Title: Structure and Logic of Conceptual Mind

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Article Category: Original contribution

Running Title: Conceptual Mind

Keywords: cohesion; contradiction; de Morgan's law; negation; proposition; reasoning; reflexive graph; syllogism; truth value object.

Word Count: 7496; Figure Count: 9

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Acknowledgements:

I dedicate the paper, with gratitude and admiration, to late Professor Baldev Raj. I'd like to thank Dr. Ruadhan O'Flanagan for suggesting the application of category theory to the study of concepts and reasoning, Professor Andrée C. Ehresmann for kindly correcting mistakes in an earlier version, and Professor F. William Lawvere for invaluable help in learning category theory. I am grateful for the NIAS-Mani Bhaumik and NIAS-Consciousness Studies Programme Research Fellowships.

Title: Structure and Logic of Conceptual Mind

Abstract

Mind, according to cognitive neuroscience, is a set of brain functions. But, unlike sets, our minds are cohesive. Moreover, unlike the structureless elements of sets, the contents of our minds are structured. Mutual relations between the mental contents endow the mind its structure. Here we characterize the structural essence and the logical form of the mind by focusing on thinking. Examination of the relations between concepts, propositions, and syllogisms involved in thinking revealed the reflexive graph structure of the conceptual mind. Objective logic of the conceptual mind is calculated from its structure. Noteworthy features of the logic of conceptual mind are: degrees of truth, varieties of negation, admission of contradiction, and the failure of a de Morgan's law. Furthermore, cohesion of the conceptual mind follows from its reflexive graph structure. Our characterization of the structure and logic of mind constitutes a substantial refinement of the contemporary cognitive neuroscientific conceptualization of the mind as a set.

Introduction

Mind is useful in making sense of and maneuvering through reality. As such, mind has been an object of serious study since antiquity. Carefully thinking about thinking, which takes place within our minds, led to logic (Lawvere and Rosebrugh, 2003, pp. 193-195, 239-240). Recently, cognitive neuroscience has highlighted the differences between unconscious and conscious thought (Kahneman, 2013; Kandel, 2013). Fascinating as these may be, we still do not have a clear understanding of the nature and workings of the mind (Fodor, 2006). In the present note, as part of scientifically accounting for the effectiveness of mind in the material world (Lawvere, 1980, 1994; Lawvere and Schanuel, 2009, pp. 84-85; Picado, 2007, p. 25), we address two foundational questions of the science of mind:

1. What is the structural essence of mind?
2. What is the objective logic of mind?

Our mathematical approach to the mind can be understood in terms of a simple example: WEEK. As a first approximation, a week can be mathematically described using a number: 7 (as in 7 days). A more refined description of the week is in terms of sets i.e., as a set of seven days: W = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}. Looking at this description, we notice that the days of a week, unlike the discrete elements of a set, are related to one another. For instance, Tuesday follows Monday, Wednesday follows Tuesday… We can capture this additional structure by way of describing the week as a set with added structure i.e., the set W of days equipped with an endomap w: W → W, which specifies for each day of the week the following day. Upon further examination, we realize that, unlike days in the above cyclic description, the Sunday that comes after Saturday is not the same Sunday that came before Monday. This added realization leads us to further refine our model as a product of structured sets: $w \times n$ (n: N → N, where N = {0, 1, 2…} and n(n) = n+1) to capture the spiral structure of days of the week (Lawvere and Schanuel, 2009, pp. 135-136, 239-240).

We begin with the mainstream cognitive neuroscientific conceptualization of mind: "mind is a set of processes carried out by the brain" (Kandel, 2013, p. 546; see also Bunge, 1981, p. 68; Kandel, Schwartz, Jessell, Siegelbaum, and Hudspeth, 2013, p. 5, 334, 384). In contrast to the structureless elements of a set, the contents of our minds [even when identified with neural processes] are structured. More importantly, since sets have no other property besides the number of elements that they contain, i.e. size (Lawvere and Rosebrugh, 2003, p. 1), if minds are sets, then all that we can say about minds is: mind A is bigger than mind B, mind X is smaller than mind Y, etc. However, we have many more things to say about minds (e.g., brilliant mind, clear mind), in addition to speaking about their size. Thus, the idea of mind as a set is, at best, a first approximation. Simply put, mind is much more structured than a set.

With the objective of refining the mainstream conceptualization of mind as a set, we examine the relations between mental contents, which endow mind its structure. We treat mind as a space where thinking takes place. More explicitly, we limit our consideration to the thinking part of the mind i.e., conceptual mind. Thinking involves concepts, propositions, and combinations of propositions as part of reasoning, i.e. syllogisms. Examination of the relations between concepts and propositions led us to put forth the structure of graph (Lawvere and Schanuel, 2009, pp. 141-142) as an abstract essence of the conceptual mind. In characterizing the essence (or theory) of mind, we are using the mathematical method of theorizing about objects. The mathematical method, according to F. William Lawvere, "consists of taking the main structure [of an object], in the sense that it is mainly responsible for the workings of the object, by itself as a first approximation to a theory of the object, i.e. mentally operating as though all further structure of the object simply did not exist" (Lawvere, 1972, pp. 9-10). Our mathematical characterization of conceptual mind is along the lines of Lawvere's category theoretic characterization of kinship (Lawvere, 1999).

Objective logic of a universe of discourse (e.g., sets, graphs) follows from the structural essence(s) of the universe (Lawvere and Schanuel, 2009, pp. 149-151, 339-347). Using this general method, we calculated the logic of conceptual mind from its structural essence of graph. The logic of conceptual mind, with its degrees of truth and varieties of negation, differs markedly from the Boolean logic of sets. In this context, failure of the de Morgan's law:

not (X *and* Y) = *not* (X) *or not* (Y)

is particularly noteworthy (see Lawvere and Rosebrugh, 2003, p. 200). Upon further examination, we find that conceptual mind has the added structure of reflexive graph (Lawvere and Schanuel, 2009, p. 145). We show that the conceptual mind, in light of its reflexive graph structure, is cohesive (Lawvere, 2005, 2007). In accounting for the combination of propositions as part of reasoning (syllogisms), we further refine our model of conceptual mind as an object consisting of three component sets:

(set of concepts, set of propositions, set of syllogisms) equipped with eight structural functions specifying the relations between concepts, propositions, and syllogisms. In the following, we provide an intuitively-accessible description of structural essences and the subsequent calculation of objective logic from structural essences. In our subsequent work, we plan to provide a category theoretic account of abstracting the theoretical essence(s) of minds, and of interpreting the thus abstracted essences to obtain concrete models of the mind in terms of functorial semantics (Lawvere, 2004; Posina, Ghista, and Roy, 2017).

