How Quantum Information can improve Social Welfare

Berry Groisman, Michael Mc Gettrick, Mehdi Mhalla, and Marcin Pawłowski

***Abstract*—In [1], [2], [3], [4], [5] it has been shown that quantum resources can allow us to achieve a family of equilibria that can have sometimes a better social welfare, while guaranteeing privacy. We use graph games to propose a way to build non-cooperative games from graph states, and we show how to achieve an unlimited improvement with quantum advice compared to classical advice.**

***Index Terms*—quantum information, Nash equilibrium, social welfare, graph games, conflict-of-interest games.**

I. Introduction

An important tool in analysing games is the concept of *Nash equilibrium* [6], which represents situations where no player has incentive to deviate from their strategy. This corresponds to situations observed in real life, with applications in economics, sociology, international relations, biology, etc. All equilibria do not have the same *social welfare*, i.e. the average payoff is different from one equilibrium to another. Games of incomplete information can exhibit better equilibria if players use a resource – a general correlation, Q. Such correlation can be viewed as a resource produced by a mediator to give *advice* to the players. The concept of advice generalizes the notion of Nash equilibrium to a broader class of equilibria [7]. All such equilibria can be classified according to the properties of the resource correlation. Three classes can be identified in addition to Nash equilibria (no correlation), namely general communication equilibria (Comm) [8], where Q is unrestricted, belief-invariant equilibria (BI) [9], [10], [11], [12] and correlated equilibria (Corr) [7]. The canonical versions of these equilibria form a sequence of nested sets within the set of canonical correlations:

$$\text{Nash} \subset \text{Corr} \subset \text{BI} \subset \text{Comm}.$$

It was demonstrated that there exist games where BI equilibria can outperform Corr equilibria [2] (in terms of a social welfare (SW) of a game) as well as games where BI equilibria outperform any non-BI equilibria. In [4] the work of [2] is extended into the quantum domain.

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Auletta at al. [1] introduce quantum correlated equilibria as a subclass of BI equilibria and show that quantum correlations can achieve optimal SW. This provides the link with quantum nonlocality, where quantum resources are used to produce *non-signalling* correlations. In this context, belief invariance describes the largest class of correlations that obey *relativistic causality*. The role of *quantum entanglement* as quantum-social welfare advice was further studied in [5].

A characteristic feature of belief-invariance is that it ensures privacy – the other players involved in the game have no information about the input one player sent to the resource.

To obtain the canonical form of the games, [13] show that one can suppose that the output of the correlation resource is the answer the players give by delegating the extra computation (from game question to input to the box and from output of the box to players' answer) to the mediator. Therefore, quantum equilibria can be reached in a setting where players each measure quantum systems or, equivalently, by just having a central system providing advices by measuring a quantum device.

Ref. [1] highlights several open questions. In particular,

(1) Whether any full-coordination game (a.k.a. a *non-local game* in quantum physics and computer science communities) can be converted into a conflict-of-interests game. Ref. [2] gives an example of a two-player variant of the CHSH game, while [1] extends their result to an n-player game in which there exists a BI equilibrium which is better than any Corr equilibrium.

(2) How can we get a large separation between the expected payoff for the quantum and correlated equilibrium cases, and what is the upper bound for the separation? In the case of two-player full coordination games this question was settled in [14], [15]. Are there conflict-of-interest games which exhibit large separation?

In this paper, we provide a natural way to convert graph games (and more generally stabiliser games) into conflict-of-interest games, and we show how we can create unbounded separation by increasing the number of players or using penalty techniques (a negative payoff). Some of the techniques we use, e.g. *distributed parallel repetition*, are novel in the topic of conflict-of-interest games.

An interesting feature in these games compared to the usual pseudo-telepathy scenarios studied in quantum information is the notion of *involvement* [16], [13], which allows one to define some interesting scenarios in non-cooperative games and which exhibits novel features, e.g. unlimited separation. If a player participates in the game but is not involved (on a particular round) it means that their strategy is not taken into account when determining the win/lose outcome. However, they do receive a corresponding payoff.

Using these games one can build games with bounded personal utilities v_0, v_1 on $O(log(\frac{1}{\epsilon}))$ players ensuring $\frac{CSW(G)}{QSW(G)} \leq \epsilon$, where CSW/QSW are the Classical/Quantum Social Welfares, respectively.

The paper is organized as follows. In Sec. II we describe graph games which are the underlying non-local games used to define our games. In Sec. III we define a non-collaborative game as a modification of the collaborative games by introducing unequal payoffs corresponding to answers 0 and 1 of each player, and discuss the corresponding quantum perfect strategy. We consider a particular version of graph games from the cycle on five vertices. Sec. IV discusses variations of non-collaborative games based on the cycle on five vertices. Finally, Sec. V shows how one can amplify the quantum advantage by adding a penalty for wrong answers and by increasing the number of players.

II. Graph games

Non-local games play a key role in Quantum Information theory. They can be viewed as a setting in which players that are not allowed to communicate receive some inputs and have to produce some outputs, and there is a winning/losing condition depending globally on their outputs for each input. Particular types of games are pseudo-telepathy games [17] which are games that can be won perfectly using quantum resources but that are impossible to win perfectly without communication when the players have access only to shared randomness. Multipartite collaborative games ($MCG(G)$) are a family of pseudo-telepathy games based on certain types of quantum states called *graph states*. The players are identified with vertices of the graph and have a binary input/output each with the winning/losing conditions built using the stabilisers of the graph states.

