Annals of Operations Research
https://doi.org/10.1007/s10479-020-03781-8

ORIGINAL RESEARCH

Optimised solutions to the last-mile delivery problem in London using a combination of walking and driving

Antonio Martinez-Sykora[1] · Fraser McLeod[2] · Carlos Lamas-Fernandez[3] · Tolga Bektaş[4] · Tom Cherrett[2] · Julian Allen[5]

Accepted: 29 August 2020

Abstract
Inspired by actual parcel delivery operations in London, this paper describes a two-echelon distribution system that combines the use of driving and walking as part of last-mile deliveries in urban areas for a single driver. The paper presents an optimisation model that explicitly treats and integrates the driving and walking elements, and describes a branch-and-cut algorithm that uses new valid inequalities specifically tailored for the problem at hand. Computational results based on real instances obtained from a courier operating in London are presented to show the performance of the algorithm.

Keywords Vehicle routing · Last-mile delivery · Branch-and-cut · Urban freight

1 Background and motivation

Sharp growth in e-commerce sales in developed countries worldwide over the last decade has led to substantial increases in urban freight goods transport, with associated negative impacts on road traffic, availability of kerbside space and air quality. In the UK, total measured national volumes in the parcels market increased by 10%, to 2.6 billion items, in 2018–19 (Ofcom 2019). This growth and worsening road traffic and parking conditions in city centres make parcel deliveries ever-more difficult to perform. Factors that increase vehicle use for parcel deliveries include the growing trend for same-day and 'instant' deliveries (within 2 h) leading to fragmentation of consignments and increasing the number and frequency of

✉ Tolga Bektaş
T.Bektas@liverpool.ac.uk

1 Centre for Operational Research, Management Science and Information Systems, University of Southampton, Southampton SO17 1BJ, UK

2 Faculty of Engineering and Physical Sciences, University of Southampton, Southampton SO17 1BJ, UK

3 NIHR CLAHRC Wessex, University of Southampton, Southampton SO17 1BJ, UK

4 University of Liverpool Management School, University of Liverpool, Chatham Street, Liverpool L69 7ZH, UK

5 Faculty of Architecture and the Built Environment, University of Westminster, London NW1 5LS, UK

Published online: 08 September 2020

deliveries (Dablanc et al. 2017) and the substantial number of competing freight transport operators that result in much duplication of van activity (Browne et al. 2014). Last-mile parcel delivery operations, characterised by multi-player, multi-drop vehicle rounds where kerbside access is needed are at direct odds with an infrastructure designed and legislated in favour of passenger transportation (Allen and Browne 2014).

Many of our major cities and particularly London have seen a considerable shift to walking, cycling and the use of buses over the past 20 years together with a fall in car traffic (Transport for London 2016). As a result, road space is being increasingly reallocated in favour of dedicated cycle and bus lanes, as well as pavement widening programmes, which in central London has led to a 30% decrease in road network capacity for private motorised vehicles between 1993 and 2009 (Transport for London 2013). With declining kerbside stopping locations, carriers' round efficiency declines due to the need for additional, unproductive driving whilst searching for parking locations and the potential increase in fines and general traffic disruption through illegal parking (Bates et al. 2017). With average speeds in cities also falling, for example, by 13% (from 16.6 to 14.5 mph) on local authority managed roads in London between 2015 and 2018 (Department for Transport 2020), there is growing interest in alternative methods for addressing the last-mile problem that aim to reduce reliance on road vehicles.

The study by Allen et al. (2018) on last-mile goods vehicle activity indicated that multi-drop parcel delivery drivers in central London typically walk 8 km (5 miles) while their vehicle is parked at the kerbside, taking up more than 60% of their time worked. Their observations suggest a two-echelon last-mile distribution model, with driving and walking as the higher and lower-level echelons, respectively. However, the design of such a last-mile distribution model gives rise to challenging optimisation problems, even for a single driver, in which decisions concerning the partitioning of customers that will be served either by driving or by walking, the sequence of locations to be visited in either mode, and the selection of parking points from where walking tours will start from and end at, need to be made simultaneously. In addition, the driving and walking has to be differentiated on the basis of travel time or cost between a pair of locations, which is likely to vary between these two modes.

Two-level distribution systems within the broader context of city logistics give rise to what is known as the class of two-echelon vehicle routing problems (2EVRPs) (see Cuda et al. 2015, for a survey). Whilst the definition of the 2EVRP allows for use of different types of vehicles at each echelon (e.g. Hemmelmayr et al. 2012), they are primarily differentiated by the capacities of the vehicles, but not by travel cost or time. The 2EVRP assumes the use of a set of 'satellites' at given locations, and where the first and the second echelon routes meet. The model described by Anderluh et al. (2017) that combines vans and cargo bikes defines a separate travel cost for each vehicle travelling between a pair of locations, but requires vans and cargo bikes to travel from separate depots, and assumes that the customers are already partitioned into those to be served by cargo bikes and vans. Our problem uniquely differs from the 2EVRP in that we associate different travel costs for the two different modes, and do not require the use of satellites.

There has recently been a stream of research dedicated to the combined use of walking and driving for home on-site services. We will not attempt to review this body of literature as it relates more to health care, but instead mention to the particular problem described by Fikar and Hirsch (2015) to serve as an example. Motivated by home health care services, but also applicable to others in home repair, maintenance and private tutoring, this problem concerns the use of vehicles to transport nurses, who can be dropped-off and picked-up at various locations. The nurses can choose to walk between a drop-off and pick-up location for site visits. Additional practical restrictions, such as time windows and mandatory working time and break regulations, also apply. The authors describe a heuristic algorithm and apply it on real-world data provided by the Austrian Red Cross.

More relevant to our setting in freight distribution is the truck-and-trailer routing problem (TTRP) (Derigs et al. 2013; Villegas et al. 2010) where the 'truck-and-trailer' route in the TTRP corresponds to the driving route in our setting, while the 'truck only' route corresponds to that of walking. Variants of the TTRP that take into account the time requirements on the deliveries as well as the limited carrying capacity of both modes of delivery have also been studied (Lin et al. 2011), which give rise to difficult optimization problems that have only recently been optimally solved with up to 100 customers (Parragh and Cordeau 2017). Such problems however implicitly assume the use of a single mode of transport in both layers. One study that looks at such a two-echelon distribution model where walking and driving is treated separately is by Nguyên et al. (2019) where time window constraints are also present. A similar distribution problem involving delivery to clusters of customers by a fleet of trucks operating from a single depot appears in De Grancy and Reimann (2016), where each customer is served on foot from the parking location within the cluster it belongs to, and where there are time window restrictions on the deliveries made. However the problems described by Nguyên et al. (2019) and De Grancy and Reimann (2016) both assume that delivery locations are clustered in advance and is provided as input data.

The paper by Lin (2011) studies a similar problem to the one presented here, namely a pick-up and delivery problem with two modes of delivery (e.g., a van and a foot courier) and time windows, in which the two modes are treated separately. In this problem, the upper-echelon mode of transport has capacity restrictions whereas the lower-echelon does not, which is different to our setting. A similar problem arising in the routing and scheduling of technicians of an energy provider is described by Coindreau et al. (2019), where workers are allowed to either share a vehicle or walk to get from one site to another. Each vehicle has a driver and can accommodate a worker, who can both choose to walk to a site. After any walking, the driver must return to the vehicle. If a worker is dropped-off at a particular location by one driver, they can be picked-up at another location by another driver, where the travel between the two locations is undertaken by foot by that worker. For this problem, the authors describe a formulation and a heuristic algorithm that is applied to a set of instances.

The delivery model we study here also bears resemblance to those that involve simultaneous routing of ground vehicles and drones, such as the one described in Luo et al. (2017) in which drones are used for deliveries and are carried on the ground vehicles. However, as the carriage capacity of drones is often limited to one parcel (Murray and Chu 2015), such problems assume that drones serve only one customer at a time, and return to the ground vehicle carrying it to pick up a new parcel for delivery to another customer. In addition, drones are also limited by their flight range. For this reason, the problem we study here can be regarded as a more general version of those involving drones and vehicles.

This paper contributes to the literature by (1) introducing a two-echelon last-mile delivery system that explicitly considers driving and walking decisions in an integrated manner, (2) describing an associated optimisation model, valid inequalities and a branch-and-cut algorithm to solve the model, and (3) presenting results on real instances derived from last-mile delivery operations in London. The rest of the paper is structured as follows. We present a formal description of the problem in Sect. 2. We then describe an optimisation model in Sect. 3, where we also present valid inequalities and describe the branch-and-cut algorithm. Computational results are given in Sect. 4. The paper concludes in Sect. 5.

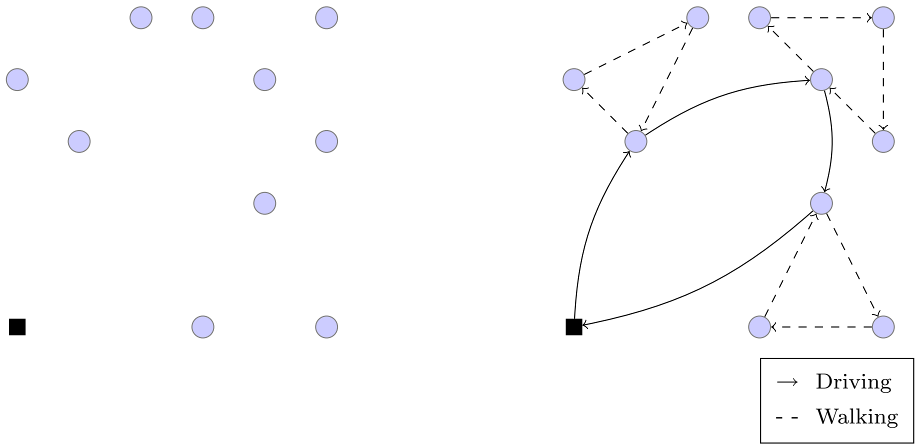

Fig. 1 A feasible solution to an instance with one depot and 10 customers, where walking is shown by the dashed lines and driving by the solid lines

2 Planning a combined driving and walking distribution model

The proposed distribution system requires explicit differentiation between walking and driving. In particular, the time required for travelling between two locations will differ between walking and driving, sometimes by a significant margin, particularly in urban areas. Furthermore, the dependency between the two echelons, one corresponding to driving and the other to walking, suggest that designing the two sets of routes separately may result in a sub-optimal solution. For a given area to be serviced by a single driver, the main decisions that need to be made in the design of a combined distribution system include:

- The locations that are to be visited by either mode of transport,
- The selection of locations where the vehicle will be parked,
- The ordering of delivery locations in both the driving route and the walking routes, bearing in mind the capacity restrictions in the latter (e.g., associated with using a backpack, a wheeled bag or a trolley).

The distribution design problem concerns a single vehicle that is pre-loaded with parcels and destined to serve a given set of customers with known locations and delivery requirements specified in terms of number and size of items. The aim is to (1) group the customers into clusters (where singleton clusters are allowed), (2) find a driving route across the clusters such that one node within each cluster is designated as the parking node, and (3) find a walking route within each cluster that starts and ends at the parking node. The objective is to minimise the total time of all travel, under the following constraints: (1) the demands of all customers must be satisfied, (2) the total weight and volume of all deliveries made within each cluster is limited to the total capacity that the driver can carry, and (3) the driving starts from and ends at a depot of a known location. Figure 1left shows an instance to the problem with the depot shown by the dark square and 10 customers shown by the small circles. A feasible solution to the problem is shown on the right of Fig. 1 which indicates the driving by the solid lines and the walking by the dashed lines.

The design of the distribution system that includes driving and walking therefore hinges on the trade-off between the time taken by the two modes, which requires the two modes to be treated separately. The model we describe in the following section explicitly addresses this particular aspect of the problem.

Time window constraints for parcel delivery are not explicitly considered here. Our analysis of a data set from a major carrier operating in London covering the period 4–9 June 2018 revealed that, of about 13,000 consignments that were delivered, only 1.6% required delivery by 9am, 2.3% by 10am and 7.2% by 12pm, whereas the rest (88.9%) could be delivered anytime until 6pm on a given day. Given the relatively small proportion of time-sensitive parcels, we assume that these could be delivered separately to ensure the delivery deadlines are met, and therefore focus our attention on parcels that do not have any time delivery constraints attached to them.

3 Mathematical model and branch-and-cut algorithm

This section presents a formulation of the problem described above along with valid inequalities. We also describe a branch-and cut algorithm to solve the formulation and in which the new valid inequalities are used.

3.1 Mathematical modelling

Let $G = (V, A)$ be a complete graph with $V = V' \cup \{0\}$ as the set of nodes, where node 0 corresponds to the depot, V' is the set of customers, and A is the arc set. Two different (non-negative) travel times are associated with each arc $(i, j) \in A$ that represent travel from node $i \in V$ to $j \in V$, namely the time c^d_{ij} of driving and the time c^w_{ij}, of walking. We allow the costs to be asymmetric. We denote by v_i (w_i) the volume (weight) of the parcels to be delivered to customer $i \in V'$.

There are no capacity or volume constraints on the vehicle as it is pre-loaded at the time of departure, but each walking route is limited to carrying parcels that total no more than Q_v units in volume and Q_w units in weight. In addition, any individual customer with demand larger than Q_v in volume or heavier than Q_w in weight must be visited and served by the vehicle. Such a customer cannot appear on a walking route, and we assume that they also cannot be a parking node. Let $r(S)$ denote the number of walking routes needed to meet the demand in a given subset S of V', calculated as $r(S) = \max\{\lceil v(S)/Q_v \rceil, \lceil w(S)/Q_w \rceil\}$ where $v(S) = \sum_{i \in S} v_i$ is the total volume of parcels to be delivered to S, and $w(S) = \sum_{i \in S} w_i$ is the total weight of parcels to be delivered to S.

The mathematical model is defined with respect to a linear multiplier $\alpha \geq 0$ to the driving time, pre-defined as an input to the optimisation problem. It inversely reflects the relative importance given to walking over driving. In particular, increasing α places less emphasis on minimising the total walking time, and puts more weight on reducing the overall driving time. This may be required if there are uncertainties on the driving times, or for when the overall vehicle mileage (and the associated fuel consumption) of the vehicle needs to be reduced.

The model uses a binary variable x^d_{ij}, which takes the value 1 if driving takes place from node $i \in V$ to node $j \in V \setminus \{i\}$, and 0 otherwise. Similarly, a binary variable x^w_{ij} takes the value 1 if walking takes place from node $i \in V$ to node $j \in V \setminus \{i\}$, and 0 otherwise. Finally, a binary variable z_i takes the value 1 if the driver parks the vehicle at the location of customer $i \in V'$ (hereafter called the *parking node*), and 0 otherwise.

Table 1 lists the parameters and the variables used in the model.