Structural Essence of Conceptual Mind

How are we going to find the structural essence of mind? The structure of an object is determined by its contents and their mutual relations. So, a first step in characterizing the structure of a given object is to find its contents and their interrelationships. If we imagine looking into minds, we might find, for example, some concepts such as DOG, GOOD, SKY, etc. in one mind X, and another such lot of concepts LINE, RED, WALK, etc. in another mind Y. If concepts in a mind are all that there are in the mind, then, with concepts as structureless elements, mind can be modeled as a set (Fig. 1a). With minds as sets, the structural essence of minds is a single-element set **1** = {•} (Fig. 2a; Lawvere, 1972, p. 135; Lawvere and Schanuel, 2009, p. 245; Reyes, Reyes, and Zolfaghari, 2004, p. 30). Simply put, having a concept is the essence in which all minds partake, and with which every mind can be constructed. There is, of course, more to a mind than the concepts that it contains. Upon looking further into our minds, we might find, in addition to concepts, a set of propositions {SKY is CLEAR, WATER is CLEAN…} in one mind X, and another set of propositions {BIRD is FLYING, BUS is RED…} in another mind Y. With both concepts and propositions represented as structureless elements, albeit of two different sets, mind can be modeled as a pair of sets: a set of concepts and a set of propositions (Fig. 1b; Reyes, Reyes, and Zolfaghari, 2004, p. 17).

Concepts and propositions are, however, not mere [unconnected] sets within our minds. Concepts and propositions in our minds are related to one another in systematic ways. The subject of a proposition is a concept (e.g., *subject* (SKY is CLEAR) = SKY); so is its predicate (*predicate* (SKY is CLEAR) = CLEAR). Thus, mind can be modeled as a pair of sets:

(set C of concepts, set P of propositions)

equipped with a parallel pair of functions:

subject, *predicate*: P → C

assigning to each proposition in the set P of propositions its subject, predicate concept in the set C of concepts. These relations between concepts and propositions endow mind the structure of irreflexive graph (Lawvere and Schanuel, 2009, pp. 141-142). In modeling minds as irreflexive graphs, concepts and propositions within a mind are displayed as dots and arrows, respectively. To each arrow representing a proposition, there is a source and a target dot representing the subject and the predicate concept, respectively, of the proposition (Fig. 1c). With minds as irreflexive graphs, the structural essence of minds is a pair of graph morphisms specifying the inclusion of concept into proposition as its subject, predicate concept (Fig. 2b; Lawvere and Schanuel, 2009, p. 150). Next, we characterize the objective logic of conceptual mind that follows from these structural essences.

Logical Form of Conceptual Mind

Objective logic of a universe of discourse is the logic intrinsic to the universe. The totality of parts of the essence of a given universe of discourse constitutes the truth value object of the universe (Reyes, Reyes, and Zolfaghari, 2004, pp. 93-101; see Appendix for the calculation of truth value object). Logical operations (*and*, *or*, *not*) can be characterized in terms of the truth value object of the universe (Lawvere and Rosebrugh, 2003, pp. 193-201; Lawvere and Schanuel, 2009, pp. 335-357).

If minds are sets (of concepts, with concepts as structureless elements; Fig. 1a), then the logic of minds is the logic of sets. The truth value object of the category of sets is a two-element set

$$\Omega = \{\text{false}, \text{true}\}$$

(Fig. 3a). The two-element truth value set can be calculated from the essence of sets, which is a single-element set $\mathbf{1} = \{\bullet\}$ (Fig. 2a). The single element set $\mathbf{1}$ has two parts ($\mathbf{0} = \{\}$, $\mathbf{1} = \{\bullet\}$), which correspond to the two elements (false, true, respectively) of the truth value set Ω (Lawvere and Schanuel, 2009, p. 343, 353; Reyes, Reyes, and Zolfaghari, 2004, pp. 95-96). These two elements are the two possible truth values (false, true) a statement (to give an illustration): 'FUNCTOR *is in* C', asserting that a concept FUNCTOR is in a part C (say, conscious part of a mind M), can take. Once we have the truth value object Ω, we can characterize logical operations (*and*, *or*, *not*) as maps to and from the truth value object (Lawvere and Schanuel, 2009, pp. 353-355). For example, the negation operation

$$not: \Omega \to \Omega$$

is an endomap on the truth value object Ω with *not* (false) = true, *not* (true) = false. In the case of sets, double negation applied to any part A (of a given object) results in the same part, i.e.

not (*not* (A)) = A

Also, note that logical contradiction, by the definition of *not* operation, equals false, i.e.

A *and not* (A) = false

(Lawvere and Schanuel, 2009, p. 355). Furthermore, the two de Morgan's laws:

not (A *and* B) = *not* (A) *or not* (B)

not (A *or* B) = *not* (A) *and not* (B)

which relate the three logical operations (*and*, *or*, *not*), are satisfied in the case of sets. These features of the logic of sets are not shared by the logic of conceptual mind.

With minds as irreflexive graphs (Fig. 1c), the first thing we notice is the degrees of truth (Fig. 3b). Consider a mind M consisting of a proposition P, say, 'SKY is BLUE'. Given a part C (say, conscious part of M), a statement—P *is in* C—can take the truth value: *true*, if P is in C. The statement is *false*, if P is not in C. In addition to these two truth values, there are three more truth values: (i) *tt* if the proposition P is not in C, but its subject and predicate concepts (SKY, BLUE) are in C, (ii) *tf* if the proposition P is not in C, but its subject (SKY) is in C, and (iii) *ft* if the proposition P is not in C, but its predicate (BLUE) is in C. The totality of these five truth values is the truth value object of conceptual minds (see Appendix). Note that these five degrees of truth correspond to the five parts of the generic proposition (e.g., SKY is BLUE). The five parts are: 1. entire proposition (SKY is BLUE), 2. subject and predicate concepts (SKY, BLUE), 3. subject (SKY), 4. predicate (BLUE), and 5. empty (Lawvere and Schanuel, 2009, pp. 344-346).

