,8].

Based on the acting load, SHM methods can be divided into two classes of static and dynamic-based methods [9]. The methods that use vibration characteristics of structures to assess the health state of structures, so-called vibration-based damage detection (VDD) [10,11]. The key premise of VDD is to estimate the modal parameters of a structure while using the analytical model that was constructed by system identification methods [12,13]. Static-based damage detection (SDD) methods rely on measuring the change in static response of structure, such as load bearing capacity, strain, deflection, and stiffness. Posenato et al. and Wu et al. [14] used strain data for damage detection of structure. Chen et al. [15] proposed a method to take advantages of stay cable force measurements and structural temperatures for damage detection. Yu et al. [16] used deflection data for damage identification in structures. Zhu et al. [17] introduced a temperature-driven method while using strain information of structures for anomaly detection.

Weigh in motion (WIM) is a widely used vehicle classification method for the health monitoring of structures. Bridge weigh in motion (BWIM) is a type of WIM technology that can identify traffic data, including speed, number of axles, axles spacing, and gross and axle weight of the passing vehicles using a series of conventional strain gauges. BWIM is particularly suitable for short-term measurements of traffic data, as it can be easily installed and detached from the bridge. Cardini and Dewolf [18] applied BWIM through using strain gauges to gain information on the quantity and weights of the trucks crossing highway bridges. Cantero et al. [19] proposed a BWIM-based damage identification method through introducing the concept of 'Virtual Axle' for deriving a damage indicator. Gonzalez and Karoumi [20] proposed a model-free damage detection method using deck accelerations response and BWIM. Kalyankar and Uddin [21] developed a three dimensional finite element model to estimate multi-vehicles-bridge interaction in a BWIM. The environmental factors are a weak point of vibration- and static-based damage detection [22]. However, in some cases, the temperature based methods shown higher sensitivity when compared to vibration based methods.

Estimations of the modal parameters in SHM are generally performed by system identification methods [23]. System identification is a mathematical procedure for establishing an analytical model based on experimental data. System identification is a mature field in SHM to extract modal parameters in VDD methods [24]. System identification methods in SHM can be classified into three categories based on their domains, including: time-domain (TD), frequency-domain (FD), and time/frequency domain (TFD) [25]. TD methods are more attractive for monitoring of civil structures, owing to the direct use of vibration signals. TD methods are generally classified within three groups of subspace system identification (SSI), natural excitation technique (NExT), and auto-regressive moving average (ARMA) [26]. The premise of the NExT method is that the response signals of a structure for ambient excitation and free-vibration have the same analytical form [27]. ARMA-based methods are popular statistical strategies for VDD of civil engineering structures. The auto regressive (AR) part ARMA models a linear function for the response time-history and the moving average (MA) section determines the moving average of the measurement response. The SSI algorithm presents a harmonious combination of algebraic, mathematical, statistical, and geometrical tools to identify the system parameters. SSI takes advantages of LS, angles between subspaces, QR decomposition, singular value decomposition (SVD), Kalman filter, and stochastic realization theory to deal with the

problem of modal parameter identification. Subspace-based methods that are used for the parameter identification of civil structures are mainly one of the following two methods of data-driven subspace system identification (SSI-DATA) and covariance-driven subspace system identification (SSI-COV) approaches [28]. The ARMA model has the most similarity to the SSI-COV model, as both methods use the correlation function of the vibration measurement in their preprocessing stage.

Recently, a large number of subspace-based methods have been applied in VDD. However, the previously conducted surveys have not kept pace with the changing environment and diversity in this field. Therefore, there is a need for a review focusing on the most important recent studies conducted in the considered area. The presented review attempts to address the available studies that employed subspace-based techniques in VDD of civil structures. This paper provides an overview of the background and new findings in SSI with a focus on both theory and practice. In addition, it describes some contributions toward the development and application of the SSI algorithm in recent years.

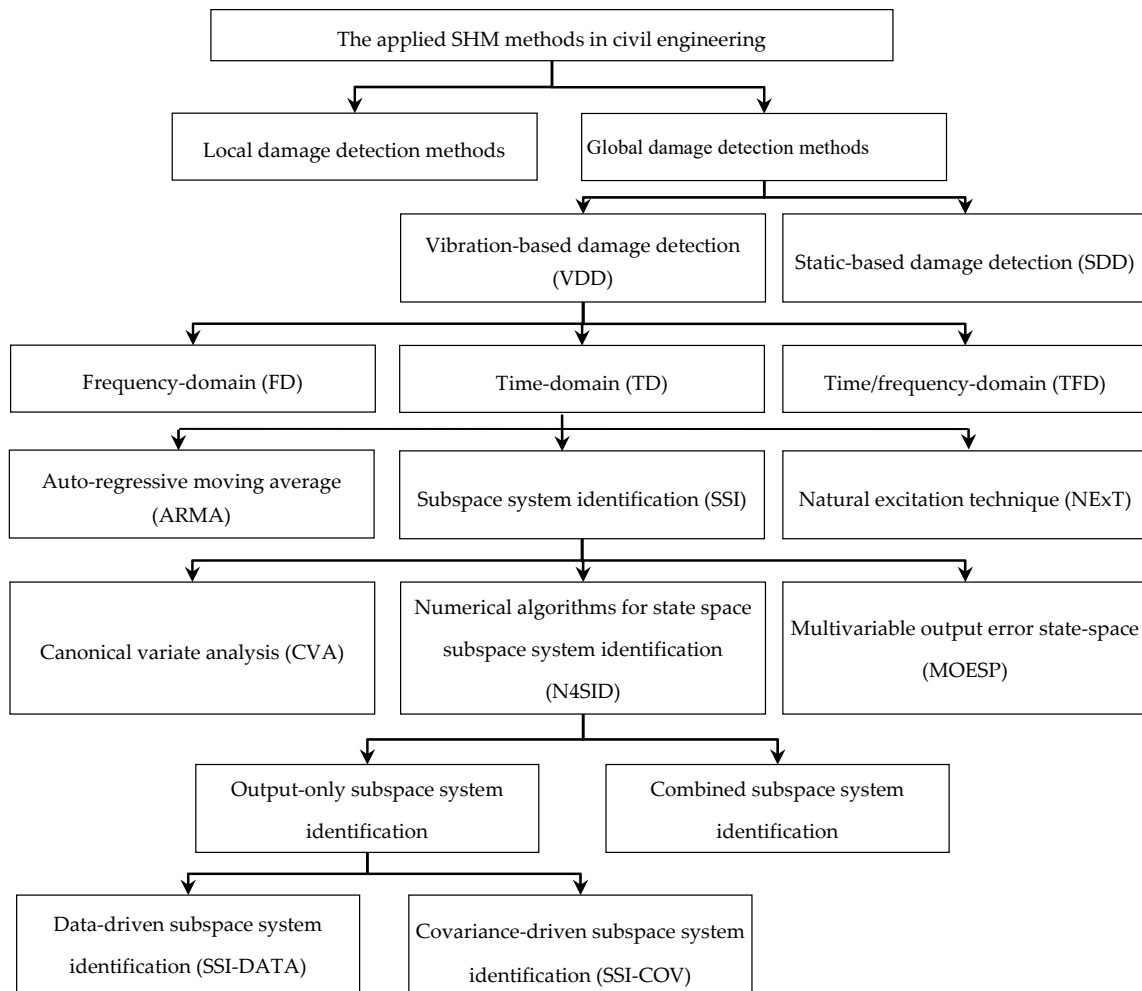
This review study is organized, as follows. In Section 2, SHM methods are outlined and the strength and drawbacks of each class are highlighted. Section 3 evaluated vibration-based damage detection (VDD) methods. The focus is on subspace system identification in the three later sections (Sections 4–6). Section 7 introduces some commercially available software that use subspace as their main constituent. Future research directions and conclusions are provided in Sections 8 and 9, respectively.

