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A COUPLED THERMO-FLEXIBLE MULTI-BODY APPROACH FOR VIRTUAL RIG TESTING OF BRAKE DISCS

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Abstract. In this paper a coupled thermo-flexible multi-body model of a test rig including a brake disc with pads is proposed. The dynamics of the system is modeled by two rotors connected via a torsional spring. The rotors represent the brake disc and the flywheel of the rig. The brake moment acting on the disc is governed by thermo-mechanical finite element analysis of the frictional contact between the pad and the disc. The finite element model is formulated by treating the disc in an Eulerian framework. The frictional power is calculated most accurately by solving Signorini's contact conditions and Coulomb's law of friction by using the augmented Lagrangian approach. This is done by including thermal expansion of the pad and the disc in the equilibrium equations and using a temperature dependent friction coefficient. The heat due to the frictional power is then transported with convection defined by the angular velocity of the disc. Distortions in the solution due to the non-symmetry of the convection matrix is stabilized by using the streamline-upwind approach. Summarized, the proposed virtual test rig of brake discs is modeled by three system of equations, i.e. the dynamic equations of the multi-body system, the energy balance of the pad and the disc, and the frictional contact of the pad and the disc. In order to obtain a robust and efficient approach, these three equation systems are solved sequentially by using Newmark's method, the trapezoidal approach and Newton's method. In such manner, temperatures, brake power and angular velocities can be generated accurately at low computational costs for different braking scenarios. This is demonstrated for a real pad-disc system to a heavy truck.

1 INTRODUCTION

In the development of disc brake systems physical rig testing of these systems is most important in order to meet design requirements. For disc brake systems to heavy trucks, the rigs for performing these experiments are large investments for the developers and consequently the tests are expensive. In addition, a test usually takes long time to set up and perform. Thus, to study several design proposals using physical rig tests quickly becomes very costly. Therefore, it would be very beneficial if virtual rig tests could be performed during the development process. In this paper, a thermo-flexible multi-body approach for performing such tests is suggested and implemented.

The dynamics of the rig is represented by a flexible multi-body model. The disc and the flywheel are modeled by two mass moments of inertia and are connected via a torsional spring. The brake moment on the disc is obtained by using a thermo-mechanical finite element model including frictional contact. Recently, Strömberg [1] developed a Eulerian framework for this class of problems. Instead of formulating the disc in a Lagrangian frame, the disc material flows through a fixed mesh, where the convective terms are defined by the angular velocity from the flexible multi-body model. Due to the fixed grid, a node-to-node contact formulation between the disc and the pad can easily be established. The contact between the disc and the pad is then solved by the augmented Lagrangian formulation using Newton's method. One of the first implementation of this approach can be found in Strömberg [2].

A similar work as presented herein, but without temperatures, can be found in Klarbring et al. [3], where the dynamic transmission error in spur gears was studied.