Table 1 A list of the parameters and the variables used in the model

Parameters	
c_{ij}^d	Driving time from node i to j on arc $(i, j) \in A$
c_{ij}^w	Walking time from node i to j on arc $(i, j) \in A$
α	Linear multiplier to the total driving time
Q_v	Maximum total volume that can be served on a given walking route
Q_w	Maximum total weight that can be served on a given walking route
$r(S)$	Number of walking routes required to serve the subset $S \subset V'$ of customers
Variables	
$x_{ij}^d \in \{0, 1\}$	Equals 1 if arc $(i, j) \in A$ is traversed by the vehicle, and 0 otherwise
$x_{ij}^w \in \{0, 1\}$	Equals 1 if arc $(i, j) \in A$ is traversed by walking, and 0 otherwise
$z_i \in \{0, 1\}$	Equals 1 if node $i \in V'$ is used as parking node, and 0 otherwise

The complete formulation is given as follows:

$$\text{Minimise} \quad \alpha \sum_{(i,j)\in A} c_{ij}^d x_{ij}^d + (1-\alpha) \sum_{(i,j)\in A} c_{ij}^w x_{ij}^w \tag{1}$$

subject to

$$\sum_{j\in V} x_{ij}^d + \sum_{j\in V'} x_{ij}^w = 1 + z_i \qquad \forall i \in V' \tag{2}$$

$$\sum_{i\in V} x_{0i}^d = 1 \tag{3}$$

$$\sum_{i\in V} x_{i0}^d = 1 \tag{4}$$

$$\sum_{i,j\in S} x_{ij}^d \leq |S| - 1 \qquad \forall S \subset V' \tag{5}$$

$$\sum_{j\in S} x_{ij}^d \geq z_i \qquad \forall i \in V' \tag{6}$$

$$\sum_{j\in V} x_{ij}^d = \sum_{j\in V} x_{ji}^d \qquad \forall i \in V' \tag{7}$$

$$\sum_{j\in S} x_{ij}^w \geq z_i \qquad \forall i \in V' \tag{8}$$

$$\sum_{j\in V'} x_{ij}^w = \sum_{j\in V'} x_{ji}^w \qquad \forall i \in V' \tag{9}$$

$$x_{ij}^w = x_{ji}^w = 0 \qquad \forall i \in V' \;\; v_i > Q_v \text{ or } w_i > Q_w \tag{10}$$

$$\sum_{i,j\in S} x_{ij}^w \leq |S| - r(S) + \sum_{i\in S} z_i \qquad \forall S \subset V' \tag{11}$$

$$x_{ij}^w + z_i + z_j \leq 2 \qquad \forall i, j \in V' \tag{12}$$

$$\sum_{k\in S} x^w_{ik} + \sum_{k,l\in S} x^w_{kl} + \sum_{k\in S} x^w_{kj} \leq |S| + 2 - z_i - z_j \qquad \forall S \subset V',\ S \neq \emptyset \tag{13}$$

$$i \in V' \setminus S, j \in V' \setminus S, i \neq j$$

$$x^d_{ij}, x^w_{ij} \in \{0, 1\} \qquad \forall (i, j) \in A \tag{14}$$

$$z_i \in \{0, 1\} \qquad \forall i \in V'. \tag{15}$$

The objective function (1) represents the total travel time parameterised with respect to α. For example, setting $\alpha = 0.5$ minimises the total travel time by giving equal importance to both driving and walking, which would effectively assume that the unit cost of driving is the same as the unit cost of walking. Constraints (2) impose degree restrictions on the nodes, with the added constraint that the parking nodes must be visited in both driving and walking routes. As a consequence of these constraints, we limit to one the number of walking routes based on the same parking node. Constraints (3) and (4) are relevant to the driving route ensuring that the vehicle leaves from and returns to the depot, where subtours are prevented by constraints (5). If a node is designated as a parking node, constraints (6) ensure that it is visited by the driver, for which route continuity is ensured by (7). Constraints (8) and (9) play a similar role for the walking routes, where customers with excess volume or weight are excluded by constraints (10). For a given subset S of customers, constraints (11) ensure that there are a sufficient number of parking nodes if they are served by a walking route. Finally, (12) and (13) are path elimination constraints that eliminate walking paths that do not start from and end at the same parking node.

Assuming that unit costs of walking and driving per unit time are available, the model above can be used to minimise the total costs (as opposed to time). In this case, the model should be run with different values of α, and after evaluating the cost of each resulting solution, the one with the minimum total cost should be chosen. This would entail solving the above formulation for as many times as the number of different values of α used. We provide numerical results on the way that the walking and driving times change with different values of α in Sect. 4.

3.2 Valid inequalities

This section presents two new sets of valid inequalities to enhance the algorithm that will be described later in this section. Both inequalities serve to improve the connectivity of two graphs, one induced by the driving and the other by the walking routes. To shed some light into the connectivity just mentioned, we present a fractional solution in Fig. 2 for an instance with seven nodes with two disconnected components, where driving (solid lines) and walking (dashed lines) takes place in each component. The solution is obtained at the root node of the branch-and-cut tree whilst solving the formulation above, where the integrality constraints (14) are relaxed, but where a number of subtour (5), walking capacity (11) and path elimination (12) and (13) constraints are added. The support graph in Fig. 2 shows that the component defined on nodes 1, 3, 4 and 5 is disconnected from the other component.

We now describe the new inequalities that are used to connect disconnected solutions such as the one shown in Fig. 2. Let $S \subset V'$ be a subset of customers with $|S| \geq 2$ and let $\gamma(S)$ denote the set of arcs with both endpoints in set S.

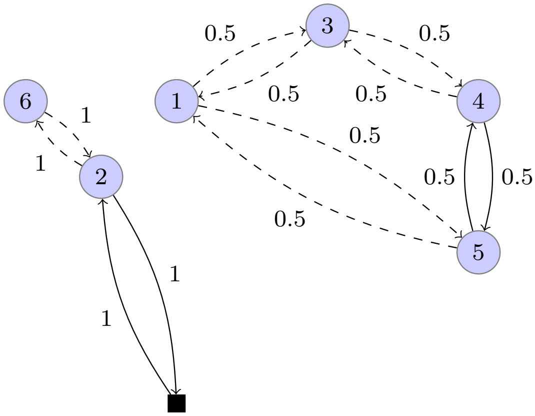

Fig. 2 A fractional solution where the numbers on the arcs correspond to the value of the x_{ij}^d variables (solid lines) and x_{ij}^w variables (dashed lines) in the fractional solution. In this solution, the subset $S^d = \{4, 5\}$ violates an extended subtour elimination constraint for the driving route and the subset $S^w = \{1, 3, 4, 5\}$ violates an extended subtour elimination constraint for the walking routes. Only node 2 is used as a rendezvous node, i.e., $z_2 = 1$

Proposition 1 *The following extended subtour inequalities for the driving routes are valid for the problem:*

$$\sum_{i \in S} \sum_{j \notin S} x_{ij}^d \geq \frac{x^d(\gamma(S))}{|S| - 1}. \tag{16}$$

Proof The inequality is trivially valid if $x^d(\gamma(S)) = 0$. If not, then $x^d(\gamma(S)) = \sum_{(i,j) \in S} x_{ij}^d \geq 1$, which means that the driver visits any customer in S from another customer in S, and which implies that there must be at least one arc from S to $V \setminus S$ used by the driver due to constraint (5). Furthermore, $x^d(\gamma(S)) \leq |S| - 1$ is always satisfied in any feasible solution where an arc with both endpoints in S is used by the driver, so the right hand side of (16) is always less than or equal to 1. □

Similar constraints on the arcs arriving to subset S can also be written as follows.

$$\sum_{i \notin S} \sum_{j \in S} x_{ij}^d \geq \frac{x^d(\gamma(S))}{|S| - 1}. \tag{17}$$

The extended subtour inequalities can also be written for the walking routes. In this case the z variables related to the parking nodes are included in the inequality since the number of arcs used in a walking route either leaving or arriving into subset $S \in V'$ will also depend on the number of nodes in S used as parking nodes.

Proposition 2 *The following extended subtour inequalities for the walking routes are valid for the problem:*

$$\sum_{i \in S} \sum_{j \notin S} x_{ij}^w + \sum_{i \in S} z_i \geq \frac{x^w(\gamma(S))}{|S| - r(S) + 1}, \tag{18}$$

$$\sum_{i \notin S} \sum_{j \in S} x_{ij}^w + \sum_{j \in S} z_j \geq \frac{x^w(\gamma(S))}{|S| - r(S) + 1}. \tag{19}$$

Proof We differentiate between the following cases:

- If all customers in S are served by the driver, i.e., no walking is required, then the right hand side of either inequality is equal to 0 and they are trivially satisfied.

– If there is a feasible closed walking path in S then $x^w(\gamma(S)) = |S|$ and $r(S) = 1$, so the right hand side of either inequality would take the value 1, which would in turn require the left hand side to attain the value of at least 1. This implies that at least one node of S is a parking node, or at least one arc either leaves S through (18) or arrives into S through (19), or both.
– If $r(S) > 1$, this means that at least two walking routes are required to serve the demand in S. In this case, the maximum value $\frac{|S|}{|S|+1-|S|/2}$ of the right hand side is given when $r(S) = |S|/2$. Then, the right hand side of either inequality could increase by at most $r(S)$, which would necessitate the use of an additional parking node, or an additional arc either leaving S through (18) or arriving into S through (19). In this case, the right hand side of either inequality is less than $|S|/2$ and the left hand side of either inequality should be greater than or equal to $|S|/2$ since that would be the minimum number of parking nodes needed.

□

It is worth highlighting that the denominator of the right hand side of constraints (18) and (19) is one unit greater than the denominator used in right hand side of (16) and (17) due to the fact that a closed route could be performed by walking in S, but not by driving. The driving must always start from and end at the depot.

Revisiting the example in Fig. 2, it can now be seen that the inequalities (16) and (17) are violated for $S = \{4, 5\}$, and the inequalities (18) and (19) are violated for $S = \{1, 3, 4, 5\}$.

3.3 Branch-and-cut algorithm

The branch-and-cut algorithm is designed to optimally solve the integer linear programming formulation presented in Sect. 3.1 and starts by solving the linear programming (LP) relaxation defined by the objective function (1) and constraints (2), (3), (4), (6), (7), (8), (9), (10). Constraints (5), (11), (13) and the integrality constraints (14) are relaxed at the root node; instead the x^w and x^d variables are restricted to be within the interval [0, 1].

Each node of the branching tree consists of a linear programme (LP) with a given set of constraints solved to optimality, and a set of separation routines which aim to find violated constraints. If a violated constraint is identified at a given node, it is added to the LP of that node and the LP is reoptimised. This process is repeated until the separation routines are not able to find any new violated inequality, following which the branch-and-cut algorithm will branch on a variable which violates the integrality constraints (14).

For a fractional solution $\bar{x}$ of the LP, we construct a support graph $\overline{G^w} = (V, \overline{A^w})$ for the walking routes where $(i, j) \in \overline{A^w}$ iff $\bar{x}^w_{ij} > 0$. Similarly, let $\overline{G^d} = (V, \overline{A^d})$ be the support graph for the driver where $(i, j) \in \overline{A^d}$ iff $\bar{x}^d_{ij} > 0$. At any node of the branching tree, one of the two cases below may arise, depending on whether the integrality conditions (14) are satisfied:

1. If the solution of an LP at a given node is integer, then we check if constraints (5), (11) and (13) are also satisfied. This can be done in linear time by shrinking the nodes in the support graphs $\overline{G^d}$ and $\overline{G^w}$. If no violated inequalities are identified, then the solution at that node would be feasible and no further branching would be needed. Otherwise, the LP is reoptimised after appending the violated inequality. We note that the addition of violated inequalities to the LP may once again make the solution fractional.
2. If the solution to the LP is fractional we use separation routines for the following inequalities:

(a) Subtour/capacity inequalities for the driving route (5).
(b) Capacity inequalities for the walking routes (11).
(c) Extended subtour inequalities for the driving and walking routes (18), (19), (16) and (17) as described in Sect. 3.2.
(d) Path elimination constraints (13).

The separation procedures used for the extended subtour elimination constraints are presented in Algorithms 1 and 2 for the driving and walking routes, respectively. In both cases we first compute the connected components of the support graph (line 2 in Algorithms 1 and 2) and check all the nodes of each connected component (lines 4–5 in Algorithms 1 and 2). Then we apply a greedy heuristic to build promising subsets S in order to identify new cuts (5), (16) and (17) for the driving route (lines 8–20 in Algorithms 1) and (11), (18) and (19) for the walking routes (lines 13–25 in Algorithm 2). Path elimination constraints for the walking routes (13) are checked only when the corresponding connected components are integer.

Algorithm 1 Finding violated extended subtour elimination constraints for the driving route

1: Compute G^d.
2: Compute the different connected components of $\overline{G}^d$.
3: **for** each connected component $\overline{G}^d_k$ of $\overline{G}^d$ **do**
4:     **if** nodes in $\overline{G}^d_k$ violate any capacity inequality (5) **then**
5:         Add inequality (5).
6:     **end if**
7:     $S \leftarrow \emptyset$
8:     **for** each node i considered in $\overline{G}^d_k$ **do**
9:         $S \leftarrow S \cup \{i\}$
10:         **while** Number of nodes in $\overline{G}^d_k$ is greater than $|S|$ **do**
11:             Compute $j' := \text{argmax}_{j \notin S} \sum_{l \in S}(x^d_{jl} + x^d_{lj})$
12:             $S \leftarrow S \cup \{j'\}$
13:             **if** inequality (5) with S is violated **then**
14:                 Add inequality (5).
15:             **end if**
16:             **if** inequalities (16) and (17) with S are violated **then**
17:                 Add violated inequalities.
18:             **end if**
19:         **end while**
20:     **end for**
21: **end for**

The separation routines shown in Algorithms 1 and 2 differ from the classical separation procedures in that one separation routine is used to find inequalities from different families. The main reason behind this idea is that we use several inequalities that use a subset S, which would need to satisfy similar conditions across the different inequalities. A detailed computational analysis of how the cutting plane approach used at each node of the branching tree enhances the branch-and-cut algorithm is presented in Sect. 4.

4 Computational experiments

The branch-and-cut algorithm was tested on a set of instances, extracted from a real data set of a courier operating in the Southwark area in London. The original data set contained

Algorithm 2 Finding violated extended subtour elimination constraints for the walking routes

Compute G^w.
Compute the different connected components of $\overline{G}^w$.
for each connected component $\overline{G}_k^w$ of $\overline{G}^w$ **do**
  if nodes in $\overline{G}_k^w$ violate any subtour inequality (11) **then**
    Add inequality (11).
  end if
  if All the arcs have a weight of 1 in $\overline{G}_k^w$ **then**
    if There is a path between two parking nodes **then**
      Add Inequality (13).
    end if
  end if
  $S \leftarrow \emptyset$
  for each node i considered in $\overline{G}_k^w$ **do**
    $S \leftarrow S \cup \{i\}$
    while Number of nodes in $\overline{G}_k^w$ is greater than $|S|$ **do**
      Compute $j' := \text{argmax}_{j \notin S} \sum_{l \in S}(x_{jl}^w + x_{lj}^w - z_{j'})$
      $S \leftarrow S \cup \{j'\}$
      if inequality (11) with S is violated **then**
        Add inequality (5).
      end if
      if inequalities (18) and (19) with S are violated **then**
        Add violated inequalities.
      end if
    end while
  end for
end for

information on 117 deliveries, including location of delivery, and volume and weight of each item delivered. The weights of the items ranged between 2 and 5 kg (with an average of 1.41 kg) and the volumes ranged between 20 and 82 L (with an average of 21.8 L), with Q_v = 200 L and Q_w = 25 kg. From this data set, we create 26 instances by randomly sampling between $|V'| = 5, 6, \ldots, 30$ unique locations, include the depot, and repeat this five times for each value of $|V'|$ to result in a total of $26 \times 5 = 130$ instances. Walking and driving times between all pairs of points were computed off-line using the Google Maps Distance Matrix API. The experiments were run using a computer with an Intel(R) Core (TM) i9-9980XE CPU at 3.00 GHz running Windows 10 with 64 bits and with 64 GB of RAM. The algorithm was coded in MVS2017 in which CPLEX 12.9 was used as the optimiser, and part of which included the use of the lazy and cut callback functions for Algorithms 1 and 2 . For comparison purposes, each instance was subject to a computational time limit of 3600 s. Testing was performed using the four following Branch-and-Cut (BC) solution strategies:

- *BC1* is a branch-and-bound algorithm that uses no cutting planes, and where the path elimination constraints (13) are separated only on integer nodes.
- *BC2* is the same as BC1 in which an initial solution (and a corresponding upper bound) is provided to the solver. The initial solution assumes that all deliveries are made by driving, obtained by the solution of the corresponding Travelling Salesman Problem, which can be solved efficiently using Concorde (http://www.math.uwaterloo.ca/tsp/concorde.html).
- *BC3* is a branch-and-cut algorithm, initialised with an upper bound as in BC2, and where Algorithms 1 and 2 are used at each node of the branching tree only for separating inequalities (5) and (11).
- *BC4* is similar to BC3 which additionally uses the extended subtour inequalities (16), (17), (18) and (19).