The combinatorial game[1] $MCG(G)$ with n players consists in asking the players questions: for each question q, each player i receives one bit q_i as input and answers one bit a_i. They can either all win or all lose depending on their answer, with winning/losing conditions described by a set $\{(q, I(q), b(q))\}$ where

- $q \in \{0,1\}^n$ is a valid question in which each player i gets the bit q_i and in the subgraph of the vertices corresponding to players receiving one, all vertices have even degree. Let $I_1 = \{i, q_i = 1\}$ and $G' = G_{|I_1}$, a question is valid if each vertex of G' has an even number of neighbors in G'.
- $I(q) \subset [n]$ is a subset of players that are called *'involved'* in the question. This set is defined using the graph structure and the question q. $I(q) = I_1 \cup \{i, N_G(i) \cap I_1 = 1 \bmod 2\}$ where $N_G(i)$ is the set of neighbors in G of the vertex i. It contains the set I_1 as well as the players j such that the vertex j has an odd number of neighbors in I_1. The sum (modulo 2) of their answers determines the winning/losing condition according to the bit $b(q)$.
- $b(q)$ is defined such that the players win the game when the question is q if the sum of the answers of the involved players is equal to the parity of the number of edges of the induced subgraph of the vertices corresponding to players receiving one: $\sum_{i \in I(q)} a_i = b(q) = |E(G')| \bmod 2$.

The losing set is the set of pairs of questions and answers for which the players lose the game $\mathcal{L} = \{(q, a), \sum_{i \in I(q)} a_i \neq b(q) \bmod 2\}$. For instance the game associated to the cycle on 5 elements $MCG(C_5)$ is defined by

- When the question is $q = 11111$ (each player has input 1), the players lose if the binary sum of their answer is 0, *i.e.* $\sum_{i=0}^{4} a_i = 0 \bmod 2$, and win otherwise.
- When the question contains 010 for three players corresponding to three adjacent vertices, the players lose if the binary sum of the answer of these three players is 1 *i.e.* $a_{i-1} + a_i + a_{i+1} = 0 \bmod 2$ when q contains $0_{i-1}1_i0_{i+1}$.
- The players win otherwise.

A variation of this game can be done by reducing the set of valid questions, for instance in the above set-up the questions of the second type have only three players 'involved', so a first version could be to chose only 5 questions of the second type and give always 0 as advice to the non-involved players. This is the game studied as an example in [13].

An important point is that the notion of involvement in MCG games is absent in unique games and introduces situations where the players might change their strategy (answer) without changing the winning/losing status of the global strategy.

To analyse these games and the strategies, one can imagine a scenario where there is one special player representing Nature who is playing against the other players. The strategy of Nature is therefore a probability distribution over the questions that we study here (as is standard in game theory) as a known function on the set of questions $w : T \to [0, 1]$ such that $\sum_{t \in T} w(t) = 1$. The games will be therefore defined by equipping the combinatorial game with a probability distribution over the questions.

III. Defining non-collaborative games

Like in multipartite collaborative graph games $MCG(G)$, we associate a non-collaborative game $NC(G)$ to each graph. We differentiate the payoff of the players using the value of their output: If the global answer wins in the non-local game, each player gets v_1 if they answer 1 and v_0 if they answer 0. If the global answer loses, they get 0.

To match the traditional terminology used in game theory the output from now on will be called *strategy*, and the input called *type*. The payoff is called *utility* and the social welfare is the average of the utilities over the players.

A non-collaborative game $NC(G)$ is thus defined from $MCG(G)$ as follows

- The considered types are $T \subset \{0,1\}^n$ where n is the number of vertices of G.
- As in MCG, to each type $t \in T$ corresponds an associated involved set $I(t)$ of players, and an expected binary answer $b(t)$.

[1] without considering probability distributions

- As in MCG, the losing set is

$$\mathcal{L} = \{(s,t), \sum_{i \in I(t)} s_i \neq b(t) \bmod 2\}.$$

We say that the players using a strategy s, given a type t, collectively win the game when the sum of the local strategies of the involved players is equal to the requested binary answer modulo 2.
- the payoff function is:

$$u_j(s|t) = \begin{cases} v_{s_j} & \text{if } (s,t) \notin \mathcal{L} \\ 0 & \text{Otherwise} \end{cases}$$

Firstly we consider the cycle on five vertices C_5. For questions which involve three players where both non-involved players have type 0 (see Figure 1), we define $NC_{00}(C_5)$ based on the non-local game $MCG(C_5)$ studied in [16], [13].

TABLE I: $NC_{00}(G)$ game (Here and in the subsequent tables the players are identified with the integers modulo 5).

Type	Involved set	Binary answer
$T_a = 11111$	$I(T_a) = \{0, 1, 2, 3, 4\}$	$b(T_0) = 1$
$T_0 = 10000$	$I(T_0) = \{0, 1, 4\}$	$b(T_0) = 0$
$T_1 = 01000$	$I(T_1) = \{0, 1, 2\}$	$b(T_1) = 0$
$T_2 = 00100$	$I(T_2) = \{1, 2, 3\}$	$b(T_2) = 0$
$T_3 = 00010$	$I(T_3) = \{2, 3, 4\}$	$b(T_3) = 0$
$T_4 = 00001$	$I(T_4) = \{3, 4, 0\}$	$b(T_4) = 0$

We consider the game with the type probability distribution $w(t) = 1/6$ for all the types.

