

A COUPLED MODE – hp FEM FOR HYDROELASTIC ANALYSIS OF SHEAR-DEFORMABLE FLOATING BODIES OF GENERAL THICKNESS IN VARIABLE BATHYMETRY

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Abstract. An efficient computational procedure is presented for the solution of coupled hydroelastic problems involving bodies of general thickness, floating over variable bathymetry regions. The problem is treated by the coupled mode system of horizontal equations derived by Athanassoulis and Belibassakis [1], for the analysis of floating, shear deformable plates or beams. The proposed beam (or plate) model is based on the addition of extra vertical elastic deformation modes, at each horizontal position along the floating body, permitting shear strain and stress to vanish on both the upper and lower boundaries and extending third-order plate theories [2]. The final coupled mode system is derived from a variational principle combining the one – field functional of the elastodynamics in the plate region with the pressure functional in the water region. The wave potential in the water column is represented by means of a local – mode series containing an extra mode, accounting for not mildly sloped bottom variations [3]. The addition of the additional modes results to increased convergence rate, enabling high accuracy with the use of a relatively small number of vertical modes. In the present work the hp-version of the Finite Element Method [4] is applied to the solution of a simplified version of the resulting system of coupled horizontal differential equations with respect to the modal amplitudes, providing good convergence rates and adaptivity capabilities, and increasing the overall efficiency of the solution strategy. Numerical results are presented demonstrating the applicability of present method.

1 INTRODUCTION

The effect of water waves on floating deformable bodies is related to both environmental and technical issues, finding important applications. A specific example concerns the interaction of waves with thin sheets of sea ice, which is particularly important in the Marginal Ice Zone

(MIZ) in the Antarctic, a region consisting of loose or packed ice floes situated between the ocean and the shore sea ice [5]. As the ice sheets support flexural-gravity waves, the energy carried by the ocean waves is capable of propagating far into the MIZ, contributing to break and melting of ice glaciers [6,7] thus accelerating global warming effects and rise in sea water level. In addition, the interaction of free-surface gravity waves with floating deformable bodies is a very interesting problem finding applications in hydrodynamic analysis and design of very large floating structures (VLFS) operating offshore (as power stations/mining and storage/transfer), but also in coastal areas (as floating airports, floating docks, residence/entertainment facilities), as well as floating bridges, floating marinas and breakwaters etc. For all the above problems hydroelastic effects are significant and should be properly taken into account. Extended surveys, including a literature review, have been presented by Kashiwagi [8], Watanabe et al [9]. A recent review on both topics and the synergies between VLFS hydroelasticity and sea ice research can be found in Squire [10].

Taking into account that the horizontal dimensions of the large floating body are much greater than the vertical one, thin-plate (Kirchhoff) theory is commonly used to model the above hydroelastic problems. Although non-linear effects are of specific importance, still the solution of the linearised problem provides valuable information, serving also as the basis for the development of weakly non-linear models. The linearised hydroelastic problem is effectively treated in the frequency domain, and many methods have been developed for its solution. These include hydroelastic eigenfunction expansion techniques [11,12,13], Boundary Element Methods [14,15], B-spline Galerkin method [16], integro-differential equations [17], Wiener-Hopf techniques [18], Green-Naghdi models [19], and others. Moreover, Meylan [20] derived a variational equation for the plate-water system by expressing the water motion as an operator equation. In the case of hydroelastic behaviour of large floating bodies in general bathymetry, a new coupled-mode system has been derived and examined by Belibassakis & Athanassoulis [3] based on local vertical expansion of the wave potential in terms of hydroelastic eigenmodes, and extending a previous similar approach for the propagation of water waves in variable bathymetry regions [21]. Similar approaches with application to wave scattering by ice sheets of varying thickness have been presented by Porter & Porter [5] based on mild-slope approximation and by Bennets et al [22] based on multi-mode expansion.

In the above models the floating body has been considered to be very thin and first-order plate theory has been applied, neglecting shear effects. In Athanassoulis & Belibassakis [1] an extension of the previous coupled-mode model is presented with application to the case of hydroelastic analysis of a thick, non-uniform, shear deformable floating elastic body, lying over variable bathymetry regions. The problem addressed therein is the scattering of linear, coupled, hydroelastic waves propagating through an inhomogeneous sea-ice environment, containing ice sheets of variable, finite thickness characterized by mildly sloped interface. The enhanced coupled-mode system of horizontal equations is obtained on the basis of higher-order theory of shear deformable plates and beams, and is derived by the local vertical expansion of the wave potential in the water region, in conjunction with an enhanced representation of the elastic displacement field in the plate, containing additional elastic vertical modes. The latter permit the shear strain and stress to vanish on both the upper and lower boundaries of the finite floating plate, and extends third-order plate theories by Reddy