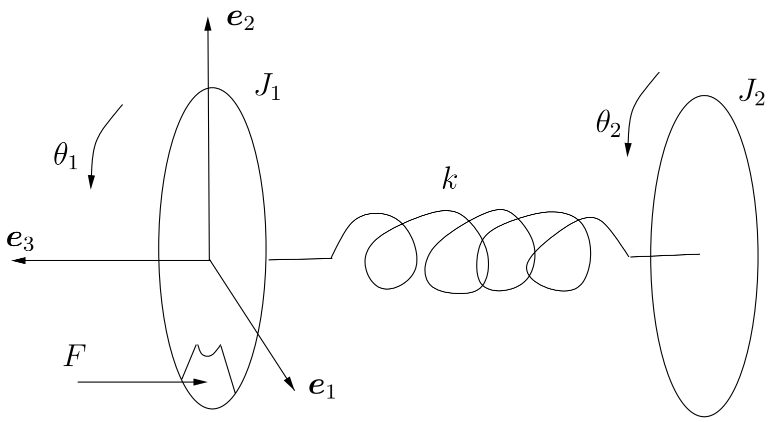


Figure 1: *The thermo-flexible multi-body model of a test rig for disc brakes.*

2 A GLOBAL FLEXIBLE DYNAMIC MODEL

Let us study the system depicted in Figure 1. The system consists of two rotors, one brake pad and a torsional spring with a stiffness k. The first rotor, with mass moment of inertia J_1, represents the brake disc and the second rotor with mass moment of inertia J_2 is the flywheel of the test rig. The two rotors are connected via the torsional spring and the kinematics of the rotors are defined by the angles $\theta_i = \theta_i(t)$ along the $\boldsymbol{e}_3$-direction as functions of time t. A brake force $\boldsymbol{F} = F(t)\boldsymbol{e}_3$ is applied on the pad which results in frictional contact between the pad and the brake disc, which in turn implies a brake moment. The frictional contact is modeled most accurately by applying a coupled thermo-mechanical finite element analysis. Details about this analysis is presented in the next section.

The frictional heating and the corresponding thermal expansion will imply that the distribution of the frictional forces will change in time. This is included in the global flexible dynamic model by letting the brake moment $\boldsymbol{M}_b = M_b(t)\boldsymbol{e}_3$ acting on the brake disc be calculated using this distribution. That is, letting r^A be the radius to a contact node A and P_t^A be the friction force at this node,

$$M_b = M_b(t) = \sum_{A=1}^{n_c} r^A P_t^A, \tag{1}$$

where n_c is the number of contact nodes.

The dynamics of the system can now be formulated as

$$\boldsymbol{J}\ddot{\boldsymbol{\theta}} + \boldsymbol{K}\boldsymbol{\theta} = \boldsymbol{M}, \tag{2}$$

where

$$\begin{aligned} \boldsymbol{J} &= \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}, \\ \boldsymbol{K} &= \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}, \\ \boldsymbol{M} &= \left\{ \begin{array}{c} M_b(t) \\ 0 \end{array} \right\}, \boldsymbol{\theta} = \left\{ \begin{array}{c} \theta_1(t) \\ \theta_2(t) \end{array} \right\}. \end{aligned} \tag{3}$$

Equation (2) is treated in time by applying the average acceleration method. Letting $\boldsymbol{\theta}_n$, $\dot{\boldsymbol{\theta}}_n$, $\ddot{\boldsymbol{\theta}}_n$ and $\boldsymbol{\theta}_{n+1}$, $\dot{\boldsymbol{\theta}}_{n+1}$, $\ddot{\boldsymbol{\theta}}_{n+1}$ represent the approximations of $\boldsymbol{\theta}(t)$ at time t_n and t_{n+1}, respectively, the procedure for this is governed by

$$\left[\boldsymbol{J} + \frac{\Delta t^2}{4}\boldsymbol{K}\right]\ddot{\boldsymbol{\theta}}_{n+1} = \boldsymbol{M}_{n+1} - \hat{\boldsymbol{M}}_n, \tag{4}$$

where

$$\begin{aligned} \hat{\boldsymbol{M}}_n &= \boldsymbol{K}\boldsymbol{\theta}_n + \Delta t\boldsymbol{K}\dot{\boldsymbol{\theta}}_n + \frac{\Delta t^2}{4}\boldsymbol{K}\ddot{\theta}_n, \\ \dot{\boldsymbol{\theta}}_{n+1} &= \dot{\boldsymbol{\theta}}_n + \frac{\Delta t}{2}(\ddot{\boldsymbol{\theta}}_n + \ddot{\boldsymbol{\theta}}_{n+1}), \\ \boldsymbol{\theta}_{n+1} &= \boldsymbol{\theta}_n + \Delta t\dot{\theta}_n + \frac{\Delta t^2}{4}(\ddot{\boldsymbol{\theta}}_n + \ddot{\boldsymbol{\theta}}_{n+1}). \end{aligned} \tag{5}$$