In addition to these degrees of truth, which distinguish the logic of conceptual minds from that of sets, conceptual minds (modeled as irreflexive graphs) admit varieties of negation. A familiar negation is the logical operation *not*, which is defined as: for any part X of an object, *not* (X) is the part of the object that is largest among all parts whose intersection with X is empty (Lawvere and Schanuel, 2009, p. 355). A different negation operation *non* can be defined dually: for any part X of an object, *non* (X) is the part of the given object that is smallest among all parts whose union with X is the entire object (Lawvere, 1986, 1991). Unlike the case of sets, where *non* and *not* are identical operations, in the case of conceptual minds (construed as irreflexive graphs), these two operations give different results (Fig. 4a). In this context, it is fascinating to note that the negation operation *non*, unlike *not*, permits logical contradiction (Fig. 4b; Lawvere, 1991, 1994; Lawvere and Rosebrugh, 2003, p. 201). Also note that, depending on the exact form of negation, double negation can result in a larger:

not (*not* (A)) > A

or a smaller:

non (*non* (A)) < A

part than the part A to which double negation is applied (Fig. 4c, d; Lawvere and Schanuel, 2009, p. 355). More importantly, one of the de Morgan's laws:

not (X *and* Y) = *not* (X) *or not* (Y)

can fail in the case of conceptual minds (Fig. 5). The other de Morgan's law:

not (X *or* Y) = *not* (X) *and not* (Y)

is valid in the case of *not*; while both laws are valid in the case of *non*. All of this logic, which distinguishes conceptual minds (irreflexive graphs) from sets, follows from merely recognizing that there are concepts and propositions within our minds, and that to each proposition there is a concept which is its subject, predicate. It is interesting to note that the category of all mathematical theories (abstract essences) of all mathematical categories also happens to be the category of graphs (Lawvere and Schanuel, 2009, p. 149).

Reasoning

In addition to the static aspects of thought (concepts, propositions), there are dynamical aspects of thinking. An elementary dynamic of the motion of thought involves combining given propositions to arrive at novel propositions as conclusions. As part of syllogistic reasoning, we compose propositions (such as):

APPLE is FRUIT ∘ FRUIT is EDIBLE = APPLE is EDIBLE

We can represent these syllogisms as commutative triangles (satisfying $f \circ g = h$, where '∘' denotes composition of propositions, which are represented by arrows f: A → B, g: B → C, and h: A → C, while A, B, and C denote concepts; Fig. 7a; Lawvere and Schanuel, 2009, pp. 16-21, 201). This composition of propositions satisfies two identity laws (exemplified by):

FRUIT is FRUIT ∘ FRUIT is EDIBLE = FRUIT is EDIBLE

APPLE is FRUIT ∘ FRUIT is FRUIT = APPLE is FRUIT

(Fig. 7b, c). Based on these observations, we can further refine our model of the conceptual mind as an object consisting of three component sets:

(set C of concepts, set P of propositions, set S of syllogisms)

which are structured by eight functions (Fig. 8).

With syllogism as the essence of conceptual mind, we can calculate the truth value object in terms of its parts. The generic syllogism (commutative triangle $f \circ g = h$) has nineteen parts: 1. $f \circ g = h$ (entire syllogism); 2. f, g, h (no syllogism, but all three propositions); 3. f, g (two propositions); 4. g, h; 5. h, f; 6. f, C (one proposition and all three concepts); 7. g, A; 8. h, B; 9. f (one proposition); 10. g; 11. h; 12. A, B, C (no syllogism, no proposition, but all three concepts); 13. A, B (two concepts); 14. B, C; 15. C, A; 16. A (one concept); 17. B; 18. C; 19. empty (no syllogism, no proposition, no concept). These nineteen parts correspond to nineteen degrees of truth ranging from FALSE to TRUE in the truth value triangle (Fig. 9; Lawvere, 1989, pp. 282-283). The truth value triangle is constructed from the incidence relations of triangles, edges, and dots using the same procedure used to calculate the truth value graph (Fig. 3b; Appendix; see also Reyes, Reyes, and Zolfaghari, 2004, pp. 93-101). The part $f \circ g = h$ (triangular surface) corresponds to TRUE, which is the truth value of, say, the statement (that a syllogism):

'APPLE is FRUIT $\circ$ FRUIT is EDIBLE = APPLE is EDIBLE' *is in* C

(where C is a given part, say, conscious part of a mind) when the syllogism is in C. The part 'empty' corresponds to FALSE, which is the truth value of the statement when the syllogism is not in C. In between, there are seventeen truth values corresponding to various scenarios such as: the syllogism is not in C, but the three propositions APPLE is FRUIT, FRUIT is EDIBLE, APPLE is EDIBLE are in C, or just one of three concepts FRUIT is in C. Thus, we find that a mere recognition of the all too clearly visible mental contents (concepts, propositions, and syllogisms) and their mutual relations reveals the rich structure and logic of the conceptual mind. The structural essences of a universe of discourse (such as graphs or minds), their extraction and subsequent interpretation to obtain models can all be given a comprehensive mathematical account in terms of functorial semantics (Lawvere, 2004; Posina, Ghista, and Roy, 2017), which we plan to present in a subsequent paper.

Cohesive Mind

We have been considering the consequences of recognizing the irreflexive graph structure of conceptual mind. Let us now refine this irreflexive graph model of the conceptual mind. If we imagine, again, looking into our minds, then we notice that, for each concept (e.g., ROSE) in a mind, there is a proposition, more specifically, an identity proposition (*identity* (ROSE) = ROSE is ROSE) in the mind. This observation suggests modeling conceptual mind as a pair of sets:

(set C of concepts, set P of propositions)

equipped with three functions:

subject: P → C

predicate: P → C

identity: C → P

with the added third function *identity* assigning to each concept in the set C of concepts its identity proposition in the set P of propositions. These three functions together constitute a reflexive graph (Fig. 1d; Lawvere and Schanuel, 2009, p. 145).

What, if any, are the implications of modeling conceptual mind as reflexive graph? An immediate consequence of refining the model of conceptual mind from irreflexive graph to reflexive graph is that it accounts for the unity or cohesiveness of mind. The cohesiveness of a universe of discourse can be assessed using the axioms of cohesion (Lawvere, 2005, 2007). One of the necessary conditions for a universe of discourse to be cohesive is that its truth value object is connected, i.e. one piece (Axiom 2 in Lawvere, 2005; Lawvere and Schanuel, 2009, pp. 358-359). Another condition of cohesion is: number of pieces of a product equals the product of pieces of the factors (Axiom 1 in Lawvere, 2005; Lawvere and Schanuel, 2009, pp. 260, 372-373). Let us now examine our models of mind in the light of these axioms. Consider our initial model of mind, wherein minds consist of concepts only. With concepts as structureless elements, minds are sets (of concepts). The truth value set {false, true}, consistent with the zero cohesion of discrete sets, is not connected (Fig. 3a). Next, consider minds consisting of propositions and concepts, along with the specification that every proposition has a subject and a predicate concept. With propositions and concepts as arrows and dots, respectively, conceptual minds are irreflexive graphs. The truth value object of irreflexive graphs is connected (Fig. 3b). However, the second condition for cohesion involving products is not satisfied in the case of irreflexive graphs (Fig. 6a). This additional condition is satisfied in case of conceptual minds, wherein for every concept (e.g., SKY) in a mind, there is an identity proposition (SKY is SKY) in the mind (Fig. 6b). Moreover, since reflexive graphs satisfy additional axioms of cohesion (Lawvere, 2005, 2007), conceptual mind, with its reflexive graph structure, is cohesive.

