VI International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2015
B. Schrefler, E. Oñate and M. Papadrakakis(Eds)

MODELLING PRESSURE CYCLE AND INTERACTION WITH REED VALVES IN A RECIPROCATING COMPRESSOR

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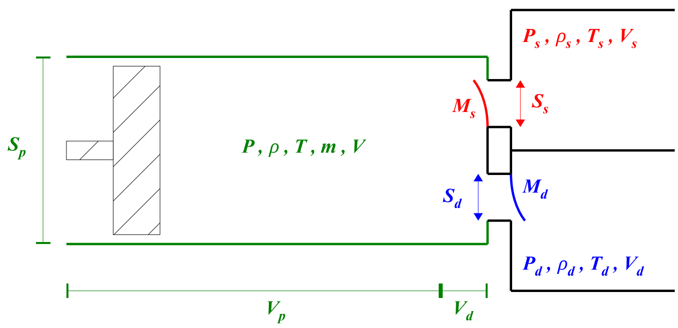
Key words: Reciprocating Compressor, Reed Valve, Pressure behaviour, Oil Stiction, Gas Inertia

Abstract. Automatic reed valves (suction and discharge) in a reciprocating compressor are noise sources due to free vibrations and structure impacts on limiters and valve seat during a pressure cycle. Understanding the noise source generation and propagation needs a well-modelled pressure cycle in the compressor. Modelling the pressure behaviour in a cylinder requires a robust thermodynamical/mechanical model in which both physics interact. A Piao-McLinden model is proposed here to simulate the behaviour of a real gas like refrigerant. The motion of the valves is formulated as a one dimensional damped mass-spring system characterized by means of a Rayleigh method. Each model is compared to experimental data before simulating the whole coupled system: compressor gas/reed valve motion.

1 INTRODUCTION

An automotive air-conditioning (AC) reciprocating compressor consists of multiple out-of-phase pistons that compress a gas provided by a suction chamber before ejecting to a discharge chamber. To do so, there exist reed valves that automatically open and close due to material characteristics (mass, stiffness, damping) as well as external parameters necessary for better compressor yield. In some conditions, the reed valve appears to be a noise source that propagates into the whole AC system and radiates to the cabin.

To understand the sound generation of the reeds, it is aimed in this paper at modelling their motion regarding the thermodynamic cycle of refrigerant 134a that is used as an input for the dynamics model of the reed. Experimental data are used as reference for the simulations.
The thermodynamics is described by the equations of Piao [1] and McLinden [2] (section 2), delivering the main thermodynamic properties of the gas (figure 1-left). The reed valves are then modeled in terms of a one-dimensional damped mass-spring systems (figure 1-right), but accounting for the 2D-beam deflection by means of a Rayleigh method (section 3). Stiction effect due to the presence of a thin lubricating oil film between the seat and the valve plate is of major importance. A simple model allows to take this phenomenon into account. Moreover, the effect of gas flow inertia that crosses a valve during its aperture is discussed. Few details of the numerical implementation are eventually introduced in section 4 before showing results for the whole interacting model during a compressor cycle (section 5).



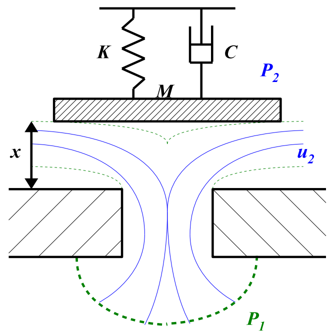


Figure 1: Left: sketch of one of the compressor cylinders with suction (s) and discharge (d) chambers (definition of the notations). Right: one-dimensional damped mass-spring system for the dynamics modelling.

2 Thermodynamical model

2.1 Piao-McLinden model

The thermodynamical model of the refrigerant R-134a is based on the equation of state of Piao *et al.* [1] and the heat capacity formulation of McLinden *et al.* [2]. Basically, it deals with the calculation of the Helmholtz function according to the equation of state of the pressure in vapor phase as well as the heat capacity at constant pressure, in order to determine all thermodynamic properties of the gas.
The equation of state for the pressure P is defined as a density power series that reads:

$$P(\rho, T) = P^* \left[\frac{T_r \rho_r}{Z^*} + \frac{b}{T_r^5} \rho_r^2 + \sum_{i=1}^{8} \rho_r^{i+1} \sum_{j=1}^{4} \frac{a_{ij}}{T_r^{j-1}} \right] \tag{1}$$