3 A LOCAL THERMO-MECHANICAL MODEL

The friction forces appearing in (1) is determined by performing a thermo-mechanical finite element analysis as outlined in this section. This is done by formulating the brake disc in an Eulerian framework. In such manner heat is transported through the mesh by convection defined by the angular velocity $\dot{\theta}_1(t)$ of the disc. The heat balance in the disc is then governed by

$$\boldsymbol{C}\dot{\boldsymbol{T}} + (\boldsymbol{O} + \boldsymbol{N} + \boldsymbol{R})\boldsymbol{T} = \boldsymbol{Q}_c, \tag{6}$$

where $\boldsymbol{C}$ is the heat capacity matrix, $\boldsymbol{O}$ represents the conduction in the disc, $\boldsymbol{N} = \boldsymbol{N}(\dot{\boldsymbol{\theta}})$ is the convection matrix and $\boldsymbol{R} = \boldsymbol{R}(\dot{\boldsymbol{\theta}})$ is a matrix of artificial conduction in accordance to the streamline-upwind approach in order to stabilize distortions in the solution caused by the nonsymmetric convection matrix. Furthermore, $\boldsymbol{T}$ is a vector of nodal temperatures and $\boldsymbol{Q}_c = \boldsymbol{Q}_c(\boldsymbol{P}_n, \boldsymbol{T}, \dot{\boldsymbol{\theta}})$ is a vector of nodal heat fluxes at the contact surface representing the contact conductance and the frictional power.

The rates of the temperature vector are treated by applying the trapezoidal rule, i.e. the following approximation is utilized ($0 \leq \alpha \leq 1$):

$$\boldsymbol{T}_{n+1} = \boldsymbol{T}_n + \Delta t \left((1-\alpha)\dot{\boldsymbol{T}}_n + \alpha \dot{\boldsymbol{T}}_{n+1} \right), \tag{7}$$

where subscripts n and $n+1$ again represent approximations of states at times t_n and t_{n+1}, respectively. Typically, we set $\alpha = 2/3$ according to the Galerkin approach.

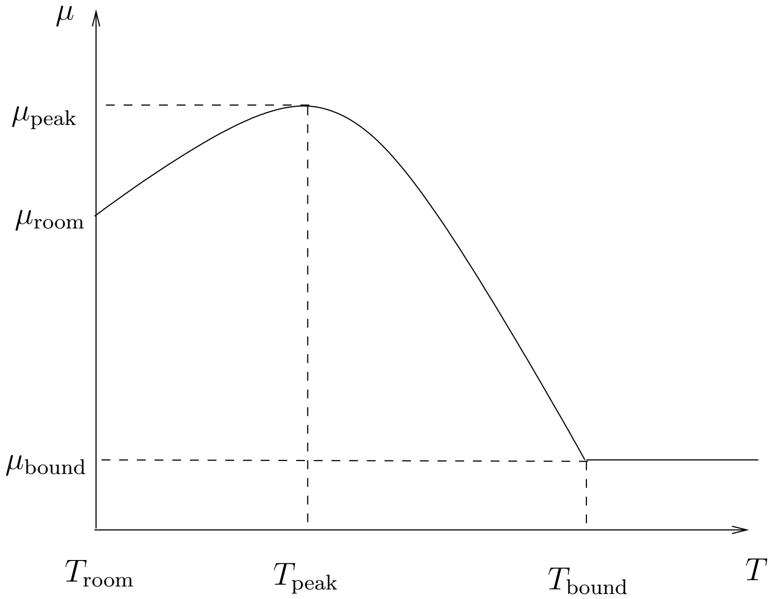


Figure 2: *The temperature dependent friction model.*

The frictional power is governed by the normal contact forces $\boldsymbol{P}_n$, a temperature dependent friction coefficient $\mu = \mu(\boldsymbol{T})$ and the tangential velocity $r^A\dot{\theta}_1$ at each contact node. That is, at a contact node A, the frictional power can be written as