Table 2 Comparison results between the four different BC strategies showing average values of five instances per line with $\alpha = 0.9$

$\|V'\|$	Best UB	BC1		BC2		BC3		BC4	
		Time (sec)	Gap (%)	Time (sec)	Gap (%)	Time (sec)	Gap (%)	Time (sec)	Gap (%)
5	1881.80	**0.03**	0.00	**0.03**	0.00	**0.03**	0.00	**0.03**	0.00
6	1929.07	**0.02**	0.00	**0.02**	0.00	0.03	0.00	0.03	0.00
7	1936.49	0.05	0.00	**0.04**	0.00	0.07	0.00	0.05	0.00
8	1971.92	**0.06**	0.00	0.08	0.00	0.08	0.00	0.07	0.00
9	1953.01	**0.08**	0.00	0.10	0.00	0.12	0.00	0.12	0.00
10	1999.72	**0.18**	0.00	0.19	0.00	0.24	0.00	0.27	0.00
11	2004.19	0.33	0.00	**0.29**	0.00	0.47	0.00	0.33	0.00
12	1999.99	0.65	0.00	0.63	0.00	0.97	0.00	**0.47**	0.01
13	2047.71	**0.97**	0.01	1.00	0.01	1.64	0.00	1.05	0.00
14	2027.06	**0.57**	0.00	0.60	0.00	1.04	0.00	0.98	0.00
15	2060.28	3.00	0.01	**2.79**	0.01	6.74	0.01	3.42	0.00
16	2066.11	2.58	0.01	2.39	0.01	5.20	0.01	**2.04**	0.01
17	2057.80	7.70	0.01	7.25	0.01	8.17	0.01	**5.36**	0.01
18	2051.55	4.95	0.01	**4.72**	0.01	6.67	0.01	6.05	0.01
19	2115.53	293.86	0.01	295.74	0.01	**137.87**	0.01	188.59	0.01
20	2126.66	65.89	0.01	66.78	0.01	**44.23**	0.01	48.13	0.01
21	2070.62	970.32	0.12	976.29	0.12	840.82	0.04	**809.77**	0.05
22	2141.56	105.08	0.01	103.43	0.01	**59.29**	0.01	61.98	0.01
23	2110.42	161.31	0.01	163.02	0.01	45.35	0.01	**44.10**	0.01
24	2144.67	436.90	0.01	439.80	0.01	301.59	0.01	**169.98**	0.01
25	2161.42	840.59	0.01	927.09	0.01	317.30	0.01	**204.54**	0.01
26	2133.17	710.63	0.01	737.86	0.01	338.76	0.01	**182.57**	0.01
27	2161.12	2054.00	0.87	2060.74	0.88	**1603.76**	0.02	1647.38	0.10
28	2162.73	3345.74	2.22	3327.57	2.11	2904.94	0.94	**2671.69**	0.55
29	2138.92	1463.26	0.39	1368.96	0.36	1309.24	0.46	**369.22**	0.01
30	2215.70	3592.44	2.84	3594.41	2.63	3601.67	3.03	**3079.53**	1.98
	Average	540.82	0.25	541.61	0.24	443.70	0.18	**365.30**	**0.11**

Table 2 shows the computational results obtained with the four different strategies, all using the setting $\alpha = 0.9$, where each line is an average of five instances. The column 'Best UB' shows, the smallest average objective value obtained by any of the four strategies. The table also presents, for each strategy, the average computational time (in seconds) required to solve the corresponding instance under column 'Time', and the average final optimality gap (in percent) under column 'Gap'. An optimality gap less than 0.01% for a given strategy indicates that all five instances with $|V'|$ customers were solved to optimality within the time limit by that strategy. The smallest average solution times in each line are indicated in bold. Further details on each strategy, such as the number of cuts added, the number of nodes in the branch and bound tree, and the lower bounds obtained at the root node are presented in the appendix, for each individual instance.

Table 3 Computational results with BC4 and $\alpha \in \{0.6, 0.8, 0.9\}$

$\lvert V' \rvert$	$\alpha = 0.60$		$\alpha = 0.80$		$\alpha = 0.90$	
	Gap/Time	D/W	Gap/Time	D/W	Gap/Time	D/W
5	0.02	100.0/0.0	0.02	93.4/6.6	0.02	2.2/97.8
6	0.01	96.8/3.2	0.05	96.8/3.2	0.04	2.0/98.0
7	0.03	100.0/0.0	0.07	71.4/28.6	0.04	3.4/96.6
8	0.03	100.0/0.0	0.13	47.6/52.4	0.05	3.6/96.4
9	0.04	96.6/3.4	0.09	89.9/10.1	0.13	2.9/97.1
10	0.03	97.4/2.6	0.16	64.8/35.2	0.30	3.5/96.5
11	0.04	100.0/0.0	0.25	49.8/50.2	0.32	3.0/97.0
12	0.04	100.0/0.0	0.29	65.3/34.7	0.47	4.9/95.1
13	0.07	100.0/0.0	0.61	46.0/54.0	1.06	1.6/98.4
14	0.11	97.8/2.2	0.44	47.8/52.2	1.00	1.5/98.5
15	0.07	99.9/0.1	1.27	57.7/42.3	3.43	3.3/96.7
16	0.09	99.9/0.1	3.12	47.8/52.2	2.00	2.8/97.2
17	0.12	100.0/0.0	1.65	46.1/53.9	5.44	1.8/98.2
18	0.10	100.0/0.0	1.85	45.8/54.2	6.08	2.0/98.0
19	0.27	100.0/0.0	4.58	31.7/68.3	185.75	3.8/96.2
20	0.21	100.0/0.0	7.49	33.0/67.0	45.80	2.4/97.6
21	0.14	100.0/0.0	2.10	51.9/48.1	0.04%	1.8/98.2
22	0.18	97.2/2.8	2.69	48.6/51.4	61.14	3.3/96.7
23	0.52	99.9/0.1	20.29	30.9/69.1	44.61	2.2/97.8
24	0.11	100.0/0.0	6.72	36.3/63.7	167.52	2.2/97.8
25	0.32	100.0/0.0	27.49	35.4/64.6	201.58	1.5/98.5
26	0.36	100.0/0.0	24.34	27.8/72.2	175.97	1.2/98.8
27	0.48	98.2/1.8	6.79	29.0/71.0	0.10%	2.3/97.7
28	0.49	98.2/1.8	42.06	48.9/51.1	0.54%	4.3/95.7
29	0.34	100.0/0.0	12.71	51.7/48.3	333.33	2.7/97.3
30	0.62	100.0/0.0	244.99	30.1/69.9	1.97%	3.8/96.2

The results shown in Table 2 suggest that, for relatively small-sized instances (i.e., those with $|V'| \leq 14$), BC1 and BC2 provide optimal solutions quicker than the other variants, but BC3, and more so BC4 present a superior performance as the instance size increases, either in terms of the computational time or the final optimality gap. Whilst a great majority of instances have been optimally solved, the difficulty of solving others of up to 30 nodes suggests that the complexity of this problem is well beyond that of the VRP. Overall, BC4 yields the smallest average optimality gap and the fastest solution time) for all instances, for which reason it is used in the remainder of the experiments.

Next, we investigate the impact of changing the multiplier α on the total driving and walking times in the resulting solutions. To this end, we test the values in the set $\alpha \in \{0.6, 0.8, 0.9\}$. The results are given in Table 3, where each row shows the average results for the group of five instances with $|V'|$ customers. The column titled 'Gap/Time' shows either the average solution time (in seconds), or the average optimality gap (appended by '%') if optimality was not attained within the one hour time limit. For gaps that are strictly positive, the best solutions obtained were used to analyse the results. The column 'D/W' shows the percentage split between the total time spent driving (D) and that of walking (W) in the resulting solutions. To provide a meaningful figures, the stem driving time, defined as the time taken to drive (and back) from the depot to the closest customer and equal to 1897 s in our data set, has been excluded in the calculation of the driving time in all instances.

As Table 3 shows, all instances using the setting $\alpha = 0.6$ have been solved to optimality. In this case, the majority of instances yielded optimal solutions that only make use of driving, which corresponds to the initial solutions used in the BC algorithm for upper bounds. The amount of driving is reduced using the settings $\alpha = 0.8$ and $\alpha = 0.9$, but this increases the difficulty of solving the model to optimality. In the case of $\alpha = 0.9$, for example, the algorithm has failed to find optimal solutions for some instances with 21, 27, 28 and 30 customers, although the optimality gap for these instances remains within 2%, and generally below 1%. The results also indicate that, using $\alpha = 0.6$, an average of 99.3% of the deliveries are done by driving, which is reduced to 51% when $\alpha = 0.8$ and further to 2.7% when $\alpha = 0.9$, showing the significant effect that the parameter α has on the resulting solutions.

For illustrative purposes, we provide a sample visualisation of solutions for an instance with 28 addresses in Fig. 3 using different values for α, namely 0.6, 0.8 and 0.9. It should be kept in mind that the routes shown in the figure are designed with respect to the capacity and volume restrictions of the items and based on street-level distance matrices.

5 Conclusions

This paper proposes that a combined driving and walking delivery system could be a viable alternative to last-mile delivery problems in urban areas to mitigate the adverse effects of more traditional means of distribution, such as traffic congestion and use of kerbside space. Our primary contribution has been to describe such a system, to propose a mathematical model along with valid inequalities, and an optimisation algorithm to prescribe solutions for such a system to operate in practice. Computational experience suggests that, for instances of up to 30 customers, the algorithm is able to either provide optimal solutions, or those that are within a reasonable gap to the optimal value. The results also suggest that the complexity of the problem increases not only with an increased number of deliveries but also when the relative weighting of one mode over the other changes. In particular, the problem becomes more difficult to solve with increased amount of walking, which may be explained by the additional constraints, such as weight and volume restrictions, required to model the walking paths.

5.1 Practical considerations

One practical consideration that may arise in the implementation of the proposed model relates to the application of the model to larger areas. In reality, carriers divide up geographical areas into sub-rounds which consist of postcode patches that are linked together by the road network. Once this segmentation is undertaken, each patch and their postcode constituent parts would be identified to create rounds using the proposed system for a single driver operating on that patch. The specific elements of the round would lend to walking routes where drop-off points for the van would be identified, and from where the walking routes would start and finish.

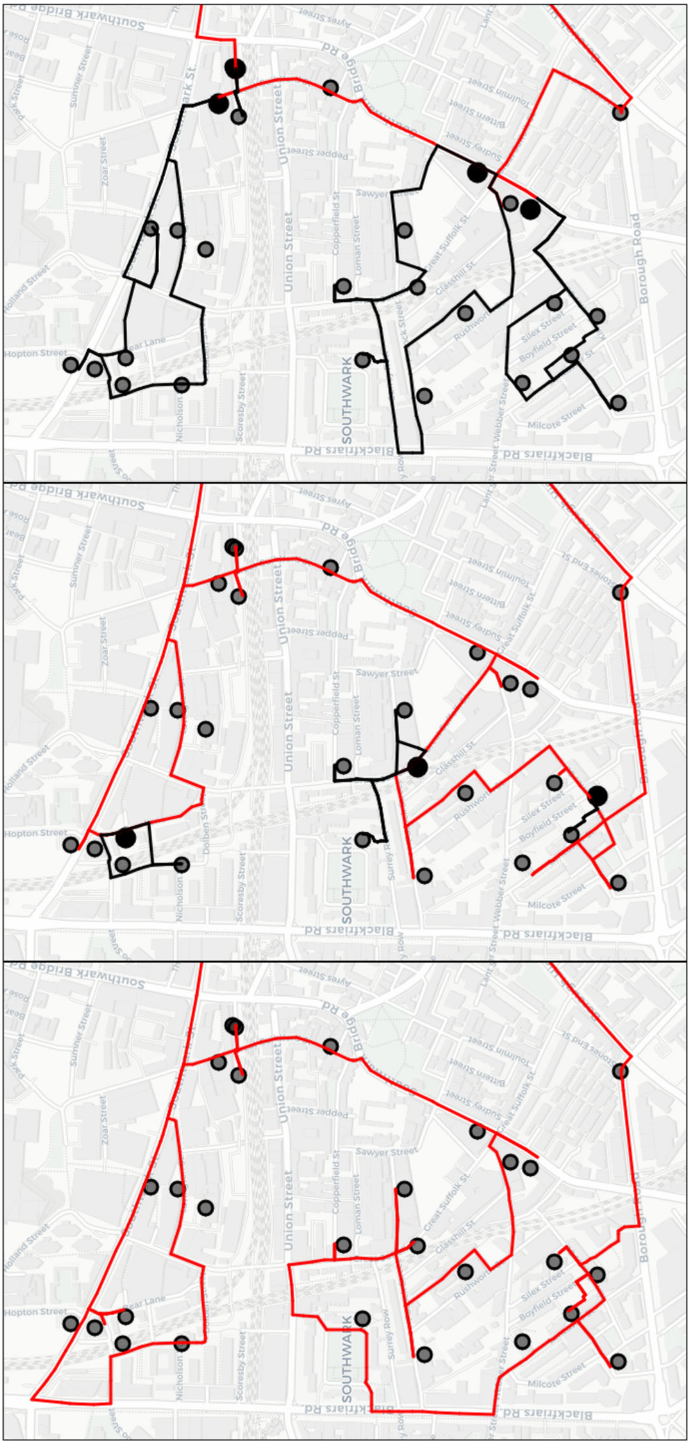

Fig. 3 Visualisation of the best solutions found for an instance with 28 customers, using the settings $\alpha = 0.6$ (left), $\alpha = 0.8$ (middle) and $\alpha = 0.9$ (right). The shaded dots represent the customer locations and the filled dots indicate the parking nodes. Red lines correspond to driving and black lines show the walking routes. The depot, located further east, is not depicted in the figures. (Leaflet, ©OpenStreetMap contributors, ©CartoDB)

One other practical aspect concerns the walking routes. Practical work undertaken by the authors (Allen et al. 2018) suggested that many drivers preferred walking as opposed to continually moving the vehicle very small distances while trying to find parking spaces. This crucially depends on the average size of parcel being delivered as the number of trips needed back and forth to the van will increase as package size increases. Our previous surveys undertaken in London showed drivers walking up to 10 km on a round. As deliveries were all within a relatively small area, it would be feasible to walk the entire round. For this reason, the amount of walking undertaken was not constrained for the application context considered here, particularly as the objective of the problem, in part, serves to minimise the length of the walking tours. Also, for larger areas (and less dense deliveries), driving would naturally take precedence as being a faster mode of delivery over longer distances. However, it is possible to introduce constraints into the model, if the walking distances were considered to be too great.