## 2. Structural Health Monitoring (SHM)

An SHM system implements strategies for the damage detection of structures [29–31]. Currently, SHM is known as a well-established tool for the diagnosis of damages in civil engineering communities and it is employed in a number of different structures, such as buildings, bridges, and dams. Structural data are collected from several points through installed sensors and they are analyzed to evaluate the health of a structure. Figure 1 shows the categorization of SHM methods with a focus on addressing key subspace-based algorithms. SHM methods are divided into local methods, which mainly rely on non-destructive evaluation (NDE) strategies and global methods. Based on the incorporated domain, the VDD strategies could be categorized as TD, FD and TFD methods. The TD methods are generally from one of the auto-regressive moving average (ARMA), natural excitation technique (NExT), or subspace system identification (SSI) families. The SSI methods are an important class of algorithm and they can be divided into three categories of canonical variate analysis (CVA), numerical algorithms for state-space subspace system identification (N4SID), and multivariable output error state-space (MOESP). N4SID is of the most favored SSI algorithm in SHM due to its capability to cope with output-only data (stochastic, unknown input). Two classes of N4SID algorithms are commonly practiced in SHM of civil engineering structures, namely the SSI-DATA method and the SSI-COV method.

NDE is a local SHM method used to perform constrained random tests to diagnose the state and severity of the possible defects. Some NDE methods are concerned with measuring defects in steel components, whereas others are designed for concrete substructures. Several NDE methods are available to identify defects in steel structures such as the ultrasonic test (UT), radiographic test (RT) [32], and eddy current test (ET) [33]. UT is an acoustic NDE method that uses ultrasonic waves passing through a structure for detecting defects. The phased array ultrasonic test (PAUT) method is a more reliable type of UT that uses a greater number of arrays in order to reliably simulate a specimen's profile [33]. A variety of methods is applied for NDE of concrete components. These methods range from the very simple strength evaluation methods, such as using rebound hammer [34], to more complex methods, such as impact-echo [35] and radiography testing [36]. Even though local SHM methods yield excellent performance for detection and localization of damages, they have some limitations and drawbacks. The main disadvantage of these techniques is that the evaluation process cannot be implemented without any prior knowledge of the approximate damage location. Moreover, in many cases, access to below of the test area is an essential requirement that is not always affordable

or practical [37]. Further detailed information on local SHM methods in civil engineering structures are provided in [38].



**Figure 1.** Classification of the structural health monitoring (SHM) methods with focus on subspace system identification (SSI) algorithm.

Researchers have proposed using global damage detection methods to identify damages in structures in order to overcome the previously mentioned limitations. Global methods are very effective choices to overcome the limitation of local SHM methods in civil engineering. In global methods, there is no limitation regarding the location of damage or even access to and preparation of the damaged area. These methods can localize and estimate the extent of damages while using the global characteristics of structures. The SDD methods require large measurement datasets for a reliable damage detection process. The targeted structure must generally be removed from its normal service in order to implement loading tests in SDD methods. Therefore, the SDD methods are not appropriate for continuous monitoring applications. When compared to VDD, the number of studies that employed SDD techniques are limited [39,40]. SDD methods are beyond the scope of this review paper and, for the sake of brevity, will not be covered in this study. A full review of the SDD has been well-documented elsewhere (for example [41]). VDD methods will be studied in the proceeding subsection.

### 3. Vibration-Based Damage Detection (VDD)

VDD is considered to be the most popular methodology in global SHM. VDD methods rely on changes in dynamic properties as an indicator of damage existence. These methods exploit observable

variations in modal parameters, such as resonant frequency, damping, and mode shape or their derivative as indicators of change in physical properties of a structure. A thorough review of the system identification methods in VDD and modal analysis is provided in Song et al. [42] and Reynders [43].

### 3.1. Frequency-Domain (FD) Methods

An FD representation of a signal is extracted from a TD response signal using a Fourier transform. A Fourier transform describes a harmonic function by a linear combination of complex exponentials. A number of enhanced algorithms have been developed in order to improve the computational efficiency of Fourier transform, which are collectively termed as a fast Fourier transforms (FFT). FFT is generally used for deriving frequency response functions (FRF), and it plays the key role for many FD damage detection approaches [44]. FD methods in VDD can be classified into three main categories, which include [26]:

1. peak picking (PP) methods;
2. complex mode indication function (CMIF) methods; and,
3. least squares complex frequency-domain (LSCF) methods.

PP methods, also referred to as basic frequency-domain (BFD), are the most typical approaches in modal testing and they initially rely on power spectral analysis and Fourier transforms. PP methods are qualitative in nature and founded upon the fact that FRF reaches an extreme approximately around the natural frequency. Furthermore, these methods could be accompanied with a half power bandwidth approach to extract damping ratio [45]. Frequency domain decomposition (FDD) is an accurate and simple technique for system identification that is widely used in modal analysis. FDD has been developed based on spectral density decomposition. The obtained spectra are a reduced form of a dynamic response for individual modes [46]. The enhanced frequency domain decomposition (EFDD) method is an extension of FDD for estimating reliable modal parameters [47].

CMIF methods can also be considered to be an extension of PP techniques. They have been widely used for the output-only identification of system parameters. CMIF is developed by performing SVD on a normal FRF matrix at each spectral line [48]. CMIF is combined with other algorithms to be used as a standalone model, such as the enhanced frequency response function (eFRF) and enhanced mode indicator function (EMIF). eFRF is the subsequent development of the CMIF method and it is used to estimate the frequencies that are associated with a particular peak in the CMIF [49]. The eFRF is rooted in the concept of physical coordinate transformation to enhance the estimation of modal parameters. EMIF could be considered as an extension of the CMIF/eFRF, which estimates modal parameters in several modes at one time. The distinctive property of this method is due to the fixity in the number of natural frequencies based on the peaks of CMIF plots.

LSCF is a fast and accurate method for estimating modal parameters. Originally, LSCF was applied to extract initial values in the maximum likelihood method. LSCF performs reliably due to its clear stabilization diagram [50]. The polyreference least-squares complex frequency-domain method (PolyMAX) is the polyreference version of the LSCF that takes advantage of the right matrix-fraction model. The main benefit of this method is that the closely spaced modal frequencies can be separated from each other [51]. El-Kafafy and Peeters [52] introduced the poly-reference least squares complex frequency-domain (pLSCF) for modal analysis. A two-step scheme is proposed to enhance the damping estimates. The proposed method can improve the processing time and accuracy of the modal identification, particularly for damping estimates.

FD methods are fast and accurate, but they suffer from some limitations in the frequency resolution of the estimated spectral data [53]. Conventional FD methods are not accurate and reliable for the analysis of non-linear and non-stationary signals. The resolution of the identified system parameters in low-frequency ranges or fewer numbers of incorporated modes is poor in these methods [54]. Moreover, the estimated damping coefficients are not accurate in the non-parametric FD methods [55].

The strong demand in the field of system identification to achieve higher accuracy and extract more information from the vibration responses led to the development of TD methods.

### 3.2. Time-Domain (TD) Methods

TD techniques rely on the fact that the vibrational properties of structures can be captured through the time-history response of a dynamic system. Hence, the extracted response in a healthy state is different from that of one that is in a damaged state. Figure 2 shows the schematic architecture of TD methods that were used for the identification of dynamic systems. Different numerical techniques, for example, FFT, SVD, least squares (LS), QR decomposition, Eigen-vector decomposition (EVD) [56], and statistical methods, were used to develop these algorithms. Observer/Kalman filter identification (OKID), NExT, and random decrement (RD) are the most common TD methods for extracting the FRF when there is no access to the input data. The input signal can be estimated using an auto-correlation or cross-correlation function [57].

