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A Mathematical Model of Dignāga's Hetu-cakra

(Within the Framework of Simon's Bounded Rationality)

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Abstract
A reasoned argument or *tarka* is essential for a wholesome vāda that aims at establishing the truth. A strong *tarka* constitutes of a number of elements including an *anumāna* based on a valid *hetu*. Several scholars, such as *Dharmakīrti, Vasubandhu and Dignāga*, have worked on theories for the establishment of a valid *hetu* to distinguish it from an invalid one. This paper aims to interpret Dignāga's *hetu-cakra,* called the wheel of grounds, from a modern philosophical perspective by deconstructing it into a simple probabilistic mathematical model. The objective is to understand how and why a *vāda* based on a probabilistically weaker *hetu* can degrade into a *Jalpa* or *vitaṇḍā*. To do so, the paper maps the concept of 'Bounded Rationality' onto the *hetu-cakra*. Bounded Rationality, an idea coined by the management thinker Herbert Simon, is often employed in understanding decision-making processes of rational agents. In the context of this paper, the concept would state that the *prativādin* and *ālocaka* (debater) may not hold unbounded information to back their *pratijñā* (proposition). The paper argues that within the probabilistically deconstructed *hetu-cakra* model, most people argue in the 'Zone of Bounded Rationality', and thus, the probability of a debate degrading into *Jalpa or vitaṇḍā* is high.

Keywords *vāda* · *debate* · *Hetu-cakra* · Bounded Rationality · Mathematical modelling

Introduction

The principal aim of a wholesome *vāda* is to resolve conflict around the pratijñā and establish 'what is true'. One's own thesis is defended by means of genuine criteria of knowledge (*pramāṇa*) and *tarka* or logic using a valid *hetu* (ground) and demonstrating it with *udāharaṇa* (corroboration) as put forth by Dasti and Phillips (2017).

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A *vāda* can degenerate into a *Jalpa* or *vitaṇḍā* when the opponent frequently and adamantly uses hetvābhāsa, i.e. faulty reasoning or fallacies. One of the ways in which the opponent can engage in frequent use of hetvābhāsa is by using, a seemingly valid, but an invalid *hetu* in the course of the debate. To understand what constitutes an invalid *hetu*, we need to trace the development of *hetu* from Nyāya sutras to Dignāga. This will allow us to delineate the subtleties involved in establishing a valid from an invalid hetu: A large part of this discussion has been borrowed from Emmanuel (2013).

As per the Nyāya Sutras, *hetu* gives the reason or evidence through the vehicle of inference (*anumāna*). It furnishes a means to prove the proposition. For instance, consider the following arguments for a famously quoted example to justify the syllogisms in Nyaya sutras:

Proposition: *Sound is non-eternal.*
Ground: *because of having the property of arising*
Corroboration: *a substance, such as pot, having the property of arising is non-eterna*l
Application: *and, likewise, sound has the property of arising*
Conclusion: *therefore, sound is non-eternal because it has the property of arising*

As per Emmanuel (2013), this can also be represented more generally in the form of:

Proposition: *p has S.*
Ground: *p has H*
Corroboration: *d has H and d has S.*
Application: *As d has H and has S, so p has H and has S*
Conclusion: *p has S*

An Evaluation of *Hetu*

Nyāya Sutras

While the *hetu* seems to be employed logically to defend the pratijñā in the above model, the *hetu* itself does not seem to be well defined. For instance, one can logically justify a wrong case in the same model with the following example:

Proposition: *This object X is a leaf.*
Ground: *X is green.*
Corroboration: *A neem tree has leaves, and the leaves are green.*
Application: *Likewise, X has the property of being green.*
Conclusion: *X is a leaf because it is green like a tree leaf.*

Clearly, the conclusion is not universally valid. A stick may be green, but that does not make it a leaf. In the above argument, the quality of greenness that defends the pratijñā is invalid because the *hetu* or ground must be directly related to the pratijñā and not merely be a correlation or co-existent. In this case, greenness is a

hetu that is co-existent with most leaves, but is not a sine qua non or causation for an object to be called a leaf as there could be leaves of different colours.