5.2 Future research directions

The nature of day-to-day parcel delivery operations within urban areas present further challenges that may form the basis of future work. First, and further to the discussion in Sect. 2, it is possible to introduce time window constraints into the model described in this paper to account for time-sensitive parcels. Such constraints, however, may result in non-linearities for particular types of formulations (Nguyên et al. 2019), and whilst they can be linearised, the additional constraints needed can significantly increase the complexity of solving the model to optimality. Alternative formulations or solution methods that can effectively deal with time window constraints is one possible avenue for further research. Second, traffic conditions within urban areas present a degree of uncertainty which impacts on driving times, whereas there is very little uncertainty associated with walking times. Incorporating such partial stochasticity falls within the class of stochastic vehicle routing problems (Gendreau et al. 2016), which are considerably more difficult to solve compared to their deterministic counterparts. Third, whilst a great majority of the deliveries are known and planned for ahead of the actual operations, there are occasional requests (e.g. collections) that dynamically arise. Accounting for such requests would result in a dynamic routing problem (Psaraftis et al. 2016), which is also significantly harder to solve than the static variants. Fourth, whereas capacity limitations have been explicitly represented in our model, this has assumed a two-dimensional treatment of the parcels. Practical requirements may dictate that both the weight and the volume of the parcels would need to be explicitly represented, and that capacity is modelled as a three-dimensional constraint. Finally, the problem as described in this paper can be extended to allow multiple vehicles for applications in larger areas.

Acknowledgements Thanks are due to three anonymous reviewers for their helpful comments on an earlier version of this paper. The authors gratefully acknowledge the EPSRC for funding this work through its financial support of Freight Traffic Control 2050 (www.ftc2050.com), EPSRC Grant Reference: EP/N02222X/1. We thank Mr Andrew Oakey for his assistance in the collection and analysis of data used in the paper.

Appendix

This section presents further details on the computational tests for the different strategies BC1, BC2, BC3 and BC4 (Tables 4, 5, 6, 7) used within the branch-and-cut algorithm as explained in Sect. 4, using $\alpha = 0.9$. For a given $|V'|$, five instances are generated, where the name of each instance follows the convention $\texttt{n}|V'|\texttt{\_r}\theta$ and where $\theta = 0, \ldots, 4$ is the instance number.

In each of the (Tables 4, 5, 6, 7), we report the total number of nodes explored in the branch-and-cut tree (Nodes), the total runtime of the algorithm (Time), the total separation time (S-Time), the total number of cuts added (Ncuts), the total number of cuts at the root node (Ncuts0), the total amount of driving (Driving, in seconds), the best lower bound (LB) and the best upper bound (UB) upon termination of the algorithm, and the final percentage optimality gap (Gap).

In Table 8 we present the full results for solving all instances with BC4, and where the value of α is varied. The results include the time for walking and driving in the solutions obtained, and include the stem driving times.

Table 4 Detailed computational results for BC1

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n5_r0	5	0	0.03	0.00	5	5	1916.30	1902.43	1902.43	0.00
n5_r1	5	0	0.02	0.00	6	6	1918.50	1867.78	1867.78	0.00
n5_r2	5	0	0.02	0.00	4	4	1918.50	1892.04	1892.04	0.00
n5_r3	5	0	0.05	0.00	8	8	1915.90	1884.92	1884.92	0.00
n5_r4	5	0	0.02	0.00	4	4	1976.40	1861.82	1861.82	0.00
n6_r0	6	0	0.00	0.00	8	8	1932.50	1925.39	1925.39	0.00
n6_r1	6	0	0.03	0.00	5	5	1918.50	1924.73	1924.73	0.00
n6_r2	6	1	0.03	0.00	7	7	1971.40	1967.81	1967.81	0.00
n6_r3	6	0	0.00	0.00	8	8	1935.90	1933.08	1933.08	0.00
n6_r4	6	0	0.02	0.00	3	3	1915.90	1894.35	1894.35	0.00
n7_r0	7	0	0.02	0.00	7	7	1932.50	1949.73	1949.73	0.00
n7_r1	7	28	0.11	0.02	14	10	1949.60	1942.76	1942.76	0.00
n7_r2	7	0	0.03	0.00	9	9	1950.90	1905.62	1905.62	0.00
n7_r3	7	104	0.06	0.02	33	18	2042.40	1973.75	1973.75	0.00
n7_r4	7	0	0.03	0.02	9	9	1915.90	1910.59	1910.59	0.00
n8_r0	8	75	0.11	0.00	38	20	1981.50	1945.33	1945.33	0.00
n8_r1	8	0	0.00	0.00	6	6	1917.70	1919.84	1919.84	0.00
n8_r2	8	0	0.03	0.02	9	9	1917.70	1940.40	1940.40	0.00
n8_r3	8	3	0.06	0.02	15	15	1985.50	2013.24	2013.24	0.00
n8_r4	8	14	0.09	0.03	33	32	2048.90	2040.79	2040.79	0.00
n9_r0	9	0	0.08	0.00	11	11	1963.20	1990.85	1990.85	0.00
n9_r1	9	26	0.08	0.00	25	14	2038.90	1977.83	1977.83	0.00
n9_r2	9	0	0.03	0.00	11	11	1918.50	1917.50	1917.50	0.00
n9_r3	9	0	0.08	0.00	12	12	1918.50	1946.38	1946.38	0.00
n9_r4	9	83	0.14	0.02	29	21	1937.50	1932.48	1932.48	0.00

Table 4 continued

Inst	$\lvert V' \rvert$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n10_r0	10	326	0.20	0.03	99	34	1941.10	1974.98	1975.11	0.01
n10_r1	10	58	0.14	0.00	26	20	2025.70	2038.03	2038.03	0.00
n10_r2	10	90	0.20	0.00	57	36	1953.10	1979.41	1979.41	0.00
n10_r3	10	194	0.22	0.02	53	26	2029.30	2005.38	2005.38	0.00
n10_r4	10	17	0.16	0.02	36	28	1933.30	2000.65	2000.65	0.00
n11_r0	11	8	0.25	0.00	22	19	2006.90	1994.25	1994.25	0.00
n11_r1	11	0	0.06	0.02	17	17	1926.10	1981.00	1981.00	0.00
n11_r2	11	116	0.20	0.02	52	26	1933.70	1961.68	1961.68	0.00
n11_r3	11	98	0.19	0.03	53	39	2010.60	2039.52	2039.59	0.00
n11_r4	11	2255	0.94	0.06	166	20	1963.20	2044.22	2044.42	0.01
n12_r0	12	16	0.14	0.00	25	22	1916.30	1963.06	1963.06	0.00
n12_r1	12	124	0.20	0.03	52	32	1980.80	1960.20	1960.35	0.01
n12_r2	12	1947	1.50	0.09	265	38	2082.90	2064.61	2064.72	0.01
n12_r3	12	48	0.17	0.05	68	35	1960.70	1983.86	1983.86	0.00
n12_r4	12	2875	1.23	0.06	254	29	2060.20	2027.74	2027.94	0.01
n13_r0	13	2435	1.36	0.16	353	42	1979.40	2059.75	2059.85	0.00
n13_r1	13	699	0.77	0.06	149	34	1935.90	2048.28	2048.47	0.01
n13_r2	13	718	0.52	0.02	87	17	1918.50	2027.01	2027.01	0.00
n13_r3	13	2967	1.77	0.09	267	63	1978.00	2049.13	2049.32	0.01
n13_r4	13	540	0.42	0.02	96	20	1919.10	2053.73	2053.89	0.01
n14_r0	14	1041	0.75	0.00	134	18	1955.40	2066.01	2066.15	0.01
n14_r1	14	220	0.36	0.00	74	36	1948.90	1981.27	1981.45	0.01
n14_r2	14	51	0.28	0.02	70	59	1916.30	1997.81	1997.81	0.00
n14_r3	14	1480	1.14	0.06	161	42	1963.20	2071.65	2071.79	0.01

Table 4 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n14_r4	14	202	0.31	0.00	82	19	1918.50	2018.11	2018.11	0.00
n15_r0	15	1485	1.53	0.02	167	38	1958.20	2040.67	2040.87	0.01
n15_r1	15	874	1.22	0.03	162	26	2019.70	2101.99	2102.16	0.01
n15_r2	15	1127	1.23	0.03	192	27	2032.30	2037.75	2037.87	0.01
n15_r3	15	3337	2.80	0.03	237	38	1943.90	2039.05	2039.25	0.01
n15_r4	15	12196	8.22	0.06	434	35	1986.50	2081.05	2081.25	0.01
n16_r0	16	531	0.84	0.00	75	52	1969.60	2027.61	2027.74	0.01
n16_r1	16	3913	4.72	0.14	720	55	2081.00	2097.80	2098.00	0.01
n16_r2	16	2576	2.42	0.11	232	33	1948.90	2055.38	2055.54	0.01
n16_r3	16	1375	1.70	0.08	206	50	1951.50	2104.13	2104.27	0.01
n16_r4	16	4039	3.20	0.09	263	22	1941.10	2044.80	2045.00	0.01
n17_r0	17	1323	1.56	0.06	198	32	1919.10	2018.12	2018.14	0.00
n17_r1	17	4042	4.50	0.09	384	41	1933.70	2070.66	2070.82	0.01
n17_r2	17	3774	3.95	0.03	317	41	1935.50	2074.70	2074.76	0.00
n17_r3	17	17317	16.67	0.16	689	32	1940.70	2075.49	2075.68	0.01
n17_r4	17	12134	11.81	0.09	544	46	2032.90	2049.44	2049.62	0.01
n18_r0	18	4914	3.84	0.06	326	75	1985.50	2056.72	2056.91	0.01
n18_r1	18	2740	2.61	0.05	251	43	1979.60	2019.97	2020.16	0.01
n18_r2	18	8497	11.80	0.14	772	43	1950.90	2084.32	2084.51	0.01
n18_r3	18	2274	2.83	0.02	298	40	1941.10	2082.60	2082.79	0.01
n18_r4	18	4030	3.69	0.05	362	42	1919.10	2013.17	2013.37	0.01
n19_r0	19	8171	9.84	0.06	508	36	1976.40	2060.91	2061.11	0.01
n19_r1	19	207659	1118.22	0.61	4234	51	1941.10	2148.13	2148.34	0.01
n19_r2	19	67393	141.16	0.25	1582	40	1988.80	2152.21	2152.42	0.01
n19_r3	19	2953	3.25	0.00	200	57	2096.20	2117.89	2118.09	0.01

Table 4 continued

Inst	$\lvert V' \rvert$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n19_r4	19	71900	196.83	0.64	2446	57	2077.20	2097.47	2097.68	0.01
n20_r0	20	6338	8.98	0.11	510	30	1980.80	2122.12	2122.33	0.01
n20_r1	20	12268	11.28	0.05	307	44	1949.70	2091.32	2091.53	0.01
n20_r2	20	14779	23.33	0.09	698	46	1999.80	2140.62	2140.82	0.01
n20_r3	20	14926	26.80	0.16	855	52	1985.50	2157.41	2157.63	0.01
n20_r4	20	83223	259.06	0.56	1847	41	1985.50	2120.79	2121.00	0.01
n21_r0	21	694064	3585.53	1.12	5716	43	1941.10	2094.81	2106.85	0.57
n21_r1	21	454141	1028.92	0.48	1997	46	1940.70	2099.89	2100.10	0.01
n21_r2	21	5342	7.47	0.16	606	42	1965.80	2074.85	2075.06	0.01
n21_r3	21	5289	6.73	0.03	367	47	1966.20	2078.30	2078.48	0.01
n21_r4	21	83934	222.95	0.33	1760	35	1960.70	1992.41	1992.61	0.01
n22_r0	22	59244	158.44	0.31	1536	48	2025.70	2152.26	2152.47	0.01
n22_r1	22	29195	69.22	0.31	1451	54	1995.40	2125.14	2125.35	0.01
n22_r2	22	53292	83.91	0.27	864	49	1940.70	2100.91	2101.12	0.01
n22_r3	22	36745	101.09	0.38	1486	51	1972.20	2144.33	2144.54	0.01
n22_r4	22	47829	112.72	0.34	1074	60	2114.80	2184.13	2184.34	0.01
n23_r0	23	25860	64.55	0.22	886	108	1940.70	2106.42	2106.63	0.01
n23_r1	23	63648	269.59	0.36	1784	49	1941.10	2131.06	2131.27	0.01
n23_r2	23	59437	271.42	0.64	2136	52	1985.50	2105.54	2105.75	0.01
n23_r3	23	55217	164.17	0.25	1762	53	1935.50	2093.94	2094.15	0.01
n23_r4	23	22366	36.83	0.17	714	52	2058.60	2114.07	2114.28	0.01
n24_r0	24	136587	645.47	0.62	2122	56	1943.90	2157.59	2157.80	0.01
n24_r1	24	74158	232.03	0.22	1641	65	2015.10	2185.20	2185.41	0.01
n24_r2	24	12222	19.55	0.22	629	39	1918.50	2065.41	2065.60	0.01
n24_r3	24	129123	955.84	0.50	3278	72	1955.00	2168.68	2168.89	0.01
n24_r4	24	69199	331.62	0.41	1855	58	2066.50	2145.64	2145.64	0.00

Table 4 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n25_r0	25	84760	336.09	0.39	1540	57	1979.60	2169.38	2169.60	0.01
n25_r1	25	52825	159.83	0.20	1604	35	1943.90	2123.22	2123.43	0.01
n25_r2	25	251248	2096.56	0.67	4798	42	1935.50	2195.75	2195.97	0.01
n25_r3	25	304961	1466.52	0.58	2381	43	1999.80	2155.03	2155.25	0.01
n25_r4	25	43602	143.97	0.22	1323	44	1935.20	2162.63	2162.84	0.01
n26_r0	26	55132	279.36	0.62	3261	45	1978.00	2173.49	2173.71	0.01
n26_r1	26	116290	412.09	0.16	1279	55	1943.90	2153.01	2153.23	0.01
n26_r2	26	35656	109.47	0.38	1287	68	1918.50	2089.05	2089.26	0.01
n26_r3	26	46560	101.83	0.19	995	64	1943.90	2127.26	2127.47	0.01
n26_r4	26	267899	2650.41	0.83	3944	43	1935.20	2121.96	2122.17	0.01
n27_r0	27	241733	3590.31	0.98	5654	53	2098.10	2165.80	2216.70	2.30
n27_r1	27	105351	550.72	0.41	1918	68	1941.10	2179.34	2179.56	0.01
n27_r2	27	129964	1041.62	0.81	2816	50	2025.90	2150.82	2151.03	0.01
n27_r3	27	237489	1499.31	0.73	2173	46	1918.50	2130.11	2130.32	0.01
n27_r4	27	246805	3588.05	1.34	6442	44	2131.30	2097.85	2140.99	2.01
n28_r0	28	324835	3585.11	1.17	5996	49	2173.50	2140.91	2237.90	4.33
n28_r1	28	346293	2385.33	0.25	2286	48	2096.70	2132.22	2132.43	0.01
n28_r2	28	315744	3591.36	0.80	4181	54	1941.10	2147.32	2157.28	0.46
n28_r3	28	304424	3584.41	1.17	6122	42	2128.80	2115.24	2188.19	3.33
n28_r4	28	269193	3582.50	1.27	6744	40	1984.30	2078.55	2141.56	2.94
n29_r0	29	73944	560.56	0.72	3211	43	1943.90	2099.33	2099.54	0.01
n29_r1	29	25806	87.92	0.39	1595	98	1918.50	2085.72	2085.93	0.01
n29_r2	29	346133	3594.48	0.64	3181	68	2143.10	2157.42	2199.63	1.92