[23] and Bickford [24] (see also [2]) to plates and beams of general shape. The above representations are used in a variational principle composed by the one-field functional of elastodynamics in the plate region, and Luke [25] pressure functional in the water region, reducing to the coupled-mode system (CMS). In the case of plates of general but slowly varying and relatively small thickness, the above shear-enhanced CMS simplifies in a form suitable for long-range, large-domain calculations, and extends the first-order model by Belibassakis & Athanassoulis [3] to shear deformable floating plates of variable finite thickness, lying over general bottom topography. In the present work, an efficient computational procedure is presented, to treat the extended coupled-mode system, using the hp-version of the Finite Element Method [4] for the solution of the resulting system of horizontal differential equations, providing very good convergence rates and adaptivity capabilities, and increasing the overall efficiency of the solution strategy. Numerical results are presented demonstrating the applicability of the proposed method.

2. FORMULATION OF THE PROBLEM

The studied environment consists of a water layer bounded above by a floating plate of general shape, as e.g., an ice sheet of variable thickness $b(x)$, and below by a rigid bottom; see Fig. 1. For simplicity we restrict ourselves to a 2D problem, however the method can be straightforward extended to 3D. Also, the upper face of the floating elastic plate is flat-horizontal (e.g. by considering that the static plate deflection to be very small), however, our analysis can be easily extended to the case of a more general upper boundary. It is also assumed that the bottom and the plate surfaces exhibit general one-dimensional variation in a subdomain of finite length. The bathymetry is characterised by straight and parallel bottom contours lying between two regions of constant but possibly different depth: $h = h_1$ (region of incidence) and $h = h_3$ (region of transmission), where $h(x)$ is the depth function. A Cartesian coordinate system is introduced, with its origin at some point on the upper elastic-plate surface (in the variable bathymetry region), the z -axis pointing upwards and the y -axis being parallel to the bottom contours. The functions $h(x)$ and $b(x)$ are smooth functions such that $h(x) = h(x_1) = h_1$ and $b(x) = b(x_1) = b_1$, for all $x \leq x_1$, and $h(x) = h(x_3) = h_3$ and $b(x) = b(x_3) = b_3$, for all $x \geq x_3$. Also, the slope of the elastic-plate deflection $w(x, y; t)$ is assumed small, so that linear theory can be applied. We consider the scattering problem of harmonic incident plane waves of angular frequency ω , under the combined effects of variable bathymetry and the infinite, floating elastic plate of general and finite thickness (b).

We shall concentrate here in the case of normally incident linear waves (as shown in Fig.1), leaving the treatment of obliquely-incident waves propagating with directions θ_1 and θ_3 with respect to the x -axis in the regions of incidence ($x \leq x_1$) and transmission ($x \geq x_3$), respectively, and more complex 3D systems to be examined in future works. For the above problem an extended coupled-mode model (eCMS) has been derived by Athanassoulis & Belibassakis [1], for the hydroelastic analysis of floating, shear deformable plate or ice sheet in general bathymetry, characterised by sloped boundaries and interfaces, taking into account finite, general thickness effects.