Vasubandhu's Critique and Issues

Emmanuel (2013) points out Vasubandhu's arguments as a counter to the above approach. Vasubandhu argues that for this argument structure (of the Nyāya Sutras) to be correct, there should be an indispensability relationship between the *hetu* and the property to be established, such as the relationship between heat and fire, such that one does not co-exist with the other meaning that their co-existence is indispensable. One can take the following example:

***Proposition**: Fire is hot.*
***Ground:** Because of emitting light.*
***Corroboration:** Whatever emits light is observed to be hot, like a light bulb.*

From this, one can abstract the following general form of argument:

***Proposition**: p has S*
***Ground:** p has H*
***Corroboration:** Whatever has H is observed to have S, like d.*

Here, d is an example of something recognized to have both H and S. The problem with the above was pointed by Katsura (1986. 165) based on the three conditions of hetu (*tri-rūpa-hetu*) which must be satisfied for it be a valid hetu: a) it should occur in the subject of an argument or p; b) the hetu or H should occur in things similar to the subject; and c) the hetu or H should not occur in things dissimilar from the subject.

However, Emmanuel (2013) and Katsura (1986. 165) point out that 'while clearly intended to define when the property to be established is indispensable for the ground', they are stated rather imprecisely for the following reasons:

First, similarity and dissimilarity are stated with two relations, though these relations are not binary relations but, rather, ternary relations. It is not contradictory for one thing to be both similar and dissimilar to another.

Second, does indispensability imply exclusivity? It is not made precise whether or not the ground occurs in some or all of the things which are similar to the subject of the argument, nor whether or not the ground does not occur in some or all of the things which are dissimilar to the subject of the argument.

Dignāga's Redressal of the Problem: Hetu-cakra (Wheel of Grounds)

Dignāga addresses the problems mentioned above using a *hetu-cakra*, called the wheel of grounds. It is a 3×3 matrix which distinguishes a proper from an improper ground and is equivalent to the last two forms of the three conditions of a ground (*tri-rūpa-hetu*). The model has been adopted from Emmanuel (2013).

Wheel of grounds			
H occurs in	All S	All s	All s
	All S`	No s`	Some s`
H occurs in	No S	No s	No s
	All S`	No s`	Some s`
H occurs in	Some S	Some s	Some s
	All S`	No s`	Some s`

Here, the *hetu* is tested against all similar or dissimilar objects and validity is assigned only on one of the nine cases in the 3 × 3 matrix. Consider the following example where we assume that there are only two kinds of tea: Indian tea and Vietnamese tea. We would like to prove that if a tea is bitter, it must be Vietnamese tea. Let us use the same model employed earlier and integrate it with *hetu-cakra*:

Proposition: *This tea (p) is Vietnamese tea (S).*
Ground: *This tea is bitter (H)*
Corroboration: *Comparison with Indian tea (S')*

Decoding the ground in hetu-cakra, we have the following '3 × 3 tea matrix':

Bitterness (H) occurs in	All Vietnamese tea All Indian Tea	All Viet. Tea No Indian Tea	All Viet. Tea Some Indian Tea
H occurs in	No Viet. Tea All Indian Tea	No Viet Tea No Indian Tea	No Viet. Tea Some Indian Tea
H occurs in	Some Viet. Tea All Indian Tea	Some Viet. Tea No Indian Tea	Some Viet. Tea Some Indian Tea

In all the above squares in the matrix, except the green box (all Vietnamese tea and no Indian tea), one cannot establish with full confidence that bitterness is a quintessential ground for a tea to be considered Vietnamese tea. For instance, if we consider Row 1 (R1), Column 3 (C3)—all Vietnamese tea, some Indian tea, it is clear that bitterness exists in Indian tea as well, and therefore, it is not unique to Vietnamese tea. Similarly, in R2, C2, since bitterness does not exist in any variety of tea, our pratijñā is held invalid. We will use this syllogism to model our tea matrix (*hetu-cakra*).

Modelling *Hetu-cakra*

Herbert Simon's Bounded Rationality (BR) Model

Let us assume that two participants are arguing over this pratijñā, and one takes a side that bitterness is unique to Vietnamese tea. To make such a pratijñā, the proponent must have either tasted all kinds of tea or have accessed information about the taste of all kinds of tea. In most cases, it is usually unlikely that one has the extent of information required to make such a perfect or highly accurate pratijñā. A rational agent would prefer to gather all the information before he makes such a claim.