Conclusions

Conceptualizing mind as a set as in “mind is a set of brain functions” (Bunge, 1981, p. 68; see also Kandel, 2013, p. 546; Kandel, Schwartz, Jessell, Siegelbaum, and Hudspeth, 2013, p. 5, 334, 384) is a first approximation. A little more realistic conception of mind would consider the distinctions between mental contents, say, by way of modeling mind as a pair of sets: (set of concepts, set of propositions). A further refinement would consider the relations between these different sets. This is exactly what we did in the present note. We modeled, via successive refinements, conceptual mind as a structure made up of three component sets: (set of concepts, set of propositions, set of syllogisms), which are equipped with eight structural functions. These structural functions specify the relations between concepts and propositions (a proposition has a concept as its subject / predicate), and the relations between propositions and syllogisms (a syllogism has a proposition as its minor / major premise / conclusion; Fig. 8). The logic of conceptual mind, which follows from its structural essences, is distinct from that of the category of sets by virtue of its degrees of truth (Fig. 3b, 9). The objective logic of conceptual mind is further distinguished from the Boolean logic of sets in light of the varieties of negation (Fig. 4a). Particularly noteworthy logical features are the admission of contradiction (Fig. 4b) and the failure of de Morgan’s law (Fig. 5).

Summing it all, our characterization of the mathematical structure and the non-Boolean logic of the conceptual mind is a refinement of the mainstream cognitive neuroscientific conceptualization of the mind as a set. Our mathematical characterization of mind can help develop definite theories of motion of thought on par with that of the mathematical theories of motion of matter. Bringing about this parity between the science of thinking and that of things is a first step towards accounting for the effectiveness of mind in the material world.

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Appendix

The truth value object of a universe of discourse is an object of the universe. For example, the truth value object of the category of sets is a two-element set **Ω** = {false, true}. Calculation of the truth value object of a category requires finding generic objects of the category. The defining property of generic objects is that any two maps in the category are equal if and only if the two maps are equal at every generic object-shaped figure. In the category of sets, a single-element set **1** = {•} is the generic object, since any two functions *f*, *g* are equal if and only if the two functions are equal at every **1**-shaped figure x, i.e., *f* = *g* if and only if *f* (x) = *g* (x), for every x. Calculation of truth value object involves enumerating parts of the generic objects. In the category of sets, the generic object **1** has two parts. They are *0*: **0** → **1**, *1*: **1** → **1**, where **0** = {}. The defining property of the truth value object **Ω** of a category is: for any object X of the category, there is a 1-1 correspondence between parts Y → X of the object X and maps from the object X to the truth value object **Ω**:

Y → X

X → Ω

Taking X = **1**, we find that, corresponding to the two parts (*0*, *1*) of the generic object **1**, there are two maps from **1** to **Ω**, which means that there are two **1**-shaped figures in **Ω**. In the category of sets, since all that there is to a set is the **1**-shaped figures in it, i.e. elements in the set, the truth value object **Ω** has two elements, **Ω** = {false, true}.

In the category of irreflexive graphs there are two generic objects:

generic dot, D = •

generic arrow, A = • → •

The generic dot D has two parts. Going by the 1-1 correspondence between parts (Y → D) of the generic dot D and maps from D to the truth value object **Ω**:

Y → D

D → Ω

there are two dot-shaped figures, i.e. there are two dots (F, T) in the truth value object **Ω** of the category of graphs. Next, the generic arrow A has five parts. Going by the 1-1 correspondence between parts (Y → A) of the generic arrow A and maps from A to the truth value object **Ω**:

$$\frac{Y \to A}{A \to \Omega}$$

there are five arrow-shaped figures, i.e. there are five arrows (*false*, *ft*, *tf*, *tt*, *true*) in the truth value object **Ω**. Now we have to determine how these two dots (F, T) and five arrows (*false*, *ft*, *tf*, *tt*, *true*) fit-together into the truth value graph **Ω**. In other words, we have to determine the incidence relations between the dots and arrows of the truth value graph **Ω**. More explicitly, we have to determine which one of the two dots is the source, target dot of each one of the five arrows. Inverse images of parts of generic objects along structural maps give the incidence relations between the generic object-shaped figures in the truth value graph. There are two structural maps *s*, *t*: D → A inserting the generic dot D into the generic arrow A as source, target dot, respectively. The inverse images of each one of the five arrows (*false*, *ft*, *tf*, *tt*, *true*; corresponding to the five parts of the generic arrow A) along the source *s*, target *t* structural maps give the source, target dot of the corresponding arrow:

1. The arrow *false* (of the truth value graph **Ω**) corresponds to the empty part of the generic arrow A, and its inverse image along the structural map *s*: D → A is the empty part of the generic dot D, i.e. the dot denoted by F (of **Ω**). Similarly, its inverse image along the structural map *t*: D → A is also the empty part of D, i.e. dot F. So, dot F is both the source and the target dot of the arrow *false* of the truth value graph **Ω**.
2. The arrow *ft* corresponds to the target dot (part) of the generic arrow A, and its inverse image along the structural map *s*: D → A is the empty part of the generic dot D, i.e. the dot denoted by F. Similarly, its inverse image along the structural map *t*: D → A is the dot (part) of D, i.e. dot T. So, the source and target dots of the arrow *ft* are the dots F and T, respectively.
3. The arrow *tf* corresponds to the source dot (part) of the generic arrow A, and its inverse image along the structural map *s*: D → A is the dot (part) of the generic dot D, i.e. the dot denoted by T. Similarly, its inverse image along the structural map *t*: D → A is the empty part of D, i.e. dot F. So, the source and target dots of the arrow *tf* are the dots T and F, respectively.
4. The arrow *tt* corresponds to the part of the generic arrow A consisting of both the source and the target dots, and its inverse image along the structural map *s*: D → A is the dot

(part) of the generic dot D, i.e. the dot denoted by T. Similarly, its inverse image along the structural map *t*: D → A is also the dot (part) of D, i.e. dot T. So, dot T is both the source and the target dot of the arrow *tt*.

5. The arrow *true* corresponds to the (entire) arrow part of the generic arrow A, and its inverse image along the structural map *s*: D → A is the dot (part) of the generic dot D, i.e. the dot denoted by T. Similarly, its inverse image along the structural map *t*: D → A is also the dot (part) of D, i.e. dot T. So, dot T is both the source and the target dot of the arrow *true*.