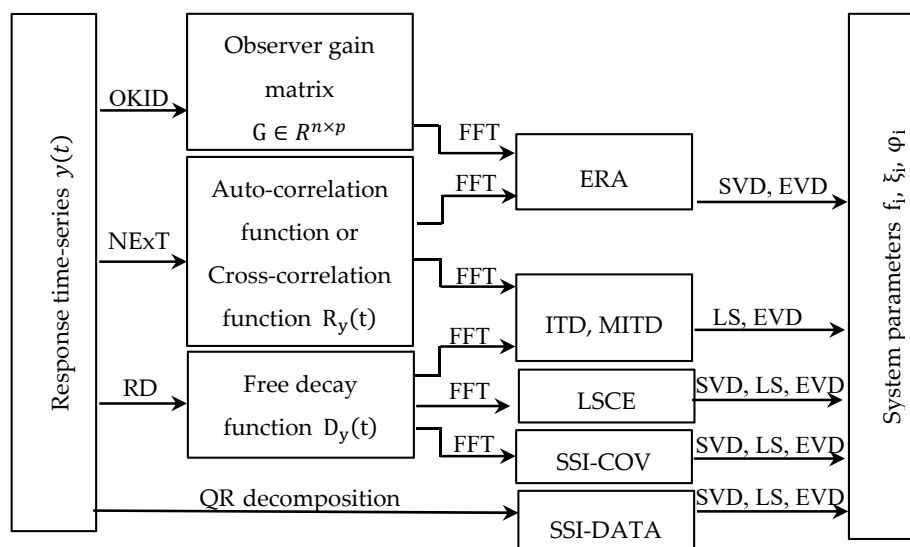


Figure 2. Schematic model for some of output-only time-domain modal identification methods.

The TD techniques in the health monitoring of civil structures can be classified into three widely known categories, which include [26]:

1. NExT methods;
2. ARMA methods; and,
3. SSI methods.

These methods are adopted for the identification of system parameters in civil engineering structures by introducing output-only extensions. NExT and SSI-COV methods are generally indirect where the SSI-DATA and ARMA methods are direct in nature, as can be seen in Figure 2. The TD methods are more appropriate for continuous monitoring compared to modal analysis methods due to direct use of response signals. Furthermore, the information extracted using TD methods is more complete than that of FD methods, particularly when a large number of modes or a large frequency range exist. A further explanation of the TD class of the identification methods are provided in the following subsections.

#### 3.2.1. Natural Excitation Technique (NExT) Methods

James et al. [58] first proposed the NExT for the modal analysis of systems exposed to noise uncertainty. The key idea behind the NExT scheme is that the cross-correlation of the response signal from random excitation can be considered as a summation of decaying sinusoids. The complex

exponential (CE) model is based on Prony's method and it was the first modal estimation method used as single-input single-output (SISO). The CE algorithm fits the curves of analytical impulse response into the experimental impulse response data [9]. The CE algorithm has been extended to the single-input multiple-output (SIMO) version by applying the LS technique. The algorithm was named the least-squares complex exponential (LSCE) [59]. The polyreference complex exponential (PRCE) is an extension of LSCE in the form of the multiple-input multiple-output (MIMO) identification of modal parameters [60].

Juang et al. [61] first proposed the Eigensystem realization algorithm (ERA) method for modal identification in the field of aerospace engineering. This method uses state-space forms together with Markov parameters in order to extract the coefficients of a dynamic system. Juang et al. [62] introduced an extension of ERA while using data correlations (ERA/DC). In the proposed method, the ERA realization-based approach is combined with a state-space correlation fit. The extracted modal parameters in this method are less sensitive to noise corruption and less prone to bias error. The improved polyreference complex exponential (IPCE) [63] is another extension to ERA and it is specifically designed for enhancing the reliability of PRCE and reducing the influence of random noise in modal identification. The IPCE technique uses correlation filtering as a pre-processing step to reduce the noise effects on measured data and minimize system order.

The Ibrahim time-domain (ITD) was of the first SIMO algorithms for estimating eigenvalues and eigenvectors in one-step. The ITD technique is reported not to be appropriate for heavy damped systems or systems with low natural frequencies [64]. The multiple references Ibrahim time-domain (MRITD) is an enhanced extension of ITD for MIMO modal analysis. The method is a high-resolution modal decomposition approach that is based on eigenanalysis [65]. OKID was originally a companion of the ERA, being denoted as OKID-ERA. The method suggested establishing a non-recursive LS observer to relate input and output data [66]. In recent years, the OKID has been introduced as a separate class of algorithms that could be combined with other models in a pairwise basis. Output-only ERA-OKID [67] and output-only observer/Kalman filter identification (O3KID) [68] methods are two recent versions of OKID for experimental modal analysis of civil engineering structures. The RD method is a TD approach for modal analysis through transforming system responses into random decrement functions [69]. RD functions are considered to be the free-vibration responses of a system. With the assumption of a zero-mean stationary Gaussian stochastic process, RD functions are proportional to correlation functions. The proposed method uses the concept of averaging to extract the random decrement signatures (RDS) of structures. Brincker [70] presented a general overview of the random decrement application in the modal analysis of structures. NExT methods provide a reliable tool for the modal identification of civil engineering structures. These methods have been implemented on several real world structures. However, the obtained results for damping ratios were less accurate than those that were obtained with other counterparts, such as the SSI and FDD algorithms [55].

### 3.2.2. Auto-Regressive Moving Average (ARMA) Methods

Auto-regressive moving average (ARMA)-based methods outperformed the purely statistical methods. The auto regressive (AR) part aims to model linear function time-history and the moving average (MA) part aims to determine the moving average of the time-series [71]. The general structure of the AR-based models depends on which of the  $A(q)$ ,  $B(q)$ ,  $C(q)$ ,  $D(q)$ , and  $F(q)$  polynomials are used in the model. Equation (1) shows the general structure of AR-based models.

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t) \quad (1)$$

Table 1 shows the most common ARMA models that were used in the system identification of civil engineering structures.

**Table 1.** Common auto-regressive moving average (ARMA) models shown with their associated polynomials.

Used Polynomials	Name of Model
$A(q), B(q)$	ARX
$A(q), B(q), C(q)$	Auto-Regressive Moving Average with eXogenous variable (ARMAX)
$A(q), C(q)$	ARMA
$A(q), B(q), D(q)$	Auto-Regressive Auto-Regressive with eXogenous input (ARARX)

Several studies have been conducted while using autoregressive-based methods for the identification of damage in structures. Table 2 presents a review of applied ARMA methods in SHM and modal analysis with a focus on the test methods, damage features, and the incorporated pattern recognition techniques.

**Table 2.** Autoregressive-based methods applied in the structural health monitoring (SHM) of civil engineering structures.