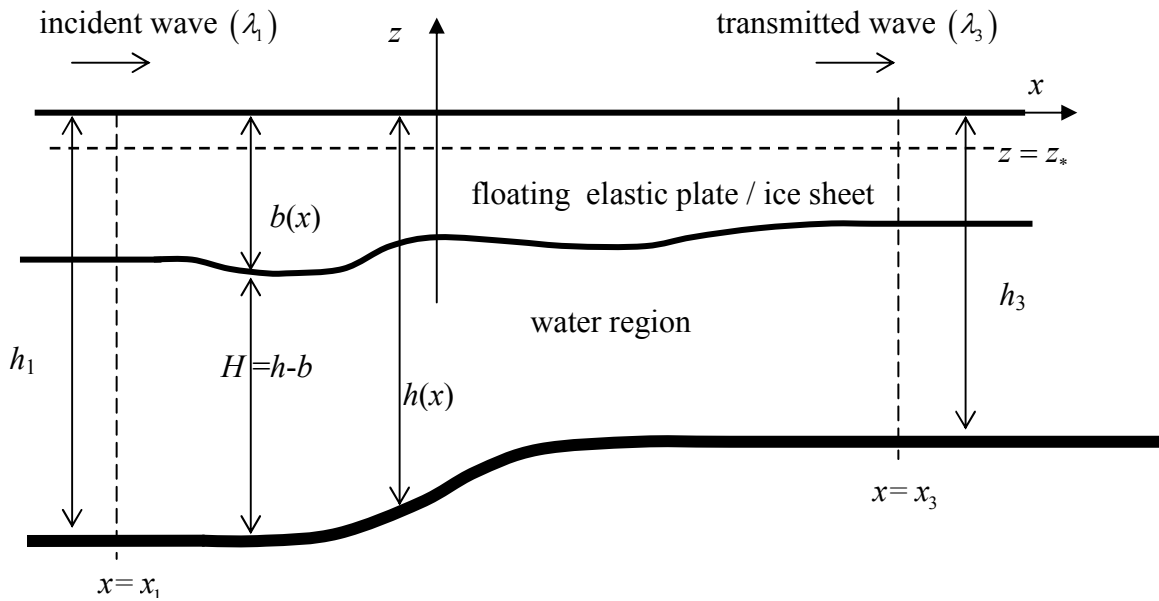


Figure 1. Floating elastic plate or ice sheet of general thickness in variable bathymetry.

The extended model is based on enhanced representations of both the elastic displacement field in the plate region and the wave potential field in the water region, permitting to model effects of shear stresses and to consistently treat the end-conditions on the non-horizontal boundaries (wetted plate surface, bottom surface).

2.1 Modal expansion of the wave potential in the water subregion

In a series of works presented by the authors, starting with the linearised water wave problem in general bathymetry (Athanassoulis & Belibassakis [22]), a vertical local mode series expansion is used to consistently represent the wave field in the water region:

$$\varphi(x, z) = \varphi_{-1}(x)Z_{-1}(z; x) + \sum_{n=0}^{\infty} \varphi_n(x)Z_n(z; x), \quad -h(x) < z < -b(x), \quad (1)$$

The major part of the set of vertical modes $\{Z_n(z; x), n = 0, 1, 2, \dots\}$ is obtained through the solution of a vertical eigenvalue problem, formulated at each horizontal position and $\varphi_{-1}(x)Z_{-1}(z; x)$ is an appropriate term, called the sloping-bottom mode, accounting for the satisfaction of the bottom boundary condition on the non-horizontal parts of the bottom. The idea of the sloping bottom mode, in conjunction with the above type of modal expansion, has been first introduced by the authors (Athanassoulis & Belibassakis [22]) in the case of water waves propagating in variable bathymetry. Since then, it has been used for many problems exhibiting similar features, such as nonlinear water waves (Belibassakis & Athanassoulis [26]), hydroacoustics (Athanassoulis *et al* [27]), and hydroelastic applications in variable bathymetry regions, formulated in the context of classical thin plate theory (Belibassakis & Athanassoulis [3]) and high-order shear deformable plate theory (Athanassoulis & Belibassakis [1]). In accordance with the latter work, the infinite set $Z_n(z; x), n = 0, 1, 2, 3, \dots$,

of functions describing the vertical structure of each mode, at each horizontal position x , are generated by

$$\partial_z^2 Z_n(z) - \kappa_n^2 Z_n(z) = 0, \quad \text{in the vertical interval } -h < z < -b, \quad (2a)$$

$$\dot{Z}_n(z = -h) = 0, \quad \text{at the bottom } z = -h(x), \quad (2b)$$

$$\alpha \dot{Z}_n(z = -b) - \mu Z_n(z = -b) = 0, \quad \text{at the water-elastic body interface } z = -b(x), \quad (2c)$$

where α is a function depended on κ . The solution the above problem is given by

$$Z_n(z) = \cosh^{-1}(\kappa_n H) \cosh[\kappa_n(z+h)], \quad n = 0, 1, 2, 3, \dots, \quad (3)$$

where the eigenvalues $\{\kappa_n, n = 0, 1, 2, \dots\}$ are obtained as the roots of (local) dispersion relation:

$$\mu H = \alpha(\kappa) \kappa H \tanh(\kappa H), \quad \text{where } \alpha(\kappa) = De_v \kappa^4 (1 - \delta) + 1 - \varepsilon \left(1 + \frac{\kappa^2 b^2}{12} (1 - \delta) \right). \quad (4)$$