The quantum perfect strategy for $NC(G)$ (see [16]) is obtained when the players each have a qubit from the graph state $|G\rangle$, which is a quantum state obtained by taking one qubit in the state $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ per vertex in the graph, and then applying a controlled Z operation per edge of the graph (see [18]). Each player i measures their qubit according to their type t_i, getting a quantum advice representing their part of the quantum strategy s_i [16]. From the study of $MCG(G)$ we have

Theorem 1. *If all the players collaborate (follow the quantum advice) then for any probability distribution over the types, the utility of each player is* $(v_0 + v_1)/2$.

Proof. The output of each quantum measurement provides uniformly all the possible answers. □

A. Is the quantum pseudo-telepathy solution a Nash equilibrium?

As the players now have an incentive to answer 1, they can sacrifice always getting a good answer to maximize their utility. Indeed, in the previous game, each player is always involved when they get type 1 and with probability $1/2$ when they get type 0; getting the wrong answer in that case only costs v_0.

Without loss of generality we consider $v_1 \geq v_0$. The players now have an incentive to answer 1, because they might be able to maximize their utility by allowing the non-zero probability of a wrong answer. Indeed, in the previous game, $NC_{00}(C_5)$, if the player gets type 1 then they are certain that they are involved, and they won't gain by defecting (not following advice). However, if their type is 0, then the probability of them being involved is $1/2$, and so there is a fifty percent chance that they will benefit from always answering 1 while not compromizing the winning combination. Getting the wrong answer in that case only costs v_0.

Theorem 2. *Let $p^{(i)}_{inv}(t_i, s_i)$ be the probability for the player i who gets type t_i and advice s_i to be involved.*

Then, in $NC(G)$, the quantum advice gives a belief-invariant Nash equilibrium iff

$$\frac{v_0}{v_1} \geq (1 - p),$$

where

$$p = \min_i \min_{t_i} p^{(i)}_{inv}(t_i, 0).$$

Proof. If the advice is $s_i = 1$ then the winning payoff is already v_1. Consider the case when player i is given the advice $s_i = 0$ (which would lead to payoff v_0 in the winning case). If the player defects then the difference of utility is $-v_0 p^{(i)}_{\text{inv}}(t_i, 0) + (1 - p^{(i)}_{\text{inv}}(t_i, 0))(v_1 - v_0)$. So the strategy is a Nash-equilibrium when $(1 - p^{(i)}_{\text{inv}}(t_i, 0))v_1 \leq v_0$, i.e $v_0/v_1 \geq 1 - p^{(i)}_{\text{inv}}(t_i, 0)$. This inequality has to hold for all types and all players. □

For $NC_{00}(C_5)$, $p^{(i)}_{\text{inv}}(0, 0) = 1/2$ and therefore the quantum nonlocal strategy is an equilibrium only when $v_0/v_1 \geq 1/2$.

One important characteristic of an equilibrium is the *Social Welfare*, which is the average utility of the players[2].

As a direct consequence of Theorem 1 the average social welfare of the quantum strategy is independent on the graph

$$QSW(NC(G)) = \frac{v_0 + v_1}{2}.$$

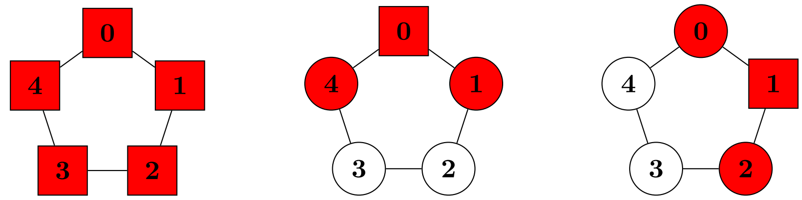
Note that the non collaborative games defined have a special feature that we call *guaranteed value*: in any run of the game players following the quantum strategy receive their expected payoff with probability 1.

IV. Some versions of $NC(C_5)$

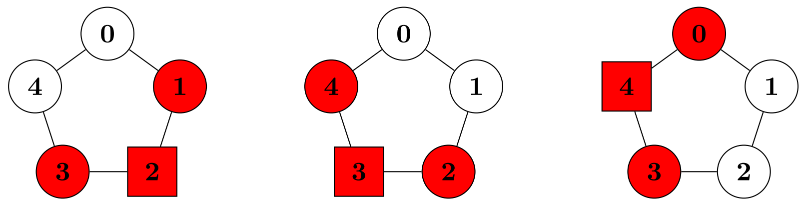
In this section we study the game $NC_{00}(C_5)$ and then introduce a number of modifications in order to improve the quantum advantage (ratio of quantum social welfare to correlated social welfare) and also to symmetrize the game such that the players get 0 and 1 with same probability or have the same probability of being involved regardless of whether their type is 0 or 1.

[2] Note that other definitions of social welfare exist in the literature.

Fig. 1: $NC_{00}(C_5)$: Square nodes indicate a 1 in the associated type, while circular nodes indicate a 0. *Involved* players in each case are shaded in **red**.



(a) $T_a = 11111$, $I = \{0, 1, 2, 3, 4\}, b = 1$ (b) $T_0 = 10000$, $I = \{4, 0, 1\}, b = 0$ (c) $T_1 = 01000$, $I = \{0, 1, 2\}, b = 0$



(d) $T_2 = 00100$, $I = \{1, 2, 3\}, b = 0$ (e) $T_3 = 00010$, $I = \{2, 3, 4\}, b = 0$ (f) $T_4 = 00001$, $I = \{3, 4, 0\}, b = 0$

A. Study of $NC_{00}(C_5)$

Pure Nash equilibria can be described by local functions: each player having one local type bit and one strategy bit to produce, can locally act as follows:

- $0 \to 0$, $1 \to 0$ constant function 0 denoted **0**
- $0 \to 1$, $1 \to 1$ constant function 1 denoted **1**
- $0 \to 0$, $1 \to 1$ Identity function denoted **Id**
- $0 \to 1$, $1 \to 0$ NOT function denoted **Not**

The set of pure Nash equilibria depends on the ratio v_0/v_1. There are 20/25/40 pure Nash equilibria (4/4/6 up to symmetry) when v_0/v_1 lies within the interval $[0, 1/3]$, $[1/3, 1/2]$ or $[1/2, 1]$ respectively (see Table II).