where coefficients a_{ij} and b are given in table 1 in appendix, superscript * means *critical value*, $\rho_r = \rho/\rho^*$ and $T_r = T/T^*$ are *reduced* density and temperature respectively. $Z^* = P^*/(r\rho^* T^*)$ is the compressibility factor of the gas in which r is the gas specific constant.
The specific Helmholtz function $f(\rho, T)$ is then calculated since $df = P/(\rho^* \rho_r^2) d\rho_r$, using Eq.1 and the formulation for the heat capacity at constant pressure proposed by McLinden *et al.* [2].
The first law of thermodynamics is then stated for a single compressor cylinder of volume V and the variation of temperature w.r.t. time is obtained:

$$\frac{dT}{dt} = \frac{1/V}{\rho\left(\frac{\partial h}{\partial T}\right)_\rho - \left(\frac{\partial P}{\partial T}\right)_\rho} \left\{ \frac{dQ}{dt} + \Phi - h\frac{dm}{dt} - V\frac{d\rho}{dt}\left[\rho\left(\frac{\partial h}{\partial \rho}\right)_T - \left(\frac{\partial P}{\partial \rho}\right)_T\right]\right\} \tag{2}$$

where $h(\rho, T)$ is the specific enthalpy of the gas in the compressor cylinder, dQ/dt is the heat transfer by conduction through the cylinder walls, m is the mass of gas in the cylinder volume V, and $\Phi = q_s h_s + q_d h_d$ is the total heat flux crossing the suction and discharge valves determined from the mass flows q_s and q_d and the specific enthalpy of each chamber.

2.2 Validation

A validation of the thermodynamical model alone has been conducted by considering the compression stage in the whole compressor cycle. In this way, Eq.2 is simplified since m becomes constant and $\Phi = 0$ (neither entering nor ejecting flows). Initial values for the thermodynamic variables (P, ρ, T, h) are taken as in suction phase (P_s, ρ_s, T_s, h_s). The calculated pressure prediction is eventually compared to measurements in figure 2, for two compressor rotation speeds in RPM (*Rotation Per Minute*). Results in pressure, for given cylinder volume $V(t)$ as function of the time t, show very good agreement between model and experiments. Similar results are found for expansion stage if initial conditions match with discharge characteristics.

3 Valve motion

Since both suction and discharge valves have a similar behaviour, only the modelling of discharge valve is introduced here. Equivalent formulations for suction valve are then implemented in the whole model simulations.
A valve consists of a reed that automatically opens and closes during a pressure cycle due to its mechanical characteristics (mass, stiffness, intrinsic damping) on one hand, some external paramaters like the presence of structural boundaries (up and down limiters) or lubricating oil on the other hand. The reed characteristics are accounted for by means of a Rayleigh method — equality of kinetic energies with effective and real mass — to be implemented into a simplified one-dimensional damped mass-spring system as shown in figure 1, where x represents the displacement of the valve plate.

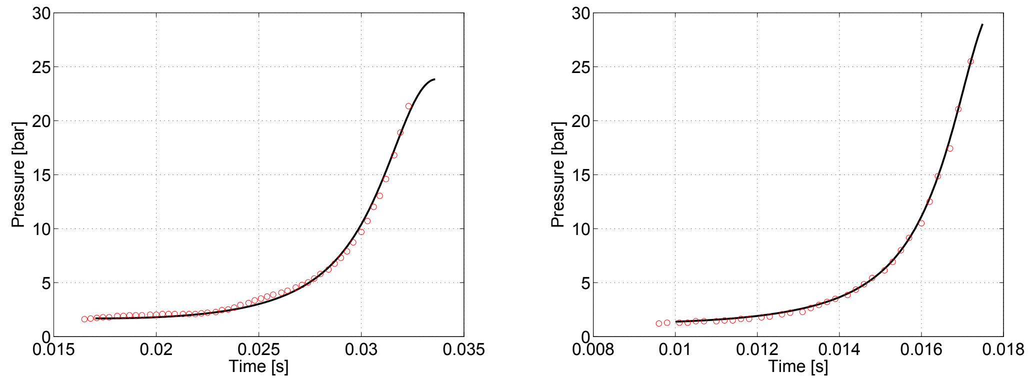


Figure 2: Pressure behaviour during compression stage at 1700 RPM (left) and 3200 RPM (right). Comparison between model prediction (line) and experiments (symbols).