Table 4 continued

Inst	$\lvert V' \rvert$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n29_r3	29	94955	380.80	0.31	1406	53	1976.50	2150.03	2150.24	0.01
n29_r4	29	328574	2692.53	0.34	1726	54	1985.50	2165.40	2165.62	0.01
n30_r0	30	348508	3585.98	1.20	5945	55	2138.60	2163.37	2285.20	5.33
n30_r1	30	284274	3590.09	0.61	3390	67	1976.50	2161.85	2186.39	1.12
n30_r2	30	250726	3588.77	1.17	4745	42	2135.70	2136.07	2195.13	2.69
n30_r3	30	387307	3605.77	0.77	3765	47	2153.90	2127.94	2211.21	3.77
n30_r4	30	300900	3591.59	0.72	4343	51	2019.30	2176.39	2204.98	1.30
Avg.		68767.31	540.82	0.25	1205.91	38.04				0.25

Table 5 Detailed computational results for BC2

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n5_r0	5	0	0.02	0.00	5	5	1916.30	1902.43	1902.43	0.00
n5_r1	5	0	0.05	0.00	6	6	1918.50	1867.78	1867.78	0.00
n5_r2	5	0	0.02	0.00	4	4	1918.50	1892.04	1892.04	0.00
n5_r3	5	0	0.05	0.00	8	8	1915.90	1884.92	1884.92	0.00
n5_r4	5	0	0.03	0.00	4	4	1976.40	1861.82	1861.82	0.00
n6_r0	6	0	0.03	0.02	8	8	1932.50	1925.39	1925.39	0.00
n6_r1	6	0	0.00	0.00	5	5	1918.50	1924.73	1924.73	0.00
n6_r2	6	1	0.00	0.00	7	7	1971.40	1967.81	1967.81	0.00
n6_r3	6	0	0.05	0.02	8	8	1935.90	1933.08	1933.08	0.00
n6_r4	6	0	0.00	0.00	3	3	1915.90	1894.35	1894.35	0.00
n7_r0	7	0	0.02	0.00	7	7	1932.50	1949.73	1949.73	0.00
n7_r1	7	28	0.08	0.00	14	10	1949.60	1942.76	1942.76	0.00
n7_r2	7	0	0.03	0.00	9	9	1950.90	1905.62	1905.62	0.00
n7_r3	7	104	0.08	0.02	33	18	2042.40	1973.75	1973.75	0.00
n7_r4	7	0	0.02	0.00	9	9	1915.90	1910.59	1910.59	0.00
n8_r0	8	75	0.16	0.02	38	20	1981.50	1945.33	1945.33	0.00
n8_r1	8	0	0.03	0.00	6	6	1917.70	1919.84	1919.84	0.00
n8_r2	8	0	0.05	0.02	9	9	1917.70	1940.40	1940.40	0.00
n8_r3	8	3	0.03	0.00	15	15	1985.50	2013.24	2013.24	0.00
n8_r4	8	14	0.12	0.00	33	32	2048.90	2040.79	2040.79	0.00
n9_r0	9	0	0.11	0.02	11	11	1963.20	1990.85	1990.85	0.00
n9_r1	9	26	0.12	0.02	25	14	2038.90	1977.83	1977.83	0.00
n9_r2	9	0	0.05	0.00	11	11	1918.50	1917.50	1917.50	0.00
n9_r3	9	0	0.08	0.00	12	12	1918.50	1946.38	1946.38	0.00
n9_r4	9	83	0.14	0.02	29	21	1937.50	1932.48	1932.48	0.00

Table 5 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n10_r0	10	326	0.31	0.02	99	34	1941.10	1974.98	1975.11	0.01
n10_r1	10	58	0.12	0.02	26	20	2025.70	2038.03	2038.03	0.00
n10_r2	10	90	0.22	0.03	57	36	1953.10	1979.41	1979.41	0.00
n10_r3	10	194	0.14	0.02	53	26	2029.30	2005.38	2005.38	0.00
n10_r4	10	17	0.16	0.00	36	28	1933.30	2000.65	2000.65	0.00
n11_r0	11	8	0.16	0.02	22	19	2006.90	1994.25	1994.25	0.00
n11_r1	11	0	0.03	0.00	17	17	1926.10	1981.00	1981.00	0.00
n11_r2	11	116	0.14	0.00	52	26	1933.70	1961.68	1961.68	0.00
n11_r3	11	98	0.22	0.03	53	39	2010.60	2039.52	2039.59	0.00
n11_r4	11	2255	0.91	0.08	166	20	1963.20	2044.22	2044.42	0.01
n12_r0	12	16	0.14	0.00	25	22	1916.30	1963.06	1963.06	0.00
n12_r1	12	124	0.20	0.00	52	32	1980.80	1960.20	1960.35	0.01
n12_r2	12	1947	1.28	0.08	265	38	2082.90	2064.61	2064.72	0.01
n12_r3	12	48	0.22	0.02	68	35	1960.70	1983.86	1983.86	0.00
n12_r4	12	2875	1.31	0.05	254	29	2060.20	2027.74	2027.94	0.01
n13_r0	13	2435	1.55	0.06	353	42	1979.40	2059.75	2059.85	0.00
n13_r1	13	699	0.72	0.05	149	34	1935.90	2048.28	2048.47	0.01
n13_r2	13	718	0.59	0.02	87	17	1918.50	2027.01	2027.01	0.00
n13_r3	13	2967	1.81	0.05	267	63	1978.00	2049.13	2049.32	0.01
n13_r4	13	540	0.34	0.03	96	20	1919.10	2053.73	2053.89	0.01
n14_r0	14	1041	0.84	0.03	134	18	1955.40	2066.01	2066.15	0.01
n14_r1	14	220	0.34	0.00	74	36	1948.90	1981.27	1981.45	0.01
n14_r2	14	51	0.36	0.03	70	59	1916.30	1997.81	1997.81	0.00
n14_r3	14	1480	1.11	0.02	161	42	1963.20	2071.65	2071.79	0.01
n14_r4	14	202	0.33	0.02	82	19	1918.50	2018.11	2018.11	0.00

Table 5 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n15_r0	15	1485	1.53	0.03	167	38	1958.20	2040.67	2040.87	0.01
n15_r1	15	874	1.12	0.03	162	26	2019.70	2101.99	2102.16	0.01
n15_r2	15	1127	1.22	0.02	192	27	2032.30	2037.75	2037.87	0.01
n15_r3	15	3337	2.31	0.05	237	38	1943.90	2039.05	2039.25	0.01
n15_r4	15	12196	7.75	0.12	434	35	1986.50	2081.05	2081.25	0.01
n16_r0	16	531	0.64	0.00	75	52	1969.60	2027.61	2027.74	0.01
n16_r1	16	3913	4.61	0.14	720	55	2081.00	2097.80	2098.00	0.01
n16_r2	16	2576	2.23	0.08	232	33	1948.90	2055.38	2055.54	0.01
n16_r3	16	1375	1.56	0.06	206	50	1951.50	2104.13	2104.27	0.01
n16_r4	16	4039	2.89	0.06	263	22	1941.10	2044.80	2045.00	0.01
n17_r0	17	1323	1.47	0.00	198	32	1919.10	2018.12	2018.14	0.00
n17_r1	17	4042	4.42	0.16	384	41	1933.70	2070.66	2070.82	0.01
n17_r2	17	3774	3.69	0.09	317	41	1935.50	2074.70	2074.76	0.00
n17_r3	17	17317	15.52	0.11	689	32	1940.70	2075.49	2075.68	0.01
n17_r4	17	12134	11.14	0.12	544	46	2032.90	2049.44	2049.62	0.01
n18_r0	18	4914	3.69	0.06	326	75	1985.50	2056.72	2056.91	0.01
n18_r1	18	2740	2.42	0.00	251	43	1979.60	2019.97	2020.16	0.01
n18_r2	18	8497	11.28	0.22	772	43	1950.90	2084.32	2084.51	0.01
n18_r3	18	2274	2.70	0.05	298	40	1941.10	2082.60	2082.79	0.01
n18_r4	18	4030	3.50	0.12	362	42	1919.10	2013.17	2013.37	0.01
n19_r0	19	8171	9.22	0.14	508	36	1976.40	2060.91	2061.11	0.01
n19_r1	19	207659	1123.62	0.97	4234	51	1941.10	2148.13	2148.34	0.01
n19_r2	19	67393	142.50	0.25	1582	40	1988.80	2152.21	2152.42	0.01
n19_r3	19	2953	3.16	0.05	200	57	2096.20	2117.89	2118.09	0.01
n19_r4	19	71900	200.19	0.69	2446	57	2077.20	2097.47	2097.68	0.01

Table 5 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n20_r0	20	6338	9.02	0.09	510	30	1980.80	2122.12	2122.33	0.01
n20_r1	20	12268	11.38	0.05	307	44	1949.70	2091.32	2091.53	0.01
n20_r2	20	14779	23.28	0.12	698	46	1999.80	2140.62	2140.82	0.01
n20_r3	20	14926	27.19	0.17	855	52	1985.50	2157.41	2157.63	0.01
n20_r4	20	83223	263.06	0.38	1847	41	1985.50	2120.79	2121.00	0.01
n21_r0	21	667101	3584.61	0.88	5694	43	1941.10	2094.51	2106.85	0.59
n21_r1	21	454141	1054.73	0.47	1997	46	1940.70	2099.89	2100.10	0.01
n21_r2	21	5342	7.66	0.17	606	42	1965.80	2074.85	2075.06	0.01
n21_r3	21	5289	6.62	0.06	367	47	1966.20	2078.30	2078.48	0.01
n21_r4	21	83934	227.83	0.47	1760	35	1960.70	1992.41	1992.61	0.01
n22_r0	22	59244	158.08	0.34	1536	48	2025.70	2152.26	2152.47	0.01
n22_r1	22	29195	66.56	0.34	1451	54	1995.40	2125.14	2125.35	0.01
n22_r2	22	53292	79.09	0.25	864	49	1940.70	2100.91	2101.12	0.01
n22_r3	22	36745	99.78	0.31	1486	51	1972.20	2144.33	2144.54	0.01
n22_r4	22	47829	113.62	0.27	1074	60	2114.80	2184.13	2184.34	0.01
n23_r0	23	25860	65.08	0.14	886	108	1940.70	2106.42	2106.63	0.01
n23_r1	23	63648	272.20	0.31	1784	49	1941.10	2131.06	2131.27	0.01
n23_r2	23	59437	275.69	0.59	2136	52	1985.50	2105.54	2105.75	0.01
n23_r3	23	55217	165.08	0.44	1762	53	1935.50	2093.94	2094.15	0.01
n23_r4	23	22366	37.03	0.20	714	52	2058.60	2114.07	2114.28	0.01
n24_r0	24	136587	650.56	0.67	2122	56	1943.90	2157.59	2157.80	0.01
n24_r1	24	74158	234.36	0.30	1641	65	2015.10	2185.20	2185.41	0.01
n24_r2	24	12222	19.89	0.11	629	39	1918.50	2065.41	2065.60	0.01
n24_r3	24	129123	969.58	0.58	3278	72	1955.00	2168.68	2168.89	0.01

Table 5 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n24_r4	24	69199	324.62	0.56	1855	58	2066.50	2145.64	2145.64	0.00
n25_r0	25	84760	348.44	0.30	1540	57	1979.60	2169.38	2169.60	0.01
n25_r1	25	52825	176.52	0.34	1604	35	1943.90	2123.22	2123.43	0.01
n25_r2	25	251248	2343.88	0.89	4798	42	1935.50	2195.75	2195.97	0.01
n25_r3	25	304961	1614.28	0.59	2381	43	1999.80	2155.03	2155.25	0.01
n25_r4	25	43602	152.36	0.36	1323	44	1935.20	2162.63	2162.84	0.01
n26_r0	26	55132	289.66	0.81	3261	45	1978.00	2173.49	2173.71	0.01
n26_r1	26	116290	420.16	0.14	1279	55	1943.90	2153.01	2153.23	0.01
n26_r2	26	35656	111.66	0.27	1287	68	1918.50	2089.05	2089.26	0.01
n26_r3	26	46560	104.34	0.20	995	64	1943.90	2127.26	2127.47	0.01
n26_r4	26	267899	2763.47	0.86	3944	43	1935.20	2121.96	2122.17	0.01
n27_r0	27	231836	3589.55	1.05	5619	53	2082.00	2163.03	2215.21	2.36
n27_r1	27	105351	560.08	0.39	1918	68	1941.10	2179.34	2179.56	0.01
n27_r2	27	129964	1065.00	0.48	2816	50	2025.90	2150.82	2151.03	0.01
n27_r3	27	237489	1504.34	0.66	2173	46	1918.50	2130.11	2130.32	0.01
n27_r4	27	242821	3584.73	1.11	6390	44	2131.30	2097.48	2140.99	2.03
n28_r0	28	345840	3586.30	0.83	6158	49	2173.50	2143.29	2237.90	4.23
n28_r1	28	346293	2280.66	0.39	2286	48	2096.70	2132.22	2132.43	0.01
n28_r2	28	372636	3599.73	0.66	4242	54	1941.10	2149.57	2157.28	0.36
n28_r3	28	327036	3586.91	1.16	6286	42	2128.80	2118.26	2188.19	3.20
n28_r4	28	289956	3584.25	1.27	6777	40	1984.30	2082.43	2141.56	2.76
n29_r0	29	73944	492.17	0.47	3211	43	1943.90	2099.33	2099.54	0.01
n29_r1	29	25806	77.61	0.28	1595	98	1918.50	2085.72	2085.93	0.01
n29_r2	29	369021	3594.84	0.67	3196	68	2143.10	2160.38	2199.63	1.78
n29_r3	29	94955	349.16	0.23	1406	53	1976.50	2150.03	2150.24	0.01
n29_r4	29	328574	2331.03	0.28	1726	54	1985.50	2165.40	2165.62	0.01

Table 5 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n30_r0	30	371296	3587.78	1.06	6033	55	2138.60	2165.64	2285.20	5.23
n30_r1	30	307561	3590.92	0.58	3390	67	1976.50	2168.64	2186.39	0.81
n30_r2	30	269780	3590.61	0.72	4812	42	2135.70	2139.18	2195.13	2.55
n30_r3	30	424570	3607.73	0.64	3822	47	2141.40	2131.68	2206.80	3.40
n30_r4	30	314690	3594.98	0.61	4332	51	2019.30	2179.08	2204.98	1.17
Avg.		70455.75	541.61	0.24	1209.96	38.04				0.24