Thus, we obtain the truth value graph Ω of the category of irreflexive graphs (Fig. 3b). Along similar lines, the truth value triangle (Fig. 9) is calculated.

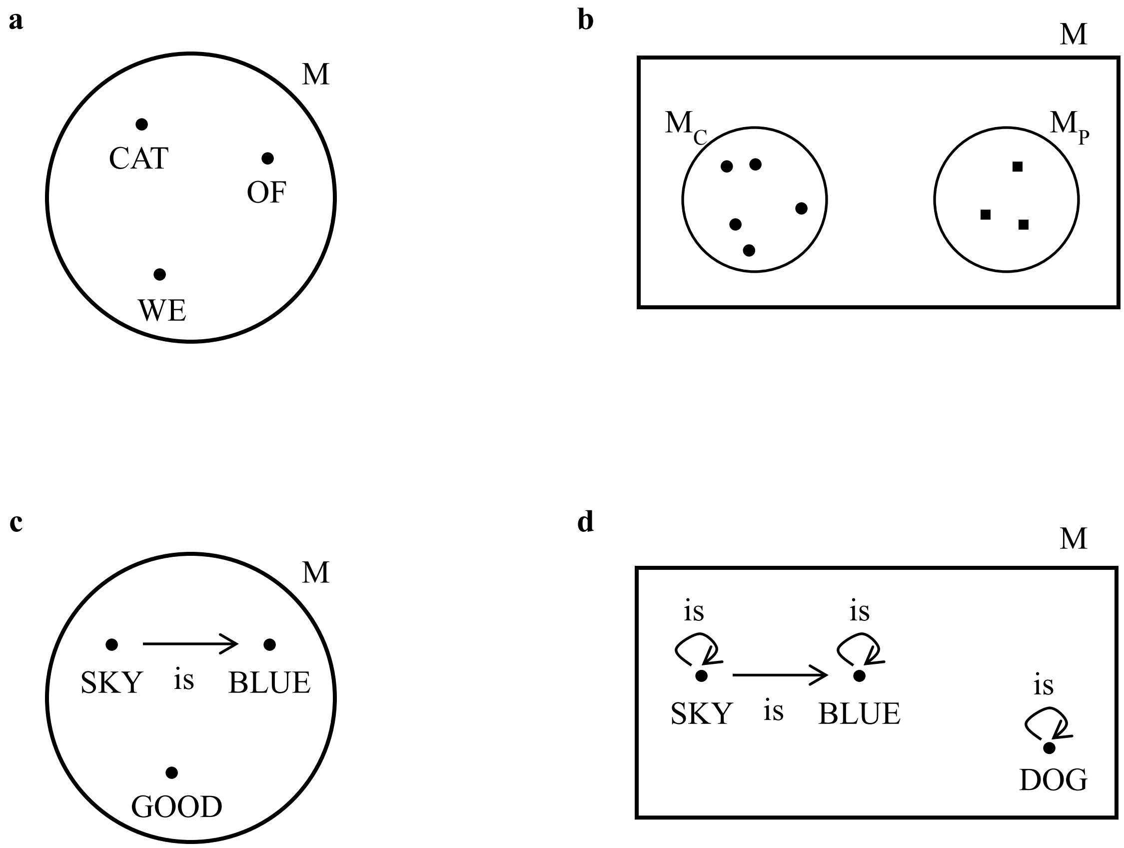

Fig. 1 Modeling Mind. **(a)** If minds consist of concepts only, then we can model mind as a set of concepts. In this model, concepts are construed as structureless elements. As an illustration, M = {CAT, WE, OF} is a mind consisting of three concepts CAT, WE, and OF (depicted as dots within a circle denoting the mind M). **(b)** A mind M modeled as a pair of sets: (a set M_C of concepts, a set M_P of propositions). Here, both concepts and propositions are construed as structureless elements, albeit of two different sets. **(c)** A mind M consisting of a proposition SKY is BLUE and a concept GOOD is modeled as an irreflexive graph. Here, concepts and propositions are displayed as dots and arrows, respectively. Note that the subject, predicate concepts (SKY, BLUE) of the proposition (SKY is BLUE) are depicted as the source, target dots integral to the arrow representing the proposition. **(d)** A mind M consisting of a proposition SKY is BLUE and a concept DOG is modeled as a reflexive graph. In this reflexive graph model, for each concept (e.g., SKY) in a mind, there is an identity proposition (SKY is SKY) in the mind. Note that concepts are displayed as loops (arrows with target dot same as the source dot).

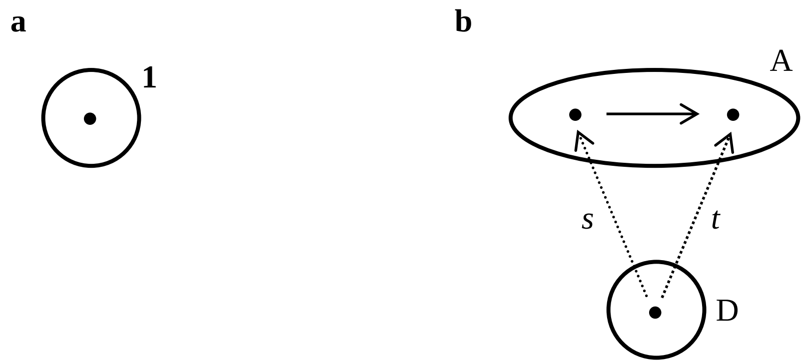

Fig. 2 Abstract Essence of Minds. (a) With mind as a set (of concepts), the structural essence of minds is a set (mind) consisting of one element (concept) i.e., a single-element set **1** = {•}. **(b)** With minds modeled as irreflexive graphs (Fig. 1c), the structural essence of minds consists of two graphs: concept (depicted as dot D) and proposition (depicted as arrow A), along with two morphisms s: D → A, t: D → A. These two morphisms specify the inclusion of concept (dot D) into proposition (arrow A) as its subject, predicate concept (source, target dot).