We can see that most of these equilibria (all of them when $v_0/v_1 \geq 1/2$) correspond to local functions winning for the 5 types.

When $v_0 = 2/3$ and $v_1 = 1$ then the quantum social welfare of the pseudotelepathy strategy is $QSW = 0.83$ whereas the best classical social welfare $CSW = 0.77$.

As noted in section III-A the probability of being involved in NC_{00} is $p(1, s) = 1$ and $p(0, s) = 1/2$ and the quantum pseudotelepathy measurements strategy is an equilibrium if $v_0/v_1 \geq 1/2$.

Similar behavior can be seen with Pareto equilibria (ones in which local utility cannot improve without reducing the outcome of someone else): see Appendix.

Recall that the characteristic feature of $NC_{00}(C_5)$ is that each player has unequal probabilities of getting different types. The game can be symmetrized by changing the types of the non-involved players from 00 to 01, as shown in the next section.

B. Comments on $NC_{01}(C_5)$

We define a second variant from $MCG(C_5)$: $NC_{01}(C_5)$ where any player gets the types 0 and 1 with probability 1/2 by adding an extra 1 for a non-involved player in the types so that $T_i = 0_{i-1}1_i0_{i+1}1_{i+2}0_{i+3}$: see Table III.

If the type probability distribution is $w(t) = 1/6$ for all the types, then one can see that any player is involved with probability 2/3 whether their input is 0 or 1, i.e. $p_{\text{inv}}^{(i)}(0, 0) = p_{\text{inv}}^{(i)}(1, 0) = 2/3$. Hence, by Theorem 2, the quantum strategy of MCG produces a Nash equilibrium iff $v_0/v_1 \geq 1/3$. Thus, one of the benefits of this variant is that quantum Nash equilibria exist at a lower ratio v_0/v_1.

Note that in this version each player is getting a perfect random bit as advice: $p(a = 1) = p(a = 0) = 1/2$.

When $v_0 = 2/3$ and $v_1 = 1$ then the quantum social welfare of the pseudotelepathy strategy is $QSW = 0.83$ whereas the best classical social welfare is $CSW = 0.78$.

C. Comments on $NC_{00,0}(C_5)$

A modification of a different kind consists in adding more questions from the stabiliser. As the first example of this kind we define a game $NC_{00,0}(C_5)$, where the additional family of questions has four involved players with the non-involved player getting type 0, as specified by Table IV.

For $v_1 = 1$, $v_0 = \frac{2}{3}$, and the probability distribution $w(T_a) = \frac{3}{13}$, $w(T_{i_1}) = w(T_{i_2}) = \frac{1}{13}$ we get a CSW of 0.72 versus a QSW of 0.83.

Note that each player gets types 0 and 1 with different probabilities. In fact, it is simple to show that no choice of w_1, w_2 and w_3 can make these probabilities equal. However, it is possible to modify the set of types so that equality becomes possible, as shown in the following.

D. Comments on $NC_{00,01,0}(C_5)$

We increase the set of types using other questions from the stabiliser: We define a game $NC_{00,01,0}(C_5)$ for which with a suitable choice of probability distribution the players get 0 and 1 with the same probability, as specified by Table V.

TABLE II: Nash equilibria for three intervals of the value v_0/v_1. Note that the critical values 1/2 and 1/3 have union of both tables as equilibria.

Local functions					Players utility $[\times 6]$					$SW[\times 30]$
$v_0/v_1 \leq 1/3$										
Id	**1**	**1**	**1**	**1**	$2v_0 + v_1$	$3v_1$	$3v_1$	$3v_1$	$3v_1$	$2v_0 + 13v_1$
Not	**Not**	**1**	**1**	**1**	$2v_0 + v_1$	$2v_0 + v_1$	$3v_1$	$3v_1$	$3v_1$	$v_0 + 11v_1$
Not	**1**	**Not**	**1**	**1**	$2v_0 + v_1$	$3v_1$	$2v_0 + v_1$	$3v_1$	$3v_1$	$4v_0 + 11v_1$
Not	**Not**	**Not**	**Not**	**1**	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$5v_1$	$8v_0 + 17v_1$
$1/3 \leq v_0/v_1 \leq 1/2$										
1	**Not**	**1**	**1**	**0**	$5v_1$	$2v_0 + 3v_1$	$5v_1$	$5v_1$	$5v_0$	$7v_0 + 18v_1$
Id	**Id**	**1**	**1**	**1**	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$5v_1$	$5v_1$	$5v_1$	$6v_0 + 19v_1$
Not	**Not**	**1**	**1**	**1**	$2v_0 + v_1$	$2v_0 + v_1$	$3v_1$	$3v_1$	$3v_1$	$4v_0 + 11v_1$
Not	**Not**	**Not**	**Not**	**1**	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$5v_1$	$8v_0 + 17v_1$
$v_0/v_1 \geq 1/2$										
Not	**Id**	**1**	**1**	**0**	$2v_0 + 3v_1$	$4v_0 + v_1$	$5v_1$	$5v_1$	$5v_0$	$11v_0 + 14v_1$
1	**Not**	**1**	**1**	**0**	$5v_1$	$2v_0 + 3v_1$	$5v_1$	$5v_1$	$5v_0$	$7v_0 + 18v_1$
Id	**Id**	**1**	**1**	**1**	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$5v_1$	$5v_1$	$5v_1$	$6v_0 + 19v_1$
Not	**Not**	**1**	**Id**	**1**	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$5v_1$	$4v_0 + v_1$	$5v_1$	$8v_0 + 17v_1$
Not	**Not**	**Not**	**Not**	**1**	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$5v_1$	$8v_0 + 17v_1$
Not	**Id**	**Not**	**Id**	**Id**	$2v_0 + 3v_1$	$4v_0 + v_1$	$2v_0 + 3v_1$	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$14v_0 + 11v_1$