Method/Reference	Test Model	Damage Feature	Pattern Recognition
AR [72]	Progressive failure test of retrofitted reinforced concrete bridge column	AR coefficients	X-bar control chart
AR-ARX [73]	Laboratory-scale 8- DOF mass-spring system	Residual error between prediction and reference signal	Multi-paradigm statistical pattern recognition
ARMAV [74]	Numerical model of a simply supported beam and a real-size steel-quake structure	Changes in natural frequency	Confidence interval and normal distribution of random variables
AR-ARX [75]	A simulated three-story frame structure	Residual errors	Sequential Probability Ratio Test (SPRT)
VFAR [76]	Numerical simulations of a six-story shear building and a three-span pre-stressed box-girder bridge	Natural frequency and damping ratio	-
VBAR [77]	Numerical model of a lumped mass system and experimental model of a steel cantilever beam	Natural frequency	-
ARMAV [78]	Steel-quake structure	Natural frequencies and damping ratio	-
TV-ARMAV [79]	Supported beam with a moving mass	Natural frequencies and mode shapes and damping ratio	-
ARMA [80]	Analytical and experimental results of the ASCE benchmark structure	First three AR components	Multi-paradigm statistical signal processing approach combined with the pattern classification
ARMA [81]	IASC-ASCE four-story experimental benchmark structure, Z24 bridge and the Malaysia-Singapore Second Link Bridge	Normalized AR coefficient	T-test statistical control chart
ARMA [82]	Simulated ASCE benchmark structure	GMM	Mahalanobis criterion function
ARIMA [83]	Data from Malaysia-Singapore Second Link bridge	Coefficients of the ARIMA model	Statistical outlier detection
TARMA [84]	Laboratory-scale steel beam with a moving mass	Natural frequencies	-

The structures range from a very simple model of a mass spring system to more elaborate real world structures, as shown in Table 2. In many cases, the coefficients of the incorporated AR-based algorithm are directly used as damage features. For modal analysis, the extracted system parameters



are utilized for the evaluation of dynamic behavior in a structure. Using modal parameters as damage features, the variation of the extracted parameters for the reference and actual state could be considered for the condition assessment of a structure. The obtained results for modal parameters are more robust when compared to other TD methods. However, higher scatterings in damping ratios are observed and the reported error is much more than other TD methods, such as SSI.

### 3.2.3. Subspace System Identification (SSI) Methods

Parametric TD methods provide a powerful and versatile mathematical framework for analyzing dynamic systems. Among all system identification methods, subspace-based techniques are the most remarkable achievement in the field of control and system identification. Meanwhile, many research studies on system identification have been concentrated on subspace methods in recent years. The subspace approach is a suitable technique for estimating the state-space model of a dynamic system. The SSI is a numerically reliable algorithm and it estimates models with good quality, particularly for multivariable systems [85,86]. The state-space form of the equation of motion can be written as Equation (2).

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) + w(t) \\ y(t) = Cx(t) + Du(t) + v(t) \end{cases} \quad (2)$$

where  $x(t+1)$ ,  $u(t+1)$ ,  $x(t)$ , and  $u(t)$  are state vectors and scalars at time instant of  $t+1$  and  $t$ , respectively.  $y_k$  is output vector,  $A$ ,  $B$ ,  $C$ , and  $D$  are system, control, output, and feedback matrices, respectively.  $w(t)$  and  $v(t)$  are measurement and process noise, respectively. Most subspace algorithms reported in the literature are closely related to the LS-based methods [87]. In the first step, an oblique projection is calculated and it is pre- and post-multiplied by appropriate weight matrices to infer the system order and state sequence. In the second step, a geometrical projection is adapted in order to eliminate the dependence of the SSI algorithm on future output. In the third step, LS is deployed to drive the  $A$  and  $C$  matrices. Finally, the Kalman predictor is used to estimate the system model by inferring the Kalman gain  $K$  of the state-space model. In a general sense, the most researched subspace methods in the field of system identification can be classified within the following three main categories [87]:

- CVA methods;
- MOESP methods; and,
- N4SID methods.

Larimore and Wallace [88] proposed CVA methods that are based on Markov parameters for TD system identification. The study continues with the same principles as the pioneering activities of Akaike [89] in a statistical setting. SVD is used as a tool to extract the incorporated canonical variates. Verhaegen [90] proposed the MOESP method for the identification of the multivariable state-space model from noisy input-output data. The MOESP subspace algorithm is known for two characteristics, those of the reduced-size Hankel matrix and the extended observability matrix. The method was not applicable for stochastic systems. Van Overschee and De Moor [91] unified proposed subspace schemes into a pragmatic approach, referred to as N4SID. The algorithm was analytically robust and reliable due to the use of SVD and QR decomposition. Based on the way, the subspace algorithm deals with the measurement time history data; they can be divided into the two categories of SSI-DATA and SSI-COV. In the next two subsections, a review on application of the SSI-DATA and SSI-COV algorithms in the SHM of civil engineering structures is outlined.

#### Data-Driven Stochastic Subspace Identification Method (SSI-DATA)

SSI-DATA is a method for identifying modal parameters by the direct use of measured response time-history [92]. Overschee et al. [93] introduced a subspace algorithm using power spectrum data. The state-space coefficients were derived using inverse discrete Fourier transform. The computational complexity in this method is higher when compared to that of direct subspace methods or FDD.

Table 3 shows some examples of the methods that use SSI-DATA for damage detection and modal analysis.

**Table 3.** Some examples of schemes that use data-driven subspace system identification (SSI-DATA) in their damage identification process.

Method/Reference	Damage Feature	Pattern Recognition	Test Model	Specification
SSI-DATA [94]	Eigensolutions	Statistical local approach	Analytical model	Application of a combined SSI
SSI-DATA [95]	Residual error	Statistical process control	Aircraft model and real bridge structure	Kalman model is extracted to represent the reference state of the structure.
Null-space based algorithm [96]	Residue matrix of orthonormality between subspaces	Statistical process control	Aircraft model and steel lighting device	Subspace analysis of the Hankel matrices is used for extracting features.
CSI/ref [97]	Modal parameters	-	Z24 bridge benchmark structure	Stabilization diagram is adopted for post processing of modal data.
McKelvey FD SSI [98]	Modal parameters	-	Uncontrolled cantilever plate	Natural frequency was predicted with an average error of 3.2% and damping ratio had an average error of 2.8%
KPCA subspace based damage detection [99]	Subspace angles	Through mapping the Subspace angles	A beam with non-linear stiffness	KPCA performs damage detection problem without modal identification.
CC-SSI [100]	Modal parameters	-	Ariane 5 launch vehicle	Clear results even in the case of non-stationary data obtained using this algorithm
RSSI [101]	Modal parameters	-	3-story steel frame and 2-story reinforced concrete frame	In this method the SVD algorithm is replaced by an advanced algorithm to update LQ decomposition.
Improved SSI [102]	Modal parameters	-	A numerical example of 7 DOF and an experimental model of Chaotianmen bridge	Less computing time due to not having QR decomposition CH matrix is constructed as a replacement for Hankel matrix. Spurious modes are removed using the model similarity index
SSA [103]	Subspace-based and nullspace-based damage indices	SVD for change-point detection	6-story steel frame	SSA algorithm uses SVD for change-point detection
Fast CC-SSI [104]	Modal parameters	-	Operational data from a ship	Fast multi-order computation
ECCA-based SSI algorithm [105]	Modal parameters	-	FE model and experimental wind tunnel bridge model	Enhanced results are achieved for weakly excited modes and noisy response signals

### Covariance-Driven Stochastic Subspace Identification (SSI-COV) Method

SSI-COV is a parametric output-only method that is used for modal analysis. The method use the vibration response time-history to extract the state-space model of a dynamic system. SSI-COV is a two-step procedure that utilizes the correlation function of vibration time-history. The system order is the only user defined parameter in SSI and it must be carefully chosen to obtain meaningful results. In these methods, the applied excitation is considered as white noise and it is equal to the covariance of the measured response time-history.

Table 4 shows some SSI-COV algorithms that are applied for the damage detection of civil engineering structures. The damage indices in these methods are either modal parameters or a novel indicator for detecting changes that are caused by damage. In the previous two subsections, the application of SSI-DATA and SSI-COV algorithms for the damage detection of civil engineering structures was presented. In the following section, a comparison between the SSI algorithm and other key system identification methods in TD and FD is presented.