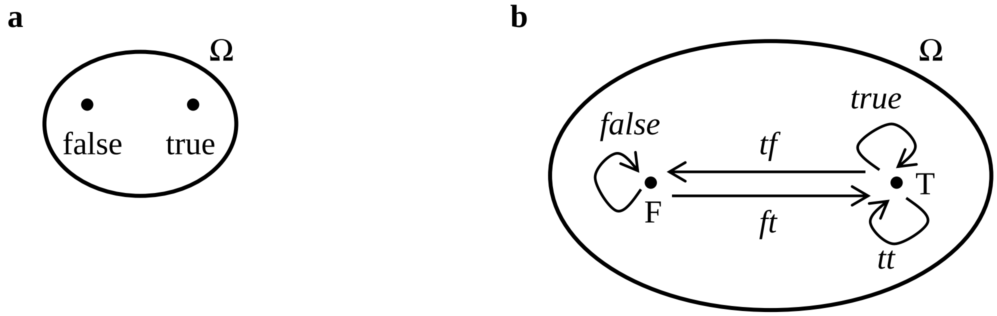

Fig. 3 Degrees of Truth. (a) With minds as sets, the truth value object of minds is a two-element set **Ω** = {false, true}. **(b)** With minds as irreflexive graphs, the truth value object **Ω** is an irreflexive graph consisting of five arrows (corresponding to the five degrees of truth at the level of propositions, which are displayed as arrows) and two dots (corresponding to the two truth values at the level of concepts, which are displayed as dots). The five arrows (*false*, *ft*, *tf*, *tt*, *true*) correspond to the five possible truth values a statement—P *is in* C—asserting the inclusion of a proposition P in a part C (of a mind) can take. If P is in C, then the truth value of the statement 'P *is in* C' is *true*; if P is not in C, then the truth value of 'P *is in* C' is *false*. In addition to these two truth values (*false*, *true*), there are three more truth values: (i) *tt* is the truth value of 'P *is in* C', if P is not in C, but both its subject and predicate concepts are in C, (ii) *tf* is the truth value of 'P *is in* C', if P is not in C, but its subject is in C, and (iii) *ft* is the truth value of 'P *is in* C', if P is not in C, but its predicate is in C. The two dots (F, T) in the truth value graph correspond to the two possible truth values (as in the case of sets) a statement asserting the inclusion of a concept (dot) in a part (of a mind) can take. The truth value graph is constructed based on the incidence relations (of dots and arrows) calculated as inverse images, along structural maps, of parts of the generic arrow (see Appendix).

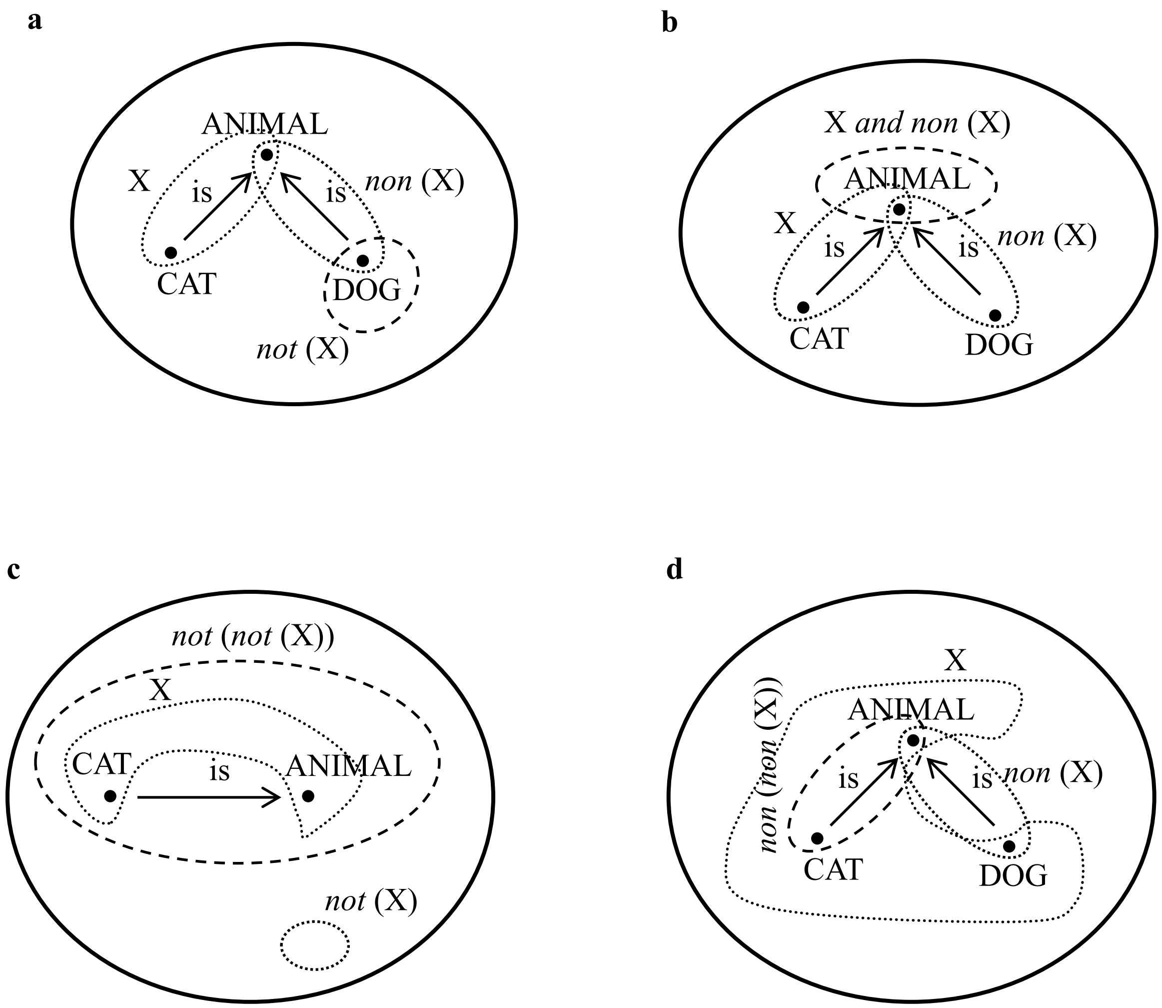

Fig. 4 Varieties of Negation. **(a)** Consider a mind consisting of two propositions: 'CAT is ANIMAL' and 'DOG is ANIMAL'. Next, consider a part X = 'CAT is ANIMAL' of the given mind. *not* (X) is the largest part among all parts of the mind whose intersection with the part X is empty, which means *not* (X) = DOG. *non* (X) is the smallest among all parts whose union with X is the entire mind. So, *non* (X) = 'DOG is ANIMAL'. **(b)** Again, let X = 'CAT is ANIMAL'. *non* (X) = 'DOG is ANIMAL'. X *and non* (X) = ANIMAL. Thus, logical contradiction 'X *and non* (X)' extracts from X (from the proposition 'CAT is ANIMAL') its boundary (the concept ANIMAL). **(c)** Consider a mind consisting of a proposition 'CAT is ANIMAL'. Let X denote a part (of the mind) consisting of two concepts: CAT, ANIMAL. Then, *not* (X) is the largest among all parts whose intersection with X is empty. So, *not* (X) is empty.