Table 6 Detailed computational results for BC3

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n5_r0	5	0	0.00	0.00	5	5	1916.30	1902.43	1902.43	0.00
n5_r1	5	0	0.08	0.02	6	6	1918.50	1867.78	1867.78	0.00
n5_r2	5	0	0.03	0.00	4	4	1918.50	1892.04	1892.04	0.00
n5_r3	5	0	0.05	0.02	8	8	1915.90	1884.92	1884.92	0.00
n5_r4	5	0	0.00	0.00	4	4	1976.40	1861.82	1861.82	0.00
n6_r0	6	4	0.06	0.02	11	11	1932.50	1925.39	1925.39	0.00
n6_r1	6	0	0.02	0.02	5	5	1918.50	1924.73	1924.73	0.00
n6_r2	6	1	0.02	0.00	8	8	1971.40	1967.81	1967.81	0.00
n6_r3	6	0	0.02	0.02	8	8	1935.90	1933.08	1933.08	0.00
n6_r4	6	0	0.02	0.00	3	3	1915.90	1894.35	1894.35	0.00
n7_r0	7	0	0.03	0.00	7	7	1932.50	1949.73	1949.73	0.00
n7_r1	7	24	0.09	0.03	15	10	1949.60	1942.76	1942.76	0.00
n7_r2	7	0	0.05	0.02	9	9	1950.90	1905.62	1905.62	0.00
n7_r3	7	139	0.16	0.09	47	20	2042.40	1973.75	1973.75	0.00
n7_r4	7	0	0.03	0.00	9	9	1915.90	1910.59	1910.59	0.00
n8_r0	8	17	0.09	0.03	16	11	1981.50	1945.33	1945.33	0.00
n8_r1	8	0	0.03	0.00	6	6	1917.70	1919.84	1919.84	0.00
n8_r2	8	0	0.02	0.00	9	9	1917.70	1940.40	1940.40	0.00
n8_r3	8	17	0.16	0.02	43	33	1985.50	2013.24	2013.24	0.00
n8_r4	8	41	0.12	0.02	21	13	2048.90	2040.79	2040.79	0.00
n9_r0	9	0	0.08	0.02	10	10	1963.20	1990.85	1990.85	0.00
n9_r1	9	31	0.09	0.02	22	14	2038.90	1977.83	1977.83	0.00
n9_r2	9	0	0.06	0.02	9	9	1918.50	1917.42	1917.50	0.00
n9_r3	9	0	0.09	0.02	12	12	1918.50	1946.38	1946.38	0.00
n9_r4	9	88	0.27	0.05	37	11	1937.50	1932.48	1932.48	0.00

Table 6 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n10_r0	10	245	0.41	0.17	91	24	1941.10	1974.95	1975.11	0.01
n10_r1	10	18	0.12	0.02	32	27	2025.70	2038.03	2038.03	0.00
n10_r2	10	131	0.27	0.12	61	24	1953.10	1979.41	1979.41	0.00
n10_r3	10	156	0.28	0.12	61	26	2029.30	2005.38	2005.38	0.00
n10_r4	10	15	0.14	0.05	23	17	1933.30	2000.65	2000.65	0.00
n11_r0	11	8	0.16	0.00	22	19	2006.90	1994.25	1994.25	0.00
n11_r1	11	0	0.06	0.00	18	18	1926.10	1981.00	1981.00	0.00
n11_r2	11	173	0.41	0.09	89	39	1933.70	1961.68	1961.68	0.00
n11_r3	11	240	0.44	0.16	76	20	2010.60	2039.54	2039.59	0.00
n11_r4	11	1104	1.31	0.73	194	29	1963.20	2044.37	2044.42	0.00
n12_r0	12	19	0.20	0.05	25	22	1916.30	1963.06	1963.06	0.00
n12_r1	12	324	0.88	0.25	118	21	1980.80	1960.35	1960.35	0.00
n12_r2	12	1197	1.77	0.69	195	27	2082.90	2064.72	2064.72	0.00
n12_r3	12	0	0.22	0.00	23	23	1960.70	1983.86	1983.86	0.00
n12_r4	12	1600	1.80	0.86	194	23	2060.20	2027.88	2027.94	0.00
n13_r0	13	2517	3.16	1.28	291	31	1979.40	2059.71	2059.85	0.01
n13_r1	13	813	1.22	0.38	131	23	1935.90	2048.47	2048.47	0.00
n13_r2	13	1025	1.19	0.55	96	23	1918.50	2026.83	2027.01	0.01
n13_r3	13	1459	2.05	0.84	259	35	1978.00	2049.20	2049.32	0.01
n13_r4	13	206	0.58	0.16	146	58	1919.10	2053.89	2053.89	0.00
n14_r0	14	1056	1.64	0.61	159	34	1955.40	2066.15	2066.15	0.00
n14_r1	14	194	0.70	0.14	77	38	1948.90	1981.45	1981.45	0.00
n14_r2	14	45	0.72	0.03	42	33	1916.30	1997.81	1997.81	0.00
n14_r3	14	1246	1.88	0.83	206	43	1963.20	2071.78	2071.79	0.00
n14_r4	14	44	0.25	0.03	23	14	1918.50	2018.11	2018.11	0.00
n15_r0	15	4678	5.92	2.41	257	27	1958.20	2040.67	2040.87	0.01

Table 6 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n15_r1	15	1197	2.53	1.00	321	30	2019.70	2102.16	2102.16	0.00
n15_r2	15	3002	3.81	1.52	173	35	2032.30	2037.67	2037.87	0.01
n15_r3	15	4593	6.30	2.66	369	56	1943.90	2039.06	2039.25	0.01
n15_r4	15	9789	15.16	5.59	676	57	1986.50	2081.04	2081.25	0.01
n16_r0	16	508	1.05	0.42	91	26	1969.60	2027.74	2027.74	0.00
n16_r1	16	4096	7.84	2.28	596	43	2081.00	2097.80	2098.00	0.01
n16_r2	16	6040	7.89	3.23	239	44	1948.90	2055.35	2055.54	0.01
n16_r3	16	1953	3.28	1.14	215	44	1951.50	2104.08	2104.27	0.01
n16_r4	16	4369	5.94	2.27	283	42	1941.10	2044.83	2045.00	0.01
n17_r0	17	924	1.80	0.58	154	26	1919.10	2018.01	2018.14	0.01
n17_r1	17	931	2.22	0.69	240	40	1933.70	2070.72	2070.82	0.00
n17_r2	17	1826	3.83	1.30	340	39	1935.50	2074.58	2074.76	0.01
n17_r3	17	4017	7.12	2.11	345	52	1940.70	2075.51	2075.68	0.01
n17_r4	17	12540	25.89	6.41	701	42	2032.90	2049.43	2049.62	0.01
n18_r0	18	4401	6.48	2.50	313	52	1985.50	2056.71	2056.91	0.01
n18_r1	18	2727	4.56	1.72	297	55	1979.60	2019.97	2020.16	0.01
n18_r2	18	6632	14.23	4.52	754	34	1950.90	2084.31	2084.51	0.01
n18_r3	18	824	1.88	0.62	153	32	1941.10	2082.62	2082.79	0.01
n18_r4	18	2992	6.19	2.25	509	44	1919.10	2013.20	2013.37	0.01

Table 6 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n19_r0	19	3328	7.52	2.22	556	41	1976.40	2060.99	2061.11	0.01
n19_r1	19	122069	536.62	67.11	3051	44	1941.10	2148.13	2148.34	0.01
n19_r2	19	19433	55.05	11.73	1221	56	1988.80	2152.20	2152.42	0.01
n19_r3	19	2313	4.03	1.05	166	22	2096.20	2117.94	2118.09	0.01
n19_r4	19	30238	86.14	16.62	1629	57	2077.20	2097.48	2097.68	0.01
n20_r0	20	4637	10.77	3.00	465	59	1980.80	2122.12	2122.33	0.01
n20_r1	20	8554	16.02	4.98	427	78	1949.70	2091.32	2091.53	0.01
n20_r2	20	7172	18.91	4.88	904	61	1999.80	2140.61	2140.82	0.01
n20_r3	20	7718	17.89	4.11	604	56	1985.50	2157.41	2157.63	0.01
n20_r4	20	38418	157.55	24.38	1696	60	1985.50	2120.79	2121.00	0.01
n21_r0	21	709757	3586.39	396.45	5494	33	1941.10	2103.69	2106.85	0.15
n21_r1	21	135036	280.38	59.67	1488	47	1940.70	2099.89	2100.10	0.01
n21_r2	21	14739	38.12	8.30	873	47	1965.80	2074.85	2075.06	0.01
n21_r3	21	4990	9.72	2.53	320	42	1966.20	2078.29	2078.48	0.01
n21_r4	21	92466	289.48	46.22	1500	36	1960.70	1992.41	1992.61	0.01
n22_r0	22	32493	110.36	17.70	1623	61	2025.70	2152.25	2152.47	0.01
n22_r1	22	8328	21.91	5.94	949	44	1995.40	2125.16	2125.35	0.01
n22_r2	22	12149	28.38	6.50	688	49	1940.70	2100.92	2101.12	0.01
n22_r3	22	15368	45.39	9.38	874	49	1972.20	2144.34	2144.54	0.01
n22_r4	22	28060	90.42	14.62	1310	60	2114.80	2184.13	2184.34	0.01
n23_r0	23	12439	38.06	7.05	1207	58	1940.70	2106.42	2106.63	0.01
n23_r1	23	14533	48.05	8.64	1195	44	1941.10	2131.06	2131.27	0.01
n23_r2	23	36127	99.89	17.27	867	56	1985.50	2105.54	2105.75	0.01
n23_r3	23	11643	29.42	7.31	978	38	1935.50	2093.95	2094.15	0.01

Table 6 continued

Inst	$\lvert V' \rvert$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n23_r4	23	4463	11.31	3.03	552	61	2058.60	2114.07	2114.28	0.01
n24_r0	24	57626	214.30	29.84	1865	56	1943.90	2157.59	2157.80	0.01
n24_r1	24	41863	180.97	23.33	1938	47	2015.10	2185.19	2185.41	0.01
n24_r2	24	3811	8.39	2.30	328	38	1918.50	2065.45	2065.60	0.01
n24_r3	24	119757	1026.14	71.16	3488	48	1955.00	2168.67	2168.89	0.01
n24_r4	24	20366	78.14	12.97	1189	68	2066.50	2145.43	2145.64	0.01
n25_r0	25	77879	450.34	40.44	1543	50	1979.60	2169.39	2169.60	0.01
n25_r1	25	10286	41.25	6.80	925	45	1943.90	2123.22	2123.43	0.01
n25_r2	25	39934	293.23	26.92	2992	49	1935.50	2195.75	2195.97	0.01
n25_r3	25	131710	617.38	72.66	1564	68	1999.80	2155.03	2155.25	0.01
n25_r4	25	32218	184.30	22.36	1969	43	1935.20	2162.63	2162.84	0.01
n26_r0	26	20468	75.19	11.72	975	61	1978.00	2173.49	2173.71	0.01
n26_r1	26	41418	159.94	23.00	1194	54	1943.90	2153.01	2153.23	0.01
n26_r2	26	14789	44.48	9.17	742	45	1918.50	2089.05	2089.26	0.01
n26_r3	26	24755	65.48	14.12	720	49	1943.90	2127.26	2127.47	0.01
n26_r4	26	168756	1348.72	98.31	3718	46	1935.20	2121.96	2122.17	0.01

Table 6 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n27_r0	27	193095	1887.36	107.00	4346	52	2092.90	2211.80	2212.02	0.01
n27_r1	27	17539	84.72	10.75	1434	43	1941.10	2179.35	2179.56	0.01
n27_r2	27	128632	1002.86	69.67	3167	38	2025.90	2150.82	2151.03	0.01
n27_r3	27	230582	1450.52	146.33	3164	56	1918.50	2130.11	2130.32	0.01
n27_r4	27	534445	3593.33	314.50	4341	42	1935.50	2131.80	2132.65	0.04
n28_r0	28	213273	3595.42	191.81	5588	48	2117.60	2154.80	2224.89	3.15
n28_r1	28	209139	2059.55	119.31	2635	48	2096.70	2132.22	2132.43	0.01
n28_r2	28	186847	1696.41	117.95	3654	45	1940.70	2156.93	2157.14	0.01
n28_r3	28	164404	3581.77	131.28	10097	51	1980.60	2166.59	2177.21	0.49
n28_r4	28	178658	3591.56	151.80	6792	51	2048.90	2122.20	2144.80	1.05
n29_r0	29	33819	253.38	26.05	2284	53	1943.90	2099.33	2099.54	0.01
n29_r1	29	5646	19.58	3.25	637	48	1918.50	2085.73	2085.93	0.01
n29_r2	29	288709	3602.06	253.80	3594	60	2136.20	2149.87	2200.05	2.28
n29_r3	29	32728	116.41	20.09	856	42	1976.50	2150.03	2150.24	0.01
n29_r4	29	217337	2554.77	133.66	2706	51	1985.50	2165.40	2165.62	0.01
n30_r0	30	198700	3599.38	205.92	7687	64	2219.20	2154.03	2304.01	6.51
n30_r1	30	244504	3611.44	225.16	3696	52	1976.50	2137.58	2189.65	2.38
n30_r2	30	154694	3593.95	147.12	6785	63	2115.80	2142.41	2194.19	2.36
n30_r3	30	201907	3608.05	188.33	5189	48	2128.40	2162.12	2212.31	2.27
n30_r4	30	208430	3595.55	176.81	4875	51	2019.30	2169.46	2204.98	1.61
Avg.		45518.70	443.70	30.85	1148.66	36.83				0.18

Table 7 Detailed computational results for BC4

Inst	$\lvert V' \rvert$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n5_r0	5	0	0.03	0.02	5	5	1916.30	1902.43	1902.43	0.00
n5_r1	5	0	0.05	0.00	10	10	1918.50	1867.78	1867.78	0.00
n5_r2	5	0	0.02	0.02	6	6	1918.50	1892.04	1892.04	0.00
n5_r3	5	6	0.03	0.00	10	8	1915.90	1884.92	1884.92	0.00
n5_r4	5	0	0.00	0.00	4	4	1976.40	1861.82	1861.82	0.00
n6_r0	6	4	0.06	0.00	21	21	1932.50	1925.39	1925.39	0.00
n6_r1	6	0	0.02	0.00	7	7	1918.50	1924.73	1924.73	0.00
n6_r2	6	1	0.03	0.00	8	8	1971.40	1967.81	1967.81	0.00
n6_r3	6	0	0.02	0.00	11	11	1935.90	1933.08	1933.08	0.00
n6_r4	6	0	0.02	0.00	3	3	1915.90	1894.35	1894.35	0.00
n7_r0	7	0	0.02	0.00	9	9	1932.50	1949.73	1949.73	0.00
n7_r1	7	0	0.08	0.00	17	17	1949.60	1942.76	1942.76	0.00
n7_r2	7	0	0.00	0.00	11	11	1950.90	1905.62	1905.62	0.00
n7_r3	7	78	0.11	0.05	55	35	2042.40	1973.75	1973.75	0.00
n7_r4	7	5	0.03	0.00	34	33	1915.90	1910.59	1910.59	0.00
n8_r0	8	15	0.09	0.05	27	27	1981.50	1945.33	1945.33	0.00
n8_r1	8	0	0.05	0.00	8	8	1917.70	1919.84	1919.84	0.00
n8_r2	8	0	0.05	0.00	17	17	1917.70	1940.40	1940.40	0.00
n8_r3	8	0	0.08	0.00	17	17	1985.50	2013.24	2013.24	0.00
n8_r4	8	16	0.09	0.02	20	14	2048.90	2040.79	2040.79	0.00
n9_r0	9	0	0.06	0.00	16	16	1963.20	1990.85	1990.85	0.00
n9_r1	9	13	0.08	0.00	27	14	2038.90	1977.83	1977.83	0.00
n9_r2	9	0	0.02	0.00	13	13	1918.50	1917.50	1917.50	0.00
n9_r3	9	4	0.05	0.00	30	30	1918.50	1946.38	1946.38	0.00
n9_r4	9	154	0.39	0.14	105	25	1937.50	1932.48	1932.48	0.00