**Table 4.** Some examples of the schemes that use covariance-driven subspace system identification (SSI-COV) in their damage identification process.

Method/Reference	Damage Feature	Pattern Recognition	Test Model	Specification
SSI-COV [106]	Subspace residual	Sensitivity analysis	Numerical analysis	Damage localization using the FE model
EMD-SSI-COV [107]	Intrinsic mode functions	Sifting process	Beichuan arch bridge	Capable of dealing with non-stationary signals
BSSI-COV [108]	Curvature evolution index	Stabilization diagram	Numerical simulation of an 7-DOF MSD model	Suitable for identifying weakly excited modes
SSI-COV [109]	Natural frequencies	Fuzzy clustering	Z24 Bridge and Republic Plaza Office Tower	No need for mode shape data
SSI-COV [110]	Residual of the observability null-space	Averaging operation	FE model of a bridge deck and laboratory test of a clamped beam within a climatic chamber	Temperature effect is evaluated using an averaging operation
SSI-COV [111]	Subspace residual	$\chi_2$ test	Workbench and instrumented beam	Handling the temperature effect is a target in this paper
SSI-COV [112,113]	State space matrices	Mahalanobis and Euclidean distance decision functions	Two simulated beam model, a laboratory-scale reinforced concrete beam and a full-scale arch bridge structure	The method was capable of locating damage in beam structures.
Recursive SSI-COV-IV [114]	Modal parameters	-	Numerical models of a Single Degree of Freedom (SDOF) structure and an ASCE benchmark steel frame structure	Model identification was conducted for a system with time-varying measurement noise
RD-SSICov [115]	Modal parameters	Modal strain energy method	Numerical model and a small scale wind turbine tower	The RD method was selected in this study for its noise reduction capabilities.
SSI-COV [116]	Modal parameters	Local optimization methods of NMA, GA, HS and PSO	Numerical examples and a laboratory model of cantilever beams	The model is appropriate to handle incomplete measurement data and truncated mode shapes
SSI-COV [117]	Curvature change, novelty index and PCA-SSA (for localization) modal updating and stiffness reduction (for quantification)	Novelty analysis	Laboratory-scale bridge foundation scouring	Detection, localization and quantification was conducted in a unified and online recursive subspace algorithm
SSI-COV [118]	Eigenfrequencies, damping ratios and mode shapes	Stabilization diagram	Experimental models of a damaged pre-stressed concrete bridge and a mid-rise building	Uncertainty quantification and effect of different setups in modal parameters was evaluated in this study

#### 4. Comparison between SSI and Other System Identification Algorithms

The SSI-DATA is a direct method and it does not need any data pre-processing to calculate correlation functions or spectrum analysis. On the other hand, the subspace identification algorithm uses QR factorization, SVD decomposition, and LS robust numerical techniques in the analysis process [119]. Several comparative studies are presented below in order to evaluate the advantages and drawbacks of the subspace identification algorithm when compared to other time and frequency domain damage identification methods in SHM. Table 5 presents a comparison between SSI method and other system identification algorithms.

**Table 5.** Comparison between subspace system identification (SSI) method and other system identification algorithms.

Reference	Identification Method	Advantages and Drawbacks
Magalhães et al. [55]	SSI PP	<ul style="list-style-type: none"> <li>• Generally, there was a good agreement between the obtained modal frequencies and mode shape in all algorithms, however the damping ratio shows the most significant scatter.</li> <li>• The PP method is an appropriate technique for modal identification of closely spaced modes.</li> <li>• SSI and FDD is a more appropriate choice for modal identification where closely spaced frequencies are not of the same type (e.g., bending or torsion).</li> <li>• PP and FDD are not suitable tools for identification of the modal parameters in cable-stayed bridges. Further analysis must be undertaken to successfully discriminate the vibrational influence of the cables from the global bridge frequencies.</li> </ul>
	FDD	<ul style="list-style-type: none"> <li>• The FDD and SSI yielded unreliable results for the second pair of closely spaced frequencies due to the coincidence of the modes and coupling effect in ambient excitation.</li> <li>• Estimation of damping ratios with wind-induced excitation has shown a significant level of scatter. Further research on ambient vibration testing is needed to approach a reliable identification of damping coefficients.</li> <li>• All the above-mentioned algorithms require human judgment during implementation. The user defined parameters for the SSI, FDD and PP are system order, singular values, and global modes, respectively.</li> </ul>
Magalhães et al. [120]	SSI-COV p-LSCF	<ul style="list-style-type: none"> <li>• Performance of both SSI-COV and p-LSCF was found to be good in the identification of natural frequencies and mode shapes.</li> <li>• It is demonstrated that p-LSCF can provide better estimation for the daily variations of damping coefficients compared to SSI.</li> </ul>
Wang et al. [121]	SSI ERA ARMA	<ul style="list-style-type: none"> <li>• All of the modal identification methods perform well when the responses are exposed to noise.</li> <li>• Model order is an important parameter in extracting correct modal parameters in all methods.</li> <li>• Significant errors are observed in the damping coefficient when system order varies.</li> <li>• Damping coefficient is sensitive to noise pollution, specifically for the first mode.</li> </ul>
	ITD	<ul style="list-style-type: none"> <li>• More stable results for modal identification were obtained by the SSI method in numerical simulations. However, the ERA method outperforms other algorithms for field testing.</li> <li>• ARMA shows the worst results for both field and numerical experiments.</li> </ul>
Moaveni et al. [122]	MNEXT-ERA	<ul style="list-style-type: none"> <li>• Comparing output-only methods of SSI-DATA, MNEXT-ERA and EFDD, it can be inferred that:</li> <li>• The identified natural frequencies using these methods display good agreement in each damage scenario.</li> </ul>
	SSI-DATA	<ul style="list-style-type: none"> <li>• The identified damping coefficient shows larger variability but still in a reasonable range.</li> </ul>
	EFDD	<ul style="list-style-type: none"> <li>• Lower damping ratios are obtained through FDD than the results identified using SSI-DATA and MNEXT-ERA.</li> <li>• The identified mode shapes using SSI-DATA were confirmed to be the most reliable among all methods.</li> </ul>
	DSI OKID-ERA	<ul style="list-style-type: none"> <li>• Comparing input-output methods of DSI, OKID-ERA, and GRA, the following were observed:</li> <li>• The identified mode-shapes in all of the above-mentioned algorithms are close to the actual mode shape of the structure.</li> </ul>
Cunha et al. [123]	GRA	<ul style="list-style-type: none"> <li>• In general, the calculated mode shapes by the input-output methods are more accurate than those obtained from output-only methods.</li> <li>• The mode-shape parameters obtained from the GRA appear to be in good agreement with the actual mode shapes of the structure.</li> </ul>
	SSI-COV	<ul style="list-style-type: none"> <li>• The same result in terms of damage detection was obtained using both the SSI-COV and FDD procedures.</li> </ul>
	FDD	<ul style="list-style-type: none"> <li>• In terms of modal parameters, perfect results were extracted through using both methods.</li> </ul>

Table 5. Cont.