Since negating the empty part gives the proposition ‘CAT is ANIMAL’, double negation of X, i.e., *not* (*not* (CAT, ANIMAL)) is the entire proposition ‘CAT is ANIMAL’, which is bigger than X (i.e. both the concepts CAT, ANIMAL). **(d)** Consider a mind consisting of two propositions: ‘CAT is ANIMAL’, ‘DOG is ANIMAL’. Let X denote a part (of the given mind) consisting of the proposition ‘CAT is ANIMAL’ and the concept DOG. *non* (X) is the smallest of all parts whose union with X is the entire mind. So, *non* (X) = ‘DOG is ANIMAL’. Since *non* (DOG is ANIMAL) = ‘CAT is ANIMAL’, double negation of X, i.e., *non* (*non* (CAT is ANIMAL, DOG)) = ‘CAT is ANIMAL’, which is smaller than X (i.e., the proposition ‘CAT is ANIMAL’, along with the concept DOG).

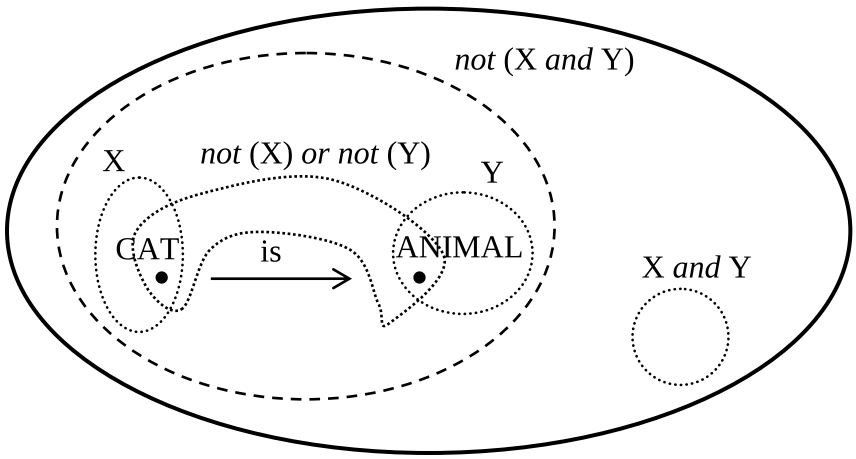

Fig. 5 Failure of de Morgan's Law. Consider a mind consisting of one proposition: 'CAT is ANIMAL'. Let X = CAT and Y = ANIMAL. X *and* Y is empty. *not* (X *and* Y) = 'CAT is ANIMAL'. *not* (X) = ANIMAL, while *not* (Y) = CAT. *not* (X) *or not* (Y) is both concepts CAT, ANIMAL. Since *not* (X *and* Y) $\neq$ *not* (X) *or not* (Y), the de Morgan's law: *not* (X *and* Y) = *not* (X) *or not* (Y) fails.

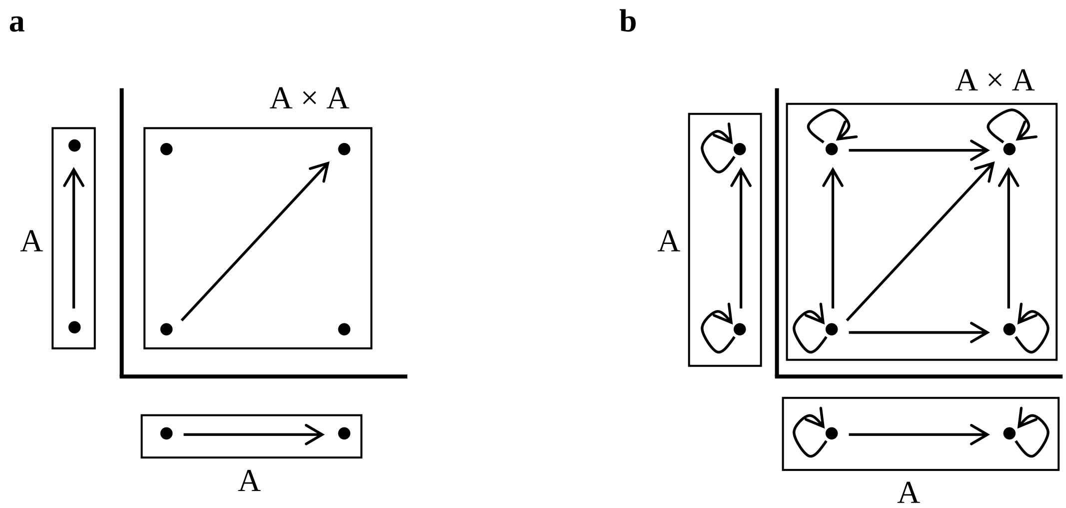

Fig. 6 Cohesion of Conceptual Mind. **(a)** In the irreflexive graph model of conceptual mind (Fig. 1c), a proposition A is an arrow along with its source and target dots representing the subject and predicate concepts of the proposition. Since the subject and predicate concepts of a proposition are integral to the proposition, the arrow A along with its source and target dots constitutes one connected piece. The product A × A consists of one arrow along with its source and target dots and, in addition to these two dots integral to the arrow, two more disconnected dots. Thus, the product consists of three pieces (one arrow plus two disconnected dots). Hence, the number of pieces of the product is not equal to the product of pieces of the factors ($3 \neq 1 \times 1$; Lawvere and Schanuel, 2009, pp. 260, 372-373), thereby failing to satisfy the product condition for cohesion. **(b)** In the reflexive graph model of conceptual mind (Fig. 1d), for every concept (depicted as a dot), there is an identity proposition (depicted as a loop on the dot). Consider a proposition A (an arrow with loops representing its subject and predicate concepts), which is one piece. The product A × A is also one connected piece. Hence, the number of pieces of the product is equal to the product of pieces of the factors ($1 = 1 \times 1$), thereby satisfying the product condition for cohesion.