In Eq. (4), $\mu = \omega^2 / g$ is the frequency parameter, $H = h - b$ is the thickness of the water layer, $D = Eb^3 / (12\rho g(1 - \nu^2))$ denotes the plate flexural rigidity (with E Young's modulus and ν Poisson's ratio). Moreover, ρ is the water density and g acceleration of gravity, $\varepsilon = m\omega^2 / \rho g$ the plate mass coefficient (with m the plate mass distribution per horizontal area). Parameter $e_v = (1 + \nu^2(1 - 2\nu))^{-1}$ is a material constant, involved in the expression of $\sigma_{xx} = Ee_v(1 - \nu^2)^{-1} \varepsilon_{xx}$. Additionally, Xb^{-2} and δ are newly introduced non-dimensional parameters given as follows:

$$\delta = 0.0135 \left(0.0135 + (X/b^2) \right)^{-1}, \quad \frac{X}{b^2} = \frac{(2\kappa^2 b^2 e_v)^{-1} 0.132(1 - \nu)}{(1 - (1/12)(\varepsilon/d)(\kappa H)^{-2})} \quad \text{and } d = De_v H^{-4}. \quad \text{More details}$$

can be found in Athanassoulis & Belibassakis[1]. We note here that the above system (Eqs.3,4) with $\alpha(\kappa) \rightarrow (D\kappa^4 + 1 - \varepsilon)$, which is asymptotically obtained for small plate thickness ($\kappa b \rightarrow 0$, $\delta \rightarrow 0$), reduces exactly to the standard hydroelastic relations based on thin-plate theory (Athanassoulis & Belibassakis [3]). On the basis of the above complete expansions of the wave potentials in the two semi-infinite strips ($x \leq x_1$) and ($x \geq x_3$) are obtained (see also Belibassakis & Athanassoulis [1]), describing plane hydroelastic waves in these regions. Demonstrative results concerning the phase speed of propagating hydroelastic waves in homogeneous floating shear deformable plate of constant thickness, non-dimensionalised with respect to the phase speed of linearised water waves in shallow conditions (C/\sqrt{gH}), are shown in Fig.2. We have used a density ratio $\rho_E / \rho = 923/1025$ corresponding to ice/water. In this case the plate has uniform finite thickness $b/h=0.5$, and Poisson ratio $\nu=0.3$. Results are presented for three representative values of flexural rigidity $d=1, 10, 100$ and three values of $\varepsilon=0, 0.5, 1$, for shoaling ratio κH ranging from very shallow to very deep water conditions ($0 < \kappa H < 16$).

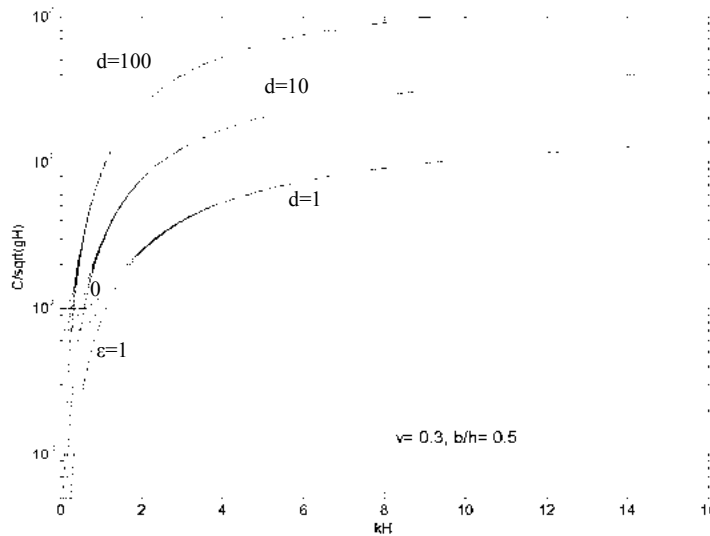


Figure 2. Phase speed of harmonic flexural waves for shear deformable plate of uniform finite thickness $b/h=0.5$, and Poisson ratio $\nu=0.3$, for various values of ε , d and shoaling ratio (κH).

We clearly observe that the effect of flexural rigidity that leads to significant increase of phase speed. Also, in the limit of very shallow conditions $\kappa H \ll 1$, we observe in this figure that the effect of mass parameter ε becomes important.

2.2 The extended coupled-mode system (eCMS)

The eCMS of horizontal differential equations has been obtained by means of a variational principle composed by the one-field functional of elastodynamics in the plate region (see, e.g. [28]), and the Luke's [25] pressure functional in the water region. The wave potential in the water column is represented by means of the local mode series expansion Eq. (1), and an enhanced fourth-order vertical expansion of the elastic displacement field in the floating plate is used containing additional elastic vertical modes, permitting the shear strain and stress to vanish on both the upper and lower boundaries of the thick floating plate. More details can be found in Athanassoulis & Belibassakis [1]. In the case of plates of general, finite thickness, but slowly varying characteristics, elimination relations are approximately derived between the vertical plate deflection (w) and the rest of the elastic displacement modes. In this case the eCMS takes the following form