Reference	Identification Method	Advantages and Drawbacks
Liu et al. [54]	SSI and ERA	<ul style="list-style-type: none"> <li>• Both the ERA and SSI methods are accurate and provide stable results for modal parameters. The methods are extremely sensitive to the size of the Hankel matrix.</li> <li>• The work burden of the algorithms is large and usually needs to perform multiple analyses.</li> </ul>
	PolyMAX	<ul style="list-style-type: none"> <li>• The PolyMAX and PolyLSCF methods are fast but the operator needs to have a high qualification.</li> <li>• The algorithm is sensitive to the size of the Hankel matrix and sometimes needs to perform multiple analyses.</li> </ul>
	PPM	<ul style="list-style-type: none"> <li>• PPM is a simple and less demanding algorithm.</li> <li>• The algorithm can identify modal parameters in the pre-defined frequency range.</li> </ul>
	PZM	<ul style="list-style-type: none"> <li>• PZM and PolyLSCF are basically similar and the obtained results can complement each other.</li> <li>• PZM uses a non-power spectrum for modal frequencies and damping.</li> </ul>
	EFDD and FSDD	<ul style="list-style-type: none"> <li>• EFDD and FSDD are fast and easy for modal parameter identification under ambient excitation.</li> <li>• The estimation precisions in these methods depend on the accuracy of the power spectral density.</li> <li>• Both methods are sensitive to noise and leakage error.</li> </ul>
	WT	<ul style="list-style-type: none"> <li>• The accuracy of the WT is relatively high.</li> </ul>
Kim and Lynch [53]	SSI	<ul style="list-style-type: none"> <li>• The WT is not conducive to the real-time monitoring due to analysis load.</li> <li>• Similar performance was confirmed for the SSI and FDD methods.</li> </ul>
	FDD	<ul style="list-style-type: none"> <li>• The FDD method slightly suffered from resolution problems due to the limited number of data points.</li> <li>• Though SSI has no resolution problems, more scattered data are generated when dealing with output-only modal analysis.</li> </ul>
Ceravolo and Abbiati [124]	SSI-DATA	<ul style="list-style-type: none"> <li>• All three output-only methods of SSI-DATA, AR and ERA/RDS show high robustness to modal frequency estimation in non-stationary conditions.</li> <li>• By using the same methods, damping estimates were very scattered.</li> </ul>
	AR	
	ERA/RDS	<ul style="list-style-type: none"> <li>• The estimation error of modal frequency in all methods remained less than 1.5%.</li> <li>• The error in estimating damping ratio ranged from 15% (in SSI) to 50% (in AR).</li> </ul>

Table 5 shows the results of a comparison between several TD, FD, and time-frequency domain methods for damage detection and modal analysis of civil engineering structures. The obtained results confirm the reliable performance of the TD methods. It was shown that SSI is a powerful tool for modal identification, in which closely spaced frequencies are not of the same type (e.g., bending or torsion) [55]. It was reported that the identified mode shapes using SSI-DATA were the most reliable when compared to MNExT-ERA or EFDD [122]. Moreover, fewer errors were obtained in estimating the damping ratio using the SSI algorithm as compared to ARMA [124]. However, SSI has some disadvantages, such as requiring human judgment of system order for implementation [55]. On the other hand, the work burden of the SSI algorithm is large and it usually needs to perform multiple analyses [54].

## 5. Challenges of SSI in Practical Application

Research on the application of subspace methods for the damage detection of civil structures emerged in the mid-1990s. Most of the methods used in SHM presume a parametric model of a dynamic system in order to characterize structural behavior under an applied excitation load. However, civil engineering structures still have many challenges to achieve a robust SHM model. The size of civil structures does not permit a large number of sensors to be mounted on a structure. Moreover, forced-vibration is not considered to be practical due to interruption in the serviceability of structures. On the other hand, civil structures are complex in terms of geometry and their material properties involve a large range of uncertainty due to operational and environmental factors [125]. In Table 6, the problems that are faced with practical implementation of SHM systems using the SSI algorithm and the researches to resolve the associated problems are presented.

Table 6 reviews the challenges of implementing the subspace algorithm in real-world applications. In practical applications of SHM, the response signal of structures is generally in the form of a non-Gaussian random signal. In such conditions, deterministic techniques result in unreliable system models. On the other hand, data loss or corruption caused by failure or loss of sensing, transmission, or storage devices during their normal use is a concern for a reliable damage detection scheme. Consequently, appropriate procedures must be considered in order to deal with the uncertainty that is caused by such instrumental failures. The accuracy of an identification algorithm is due not only to its insensitivity to environmental variation and instrumental failure, but also to the inherent performance of the estimation scheme. Studies have been conducted to improve the performance of the SSI method and enhance the modal identification process itself. The inherent performance of an estimation scheme to deal with problems, such as the short-length of a signal, non-stationarity measurement data, system non-linearity, leakage error, or different measurement setups remains a challenge for a reliable SHM. In some cases, resolving the problem and increasing the accuracy demands exhaustive expert assistance and time-consuming computation burdens. Gluing the non-simultaneously measured set-ups of sensor data is another controversial issue that needs to be considered before applying any identification platform. The aforementioned drawbacks are the topic of ongoing research in the field of SHM.

**Table 6.** The challenges of SSI in practical applications.

Reference	Method	Concern
Peeters and De Roeck [126]	SSI	Dealing with the sensor data of different measurement setups.
Pridham and Wilson [127]	Correlation-driven subspace algorithm	The effect of sample size and the dimension of the Hankel matrix on the estimated damping coefficient
Pan [128]	-	Discriminates the effects of epistemic uncertainty
Benveniste and Mevel [129]	SSI	Consistency of the subspace algorithm against non-stationarities in the measurement data
Reynders et al. [130]	SSI	Removing bias errors in ambient vibrations
Brasiliano et al. [131]	SSI-COV and SSI-DATA	Effect of non-structural elements in the vibrational behavior of bridge structures.
Alicioğlu & Luş [132]	SSI-COV and SSI-DATA techniques through BR and CVA.	The effect of structural complexity and ambient conditions on modal identification.
Marchesiello et al. [133]	ST-SSI	Time-variant identification
Balmès et al. [134]	Extended SSI-COV-based	Handling temperature effect in the identified modal parameters.
Wang et al. [135]	SSI together with ARX	Damage detection of Hammerstein systems or non-linearity introduced to a linear dynamic system through piece-wise constant inputs
Magalhães et al. [136]	EFDD and SSI-COV	Quality of the extracted damping ratios considering the proximity of natural frequencies, non-proportional damping and accuracy of the identification algorithms
Döhler et al. [137]	SSI-COV	Multiple non-simultaneously recorded measurement setups.
Carden and Mita [138]	A combined SSI-DATA method and first order perturbation technique	The challenges faced in extracting the accurate confidence interval of modal parameters in civil engineering structures.
Döhler and Mevel [139]	SSI- a modular and scalable approach	Merging sensor data by applying a modular approach.
Döhler et al. [140]	First-order SSI method perturbation analysis	Evaluation of the statistical uncertainty in a multi-setup configuration
Loh and Chen [141]	Hybrid RSSI-COV, NLPCA and AANN	Distinguishing the damage abnormality from those caused by environmental and operational variations
Brehm et al. [142]	Power spectral amplitudes and FE model	Determination of the optimum position of the reference sensor
Cara et al. [143]	Kalman filter SSI algorithm	Contribution of the modal parameters in each mode to the recorded measurement data.
Ashari and Mevel [144]	SSI-based algorithm	Injecting auxiliary input to extract the unexcited modes
Tondreau and Deraemaeker, [145]	Monte-Carlo and SSI	Introducing uncertainty into an FE model
Rainieri and Fabbrocino [146]	SSI	Influence of the system order and number of block rows in identification accuracy
Cho et al. [147]	SDSI	Implementation of VDD using wireless sensor networks
Markovsky [148]	SSI	Influence of missing measurement data in dynamic system
Banfi & Carassale [149]	SSI, SRA and FDD	Effect of uncertainty in determining modal parameters
Spiridonakos et al. [150]	PCE	Humidity and temperature effects
Huynh et al. [151]	A combined SSI algorithm and short-time Fourier transform	Structural analysis of a cable-stayed bridge under typhoons with various wind speeds.
Ren et al. [152]	Improved SSI algorithm	Eliminating spurious modes caused by non-white noise

## 6. Application of Subspace Identification in Civil Engineering Structures

Currently, stochastic subspace methods are widely accepted tools in civil engineering communities. Large number of SHM methods is operating in structures that are subjected to dynamic vehicular, seismic, wind, or impact loading. In this study, the application of the SSI algorithm in civil engineering structures is investigated in the following three categories:

1. beams and two dimensional (2D) frames;
2. three dimensional (3D) frames and building structures; and,
3. bridges and other structures.