Herbert Simon, an influential management thinker, points out this issue in obtaining perfect information for an agent. He states that our decisions are rationally bounded, meaning that we make decisions that are rational, but within the limits of the information available to us and our mental capabilities. In the above case, a proponent (agent) can only acquire a limited amount of information to make a rational pratijñā. For instance, one can make a claim in R3, C3 (some Vietnamese tea, some Indian tea) much more easily than that in the middle of the matrix, R2, C2 (no Vietnamese tea, no Indian tea) because the latter necessitates access to nearly infinite or unbounded amount of information. Based on the information required, let us transform the tea matrix into zones of bounded and unbounded rationality. As clear from the discussion above, the zone of confidence is when one can claim his pratijñā with full confidence such as in R1, C2. On the other hand, the least certain or the zone of ignorance would be R3, C3 where the proponent only holds information about some kinds of both Vietnamese tea and Indian tea. Concerning the zones in between, where one needs to know the taste of all kinds of tea, we can classify them under Zones of Unbounded Rationality (UBR), and the rest can be classified under Zones of Bounded Rationality (BR). This is because less information is required to make such claims in the latter case.

Knowledge of H	Zone of Unbounded Rationality (UBR)	**Zone of UBR (Confidence)**	Zone of Bounded Rationality (BR)
Knowledge of H	Zone of UBR	Zone of UBR	Zone of BR
Knowledge of H	Zone of BR	Zone of BR: Certainty	Zone of BR: Ignorance

Mathematical Modelling of *Hetu-cakra*

Let us consider that an agent wishes to hold perfection or unbounded information in order to conduct a debate successfully and this optimum ideal be defined as U*. But, due to the nature of Bounded Rationality of an agent, as posited by Simon, one can only attain a satisfactory result defined here as A. Let us define the difference between U* and A (i.e. U*- A) to be the irrationality coefficient (assumed to be ε). The value of ε tends to go down in a debate as the agent gathers more and more information and progresses from a satisfactory status A to the optimum ideal U*. The lower the irrationality coefficient, the more likely it is that the agent holds more information and puts forth a more substantial *pratijñā* with a well backed *hetu* leading to a good debate.

To put it shortly, $\varepsilon = U^* - A$, where U* is the optimum result and A is the aspiration or satisfactory result. Satisficing options S(ε) can be defined as a set of all those options **s** such that $\mathbf{U(s) \geq U^* - \varepsilon}$**.** Several scholars such as Bubelis (1979) prefer to call this model as the epsilon-equilibrium, or near Nash equilibrium, which is used frequently in economics and game theory. We would not be using the equilibrium-related properties of the equation and confine ourselves to a rudimentary reading of the equation.

In order to understand what characterizes satisfactoriness in a debate, I propose to break down ε (satisfactoriness) into two components:

1. *Confidence factor (c)* With what confidence can the proposition be held valid? Only for cases when the hetu is universally valid, *c* tends to infinity, else *c* tends to a minimum value or even zero.
2. *Information factor (i)* When making the assertion, how much information does the debater need to gather to make a conclusive statement? For cases needing unbounded or perfect information, *i* tends to infinity, and in other cases, such as attaining only a satisfactory level of information, *i* tends either to a minimum value or zero.

$\boldsymbol{\varepsilon = f(i, c)}$**.** The more information is required and the higher is the confidence, the closer A would move to U* and ε would be minimized. The lesser the information base and the confidence factor, the farther apart A will be from U* and would push the value of ε higher.

Let us model our tea matrix as a function of *i* and *c*. The tea matrix has been redrawn for a convenient reference.

Bitterness (H) occurs in	All Vietnamese tea All Indian Tea	All Viet. Tea No Indian Tea	All Viet. Tea Some Indian Tea
H occurs in	No Viet. Tea All Indian Tea	No Viet Tea No Indian Tea	No Viet. Tea Some Indian Tea
H occurs in	Some Viet. Tea All Indian Tea	Some Viet. Tea No Indian Tea	Some Viet. Tea Some Indian Tea

Modelling this matrix based on information factor, we get the following matrix. Anywhere the agent requires to taste all kinds of tea to arrive at a conclusion that all or none of a particular kind of tea tastes bitter, the information factor would tend towards infinity because one practically needs to go through all tea samples or at least research about all tea samples to make a corresponding statement. So, R1, C1 to R2, C2, *i* factor tends to infinity. In all other cases (except R3, C3), *i* factor rests between a minimum value and infinity because the agent does not need to taste all varieties to make that statement, such as some Vietnamese tea is bitter, but no Indian tea is bitter (R3, C2).