$$\sum_{n=-1}^{\infty} a_{mn}(x) \frac{\partial^2 \varphi_n(x)}{\partial x^2} + b_{mn}(x) \frac{\partial \varphi_n}{\partial x} + c_{mn}(x) \varphi_n(x) = i\omega w(x), \quad m = -1, 0, 1, \dots, \quad (6)$$

in conjunction with the following equation providing the coupling between the water-wave modes (φ_n) and the elastic plate deflection (w):

$$\frac{\partial^2}{\partial x^2} \cdot \left(dH^4 (1-\delta) \frac{\partial^2 w}{\partial x^2} \right) - \varepsilon \frac{b^2}{12} \frac{\partial^2 w}{\partial x^2} + (1-\varepsilon)w = \frac{i\mu}{\omega} \sum_{n=0}^{\infty} \varphi_n(x) . \quad (7)$$

In the above equations, the x -dependent coefficients $a_{mn}(x)$, $b_{mn}(x)$ and $c_{mn}(x)$ are given by the following expressions

$$a_{mn}(x) = \langle Z_n, Z_m \rangle_{-h}^{-b}, \quad (8a)$$

$$b_{mn}(x) = 2 \left\langle \frac{\partial Z_n}{\partial x}, Z_m \right\rangle_{-h}^{-b} + \frac{dh}{dx} Z_n(z = -h; x) Z_m(z = -h; x) - \frac{db}{dx} Z_n(z = -b; x) Z_m(z = -b; x), \quad (8b)$$

$$c_{mn}(x) = \left\langle \frac{\partial^2 Z_n}{\partial x^2} + \frac{\partial^2 Z_n}{\partial z^2} - q^2 Z_n, Z_m \right\rangle_{-h}^{-b} - \left(\frac{\partial Z_n(z = -b; x)}{\partial z} + \frac{db}{dx} \frac{\partial Z_n(z = -b; x)}{\partial x} \right) Z_m(z = -b; x) + \left(\frac{\partial Z_n(z = -h; x)}{\partial z} + \frac{dh}{dx} \frac{\partial Z_n(z = -h; x)}{\partial x} \right) Z_m(z = -h; x), \quad (8c)$$

where $\langle f, g \rangle_{-h}^{-b} = \int_{z=-h(x)}^{z=-b(x)} f(z)g(z)dz$. After solving the system of Eqs. (6), (7) the wave characteristics can be obtained all over the domain by means of the calculated wave modes $\varphi_n(x)$, $n = -1, 0, 1, 2, 3, \dots$, using the expansion (1). Also, the elastic strain and stress distributions in the thick plate cross section are obtained from the solution, using expressions connecting the vertical deflection (w) and the rest of the shear deformable plate modes (see Athanassoulis & Belibassakis [1]).

The eCMS is supplemented by specific boundary conditions ensuring complete matching between the wave and the elastic fields at the two vertical interfaces (at $x = x_1$ and $x = x_3$) separating the variable bathymetry inhomogeneous subdomain from the regions of incidence ($x \leq x_1$) and transmission ($x \geq x_3$), respectively. More specifically at the left interface

$$\varphi'_0(x_1) + i\kappa_0^{(1)}\varphi_0(x_1) = 2i\kappa_0^{(1)}\exp(i\kappa_0^{(1)}x_1), \quad \varphi'_n(x_1) + i\kappa_n^{(1)}\varphi_n(x_1) = 0, \quad n=1,2,3.., \quad (9a)$$

$$\varphi''_n(x_1) + (\kappa_n^{(1)})^2 \varphi_n(x_1) = 0, \quad \text{at } x = x_1, \quad \text{and} \quad (9b)$$

$$w(x_1) = \frac{i}{\omega} \sum_{n=0}^{\infty} \kappa_n^{(1)} \tanh(\kappa_n^{(1)}H_1) \varphi_n(x_1), \quad (9c)$$

where a prime denotes x -differentiation. Moreover on the right vertical interface

$$\varphi'_n(x_3) - i\kappa_n^{(3)}\varphi_n(x_3) = 0, \quad n=0,1,2,3.., \quad \text{and} \quad \varphi''_n(x_3) + (\kappa_n^{(3)})^2 \varphi_n(x_3) = 0, \quad \text{at } x = x_3, \quad (10a,b)$$

$$w(x_3) = \frac{i}{\omega} \sum_{n=0}^{\infty} \kappa_n^{(3)} \tanh(\kappa_n^{(3)}H_3) \varphi_n(x_3). \quad (10c)$$

In the above equations $\kappa_n^{(m)}$, $m=1,3$ denote eigenvalues obtained from the extended hydroelastic dispersion relation (4) formulated at the constant depth and plate thickness

subregions, respectively. The forcing of the eCMS appears only Eq.(9a) and is associated with the mode representing the incident wave exciting the hydroelastic waveguide.