TABLE III: $NC_{01}(G)$ game

Type	Involved set	Binary answer
$T_a = 11111$	$I(T_a) = \{0,1,2,3,4\}$	$b(T_a) = 1$
$T_0 = 10100$	$I(T_0) = \{0,1,4\}$	$b(T_0) = 0$
$T_1 = 01010$	$I(T_1) = \{0,1,2\}$	$b(T_1) = 0$
$T_2 = 00101$	$I(T_2) = \{1,2,3\}$	$b(T_2) = 0$
$T_3 = 10010$	$I(T_3) = \{2,3,4\}$	$b(T_3) = 0$
$T_4 = 01001$	$I(T_4) = \{3,4,0\}$	$b(T_4) = 0$

We consider this game with type probability distributions given by $w(T_a) = 3/13$, $w(t_{i_1}) = 1/26$, $w(T_{i_2}) = 1/26$ and $w(T_{i_3}) = 1/13$.

The involvement probabilities satisfy $P_{inv}(1) > P_{inv}(0) = 8/13$ and the best classical Social Welfare with $v_0 = 2/3$, $v_1 = 1$ is $CSW = 0.72$ versus a QSW of 0.83.

Note that even though the types T_{i_2} and T_{i_3} are similar, the involved sets and thus the utilities are different. However, if one wants to restrict to scenarios in which the utility can be deterministically determined from the type, one can just add an extra player with a type allowing to distinguish the different cases and with utility being the average utility of the other players independently of his/her action.

V. QUANTUM VS CORRELATION SEPARATION

In [1] it is asked as an open question whether the separation between classical and quantum social welfare is bounded. We show in this section how two families of amplification techniques can increase the separation by adding a penalty for wrong answers and then by increasing the number of players.

A. *Wrong answer penalty*

A possible technique is to penalize bad answers more, using the fact that classical functions always produce a bad answer for some question. Instead of getting 0 when losing we generalize so that each player gets $-N_g v_1$ if they answer 1 and $-N_g v_0$ if they answer 0, where N_g can be seen as the penalty for giving a wrong answer. If $\delta_{(s,t),\mathcal{L}} = 1$ if $(s,t) \in \mathcal{L}$ and 0 otherwise, and N_g is a positive number, then

$$u_j(s|t) = (-N_g)^{\delta_{(s,t),\mathcal{L}}} v_{s_j} .$$

For $NC_{01}(C_5)$, as soon as $N_g > 3v_1$ there exists only two classical Nash equilibria:

- All 0 with a social welfare of $\frac{-N_g v_0 + 5v_0}{6}$ and
- All NOT with a social welfare of $\frac{-N_g v_0 + 2v_0 + 3v_1}{6}$.

Therefore the classical social welfare decreases linearly with the penalty while the quantum average social welfare remains $\frac{v_1+v_0}{2}$.

B. *Distributed parallel repetition*

The distributed parallel composition of nonlocal games appears in [19] for the study of non-signalling correlations and also in [20] where it is called k-fold repetition. k groups

TABLE IV: $NC_{00,0}(G)$ game.

Type	Involved set	Binary answer
$T_a = 11111$	$I(T_a) = \{0,1,2,3,4\}$	$b(T_0) = 1$
$T_{i_1} = 0_{i_1-2}0_{i_1-1}1_{i_1}0_{i_1+1}0_{i_1+2}$ $i_1 \in \{0,\ldots,4\}$	$I(T_{i_1}) = \{i_1-1, i_1, i_1+1\}$	$b(T_{i_1}) = 0$
$T_{i_2} = 0_{i_2-1}1_{i_2}0_{i_2+1}1_{i_2+2}0_{i_2+3}$ $i_2 \in \{0,\ldots,4\}$	$I(T_{i_2}) = \{i_2-1, i_2, i_2+2, i_2+3\}$	$b(T_{i_2}) = 0$

TABLE V: $NC_{00,01,0}(G)$ game.

Type	Involved set	Binary answer
$T_a = 11111$	$I(T_a) = \{0,1,2,3,4\}$	$b(T_0) = 1$
$T_{i_1} = 0_{i_1-2}0_{i_1-1}1_{i_1}0_{i_1+1}0_{i_1+2}$ $i_1 \in \{0,\ldots,4\}$	$I(T_{i_1}) = \{i_1-1, i_1, i_1+1\}$	$b(T_{i_1}) = 0$
$T_{i_2} = 0_{i_2-1}1_{i_2}0_{i_2+1}1_{i_2+2}0_{i_2+3}$ $i_2 \in \{0,\ldots,4\}$	$I(T_{i_2}) = \{i_2-1, i_2, i_2+1\}$	$b(T_{i_2}) = 0$
$T_{i_3} = 0_{i_3-1}1_{i_3}0_{i_3+1}1_{i_3+2}0_{i_3+3}$ $i_3 \in \{0,\ldots,4\}$	$I(T_{i_3}) = \{i_2-1, i_2, i_2+2, i_2+3\}$	$b(T_{i_3}) = 0$

of players play at the same time and they win collectively if all the groups win their game.