3.1 Formulation

The dynamics equation of the one-dimensional system satisfied by displacement x reads:

$$\ddot{x} + \frac{\omega_0}{Q_0}\dot{x} + \omega_0^2 x = \frac{F_p}{M} + \frac{F_{stic}}{M} \tag{3}$$

where $\omega_0 = \sqrt{K/M}$ is the angular frequency with $K \approx 8 \cdot 10^3$ $N.m^{-1}$ the spring stiffness and $M \approx 1.5 \cdot 10^{-4}$ kg the effective mass — in terms of the Rayleigh method — in order to consider the deflection of a real beam, Q_0 is a quality factor related to the damping coefficient $C = \omega_0 M/Q_0$, and F_p and F_{stic} are the pressure and stiction forces respectively. Besides, solving this equation suggests some complications due to the presence of structural obstacles and lubricating oil in a real compressor.

3.2 Structural boundaries

A first correction on displacement is necessary because of structural boundaries such as the valve seat and the limiter of height x_L. Since the resolution of the whole system {Thermodynamics+Mechanics} is performed step-by-step, this is simply done by imposing the opposite value of the valve plate velocity at previous step, corrected by a coefficient of restitution $0 < \eta < 1$ representing the absorbed momentum when the valve impacts a boundary. η is commonly defined as the square root of the before/after impact kinetic energy ratio. In other words, for calculated displacement x_i at time-step i:

$$\text{if} \begin{cases} x_i < 0 & \Rightarrow \quad x_i = 0 \quad , \quad \dot{x}_i = -\eta \dot{x}_{i-1} \\ x_i > x_L & \Rightarrow \quad x_i = x_L \quad , \quad \dot{x}_i = -\eta \dot{x}_{i-1} \end{cases}$$

According to Cross [3], it is taken here a coefficient of restitution for steel/steel impact of $\eta = 0.85$.

3.3 Gas inertia

The inertia of the fluid that crosses a valve during its aperture generates a pressure force which aims at keeping the valve opened instead of closing quickly as it is shown in figure 4 after the first maximum of the curves, whereas it is smoother in the experiments. A simple model for the correction term on pressure is performed here, from the unsteady Bernoulli's equation expressed here from the cylinder (point 1) to the discharge chamber (point 2) as in figure 1-right:

$$\rho_2 \frac{d\varphi}{dt} + \rho_2 \frac{u_2^2}{2} + P_2 = P_1 \tag{4}$$

with the potential $\varphi = \int_1^2 u ds \approx L_{eff} u_2$ in which u_2 is the gas jet speed and $L_{eff} \approx 1.2{\cdot}10^{-2}\ m$ is an effective length that is as the same order as the sum of the valve seat orifice and thickness. u_1, the gas flow speed at point 1 is neglected because the cylinder cross-section S_1 is much higher than the jet cross-section S_2. Thus, the correction corresponds to the time-derivative $\rho_2 L_{eff} du_2/dt$. A similar correction can be also applied at the suction valve but it has not been implemented here since the inertia correction is directly related to pressure difference between both chambers and the suction valve involves a much lower difference.

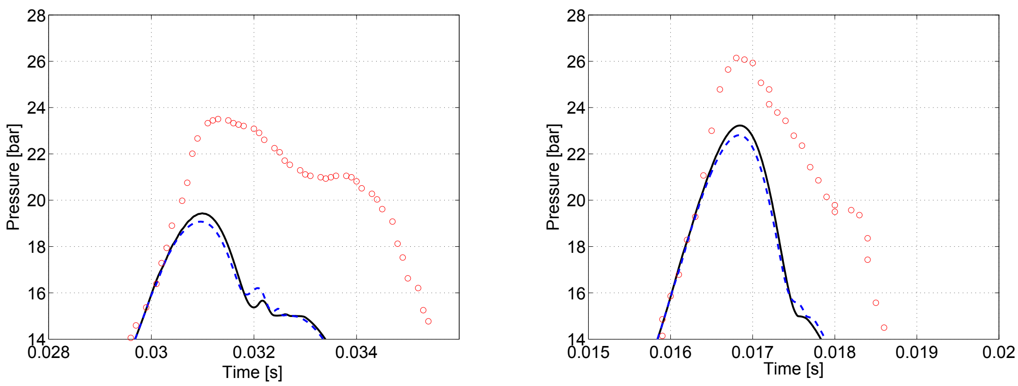


Figure 3: Inertia effect at 1700 RPM (left) and 3200 RPM (right). Experiments (symbols) *vs* model without inertia (—) and with inertia (−−)

Figure 3 shows that inertia effect is not so important whatever the compressor rotation speed. It apperas the maximum of pressure reached at the opening of the valve is slightly reduced and furthermore the decreasing slope during the valve aperture is attenuated. The time the valve closes is *a priori* not influenced, justified since the pressure difference is supposed to be equal or close to zero upon closing.