2.3 Reformulation as a second-order system

The discrete version of the present CMS is obtained by truncating the local-mode series (1) to $n=N$ keeping the first $N+2$ modes $\{\varphi_n(x), n=-1, 0, 1, \dots, N\}$. Subsequently, by setting $\varphi_{N+1}(x) = w(x)$ and $\varphi_{N+2}(x) = dH^4(1-\delta)w(x)''$, Eq.(7) is equivalently written as follows

$$dH^4(1-\delta)\varphi_{N+1}''(x) - \varphi_{N+2}(x) = 0, \quad (11a)$$

$$\varphi_{N+2}''(x) + \frac{\varepsilon b^2}{12dH^4}\varphi_{N+2}(x) + (1-\varepsilon)\varphi_{N+1}(x) - \frac{i\mu}{\omega} \sum_{n=-1}^N \varphi_n(x) = 0. \quad (11b)$$

Thus, the present eCMS is put in the following, second-order form

$$\sum_{n=-1}^{N+2} \left(\widehat{a}_{mn}(x)\varphi_n'' + \widehat{b}_{mn}(x)\varphi_n' + \widehat{c}_{mn}(x)\varphi_n \right) = 0, \quad m = -1, 0, 1, \dots, N, N+1, N+2. \quad (12)$$

where the definition of general $\widehat{a}_{mn}, \widehat{b}_{mn}, \widehat{c}_{mn}$ coefficients is obtained from Eqs. (8) and (11).

3. CMS SOLUTION WITH hp-ADAPTIVE FINITE ELEMENTS

Assuming that the matrix \widehat{a}_{mn} is invertible, the coupled mode system is written in the form:

$$-\mathbf{u}'' + \mathbf{B}\mathbf{u}' + \mathbf{C}\mathbf{u} = 0, \quad x \in (x_1, x_3) \quad \text{where } \mathbf{B} = -\widehat{\mathbf{a}}^{-1}\widehat{\mathbf{b}} \quad \text{and} \quad \mathbf{C} = -\widehat{\mathbf{a}}^{-1}\widehat{\mathbf{c}}, \quad (13)$$

where $\mathbf{u}(x)$ is the vector of unknown modal amplitudes of dimension $M = N + 4$. The system is accompanied by the boundary conditions

$$\mathbf{u}' + \mathbf{S}_1\mathbf{u} = \mathbf{T}_1, \quad \text{at } x = x_1, \quad \text{and} \quad \mathbf{u}' + \mathbf{S}_3\mathbf{u} = \mathbf{T}_3 = 0, \quad \text{at } x = x_3, \quad (14)$$

where the coefficients $\mathbf{S}_n, n=1, 3$, and the forcing \mathbf{T}_1 are obtained from Eqs.(9) and (10). The weak formulation of boundary value problem (13),(14) consists of finding $\mathbf{u} \in V$, where V

is the Cartesian product of Sobolev spaces $V = [H^1(x_1, x_3)]^M$ defined over \mathbb{C} , such that

$$r(\mathbf{w}, \mathbf{u}) = \mathbf{w}^*(x_3)\mathbf{T}_3 - \mathbf{w}^*(x_1)\mathbf{T}_1 = F(\mathbf{w}), \quad \forall \mathbf{w} \in V, \quad (15a)$$

where \mathbf{w}^* denotes the complex conjugate of $\mathbf{w} \in V$ and $r(\cdot, \cdot): V \times V \rightarrow \mathbb{C}$ is the following continuous sesquilinear form

$$r(\mathbf{w}, \mathbf{u}) = \int_{x_1}^{x_3} \mathbf{w}^* \mathbf{u}' dx + \int_{x_1}^{x_3} \mathbf{w}^* \mathbf{B}\mathbf{u}' dx + \int_{x_1}^{x_3} \mathbf{w}^* \mathbf{C}\mathbf{u} dx + \mathbf{w}^*(x_3)\mathbf{S}_3\mathbf{u}(x_3) - \mathbf{w}^*(x_1)\mathbf{S}_1\mathbf{u}(x_1). \quad (15b)$$