Beams and two-dimensional (2D) models attempt to explain the performance of the SHM algorithm while using numerical FE or laboratory-scale models [153]. Many of the studies on these structures are generally academic in nature. Three-dimensional (3D) frames and building structures are generally used to investigate the practical aspects of SHM. Bridges are spectacular due to their specific loading conditions when compared to building-type structures [154]. Vehicular and pedestrian loading, together with exposure to environmental variations due to temperature and wind precipitation, have been the focus of the analysis of bridge structures in many pioneering studies of SHM [155]. Tables 7–9 show some application of SSI algorithms for damage detection of 2D frames, 3D frames, and buildings and bridges, and other structures, respectively.

**Table 7.** Application of SSI algorithms for damage detection of beams and two-dimensional (2D) frames.

Reference	Applied Damage Detection Technique	Test Structure	Damage Configuration	Damage Identification Results
Vanlanduit et al. [156]	Subspace theory and RSVD	Laboratory-scale aluminum beam	(1) 30% and 50% saw cut in the middle (2) A beam with fatigue crack under several geometrical and damping conditions.	Reliable results were obtained by applying the introduced RSVD method.
Naseralavi et al. [157]	Theory of subspaces and Kernel parallelization	Three simulations of 1D and 2D case studies	Reduction in Young's modulus ranging from 10% to 30% correlated with the extent of the damage The crack-like damage simulated by the loosened 3 to 7 number of bolts, which correlates with a reduction of bending stiffness from 3% to 30%, respectively.	Most single and multiple damage scenarios
Döhler and Hille [158]	Subspace-based	(1) Numerical simulations (2) A 1D steel frame structure		All damage cases were clearly detected under variable excitation using the new test method
Banan and Mehdi-Pour, [159]	Concept of subspace rotation and Monte Carlo simulation	A simulation model of a braced-frame structure	Single damage case with a 25%, 50%, and 95% reduction in Young's Modulus	Damages were detected at noise ratios less than 5% but for a noise ratio of 20%, the results were unreliable for all damage rates.



**Table 8.** Application of SSI algorithms for damage detection of three-dimensional (3D) frames and buildings.

Reference	Applied Damage Detection Technique	Test Structure	Damage Configuration	Damage Identification Results
Siegert et al. [160]	Statistical subspace-based algorithm	Laboratory-scale composite bridge deck	40% reduction of the Young modulus	Algorithm can successfully detect the damages of the structure investigated
Li and Chang [114]	SSI and IV	Numerical model of a four-story, ASCE benchmark steel frame	A damage case of 30% reduction in the stiffness of the first story	Application of the technique tracking damages in the presence of measurement noise was successful.
Huang et al. [161]	SSI and DLV	Numerical model and laboratory-scale of a five-story steel frame	Damage scenarios with single and two damage case inflicted on the first second, third and fourth stories' columns	(1) Poor results were recorded for multiple damage cases. (2) The obtained result for the ill-conditioned counterpart was effective.
Kim and Lynch [162]	SSI algorithms	Laboratory scale of a single-bay six-story steel frame structure	Six damage cases comprised of saw cuttings of 3, 6 and 9 cm of the columns on the first and second floors	All damage in the structure was easy to identify using the proposed algorithm even in the presence of noise.
Wang et al. [163]	SSI and DLV	Five-story shear frame structure with diagonal bracings.	Damage conditions simulated by partially removing some of the bracings	The proposed algorithm for damage localization was successful
Moaveni et al. [164]	Deterministic-stochastic subspace identification	Laboratory-scale, three-story, two-bay, infilled RC	Progressive damage induced by shaking table experiencing scaled earthquake records.	Natural frequencies decrease with increase in damage level during the test experiment
Weng et al. [165]	SI and model updating	Six-story steel frame structure and a two-story RC frame.	(1) Loosening the connection bolts in single and multiple damage case in 2nd, 4th and 6th stories. (2) by progressive damage through shaking table in a RC frame.	The method was capable of detecting, localizing and quantifying the damage in both steel and concrete frames.
Karami and Akbarabadi [166]	Active control strategy and SSI	Numerical model of a 5 story and a 20 story shear building	Reduction in the stories' lateral stiffness ranged from 10 to 50%	The algorithm can detect damages accurately.
Belleri et al. [167]	Combined deterministic-stochastic subspace identification	Real-size three-story half-scale precast concrete building	Base excitation leading to loss of stiffness, cracks, and failure of the connections	Damage reduces natural frequencies and increases the damping ratios
Shinagawa and Mita [168]	Subspace identification	Full-scale four-story steel building	Collapse test.	Strong correlation between the extracted features with structural damage was observed.
Zhou et al. [169]	Subspace-based methods	A six-story residential building	More than 20 earthquake aftershocks recorded after instrumentation	Results of damage detection were not very reliable for earthquake induced excitation
Yoshimoto et al. [170]	MIMO SSI method and sub-structuring	An existing 7-story base-isolated building.	Stiffness reduction in the stories	The estimated damage of the stories correlates with the reduction ratio.

**Table 9.** Application of SSI algorithms for damage detection of bridges and other structures.

Reference	Applied Damage Detection Technique	Test Structure	Damage Configuration	Damage Identification Results
Allahdadian et al. [171]	SSDD and $\chi^2$ test	Reibersdorf Bridge	Damage simulated in girder, column, bearing, deck and cap beam with the ratios varied between 20%, 40% and 80%.	Unable to detect minor damage with a ratio of 20%
Dohler et al. [172]	Statistics-based subspace and $\chi^2$ test	S101 Bridge	Progressive damage test including cutting column, column settlement, horizontal crack in column, settling of bridge deck, and cutting pre-stress tendons.	(1) All steps of the column settlement and uplifting were clearly visible (2) Cutting of the pre-stressing tendons was not detectable.
Deraemaeker et al. [173]	Automated SSI and Shewhart-T control charts	Numerical model of a 3 span bridge	Four damage scenarios simulated by stiffness reduction in three different locations.	Proposed method was capable of detecting damage scenarios
Hu et al. [174]	SSI-COV and PCA algorithm	Pedro e Ines footbridge	Several realistic damage scenarios of 5%, 10%, 15%, 20% and 30% reduction of model spring constants	The method was capable of detecting all damage scenarios
Loh and Chao [175]	SSI-COV and MSSA	Laboratory-scale bridge	Displacement and lowering of a pier due to scouring	Localization and quantification were successful
Mevel and Goursat [176]	SSI	Z24 Bridge	Pier settlement of 20mm and 80 mm.	The efficiency of the proposed method was confirmed
Lin et al. [113]	Statistics-based SSI	(1) Laboratory-sale pre-stressed RC beams (2) a full-scale RC arch bridge	Gradually releasing the pre-stressed tendon from 17% to 73%	Algorithm was successful in detecting damage
Kullaa [177]	Subspace and stabilization diagram and control chart	Z24 Bridge	Pier settlements of 40mm and 95mm	Multivariate control charts have better performance in damage detection compared to univariate methods
Reynders, and De Roeck [178]	SSI algorithm and KPCA	Z24	Pier settlement, tilt of foundation, concrete spalling, landslide of abutment, failure of concrete hinge, failure of anchor heads and rupture of tendons were introduced to the structure	The proposed algorithm was capable of detecting imperfection in most cases
Nguyen et al. [179]	NSA and EPCA	Numerical model of Champangshiehl Bridge	Several intentionally created cuttings in pre-stressed tendons	The obtained result was encouraging.