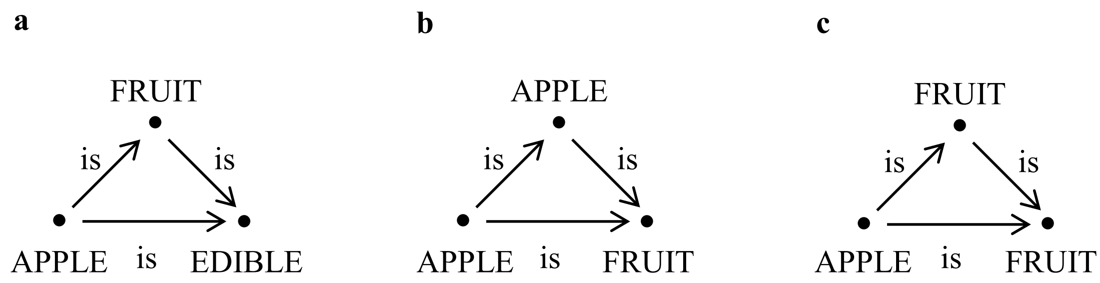

Fig. 7 Syllogisms as Commutative Triangles. **(a)** A pair of successive propositions: (APPLE is FRUIT, FRUIT is EDIBLE), wherein second proposition's subject (FRUIT) is same as the first proposition's predicate (FRUIT), can be composed to obtain a composite proposition: APPLE is FRUIT ∘ FRUIT is EDIBLE = APPLE is EDIBLE. Composition of propositions (as in this syllogism) can be modeled as a commutative triangle, with concepts as dots and propositions as arrows. **(b)** Syllogisms satisfy two identity laws: left and right identity laws. Left identity law i.e. composing a proposition with the identity proposition of its subject concept results in the proposition (as in): APPLE is APPLE ∘ APPLE is FRUIT = APPLE is FRUIT. **(c)** Right identity law i.e. composing a proposition with the identity proposition of its predicate concept results in the proposition: APPLE is FRUIT ∘ FRUIT is FRUIT = APPLE is FRUIT.

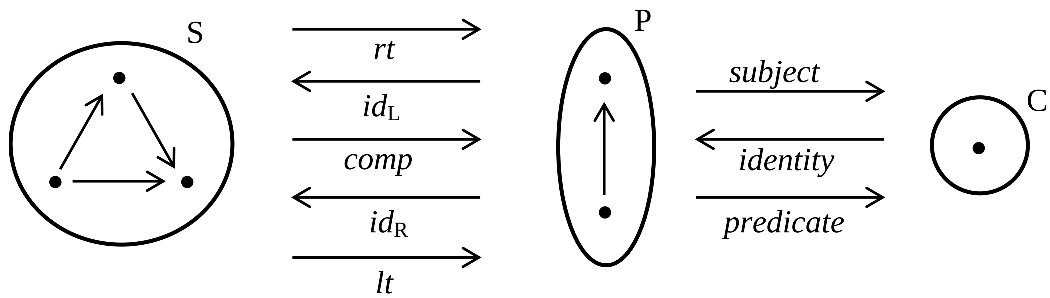

Fig. 8 Model of the Conceptual Mind. Conceptual mind consists of three components sets: 1. a set C of concepts (dots), 2. a set P of propositions (arrows, with a source and a target dot), and 3. a set S of syllogisms (commutative triangles formed of three arrows and three dots). (For the sake of clarity, only one generic element of each one of the three sets C, P, and S is displayed.) These three sets are structured by eight functions. The structural function *identity* from the set C of concepts to the set P of propositions inserts each concept (e.g., FRUIT) in the set of concepts into the set of propositions as an identity proposition (FRUIT is FRUIT). The functions *subject*, *predicate* from the set P of propositions to the set C of concepts assign to each proposition (e.g., 'SKY is CLEAR') its subject, predicate concept (SKY, CLEAR), respectively. The structural functions *lt*, *rt*, and *comp* from the set S of syllogisms to the set P of propositions extract a proposition from a syllogism (e.g., *lt* (APPLE is FRUIT ∘ FRUIT is EDIBLE = APPLE is EDIBLE) = APPLE is FRUIT). The functions id_L and id_R from the set P of propositions to the set S of syllogisms insert propositions as identity syllogisms (e.g., id_L (APPLE is FRUIT) = (APPLE is APPLE ∘ APPLE is FRUIT = APPLE is FRUIT).

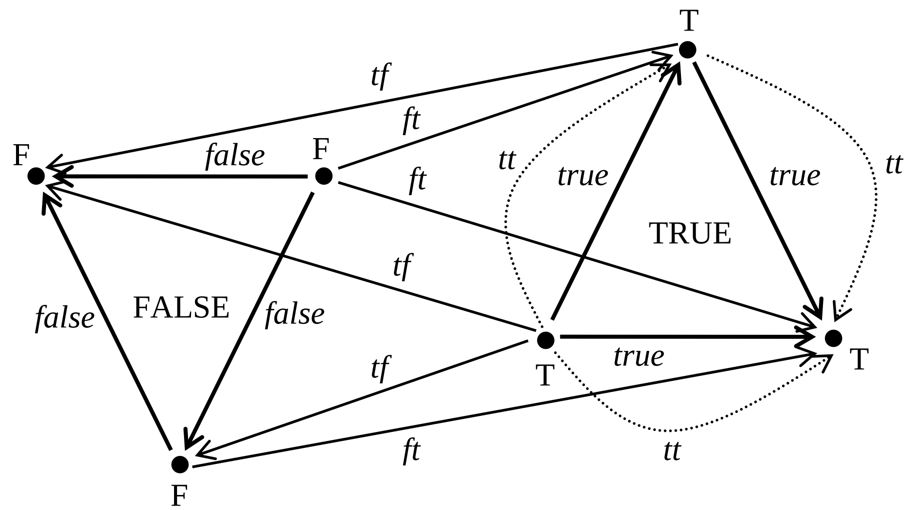

Fig. 9 Truth Value Triangle. Triangulated surface of the truth value triangle is calculated based on the nineteen parts of a generic syllogism (commutative triangle $f \circ g = h$). They are: $\{\{f \circ g = h\}, \{f, g, h\}, \{f, g\}, \{g, h\}, \{h, f\}, \{f, \mathrm{C}\}, \{g, \mathrm{A}\}, \{h, \mathrm{B}\}, \{f\}, \{g\}, \{h\}$, {A, B, C}, {A, B}, {B, C}, {C, A}, {A}, {B}, {C}, {}}. The nineteen degrees of truth corresponding to these nineteen parts are displayed as nineteen triangles. The triangular surface TRUE corresponds to the truth value of a statement (that a syllogism): 'APPLE is FRUIT ∘ FRUIT is EDIBLE = APPLE is EDIBLE' *is in* C (where C is a given part, say, conscious part of a mind) when the syllogism is in C. The triangular surface FALSE is the truth value of the statement when the syllogism is not in C. In between these two extremes, there are seventeen degrees of falsity corresponding to various scenarios such as: the syllogism is not in C, but all the three propositions APPLE is FRUIT, FRUIT is EDIBLE, APPLE is EDIBLE are in C (triangle formed by the three arrows labeled *true*), or just one of three concepts FRUIT is in C (triangle formed by the three arrows labeled *ft*, *tf*, *false*).