Assuming that $r(\cdot, \cdot)$ is V -elliptic (i.e. $\exists \mu \in \mathbb{R}^+$ such that $r(\mathbf{u}, \mathbf{u}) \geq \mu \|\mathbf{u}\|_V^2$, with ellipticity constant μ) direct application of the Lax – Milgram lemma guarantees the existence of a

unique $\mathbf{u} \in V$, solution to variational problem (15), $\forall F \in V'$ (the dual of V). Further, we have the a priori estimate

$$\|\mathbf{u}\|_V \leq \frac{1}{\mu} \|F\|_{V'} \quad . \quad (16)$$

Finite element approximation of solutions of the variational problem are constructed by considering the subspaces $V^h \subset V$ and finding $\mathbf{u}^h \in V^h$ such that $r(\mathbf{w}^h, \mathbf{u}^h) = \mathbf{w}^h(x_3)\mathbf{T}_2 - \mathbf{w}^h(x_1)\mathbf{T}_1$ for all $\mathbf{w}^h \in V^h$. For the implementation of the Finite Element Method, we assume a partition of the interval $[x_1, x_3]$ of the form $x_1 = s_1 < s_2 < \dots < s_{N+1} = x_3$, $N \in \mathbb{N}$. Let $P_p(s)$ be a polynomial of degree p . We now define the subspaces $V^h \subset V$ as $V^h \doteq \left\{ \mathbf{u}^h \in [H^1(x_1, x_3)]^M : u_j^h|_{[s_i, s_{i+1}]} \equiv P_p(s), i=1, 2, \dots, N, j=1, 2, \dots, M, p \in \mathbb{N} \right\}$. From standard theory (e.g., Babuška & Ihlenburg [29]) we expect the following estimate of the error associated with the present approximate solution

$$\|\mathbf{u} - \mathbf{u}^h\|_V \leq ch^p \|\mathbf{u}\|_{[H^2(x_1, x_3)]^M} \quad , \quad (17)$$

holding for some positive constant c that does not depend on h or p .

4 NUMERICAL EXAMPLES AND DISCUSSION

In order to illustrate the applicability of our method, a specific example is presented in Figs. 3-5 concerning the propagation of harmonic hydroelastic waves of period $T=10\text{sec}$ ($\omega=0.628\text{rad/sec}$) on floating ice sheet, characterised by modulus of elasticity $E=5\text{GPa}$, Poisson's ratio $\nu=0.3$ and values of ice/water densities $923/1025 \text{ kg/m}^3$ (see also Squire *et al* [6]). In this case, except of uniform ice sheet of finite thickness $b=1\text{m}$, in semi-infinite strips of constant depths $h=13\text{m}$ and $h=7\text{m}$, respectively, we also consider the effect of inhomogeneous ice thickness in $100\text{m} < x < 400\text{m}$, with specific form as shown in Fig.3.

In this case the beam thickness varies from 1m at the ends of the domain to 3m in the middle part. We also consider the effect of an underwater shoal, extending from $x=0\text{m}$ to 500m , connecting two regions of constant but different depths: the left region of wave-incidence, where $h_1=13\text{m}$, and the region of transmission (right half strip), where $h_3=7\text{m}$. In the latter case, the average and maximum values of the slope of the bottom profile are 1.2% and 6%, respectively. Present method results have been obtained by retaining 5 modes in the local series Eq. (1), which was found to be enough for numerical convergence. Furthermore numerical results are based on a discretization using $N=251$ elements for subdivision of the segment from $x_1=0$ to $x_3=500\text{m}$, and $p=1$, which is shown to provide reasonable accuracy. Future work will focus on the detailed investigation of the rates of convergence and evaluation of the efficiency of present FEM scheme for higher p , demonstrating the overall robustness of the solution procedure.

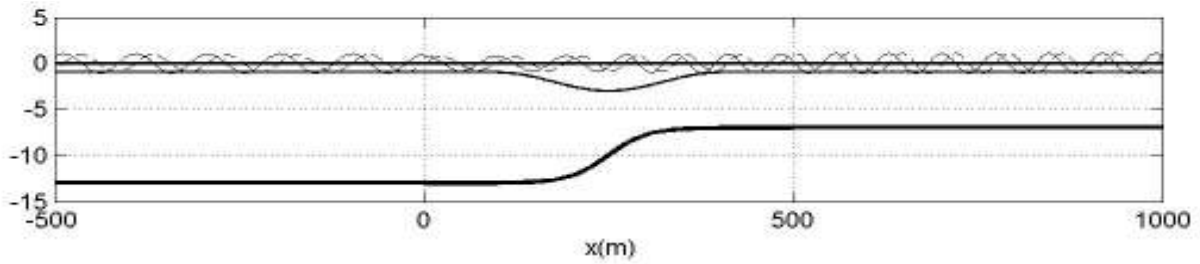


Figure 3. Real part (solid line) and imaginary part (dashed line) of the wave field φ at the top of the water layer. The middle subregion containing floating body and bottom inhomogeneity extends from $x_1 = 0$ to $x_3 = 500m$.