$$Q_f^A = \mu(T^A)P_n^A r^A \dot{\theta}_1. \tag{8}$$

The temperature dependent friction model is defined by the friction coefficient μ_{room} at the room temperature T_{room}, a peak value μ_{peak} at a certain temperature T_{peak}, and a lower bounded value μ_{bound} at high temperatures above T_{bound}. These assumptions are illustrated by Figure 2. A quadratic regression model,

$$\mu(T) = \mu_0 + \mu_1 T + \mu_2 T^2, \tag{9}$$

is then fitted according to the values obtained from experiments of μ_{room}, T_{room}, μ_{peak}, T_{peak}, μ_{bound} and T_{bound}.

The contact forces are obtained by solving equilibrium for linear elastic bodies with thermal expansion included. In addition, the contact forces are in accordance to Signorini's unilateral contact conditions and Coulomb's law of friction for global sliding. Equilibrium of the disc reads

$$\boldsymbol{K}\boldsymbol{d} + \hat{\boldsymbol{K}}\boldsymbol{T} + (\boldsymbol{C}_n + \boldsymbol{C}_t(\boldsymbol{T}))^T \boldsymbol{P}_n = \boldsymbol{F}_\omega, \tag{10}$$

where $\boldsymbol{d}$ is the displacement vector, $\boldsymbol{K}$ is the stiffness matrix, $\hat{\boldsymbol{K}}$ represents the thermal expansion properties, $\boldsymbol{C}_n$ contains normal directions, $\boldsymbol{C}_t(\boldsymbol{T})$ is defined by the tangential sliding directions as well as the temperature dependent friction model in (9), and, finally, $\boldsymbol{F}_\omega = \boldsymbol{F}_\omega(\theta_1)$ is a vector of centripetal forces.

Signorini's contact conditions are treated by the well-known augmented Lagrangian approach. By letting $\boldsymbol{d}_n = \boldsymbol{C}_n\boldsymbol{d}$, this can be written as

$$\boldsymbol{P}_n = \frac{\boldsymbol{P}_n + r\boldsymbol{d}_n + |\boldsymbol{P}_n + r\boldsymbol{d}_n|}{2}, \tag{11}$$

where $r > 0$ is a penalty coefficient and it is assumed that the initial gaps between the pad and the disc are negligible.

4 A SEQUENTIAL APPROACH

Summarized, for a given state defined by $\boldsymbol{d}_n$, $\boldsymbol{P}_n$, $\boldsymbol{T}_n$, $\boldsymbol{\theta}_n$, $\dot{\boldsymbol{\theta}}_n$, $\ddot{\boldsymbol{\theta}}_n$ at a time t_n, our coupled thermo-flexible system is treated by solving the following equations sequentially in order to find the next state at time t_{n+1}:

$$\begin{aligned} \boldsymbol{h}(\boldsymbol{d}_{n+1}, \boldsymbol{P}_{n+1}; \boldsymbol{T}_n, \dot{\boldsymbol{\theta}}_n) &= \boldsymbol{0}, \\ \boldsymbol{J}_{\text{eff}}\ddot{\boldsymbol{\theta}}_{n+1} &= \boldsymbol{M}_{\text{eff}}(\boldsymbol{P}_{n+1}, \boldsymbol{\theta}_n, \dot{\boldsymbol{\theta}}_n, \ddot{\boldsymbol{\theta}}_n), \\ \boldsymbol{A}(\boldsymbol{P}_{n+1}, \dot{\boldsymbol{\theta}}_{n+1})\boldsymbol{T}_{n+1} &= \boldsymbol{Q}_{\text{eff}}(\boldsymbol{P}_{n+1}, \boldsymbol{T}_n, \dot{\boldsymbol{\theta}}_{n+1}). \end{aligned} \tag{12}$$