In most structures, damage occurs in the form of a reduction in the cross-section of structural members. Partial reductions in 2D structures are usually detectable with a high level of precision. The robust applicability of the proposed methods to solve the problem of detecting local damage in real and complicated civil engineering structures has not been validated, even though these methods work relatively well in simple structures. Three-dimensional frames and building structures are generally complex and they pose challenges for both practitioners and researchers. Damage in building structures is usually in the form of a partial reduction in the cross-sectional area in column elements. In some cases, damage is made by opening bolts in a beam-column connection. However, beam damages are less important and they require higher detectability resolution. Bridge structures are a very important element of transportation. An in-service bridge is subject to loads, such as traffic, temperature variation, wind loading, and deterioration, under aggressive environments. Applying SHM to bridge structures poses significant challenges due to the specific types of loading and complexity of the structure. As a general conclusion, it could be derived that damage detection strategies that use modal frequencies, mode shapes or mode shape curvatures as their damage sensitive features [180] are only efficient for

the detection of global damage and are not generally sensitive enough to detect changes in the local elements of structures [181].

## 7. Application of Subspace Identification in Civil Engineering Structures

In general, an individual program or a combination of software packages implement the process of damage detection [182,183]. A structural monitoring program is considered an algorithm for analyzing response signals, extracting damage features and deploying pattern recognition paradigms, ultimately leading to damage identification [184]. Subspace methods have been used as the central part of many of the structural monitoring programs used in industry [185,186]. In this subsection, the industrial software packages used in modal identification and SHM, which have adapted SSI as their core identification process, are further investigated.

Table 10 provides some of the commercially available software that use subspace identification algorithms as their standard technique for SHM and modal analysis. Most of the available algorithms are generally used for modal analysis. The SHM algorithms are composed of (i) identification, (ii) feature extraction, and (iii) pattern recognition steps and the implementation of a unified algorithm for huge diversity of each category is quite challenging and rewarding.

**Table 10.** SSI-based commercially available software for SHM and modal analysis.

Reference	Software	Analysis Tool	Environment	Purpose
Döhler et al. [182]	ARTEMIS	CC-SSI	-	Modal identification
Y. Zhou et al. [183]	ModalVIEW	SSI, RPF and LSCF	LabVIEW	Modal identification
Hu et al. [184]	SMI	EFDD	LabVIEW	Modal identification
Hu et al. [184]	CSMI	PP, FDD and SSI	LabVIEW	Modal identification
Goursat & Mevel [186]	COSMAD	SSI-COV	Scilab	Modal identification
Chang et al. [185]	SMIT	SSI, auto-regressive-based methods and realization-based algorithms	MATLAB	SHM

## 8. Future Research Directions

In the future, researchers should focus on the identification of local damages by improving the accuracy and noise-robustness of damage identification algorithms [187,188]. Furthermore, they must think about introducing a novel platform for the implementation of commercialized SHM software that is versatile enough to deal with the diversity of techniques in a damage detection system. Providing a user-friendly platform for the implementation of the SHM algorithm will improve the general usage of SHM software in solving real-life engineering problems. Furthermore, the extensible architecture would enhance the applicability of the software by enabling users to modify the existing base code and add their own extensions. A modular and flexible architecture enables a wide variety of reported methods to deal with within an integrated framework. Enhancing the SSI properties to deal with real-world applications, such as noise inclusion, short length data, and gluing sensor data, will enhance accuracy to provide more reliable damage detection results.

## 9. Conclusions

This paper presented a review of the recent advances in subspace-based SHM methods. SSI is a powerful choice for modal identification, particularly when dealing with closely spaced frequencies or noisy response signals. The identified mode shapes using SSI-DATA were confirmed to be the most reliable when compared to MNExT-ERA and EFDD. Moreover, fewer errors were obtained in estimating the damping ratio using the SSI algorithm as compared to ARMA. However, there are some disadvantages in using SSI, such as requiring human judgment for system order and large work burden of the SSI algorithm.

In most structures, damage is imposed in the form of a reduction in the cross-section of structural members. A review on applied models used for verification of damage detection algorithms shows that damages in the 2D structures are usually detectable with a high level of precision. Three-dimensional frames and building structures are generally complex and pose challenges for both practitioners and researchers. Even though lots of proposed SHM algorithms work relatively well in simple structures, the robust application of these methods to solve the problem of local damage detection in real and complicated civil engineering structures has not been validated. The obtained results demonstrate that damage detection strategies that use vibration parameters of a dynamic system are only efficient for the detection of global damages and they are not generally sensitive enough to detect local damages in structures.

Most commercially available software is generally designed for modal analysis and cannot be directly used for the SHM of structures. The commercial modal analysis toolboxes introduced are mainly implemented while using LabVIEW, which adapts a limited number of system identification methods, for example, SSI, LSCF, FDD, and SSI. The identification of modal parameters generally starts with extracting the response signal of structures and it results in the modal parameters of natural frequency, mode shapes, and damping ratio. However, SHM algorithms are generally complicated due to the wide variety of methods reported and their inherent structures. Hence, the implementation for the large diversity found in each category within a unified algorithm is quite a challenging problem.

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## Nomenclature

AR	Auto regressive
ARMA	Auto-regressive moving average
ARMAX	Auto-Regressive Moving Average with eXogenous variable
ARX	Auto-Regressiv with eXogenous input
BFD	Basic frequency-domain
CE	Complex exponential
CMIF	Complex mode indication function
CVA	Canonical variate analysis
EFDD	Enhanced frequency domain decomposition
eFRF	Enhanced frequency response function
ERA	Eigensystem realization algorithm
ERA/DC.	ERA using data correlations
ERA-OKID-OO	Output-only ERA-OKID

ET	Eddy current test
EVD	Eigen-vector decomposition
FD	Frequency-domain
FE	Finite element
FFT	Fast Fourier transforms
FRF	Frequency response functions
IPCE	Improved polyreference complex exponential
ITD	Ibrahim time-domain
LS	Least-squares
LSCE	Least-squares complex exponential
MA	Moving average
MIMO	Multiple-input multiple-output
MOESP	Multivariable output error state-space
MRITD	Multiple references Ibrahim time-domain
N4SID	Numerical algorithms for state-space subspace system identification
NDE	Non-destructive evaluation
NExT	Natural excitation technique
O3KID	Output-only observer/Kalman filter identification
OKID	Observer/Kalman filter identification
PAUT	Phased array ultrasonic test
pLSCF	Poly-reference least squares complex frequency-domain
PolyMAX	Polyreference least-squares complex frequency-domain method
PP	Peak picking
PRCE	Polyreference complex exponential
RD	Random decrement
RDS	Random decrement signatures
RT	Radiographic test
SDD	Static-based damage detection
SHM	Structural health monitoring
SIMO	Single-input multiple-output
SISO	Single-input single-output
SSI	Subspace system identification
SSI-COV	Covariance-driven subspace system identification
SSI-DATA	Data-driven subspace system identification
SVD	Singular value decomposition
TARMA	Time-dependent auto-regressive moving average
TD	Time-domain
TFD	Time/frequency domain
UT	Ultrasonic test
VDD	Vibration-based damage detection

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