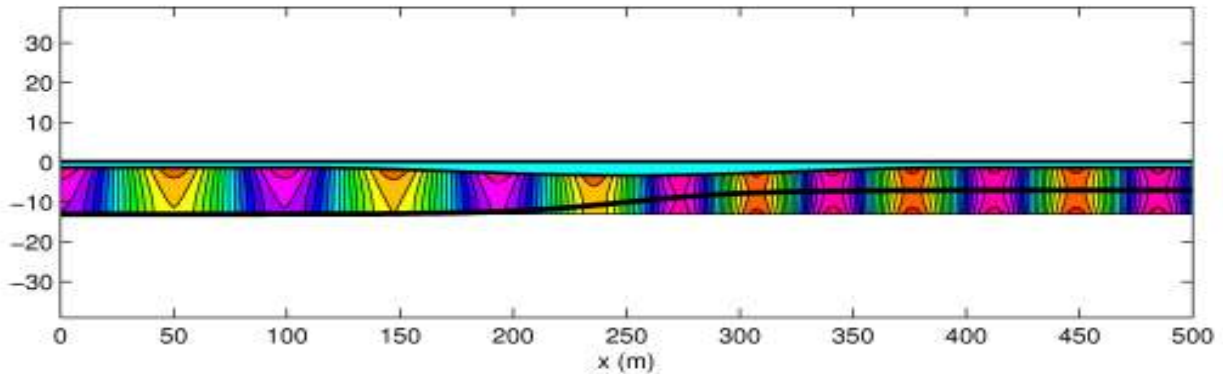


Figure 4. Plot of the wave field φ (real part) in the water layer, as calculated by the present method, using equipotential lines. The ice layer is indicated by using cyan lines. Extension of equipotential lines below the bottom profile is maintained in order to visualize the fulfilment of corresponding boundary condition both on the flat and sloping parts.

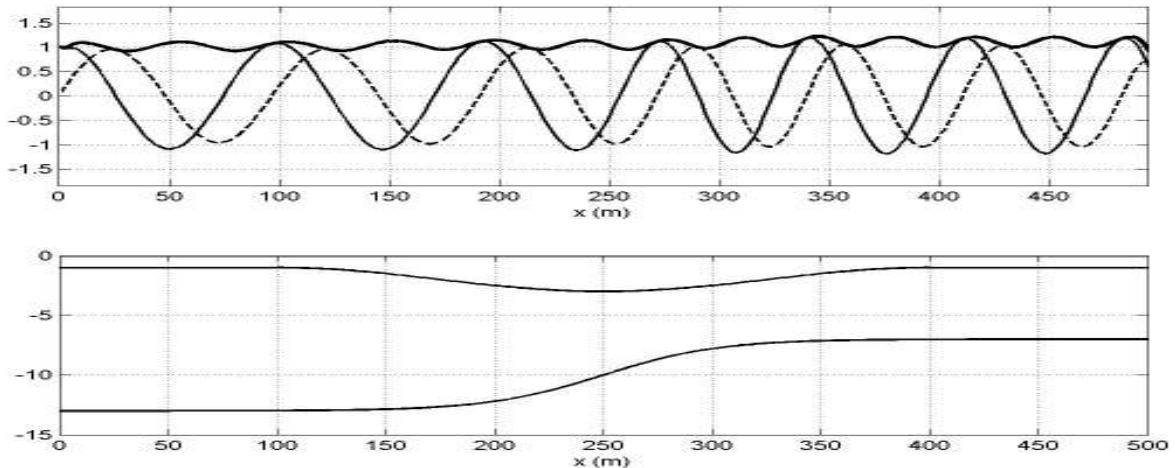


Figure 5. Real part (solid line) and imaginary part (dashed line) of the elastic deflection w , as calculated by the present method, in the middle subregion containing floating body and bottom inhomogeneity extending from $x_1 = 0$ to $x_3 = 500m$.

CONCLUSIONS

A novel coupled-mode system of horizontal differential equations has been applied to the hydroelastic analysis of large floating bodies or ice sheets of general, finite thickness, lying over variable bathymetry regions. The present method is based on the theory of shear deformable plates (or beams), and is derived by an enhanced representation of the elastic displacement field, containing additional elastic vertical modes and permitting the shear strain and stress to vanish on both the upper and lower boundaries of the thick floating plate derived by Athanassoulis and Belibassakis[1], for the analysis of floating, shear deformable floating plates or beams of general shape. The proposed plate (or beam) model is based on the addition of extra vertical elastic deformation modes, at each horizontal position along the floating body, permitting shear strain and stress to vanish on both the upper and lower boundaries and extending third-order plate theories. First numerical results are obtained by applying the hp-version of the Finite Element Method to the solution of the resulting system, indicating good convergence rates and adaptivity capabilities.

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