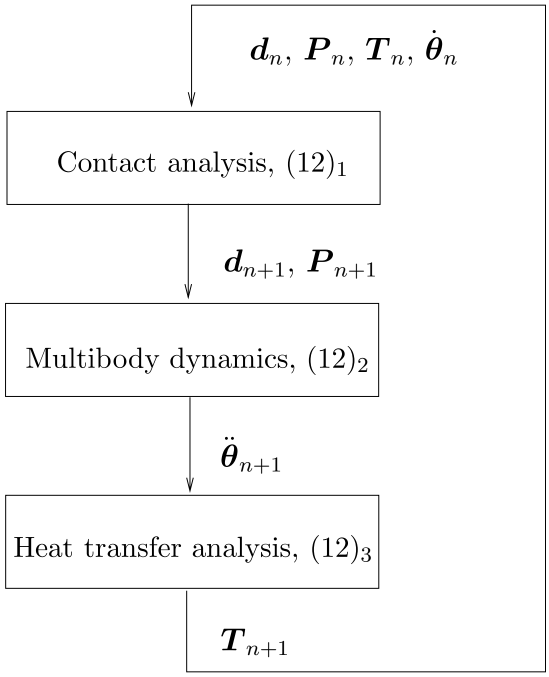


Figure 3: *The sequential approach.*

The first equation system in $(12)_1$ represents the augmented Lagrangian formulation of the thermo-mechanical contact between the pad and the disc. This was presented in more details for the disc in (10) and (11). For given $\boldsymbol{T}_n$ and $\dot{\boldsymbol{\theta}}_n$, the solution to $(12)_1$, defined by $\boldsymbol{d}_{n+1}$ and $\boldsymbol{P}_{n+1}$, is obtained by applying a non-smooth Newton method.