More formally, given a non-collaborative game NC(G) on n players with set of types T, involvement function I and expected binary answer b, where types $t_i \in T$ are picked with probability distribution $w(t_i)$, the non-collaborative game k-fold $NC(G)$ is the game on nk players with types $T' = \{t_1 \times \ldots \times t_k,\, t_1, \ldots, t_k \in T\}$. It has a losing set $\mathcal{L}' = \{(s,t), \exists j, \sum_{i \in I(t_j)} s_{i,j} \neq b(t_j) \bmod 2\}$. The utility of the player i in the group j is $v_{s_{(i,j)}}$ if $(s,t) \notin \mathcal{L}'$ and 0 otherwise. The types for each group of players are picked independently: $w'(t_1, \ldots t_k) = \pi_{1 \leq j \leq k} w(t_j)$.

Theorem 3. *There exist games with bounded personal utilities v_0, v_1 on $O(log(\frac{1}{\epsilon}))$ players ensuring $\frac{CSW(G)}{QSW(G)} \leq \epsilon$ for the ratio best classical social welfare over quantum social welfare with guaranteed value.*

Proof. It is easy to bound the utility in these settings as for any strategy in a repeated game. If a player p is involved in the strategy S_j but is not involved in the strategy S_i of another group then his utility is conditioned by the fact that the S_i strategy wins to receive a positive utility and

$$u^p(S_i \times S_j) \leq p_{win}(S_i) u^p(S_j)$$

As the quantum strategy obtained from following the nonlocal advice always wins, the QSW remains unchanged whereas the CSW decreases. For instance $CSW(k\text{−fold } NC_{00}(C_5)) = \frac{5}{6}^k CSW(NC_{00}(C_5))$.

Therefore using these games one can build games with bounded personal utilities v_0, v_1 on $O(log(\frac{1}{\epsilon}))$ players ensuring $\frac{CSW(G)}{QSW(G)} \leq \epsilon$. □

Note that the separation obtained here uses binary types and actions and implies that $O(\log n)$ players are enough to achieve a ratio of improvement of social welfare $QSW/CSW \geq n$.

VI. Conclusion

We have used properties of multipartite graph games to define conflict of interest games, and shown that by combining such games the ratio classical social welfare / quantum social welfare can go to zero.

One can easily extend to stabilizer games [20] to have any number of types and possible strategies.

As pointed out by [1], quantum advice equilibria can be reached without needing a trusted mediator, furthermore they ensure privacy as they are belief invariant. Some other features may be emphasized if we define Nash equilibria using pseudotelepathy games: such situations ensure a guaranteed utility and they are also better when analysing the maximal minimal utility. It may be interesting to investigate further how this guaranteed value property for some quantum equilibria can be used. On the other hand, it would also be interesting to investigate how relaxing the guaranteed win requirement might allow to increase the QSW even further.

The possibility of potentially unlimited improvement of social welfare while preserving belief invariance is therefore a strong motivation to consider classical payoff tables that arise for usual situations in which Nash equilibria occur and play an important role. For example, in routing problems an advice provider could use a quantum advice system as follows. To calculate the advice to send to each player, the advice provider should either (a) send a rotated qubit to each player (who will then measure their qubit to get the answer), or, in a trusted setting, (b) perform a quantum measurement and send a classical message.

APPENDIX

Pareto equilibria for NC_{00} when $v_0/v_1 \leq 1/3$, No. solutions: 121, No. distinct equilibria: 18

Local functions					Players utility [×6]					$SW[\times 30]$
Id	**1**	**1**	**0**	**0**	$3v_0+2v_1$	$5v_1$	$5v_1$	$5v_0$	$5v_0$	$13v_0+12v_1$
Not	**Not**	**Id**	**0**	**0**	v_0+2v_1	v_0+2v_1	v_0+2v_1	$3v_0$	$3v_0$	$9v_0+6v_1$
Not	**Id**	**1**	**1**	**0**	$2v_0+3v_1$	$4v_0+v_1$	$5v_1$	$5v_1$	$5v_0$	$11v_0+14v_1$
1	**Not**	**1**	**1**	**0**	$5v_1$	$2v_0+3v_1$	$5v_1$	$5v_1$	$5v_0$	$7v_0+18v_1$
1	**Not**	**Not**	**1**	**0**	$5v_1$	v_0+4v_1	v_0+4v_1	$5v_1$	$5v_0$	$7v_0+18v_1$
Id	**Not**	**1**	**Id**	**0**	$3v_0+2v_1$	v_0+4v_1	$5v_1$	$3v_0+2v_1$	$5v_0$	$12v_0+13v_1$
Not	**Id**	**Id**	**Not**	**0**	v_0+4v_1	$4v_0+v_1$	$4v_0+v_1$	v_0+4v_1	$5v_0$	$15v_0+10v_1$
Id	**1**	**1**	**1**	**1**	$2v_0+v_1$	$3v_1$	$3v_1$	$3v_1$	$3v_1$	$2v_0+13v_1$
Id	**Id**	**1**	**1**	**1**	$3v_0+2v_1$	$3v_0+2v_1$	$5v_1$	$5v_1$	$5v_1$	$6v_0+19v_1$
Not	**Not**	**1**	**1**	**1**	$2v_0+v_1$	$2v_0+v_1$	$3v_1$	$3v_1$	$3v_1$	$4v_0+11v_1$
Not	**1**	**Not**	**1**	**1**	$2v_0+v_1$	$3v_1$	$2v_0+v_1$	$3v_1$	$3v_1$	$4v_0+11v_1$
Not	**Not**	**1**	**Id**	**1**	$2v_0+3v_1$	$2v_0+3v_1$	$5v_1$	$4v_0+v_1$	$5v_1$	$8v_0+17v_1$
Not	**Not**	**Id**	**Id**	**1**	$2v_0+3v_1$	v_0+4v_1	$3v_0+2v_1$	$3v_0+2v_1$	$5v_1$	$9v_0+16v_1$
Not	**Id**	**Not**	**Id**	**1**	v_0+2v_1	$2v_0+v_1$	$2v_0+v_1$	$2v_0+v_1$	$3v_1$	$7v_0+8v_1$
Not	**Id**	**Id**	**Not**	**1**	v_0+2v_1	v_0+2v_1	v_0+2v_1	v_0+2v_1	$3v_1$	$4v_0+11v_1$
Not	**Not**	**Not**	**Not**	**1**	$2v_0+3v_1$	$2v_0+3v_1$	$2v_0+3v_1$	$2v_0+3v_1$	$5v_1$	$8v_0+17v_1$
Not	**Id**	**Not**	**Id**	**Id**	$2v_0+3v_1$	$4v_0+v_1$	$2v_0+3v_1$	$3v_0+2v_1$	$3v_0+2v_1$	$14v_0+11v_1$
Not	**Not**	**Not**	**Not**	**Not**	v_0+4v_1	v_0+4v_1	v_0+4v_1	v_0+4v_1	v_0+4v_1	$5v_0+20v_1$