3.4 Stiction effect

Stiction appears due to the presence of a thin oil film between the valve seat and the valve plate. This phenomenon is shown to have an important effect [4, 5, 6, 7] on the opening of the valve by generating by capillarity a low pressure area in the oil which keeps the valve closed. This induces an overpressure in the cylinder required to lift the valve plate and consequently a delay of the opening time.
Here, stiction is modeled as a constant pressure force applied on a ring surface of 1 mm wide around the valve orifice and keeping the valve closed. While the calculated displacement x (Eq.3) is smaller than a hundredth of the limiter position x_L — same order as the oil film thickness of Khalifa and Liu [5] — then the stiction force is applied. Otherwise, when x becomes higher, the stiction force vanishes. Such a simple model seems to be sufficient to simulate the effect of stiction observed in the measurements (figure 4). It can be noted that the oil stiction force seems to increase with the rotation speed of the compressor which agrees with Bauer's observation [4]. Moreover, the maximum amplitude of pressure is much higher than the experiments at the opening which is counterbalanced by an important deacrease, but good agreement is observed at the closing.

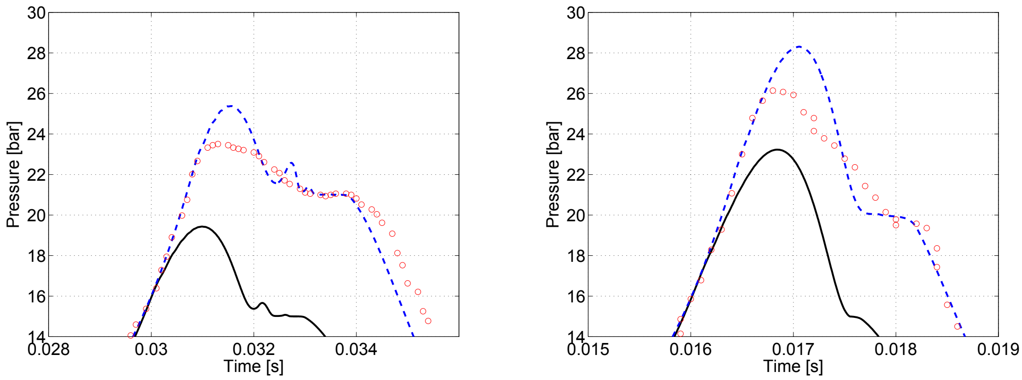


Figure 4: Stiction effect (no inertia) at 1700 RPM (left) and 3200 RPM (right). Experiments (symbols) *vs* model without stiction (—) and model with stiction (−−)

Of course, improvements can be brought by modelling this force by means of the Reynolds lubrication theory [8], as done by Pizarro *et al.* [6] who splitted the stiction force up as a combination of a viscous effect force (oil deformation), a capillary force (meniscus curvature) and an interfacial tension force. The main difference with the paper of Khalifa and Liu [5] is the latter consider an infinite oil volume whereas Pizarro *et al.* assume a finite oil volume that must be estimated.

4 Numerical implementation

Simulations have been conducted using the MATLAB R2013a calculation software, in a step-by-step resolution because of the couplings between equation variables. First of all,

the temperature variation is calculated from Eq. 2 in order to obtain the thermodynamic properties (T, ρ, P, m). Then the flow mass q is evaluated to be used as a mechanical model input. Displacement and velocity of the valve plate are then simulated according to all aforementioned conditions (stiction, inertia, limiters). The mass flow q is finally corrected so as to account for the real height x of the valve plate for the next step.
To solve the dynamics equation of valve motion (Eq.3), a classical 4^{th}-order Runge-Kutta scheme solver (RK4) is used at each time-step. Both suction and discharge valve motions are calculated independently. Time origin is taken when the cylinder volume is equal to the dead volume, so that thermodynamic properties are equal to the discharge values and the discharge valve plate is supposed opened (suction valve closed).
Due to analytical formulations, the whole calculation performs within 2 seconds for about 5000 time-steps per pressure cycle. The convergence is observed by the second cycle, because of discrepancies between assumed initial conditions and reality (especially the unknown exact position of discharge valve plate).

5 General results for a pressure cycle

Figure 5 shows the importance to model the aforementioned effects by comparing the simplest model with only structural boundaries accounted for (neither oil stiction nor gas inertia forces) and the model with all of the three conditions: limiters+stiction+inertia. The experimental data are plotted here as the reference for the models.