Information factor: H	$i \rightarrow \infty$	$i \rightarrow \infty$	$i^{\min} < \mathrm{I} < \infty$
Information factor: H	$i \rightarrow \infty$	$i \rightarrow \infty$	$i^{\min} < i < \infty$
Information factor: H	$i^{\min} < i < \infty$	$i^{\min} < i < \infty$	$i = i^{\min}$

Modelling the tea matrix on the confidence factor is simpler, and one can assert with full confidence in R1, C2 that bitterness is a valid *hetu* for our pratijñā because all Vietnamese tea is bitter and no Indian tea tastes bitter. In all other cases, except R3, C2, confidence factor *c* tends to be zero because even Indian tea tastes bitter, and therefore, bitterness is not a quintessential quality of Vietnamese tea alone. In R3, C2, there is still scope to explore more Indian tea and one may come across a variety that tastes bitter. Because the agent here does not hold

unbounded information, his confidence is not very high and he sticks to a value of $c^{\min}$.

Confidence factor: H	$c = 0$	$\boldsymbol{c \to \infty}$	$c = 0$
Confidence factor: H	$c = 0$	$c = 0$	$c = 0$
Confidence factor: H	$c = 0$	$c = c^{\min}$	$c = 0$

Based on the discussion above: *Optimum solution*: $(i, c) \to \infty$ where e is minimized; *Least satisfactory solution*: $(i, c) \sim (i^{\min}, 0)$; *Satisfactory solution*: $i^{\min} < i < \infty$ and $c = c^{\min}$. This can be tabulated in the following matrix, and we can summarize our results and derive conclusion from this modelling exercise.

Solution H	$(i\,, c) \to (\infty, 0)$	$(i\,, c) \to \infty$	$(i\,, c) \to (i^{A}, 0)$
Solution H	$(i\,, c) \to (\infty, 0)$	$(i\,, c) \to (\infty, 0)$	$(i\,, c) \to (i^{B}, 0)$
Solution H	$(i\,, c) \to (i^{D}, 0)$	$(i\,, c) \to (i^{C}, c^{\min})$	$(i\,, c) \to (i^{\min}, 0)$

* $i^{A}, i^{B}, i^{C}, i^{D} > i^{\min}$

Results and Conclusion

The first clear insight that one can gather from tabulating a hetu-cakra based on the Bounded Rationality (BR) model is that some kind of debates demand unbounded information. Since not all agents possess unbounded information, some debates should be either confined to the domain of subject experts or the scope of debate should be limited; otherwise, the debates would not be highly rational. An optimum debate is where both i and c factors are at their optimum levels. This is not very likely though.

Second, most people tend to argue in the Zone of Bounded Rationality (ZBR), and the probability of a debate degrading into *Jalpa/vitaṇḍā* is the highest in this zone precisely because the proponent does not hold enough information to substantiate his pratijñā and is likely to use tools such as quibbling to win over. When UBR is accessible due to factors such as easy availability of information, most debates

should be of the nature of *vāda* and may not degenerate into *vitaṇḍā or Jalpa.* In order to conduct a successful *vāda,* a moderator needs to be sensitive to the bounded v/s unbounded nature of a debate and should confine the debate to areas where a good *vāda* can be conducted without needing to minimize the value of ε very significantly. In other words, a satisfactory debate can be organized if the debate is organized along the lines where the need for information factor (*i*) can be reduced, and participants can focus more on advancing credible arguments rather than trying to be perfectly rational agents.

Any such mathematical modelling is a first step into decoding the complex interaction of a proponent and an opponent during the course of a debate. It allows us to examine debates from a scientific point of view. More sophisticated models can be developed extending the BR model for a general case where there are more than one *hetu* (H) and where the proposition involves *n* number of objects (p), qualities (S) and corroborations (d). However, the fundamental nature of the debate that revolves around the establishment of a valid hetu and the information required to accomplish the same would not change irrespective of the sophistication of the mathematical model.

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