Pareto equilibria for NC_{00} when $1/3 \leq v_0/v_1 \leq 1/2$, No. solutions: 91, No. distinct equilibria: 14

Local functions					Players utility [×6]					$SW[\times 30]$
Id	**1**	**1**	**0**	**0**	$3v_0+2v_1$	$5v_1$	$5v_1$	$5v_0$	$5v_0$	$13v_0+12v_1$
Not	**Id**	**1**	**1**	**0**	$2v_0+3v_1$	$4v_0+v_1$	$5v_1$	$5v_1$	$5v_0$	$11v_0+14v_1$
1	**Not**	**1**	**1**	**0**	$5v_1$	$2v_0+3v_1$	$5v_1$	$5v_1$	$5v_0$	$7v_0+18v_1$
1	**Not**	**Not**	**1**	**0**	$5v_1$	v_0+4v_1	v_0+4v_1	$5v_1$	$5v_0$	$7v_0+18v_1$
Id	**Not**	**1**	**Id**	**0**	$3v_0+2v_1$	v_0+4v_1	$5v_1$	$3v_0+2v_1$	$5v_0$	$12v_0+13v_1$
Not	**Id**	**Id**	**Not**	**0**	v_0+4v_1	$4v_0+v_1$	$4v_0+v_1$	v_0+4v_1	$5v_0$	$15v_0+10v_1$
Id	**Id**	**1**	**1**	**1**	$3v_0+2v_1$	$3v_0+2v_1$	$5v_1$	$5v_1$	$5v_1$	$6v_0+19v_1$
Not	**Not**	**1**	**1**	**1**	$2v_0+v_1$	$2v_0+v_1$	$3v_1$	$3v_1$	$3v_1$	$4v_0+11v_1$
Not	**Not**	**1**	**Id**	**1**	$2v_0+3v_1$	$2v_0+3v_1$	$5v_1$	$4v_0+v_1$	$5v_1$	$8v_0+17v_1$
Not	**Not**	**Id**	**Id**	**1**	$2v_0+3v_1$	v_0+4v_1	$3v_0+2v_1$	$3v_0+2v_1$	$5v_1$	$9v_0+16v_1$
Not	**Id**	**Id**	**Not**	**1**	v_0+2v_1	v_0+2v_1	v_0+2v_1	v_0+2v_1	$3v_1$	$4v_0+11v_1$
Not	**Not**	**Not**	**Not**	**1**	$2v_0+3v_1$	$2v_0+3v_1$	$2v_0+3v_1$	$2v_0+3v_1$	$5v_1$	$8v_0+17v_1$
Not	**Id**	**Not**	**Id**	**Id**	$2v_0+3v_1$	$4v_0+v_1$	$2v_0+3v_1$	$3v_0+2v_1$	$3v_0+2v_1$	$14v_0+11v_1$
Not	**Not**	**Not**	**Not**	**Not**	v_0+4v_1	v_0+4v_1	v_0+4v_1	v_0+4v_1	v_0+4v_1	$5v_0+20v_1$

Pareto equilibria when $\geq 1/2$, No. solutions: 81, No. distinct equilibria: 12