Table 7 continued

Inst	$\lvert V' \rvert$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n10_r0	10	257	0.36	0.17	77	32	1941.10	1975.04	1975.11	0.00
n10_r1	10	12	0.12	0.02	22	22	2025.70	2038.03	2038.03	0.00
n10_r2	10	64	0.17	0.02	64	54	1953.10	1979.41	1979.41	0.00
n10_r3	10	475	0.55	0.30	71	22	2029.30	2005.20	2005.38	0.01
n10_r4	10	0	0.14	0.02	25	25	1933.30	2000.65	2000.65	0.00
n11_r0	11	12	0.16	0.05	67	62	2006.90	1994.25	1994.25	0.00
n11_r1	11	0	0.06	0.02	20	20	1926.10	1981.00	1981.00	0.00
n11_r2	11	53	0.23	0.02	40	25	1933.70	1961.68	1961.68	0.00
n11_r3	11	199	0.47	0.11	95	62	2010.60	2039.59	2039.59	0.00
n11_r4	11	367	0.72	0.30	99	41	1963.20	2044.32	2044.42	0.00
n12_r0	12	21	0.30	0.02	42	40	1916.30	1962.94	1963.06	0.01
n12_r1	12	323	0.48	0.22	78	33	1980.80	1960.19	1960.35	0.01
n12_r2	12	593	0.97	0.23	118	51	2082.90	2064.59	2064.72	0.01
n12_r3	12	0	0.11	0.03	36	36	1960.70	1983.86	1983.86	0.00
n12_r4	12	203	0.48	0.16	75	50	2060.20	2027.79	2027.94	0.01
n13_r0	13	565	0.97	0.30	106	31	1979.40	2059.85	2059.85	0.00
n13_r1	13	476	0.95	0.33	133	46	1935.90	2048.47	2048.47	0.00
n13_r2	13	1096	1.41	0.69	126	36	1918.50	2026.84	2027.01	0.01
n13_r3	13	559	1.23	0.47	183	45	1978.00	2049.32	2049.32	0.00
n13_r4	13	361	0.70	0.22	122	38	1919.10	2053.75	2053.89	0.01
n14_r0	14	696	1.55	0.59	216	35	1955.40	2066.10	2066.15	0.00
n14_r1	14	206	0.47	0.09	95	43	1948.90	1981.45	1981.45	0.00
n14_r2	14	23	0.33	0.00	72	62	1916.30	1997.81	1997.81	0.00
n14_r3	14	1263	2.23	1.02	209	59	1963.20	2071.60	2071.79	0.01
n14_r4	14	9	0.31	0.02	29	27	1918.50	2018.11	2018.11	0.00

Table 7 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n15_r0	15	454	1.11	0.31	154	54	1958.20	2040.87	2040.87	0.00
n15_r1	15	380	1.50	0.30	239	101	2019.70	2102.16	2102.16	0.00
n15_r2	15	662	1.39	0.52	168	50	2032.30	2037.87	2037.87	0.00
n15_r3	15	1319	2.75	0.86	204	46	1943.90	2039.10	2039.25	0.01
n15_r4	15	7121	10.33	4.41	545	58	1986.50	2081.05	2081.25	0.01
n16_r0	16	353	0.81	0.20	88	26	1969.60	2027.74	2027.74	0.00
n16_r1	16	1547	3.06	1.08	417	60	2081.00	2097.83	2098.00	0.01
n16_r2	16	775	2.16	0.52	173	52	1948.90	2055.39	2055.54	0.01
n16_r3	16	765	1.42	0.27	118	41	1951.50	2104.17	2104.27	0.01
n16_r4	16	1324	2.75	0.94	282	52	1941.10	2044.84	2045.00	0.01
n17_r0	17	1178	2.92	1.02	271	65	1919.10	2017.99	2018.14	0.01
n17_r1	17	504	1.62	0.31	185	38	1933.70	2070.74	2070.82	0.00
n17_r2	17	2034	4.70	1.72	395	37	1935.50	2074.74	2074.76	0.00
n17_r3	17	2778	6.20	1.95	635	110	1940.70	2075.47	2075.68	0.01
n17_r4	17	5516	11.34	3.41	563	59	2032.90	2049.42	2049.62	0.01
n18_r0	18	1917	4.55	1.33	340	63	1985.50	2056.75	2056.91	0.01
n18_r1	18	1462	3.64	1.20	383	72	1979.60	2019.98	2020.16	0.01
n18_r2	18	3898	10.72	2.67	744	78	1950.90	2084.32	2084.51	0.01
n18_r3	18	2820	6.06	2.28	422	108	1941.10	2082.79	2082.79	0.00
n18_r4	18	2801	5.30	1.50	377	114	1919.10	2013.18	2013.37	0.01
n19_r0	19	1270	3.11	0.98	250	40	1976.40	2061.06	2061.11	0.00
n19_r1	19	198532	886.12	114.50	2968	77	1941.10	2148.13	2148.34	0.01
n19_r2	19	11880	30.08	6.48	621	67	1988.80	2152.21	2152.42	0.01
n19_r3	19	1199	3.06	0.86	311	48	2096.20	2117.89	2118.09	0.01
n19_r4	19	7473	20.59	5.02	741	112	2077.20	2097.48	2097.68	0.01

Table 7 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n20_r0	20	10384	32.50	8.19	1162	85	1980.80	2122.13	2122.33	0.01
n20_r1	20	1122	3.11	0.98	265	69	1949.70	2091.45	2091.53	0.00
n20_r2	20	8067	27.97	5.86	1121	56	1999.80	2140.61	2140.82	0.01
n20_r3	20	13522	39.52	9.70	959	90	1985.50	2157.43	2157.63	0.01
n20_r4	20	27156	137.58	17.41	2115	55	1985.50	2120.79	2121.00	0.01
n21_r0	21	343994	3585.98	219.72	5741	78	1941.10	2102.74	2106.85	0.19
n21_r1	21	121871	400.36	65.94	2269	89	1940.70	2099.89	2100.10	0.01
n21_r2	21	5275	16.08	3.73	717	52	1965.80	2074.90	2075.06	0.01
n21_r3	21	5455	12.06	3.59	394	70	1966.20	2078.29	2078.48	0.01
n21_r4	21	13772	34.38	8.31	1012	110	1960.70	1992.42	1992.61	0.01
n22_r0	22	25201	130.59	17.20	1612	80	2025.70	2152.26	2152.47	0.01
n22_r1	22	3950	11.80	2.86	530	75	1995.40	2125.17	2125.35	0.01
n22_r2	22	9651	25.08	6.34	946	98	1940.70	2100.92	2101.12	0.01
n22_r3	22	20920	70.31	13.92	1219	73	1972.20	2144.33	2144.54	0.01
n22_r4	22	20252	72.09	11.02	1043	122	2114.80	2184.13	2184.34	0.01
n23_r0	23	5828	18.56	4.77	824	75	1940.70	2106.43	2106.63	0.01
n23_r1	23	13231	43.92	7.83	810	74	1941.10	2131.06	2131.27	0.01
n23_r2	23	28512	111.70	17.81	1494	91	1985.50	2105.54	2105.75	0.01
n23_r3	23	11764	32.94	7.72	1163	96	1935.50	2093.95	2094.15	0.01
n23_r4	23	4254	13.36	2.47	723	77	2058.60	2114.08	2114.28	0.01
n24_r0	24	33886	136.72	20.27	1439	50	1943.90	2157.59	2157.80	0.01
n24_r1	24	19842	137.55	14.56	2316	78	2015.10	2185.19	2185.41	0.01
n24_r2	24	869	2.88	0.55	206	75	1918.50	2065.60	2065.60	0.00
n24_r3	24	50110	330.34	32.67	1910	106	1955.00	2168.67	2168.89	0.01
n24_r4	24	41028	242.41	29.45	1601	82	2066.50	2145.43	2145.64	0.01

Table 7 continued

Inst	$\|V'\|$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n25_r0	25	23465	150.09	15.77	1708	113	1979.60	2169.39	2169.60	0.01
n25_r1	25	14381	56.02	9.23	885	86	1943.90	2123.22	2123.43	0.01
n25_r2	25	45078	371.80	33.48	2728	62	1935.50	2195.75	2195.97	0.01
n25_r3	25	63520	364.64	38.31	1996	72	1999.80	2155.04	2155.25	0.01
n25_r4	25	14575	80.16	11.48	1425	107	1935.20	2162.65	2162.84	0.01
n26_r0	26	31391	156.44	21.80	1764	92	1978.00	2173.49	2173.71	0.01
n26_r1	26	54173	186.11	32.88	1186	88	1943.90	2153.01	2153.23	0.01
n26_r2	26	7207	30.69	6.56	979	76	1918.50	2089.05	2089.26	0.01
n26_r3	26	12290	39.88	8.31	815	164	1943.90	2127.26	2127.47	0.01
n26_r4	26	106476	499.73	60.44	1932	99	1935.20	2121.96	2122.17	0.01
n27_r0	27	210856	3593.06	148.72	4638	115	2111.00	2204.29	2212.30	0.36
n27_r1	27	18478	117.30	12.25	1995	60	1941.10	2179.34	2179.56	0.01
n27_r2	27	102569	743.00	55.33	2466	92	2025.90	2150.82	2151.03	0.01
n27_r3	27	39539	188.09	25.47	1456	52	1918.50	2130.11	2130.32	0.01
n27_r4	27	491352	3595.45	341.48	3016	61	1935.50	2129.84	2132.65	0.13
n28_r0	28	181550	3595.80	172.02	5139	89	2107.10	2178.61	2207.46	1.31
n28_r1	28	88869	951.09	62.81	2678	101	2096.70	2132.22	2132.43	0.01
n28_r2	28	188286	3591.56	162.33	6251	69	1940.70	2151.67	2157.14	0.25
n28_r3	28	131431	3586.22	124.39	7332	86	2090.80	2157.63	2183.36	1.18
n28_r4	28	121840	1633.77	88.58	4334	82	1975.10	2133.07	2133.28	0.01
n29_r0	29	30332	224.72	27.80	2690	89	1943.90	2099.33	2099.54	0.01
n29_r1	29	5973	37.39	5.91	1413	84	1918.50	2085.77	2085.93	0.01
n29_r2	29	94310	1158.19	64.11	2186	120	2143.10	2193.05	2193.27	0.01
n29_r3	29	15247	82.72	11.86	1213	69	1976.50	2150.03	2150.24	0.01
n29_r4	29	44572	343.06	31.56	1725	91	1985.50	2165.41	2165.62	0.01

Table 7 continued

Inst	$\lvert V' \rvert$	Nodes	Time	S-Time	Ncuts	Ncuts0	Driving	LB	UB	Gap (%)
n30_r0	30	145538	3592.48	159.36	7696	97	2098.10	2169.56	2299.36	5.65
n30_r1	30	96447	1012.17	67.36	2495	88	1976.50	2186.17	2186.39	0.01
n30_r2	30	160600	3592.91	119.69	5040	96	2115.80	2186.35	2192.20	0.27
n30_r3	30	202242	3603.02	199.09	3950	81	2051.40	2133.21	2218.19	3.83
n30_r4	30	224012	3597.09	149.11	4569	84	1985.50	2200.31	2203.04	0.12
Avg.		31343.12	365.30	22.77	1043.05	58.96				0.11

Table 8 Computational results with the BC4 algorithm using $\alpha \in \{0.6, 0.8, 0.9\}$

Inst	$\alpha = 0.60$			$\alpha = 0.80$			$\alpha = 0.90$		
	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking
n5_r0	0.02	2317.20	0.00	0.02	2317.20	0.00	0.00	1916.30	1777.60
n5_r1	0.00	2263.10	0.00	0.03	2232.40	118.00	0.02	1918.50	1411.30
n5_r2	0.03	2268.70	0.00	0.02	2268.70	0.00	0.02	1918.50	1653.90
n5_r3	0.03	2185.00	0.00	0.00	2185.00	0.00	0.03	1915.90	1606.10
n5_r4	0.00	2152.20	0.00	0.03	2152.20	0.00	0.03	1976.40	830.60
n6_r0	0.02	2254.70	0.00	0.03	2254.70	0.00	0.08	1932.50	1861.40
n6_r1	0.00	2384.00	0.00	0.08	2384.00	0.00	0.05	1918.50	1980.80
n6_r2	0.03	2362.20	71.00	0.06	2362.20	71.00	0.03	1971.40	1935.50
n6_r3	0.02	2312.70	0.00	0.02	2312.70	0.00	0.05	1935.90	1907.70
n6_r4	0.00	2314.00	0.00	0.06	2314.00	0.00	0.02	1915.90	1700.40
n7_r0	0.03	2427.30	0.00	0.06	2282.50	553.60	0.02	1932.50	2104.80
n7_r1	0.03	2322.30	0.00	0.11	2239.20	210.00	0.06	1949.60	1881.20
n7_r2	0.03	2247.90	0.00	0.03	2247.90	0.00	0.02	1950.90	1498.10
n7_r3	0.05	2308.10	0.00	0.09	2308.10	0.00	0.09	2042.40	1355.90
n7_r4	0.02	2311.30	0.00	0.06	2311.30	0.00	0.03	1915.90	1862.80
n8_r0	0.02	2321.70	0.00	0.14	2321.70	0.00	0.09	1981.50	1619.80
n8_r1	0.00	2435.60	0.00	0.12	1917.70	1939.10	0.03	1917.70	1939.10
n8_r2	0.05	2408.70	0.00	0.06	2408.70	0.00	0.03	1917.70	2144.70
n8_r3	0.03	2543.50	0.00	0.20	2543.50	0.00	0.06	1985.50	2262.90
n8_r4	0.03	2501.50	0.00	0.14	2380.40	362.00	0.05	2048.90	1967.80
n9_r0	0.03	2445.60	0.00	0.14	2445.60	0.00	0.08	1963.20	2239.70
n9_r1	0.05	2316.30	79.40	0.09	2269.30	249.00	0.16	2038.90	1428.20
n9_r2	0.03	2334.90	0.00	0.09	2334.90	0.00	0.05	1918.50	1908.50

Table 8 continued

Inst	$\alpha = 0.60$			$\alpha = 0.80$			$\alpha = 0.90$		
	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking
n9_r3	0.05	2373.50	0.00	0.11	2373.50	0.00	0.06	1918.50	2197.30
n9_r4	0.05	2289.20	0.00	0.02	2289.20	0.00	0.31	1937.50	1887.30
n10_r0	0.05	2363.60	75.60	0.03	2363.60	75.60	0.47	1941.10	2281.20
n10_r1	0.06	2539.80	0.00	0.19	2401.90	535.00	0.12	2025.70	2149.00
n10_r2	0.03	2404.00	0.00	0.08	2404.00	0.00	0.23	1953.10	2216.20
n10_r3	0.02	2414.00	0.00	0.06	2271.30	506.40	0.56	2029.30	1790.10
n10_r4	0.02	2563.70	0.00	0.44	2478.00	207.60	0.12	1933.30	2606.80
n11_r0	0.02	2453.80	0.00	0.27	2159.50	1153.70	0.14	2006.90	1880.40
n11_r1	0.03	2550.10	0.00	0.28	2450.60	263.00	0.06	1926.10	2475.10
n11_r2	0.02	2399.70	0.00	0.16	2349.80	135.60	0.25	1933.70	2213.50
n11_r3	0.08	2543.20	0.00	0.36	2431.50	274.20	0.47	2010.60	2300.50
n11_r4	0.05	2439.60	0.00	0.20	2297.70	391.60	0.67	1963.20	2775.40
n12_r0	0.00	2340.10	0.00	0.06	2340.10	0.00	0.28	1916.30	2383.90
n12_r1	0.03	2420.00	0.00	0.28	2355.90	176.60	0.52	1980.80	1776.30
n12_r2	0.08	2535.90	0.00	0.81	2311.00	712.00	1.00	2082.90	1901.10
n12_r3	0.05	2454.20	0.00	0.11	2449.60	13.80	0.09	1960.70	2192.30
n12_r4	0.05	2435.30	0.00	0.20	2329.20	317.60	0.47	2060.20	1737.60
n13_r0	0.05	2600.40	0.00	0.12	2260.90	932.00	1.03	1979.40	2783.90
n13_r1	0.06	2461.00	0.00	0.12	2461.00	0.00	0.94	1935.90	3061.60
n13_r2	0.05	2547.60	0.00	0.91	2314.40	768.80	1.36	1918.50	3003.60
n13_r3	0.02	2521.10	0.00	1.44	2521.10	0.00	1.23	1978.00	2691.20
n13_r4	0.20	2599.70	0.00	0.45	2282.90	1066.00	0.72	1919.10	3267.00
n14_r0	0.25	2634.80	0.00	0.80	2425.80	604.00	1.55	1955.40	3062.90
n14_r1	0.03	2430.70	0.00	0.45	2361.80	232.80	0.55	1948.90	2274.40
n14_r2	0.14	2605.20	0.00	0.58	2332.60	812.60	0.39	1916.30	2731.40