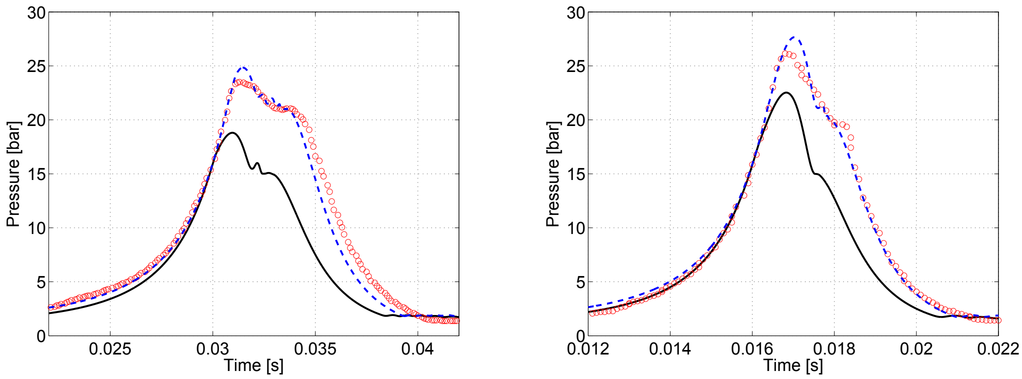


Figure 5: Comparisons between simulations (lines) and experiments (symbols) at 1700 RPM (left) and 3200 RPM (right) for a whole pressure cycle. (—) simplest model without limiters restitution, stiction nor inertia. (−−) model with limiters restitution, oil stiction and gas inertia.

Oil stiction appears to have the most significant impact on the pressure behaviour in the cylinder since it imposes a high increase by sticking the valve plate onto the seat. The inertia of the gas creates a pressure drop when it is at its maximum in the compressor cycle. This leads to a smoother slope in decreasing the pressure till the closing of the valve.

However, as observed in the previous sections, the maximum pressure is overestimated in the simulations when the valve opens, and the valve is predicted to close before the measurement while, especially at lowest compressor rotation speeds. This can be explained as the oil stiction is taken into account only for the opening but it also plays a role when the valve closes.

6 Conclusion

In automotive reciprocating compressor, all suction and discharge valves open and close successively during a pressure cycle. This generates acoustic interactions (amplification or inhibition) that appear to have an effect on the noise that propagates through the air-conditioning system and radiates to the cabin. Thus, modelling pressure and valve motion behaviours allow to discriminate the noise sources produced by the impacts of the reeds onto the limiters and the seat as well as by the vibrations of the valve plates (eigenfrequencies).

In this paper, a coupled system involving a thermodynamical model and a mechanical model has been implemented. Simulations give good agreements with experimental data by accounting for oil stiction and gas inertia phenomena. However, the latter is shown not to be of high importance compared to the other. The stiction model can also be improved according to recent studies [5, 6].

Moreover, only stiction between the valve plate and the seat at the opening has been considered here, but there likely exists oil stiction between the valve plate and the limiter. Besides, stiction should play a role at the closing, especially by inducing an accelerated decrease when oil film makes contact with the plate (oil suction) and a later opening after the plate bounded on the seat. Then, parametric sensitivity investigations must be done so as to optimize the involved variables. Since the mechanical model is based on a one-dimensional damped mass-spring system, considering a realistic two dimensional non-homogeneous beam should actually improve the predictions. Finally, the acoustic interactions (resonances) between all out-of-phase valve cycles in the suction and discharge chambers would be studied and compared to the measurements.

Acknowledgements

This work has been supported by Région Bretagne and Sanden Manufacturing Europe (SME). The authors are very grateful to the SME team for the fruitful discussions and experimental data they provided.

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Appendix

Table 1: Values of coefficients a_{ij} and b in the equation of state of Piao [1]

	a_{i1}	a_{i2}	a_{i3}	a_{i4}
a_{1j}	10.200664	-33.117138	27.006031	-9.3442006
a_{2j}	-32.533383	84.321026	-53.638675	5.1293252
a_{3j}	78.123563	-186.15733	105.08815	0
a_{4j}	-68.597038	163.7985	-89.655243	0
a_{5j}	28.411548	-68.88115	34.766061	0
a_{6j}	-4.9177332	13.40429	-5.7021512	0
a_{7j}	0	-0.61139649	0	0
a_{8j}	0.057702463	-0.077630849	0.069571809	0
b	0.043959113			