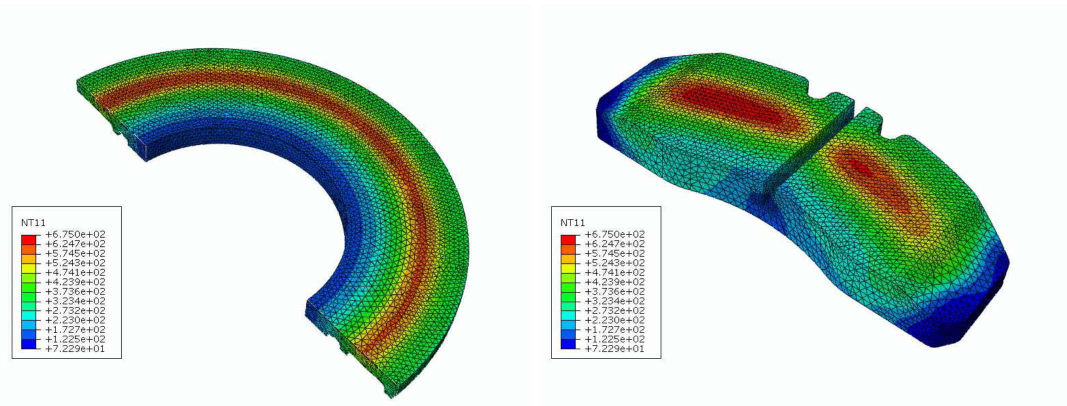


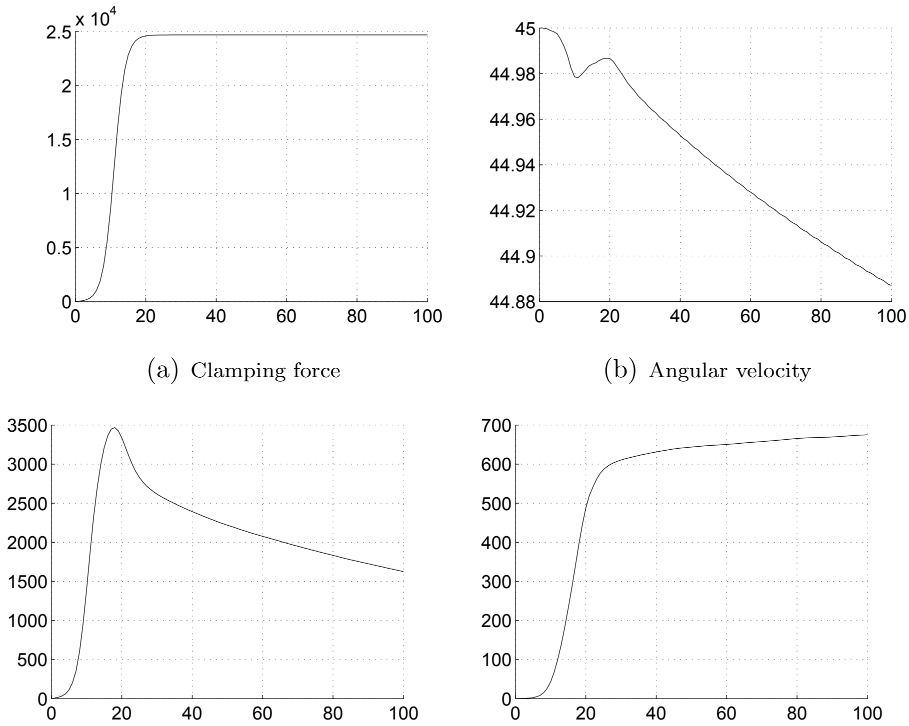
Figure 4: *A heat band on the disc and the corresponding temperatures on the pad.*

$(12)_2$ is a compact formulation of (4) using

$$\begin{aligned} \boldsymbol{J}_{\text{eff}} &= \left[\boldsymbol{J} + \frac{\Delta t^2}{4} \boldsymbol{K} \right], \\ \boldsymbol{M}_{\text{eff}}(\boldsymbol{P}_{n+1}, \boldsymbol{\theta}_n, \dot{\boldsymbol{\theta}}_n, \ddot{\boldsymbol{\theta}}_n) &= \boldsymbol{M}_{n+1} - \hat{\boldsymbol{M}}_n. \end{aligned} \tag{13}$$

Finally, $(12)_3$ represents heat balance of the system, which was discussed previously in detail for the disc, see (6) and (7). One might notice that the contact conductances have here been included in $\boldsymbol{A} = \boldsymbol{A}(\boldsymbol{P}_{n+1}, \dot{\boldsymbol{\theta}}_{n+1})$ together with $\boldsymbol{C}$, $\boldsymbol{O}$, $\boldsymbol{N}$ and $\boldsymbol{R}$.

The sequentially procedure for solving the equations in (12) is also illustrated by Figure 3.



(a) Clamping force

(b) Angular velocity

(c) Brake moment

(d) Maximum temperture

Figure 5: *History of data generated by the toolbox.*

5 NUMERICAL EXAMPLES

The thermo-flexible multi-body approach presented above is implemented in an in-house toolbox using Matlab and Fortran. A GUI is also developed such that a virtual rig test is easily set up by a user. An example of such a test is demonstrated in Figures 4 and 5. The test represents hard braking of a truck moving downhill. In a short time a heat band of high temperatures develops on the disc surface. This is shown in Figure 4, where also a plot of the corresponding temperatures on the pad is given. In addition, the toolbox also generates histories of different data such as e.g. angular velocity, braking moment and maximum temperature, see Figure 5. In a near future, the virtual tests generated with the toolbox will be compared to real experiments. This will be discussed at the conference and is a topic of a forthcoming paper.

6 CONCLUDING REMARKS

In this work a new thermo-flexible multi-body approach for simulating rig tests of disc brakes is presented. The proposed method is robust and efficient. This is demonstrated by simulating a rig test of a real disc brake to a heavy truck.

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