Table 8 continued

Inst	$\alpha = 0.60$			$\alpha = 0.80$			$\alpha = 0.90$		
	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking
n14_r3	0.11	2483.90	71.00	0.23	2380.40	436.00	2.22	1963.20	3049.10
n14_r4	0.03	2549.30	0.00	0.16	2399.90	548.80	0.31	1918.50	2914.60
n15_r0	0.02	2523.00	0.00	2.59	2523.00	0.00	1.06	1958.20	2784.90
n15_r1	0.16	2675.30	0.00	1.56	2483.50	595.10	1.48	2019.70	2844.30
n15_r2	0.05	2516.20	0.00	0.81	2245.60	982.30	1.36	2032.30	2088.00
n15_r3	0.03	2539.80	0.00	0.95	2426.50	384.80	2.66	1943.90	2897.40
n15_r4	0.11	2506.40	2.00	0.42	2501.80	15.80	10.59	1986.50	2934.00
n16_r0	0.12	2608.80	2.00	2.91	2252.00	1237.80	0.86	1969.60	2551.00
n16_r1	0.02	2542.80	0.00	0.95	2429.80	381.40	3.00	2081.00	2251.00
n16_r2	0.03	2652.80	0.00	5.69	2475.20	439.60	2.02	1948.90	3015.30
n16_r3	0.12	2718.80	0.00	3.17	2674.90	118.00	1.44	1951.50	3479.20
n16_r4	0.14	2554.80	0.00	2.89	2342.60	763.30	2.67	1941.10	2980.10
n17_r0	0.12	2527.30	0.00	1.86	2382.00	393.20	2.92	1919.10	2909.50
n17_r1	0.17	2678.10	0.00	3.19	2311.70	1341.90	1.61	1933.70	3304.90
n17_r2	0.05	2556.90	0.00	0.42	2407.40	502.60	4.80	1935.50	3328.10
n17_r3	0.09	2587.70	0.00	1.16	2483.40	274.20	6.39	1940.70	3290.50
n17_r4	0.16	2496.90	0.00	1.64	2360.70	369.20	11.50	2032.90	2200.10
n18_r0	0.05	2514.10	0.00	0.73	2335.30	676.80	4.58	1985.50	2699.60
n18_r1	0.22	2493.60	0.00	0.33	2334.20	471.80	3.59	1979.60	2385.20
n18_r2	0.03	2614.90	0.00	4.78	2469.50	535.00	10.80	1950.90	3287.00
n18_r3	0.16	2735.00	0.00	2.70	2430.30	809.00	6.03	1941.10	3358.00
n18_r4	0.06	2563.10	0.00	0.72	2389.10	431.00	5.42	1919.10	2861.80
n19_r0	0.22	2594.70	0.00	5.73	2326.40	873.80	3.19	1976.40	2823.50
n19_r1	0.27	2734.60	0.00	6.62	2504.50	619.60	875.02	1941.10	4013.50

Table 8 continued

Inst	$\alpha = 0.60$			$\alpha = 0.80$			$\alpha = 0.90$		
	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking
n19_r2	0.19	2742.70	0.00	2.70	2341.60	1156.00	28.36	1988.80	3625.00
n19_r3	0.30	2762.20	0.00	6.42	2191.20	1808.70	2.80	2096.20	2315.10
n19_r4	0.39	2546.40	0.00	1.41	2404.80	453.90	19.41	2077.20	2282.00
n20_r0	0.16	2695.70	0.00	14.36	2388.30	1116.40	30.97	1980.80	3396.10
n20_r1	0.08	2631.50	0.00	0.48	2408.30	646.60	2.98	1949.70	3368.00
n20_r2	0.52	2790.30	0.00	5.20	2215.30	1826.50	26.98	1999.80	3410.00
n20_r3	0.06	2752.70	0.00	16.16	2642.10	347.80	37.19	1985.50	3706.80
n20_r4	0.25	2621.00	0.00	1.25	2317.70	1114.40	130.89	1985.50	3340.50
n21_r0	0.25	2659.40	0.00	2.70	2328.60	1058.20	0.18%	1941.10	3598.60
n21_r1	0.06	2653.20	0.00	4.67	2307.60	1132.80	395.83	1940.70	3534.70
n21_r2	0.05	2558.40	0.00	1.05	2558.40	0.00	16.03	1965.80	3058.40
n21_r3	0.20	2603.70	0.00	0.98	2510.90	223.40	12.00	1966.20	3089.00
n21_r4	0.16	2381.50	0.00	1.11	2381.50	0.00	34.08	1960.70	2279.80
n22_r0	0.31	2682.70	0.00	0.91	2416.30	794.50	127.42	2025.70	3293.40
n22_r1	0.03	2610.80	107.00	0.78	2571.90	242.20	11.72	1995.40	3294.90
n22_r2	0.12	2624.70	0.00	3.91	2469.70	504.40	24.62	1940.70	3544.90
n22_r3	0.33	2699.50	2.00	0.30	2369.50	917.60	70.55	1972.20	3695.60
n22_r4	0.11	2717.80	0.00	7.55	2530.40	574.80	71.39	2114.80	2810.20
n23_r0	0.56	2685.00	0.00	2.34	2399.20	893.80	18.61	1940.70	3600.00
n23_r1	0.23	2676.10	0.00	9.20	2398.80	965.40	43.88	1941.10	3842.80
n23_r2	0.47	2627.00	0.00	22.25	2593.30	127.80	111.53	1985.50	3188.00
n23_r3	0.41	2806.30	0.00	37.09	2269.20	1904.50	34.78	1935.50	3522.00
n23_r4	0.94	2773.10	2.40	30.56	2285.20	1619.50	14.27	2058.60	2615.40

Table 8 continued

Inst	$\alpha = 0.60$			$\alpha = 0.80$			$\alpha = 0.90$		
	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking
n24_r0	0.12	2759.30	0.00	4.06	2533.70	573.30	139.52	1943.90	4082.90
n24_r1	0.05	2738.20	0.00	0.69	2467.40	883.50	133.56	2015.10	3718.20
n24_r2	0.23	2728.20	0.00	20.19	2342.10	1294.90	2.88	1918.50	3389.50
n24_r3	0.11	2762.50	0.00	2.66	2318.00	1396.10	324.42	1955.00	4093.90
n24_r4	0.03	2723.40	0.00	6.02	2533.80	604.00	237.23	2066.50	2857.90
n25_r0	0.14	2665.80	0.00	8.19	2564.20	387.40	147.77	1979.60	3879.60
n25_r1	0.72	2746.80	0.00	9.92	2256.10	1616.90	54.97	1943.90	3739.20
n25_r2	0.09	2839.80	0.00	110.50	2533.10	1137.30	364.45	1935.50	4540.20
n25_r3	0.30	2767.90	0.00	1.20	2516.70	717.40	360.72	1999.80	3554.30
n25_r4	0.34	2800.80	0.00	7.62	2424.30	1269.60	80.02	1935.20	4211.60
n26_r0	0.47	2872.80	0.00	19.11	2370.90	1656.10	154.70	1978.00	3935.10
n26_r1	0.41	2786.40	0.00	44.97	2242.50	1818.80	184.27	1943.90	4037.20
n26_r2	0.45	2746.00	0.00	43.72	2359.30	1349.10	30.84	1918.50	3626.10
n26_r3	0.31	2840.20	0.00	8.09	2570.80	658.30	39.77	1943.90	3779.60
n26_r4	0.16	2635.50	0.00	5.80	2387.10	854.60	470.30	1935.20	3804.90
n27_r0	0.42	2843.10	0.00	4.48	2455.30	1144.00	0.36%	2111.00	3124.00
n27_r1	0.67	2830.70	79.40	5.67	2344.10	1667.90	121.16	1941.10	4325.70
n27_r2	0.11	2710.40	0.00	8.59	2509.90	732.20	796.86	2025.90	3277.20
n27_r3	0.77	2799.40	0.00	5.67	2398.10	1341.70	198.45	1918.50	4036.70
n27_r4	0.44	2687.70	0.00	9.55	2288.40	1275.60	0.11%	1935.50	3907.00
n28_r0	0.86	2779.70	0.00	73.23	2505.50	893.70	1.31%	2107.10	3110.70
n28_r1	0.81	2687.20	73.40	2.77	2466.00	567.30	950.55	2096.70	2454.00
n28_r2	0.09	2660.10	0.00	1.02	2660.10	0.00	0.23%	1940.70	4105.10

Table 8 continued

Inst	$\alpha = 0.60$			$\alpha = 0.80$			$\alpha = 0.90$		
	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking	Gap/Time	Driving	Walking
n28_r3	0.44	2766.00	0.00	28.17	2475.40	903.80	1.13%	2090.80	3016.40
n28_r4	0.27	2673.10	0.00	105.09	2425.80	820.00	1577.36	1975.10	3556.90
n29_r0	0.33	2813.60	0.00	33.39	2580.80	629.10	207.45	1943.90	3500.30
n29_r1	0.67	2724.40	0.40	3.19	2491.50	658.60	34.53	1918.50	3592.80
n29_r2	0.28	2680.20	0.00	0.77	2557.70	347.80	1041.59	2143.10	2644.80
n29_r3	0.12	2784.50	0.40	24.28	2602.40	628.10	74.22	1976.50	3713.90
n29_r4	0.30	2722.20	0.00	1.91	2463.20	736.30	308.86	1985.50	3786.70
n30_r0	0.67	2938.90	0.00	1161.58	2388.60	1932.10	5.61%	2098.10	4110.70
n30_r1	1.03	2856.30	0.00	5.47	2377.10	1204.70	994.25	1976.50	4075.40
n30_r2	0.67	2741.30	0.00	33.27	2333.90	1506.80	0.20%	2115.80	2879.80
n30_r3	0.34	2756.40	0.00	23.45	2539.80	729.90	3.88%	2051.40	3719.30
n30_r4	0.38	2697.70	0.00	1.17	2485.50	766.90	0.15%	1985.50	4160.90

References

Allen, J., & Browne, M. (2014). Road freight transport to, from, and within London. *The London Journal*, *39*(1), 59–75.

Allen, J., Piecyk, M., Piotrowska, M., McLeod, F., Cherrett, T., Ghali, K., et al. (2018). Understanding the impact of e-commerce on last-mile light goods vehicle activity in urban areas: The case of London. *Transportation Research Part D: Transport and Environment*, *61*(Part B), 325–338.

Anderluh, A., Hemmelmayr, V. C., & Nolz, P. C. (2017). Synchronizing vans and cargo bikes in a city distribution network. *Central European Journal of Operations Research*, *25*(2), 345–376.

Bates, O., Knowles, B., & Friday, A. (2017). Are people the key to enabling collaborative smart logistics? In *Proceedings of the 2017 CHI conference extended abstracts on human factors in computing systems, ACM* (pp. 1494–1499).

Browne, M., Rizet, C., & Allen, J. (2014). A comparative assessment of the light goods vehicle fleet and the scope to reduce its CO_2 emissions in the UK and France. *Procedia-Social and Behavioral Sciences*, *125*, 334–344.

Coindreau, M.-A., Gallay, O., & Zufferey, N. (2019). Vehicle routing with transportable resources: Using carpooling and walking for on-site services. *European Journal of Operational Research*, *279*(3), 996–1010.

Cuda, R., Guastaroba, G., & Speranza, M. (2015). A survey on two-echelon routing problems. *Computers & Operations Research*, *55*, 185–199.

Dablanc, L., Morganti, E., Arvidsson, N., Woxenius, J., Browne, M., & Saidi, N. (2017). The rise of on-demand instant deliveries in European cities. *Supply Chain Forum: An International Journal*, *18*(4), 203–217.

De Grancy, G. S., & Reimann, M. (2016). Vehicle routing problems with time windows and multiple service workers: a systematic comparison between ACO and GRASP. *Central European Journal of Operations Research*, *24*(1), 29–48.

Department for Transport (2020). *Average speed on local a roads: Monthly and annual averages, statistical table CGN0501'*. Retrieved from https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/870311/cgn0501.ods.

Derigs, U., Pullmann, M., & Vogel, U. (2013). Truck and trailer routing-problems, heuristics and computational experience. *Computers & Operations Research*, *40*(2), 536–546.

Fikar, C., & Hirsch, P. (2015). A matheuristic for routing real-world home service transport systems facilitating walking. *Journal of Cleaner Production*, *105*, 300–310.

Gendreau, M., Jabali, O., & Rei, W. (2016). 50th anniversary invited article-future research directions in stochastic vehicle routing. *Transportation Science*, *50*(4), 1163–1173.

Hemmelmayr, V. C., Cordeau, J.-F., & Crainic, T. G. (2012). An adaptive large neighborhood search heuristic for two-echelon vehicle routing problems arising in city logistics. *Computers & Operations Research*, *39*(12), 3215–3228.

Lin, C. (2011). A vehicle routing problem with pickup and delivery time windows, and coordination of transportable resources. *Computers & Operations Research*, *38*(11), 1596–1609.

Lin, S.-W., Vincent, F., & Lu, C.-C. (2011). A simulated annealing heuristic for the truck and trailer routing problem with time windows. *Expert Systems with Applications*, *38*(12), 15244–15252.

Luo, Z., Liu, Z., & Shi, J. (2017). A two-echelon cooperated routing problem for a ground vehicle and its carried unmanned aerial vehicle. *Sensors*, *17*(5), 1144.

Murray, C. C., & Chu, A. G. (2015). The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery. *Transportation Research Part C: Emerging Technologies*, *54*, 86–109.

Nguyên, T. B. T., Bektaş, T., Cherrett, T. J., McLeod, F. N., Allen, J., Bates, O., et al. (2019). Optimising parcel deliveries in London using dual-mode routing. *Journal of the Operational Research Society*, *70*(6), 998–1010.

Ofcom (2019). *Annual monitoring update on the postal market—Financial year 2018–19*. Technical report. Retrieved from https://www.ofcom.org.uk/postal-services/information-for-the-postal-industry/monitoring_reports.

Parragh, S., & Cordeau, J.-F. (2017). Branch-and-price and adaptive large neighborhood search for the truck and trailer routing problem with time windows. *Computers & Operations Research*, *83*, 28–44.

Psaraftis, H., Wen, M., & Kontovas, C. (2016). Dynamic vehicle routing problems: Three decades and counting. *Networks*, *67*(1), 3–31.

Transport for London (2013). *Travel in London: Report 6*. Technical report. Retrieved from http://content.tfl.gov.uk/travel-in-london-report-6.pdf.

Transport for London (2016). *Travel in London: Report 9*. Technical report. Retrieved from http://content.tfl.gov.uk/travel-in-london-report-9.pdf.

Villegas, J., Prins, C., Prodhon, C., Medaglia, A., & Velasco, N. (2010). GRASP/VND and multi-start evolutionary local search for the single truck and trailer routing problem with satellite depots. *Engineering Applications of Artificial Intelligence*, *23*(5), 780–794.