Local functions					Players utility [×6]					$SW[\times 30]$
Id	**1**	**1**	**0**	**0**	$3v_0+2v_1$	$5v_1$	$5v_1$	$5v_0$	$5v_0$	$13v_0+12v_1$
Not	**Id**	**1**	**1**	**0**	$2v_0+3v_1$	$4v_0+v_1$	$5v_1$	$5v_1$	$5v_0$	$11v_0+14v_1$
1	**Not**	**1**	**1**	**0**	$5v_1$	$2v_0+3v_1$	$5v_1$	$5v_1$	$5v_0$	$7v_0+18v_1$
1	**Not**	**Not**	**1**	**0**	$5v_1$	v_0+4v_1	v_0+4v_1	$5v_1$	$5v_0$	$7v_0+18v_1$
Id	**Not**	**1**	**Id**	**0**	$3v_0+2v_1$	v_0+4v_1	$5v_1$	$3v_0+2v_1$	$5v_0$	$12v_0+13v_1$
Not	**Id**	**Id**	**Not**	**0**	v_0+4v_1	$4v_0+v_1$	$4v_0+v_1$	v_0+4v_1	$5v_0$	$15v_0+10v_1$
Id	**Id**	**1**	**1**	**1**	$3v_0+2v_1$	$3v_0+2v_1$	$5v_1$	$5v_1$	$5v_1$	$6v_0+19v_1$
Not	**Not**	**1**	**Id**	**1**	$2v_0+3v_1$	$2v_0+3v_1$	$5v_1$	$4v_0+v_1$	$5v_1$	$8v_0+17v_1$
Not	**Not**	**Id**	**Id**	**1**	$2v_0+3v_1$	v_0+4v_1	$3v_0+2v_1$	$3v_0+2v_1$	$5v_1$	$9v_0+16v_1$
Not	**Not**	**Not**	**Not**	**1**	$2v_0+3v_1$	$2v_0+3v_1$	$2v_0+3v_1$	$2v_0+3v_1$	$5v_1$	$8v_0+17v_1$
Not	**Id**	**Not**	**Id**	**Id**	$2v_0+3v_1$	$4v_0+v_1$	$2v_0+3v_1$	$3v_0+2v_1$	$3v_0+2v_1$	$14v_0+11v_1$
Not	**Not**	**Not**	**Not**	**Not**	v_0+4v_1	v_0+4v_1	v_0+4v_1	v_0+4v_1	v_0+4v_1	$5v_0+20v_1$

Nash equilibria for NC_{01} when $1/3 \leq v_0/v_1 \leq 1/2$, No. solutions: 76, No. distinct equilibria: 13

Local functions					Players utility [×6]					$SW[\times 30]$
1	**1**	**Id**	**0**	**0**	$5v_1$	$5v_1$	$2v_0 + 3v_1$	$5v_0$	$5v_0$	$12v_0 + 13v_1$
Not	**Id**	**1**	**1**	**0**	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$5v_1$	$5v_1$	$5v_0$	$11v_0 + 14v_1$
1	**Not**	**1**	**1**	**0**	$5v_1$	$3v_0 + 2v_1$	$5v_1$	$5v_1$	$5v_0$	$8v_0 + 17v_1$
Not	**Id**	**Id**	**1**	**0**	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$5v_1$	$5v_0$	$11v_0 + 14v_1$
1	**Not**	**Not**	**1**	**0**	$5v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$5v_1$	$5v_0$	$9v_0 + 16v_1$
1	**Not**	**1**	**Id**	**0**	$5v_1$	$2v_0 + 3v_1$	$5v_1$	$2v_0 + 3v_1$	$5v_0$	$9v_0 + 16v_1$
Id	**1**	**Not**	**Id**	**0**	$2v_0 + 3v_1$	$5v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$5v_0$	$11v_0 + 14v_1$
Id	**Id**	**1**	**1**	**1**	$3v_0 + 2v_1$	$2v_0 + 3v_1$	$5v_1$	$5v_1$	$5v_1$	$5v_0 + 20v_1$
Not	**Not**	**1**	**Id**	**1**	$2v_0 + 3v_1$	$3v_0 + 2v_1$	$5v_1$	$3v_0 + 2v_1$	$5v_1$	$8v_0 + 17v_1$
Not	**Not**	**Id**	**Id**	**1**	$3v_0 + 2v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$3v_0 + 2v_1$	$5v_1$	$10v_0 + 15v_1$
Not	**Not**	**Not**	**Not**	**1**	$3v_0 + 2v_1$	$2v_0 + 3v_1$	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$5v_1$	$11v_0 + 14v_1$
Not	**Id**	**Not**	**Id**	**Id**	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$2v_0 + 3v_1$	$14v_0 + 11v_1$
Not	**Not**	**Not**	**Not**	**Not**	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$2v_0 + 3v_1$	$10v_0 + 15v_1$

Nash equilibria for NC_{01} when $v_0/v_1 \geq 1/2$, No. solutions: 40, No. distinct equilibria: 6

Local functions					Players utility [×6]					$SW[\times 30]$
Not	**Id**	**1**	**1**	**0**	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$5v_1$	$5v_1$	$5v_0$	$11v_0 + 14v_1$
1	**Not**	**1**	**1**	**0**	$5v_1$	$3v_0 + 2v_1$	$5v_1$	$5v_1$	$5v_0$	$8v_0 + 17v_1$
Id	**Id**	**1**	**1**	**1**	$3v_0 + 2v_1$	$2v_0 + 3v_1$	$5v_1$	$5v_1$	$5v_1$	$5v_0 + 20v_1$
Not	**Not**	**1**	**Id**	**1**	$2v_0 + 3v_1$	$3v_0 + 2v_1$	$5v_1$	$3v_0 + 2v_1$	$5v_1$	$8v_0 + 17v_1$
Not	**Not**	**Not**	**Not**	**1**	$3v_0 + 2v_1$	$2v_0 + 3v_1$	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$5v_1$	$11v_0 + 14v_1$
Not	**Id**	**Not**	**Id**	**Id**	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$3v_0 + 2v_1$	$2v_0 + 3v_1$	$14v_0 + 11v_1$

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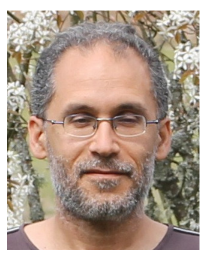

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