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ANALYSIS OF ROCK MASSIF BASED ON THE THEORY OF DAMAGE

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Abstract. This paper deals with analysis of rock massif in connection with a planned deposit of nuclear waste. This problem has emerged with renessaince of nuclear power plants which are now considered as a stable and relatively clean source of electrical energy. The deposits are located deep in rock massif and analysis of stress redistribution caused by cave drifting and temperature changes is required. Difficulties with solution of systems of non-linear algebraic equations were discovered and two variants of the arc-length method were tested.

1 Introduction

Renaissance of nuclear power plants in many countries is accompanied with solution of problem with nuclear waste. One method considered is the excavation of large underground caves where the waste can be stored with the help of very ingenious technology. The artificially made caves are located deep under the surface in rock massif. This topic is in the center of attention of many research group [1, 2, 3].

The final analysis of deposit of nuclear waste has to contain not only mechanical part but also transport processes have to be studied in connection with possible leakage. Therefore, hydro-thermo-mechanical analysis will be required. The main emphasis will be laid on the coupling of damage parameters with material diffusivity and permeability. In this preliminary study, only one way coupling, where the heat transfer influences the mechanical analysis, is taken into account.

Special attention has to be devoted to the solver of non-linear algebraic equations which are obtained by discretisation of the problem solved by the finite element method. The arc-length method is used in this paper because of the softening branch of the stress–strain diagrams. Some difficulties were observed and the classical spherical arc-length method was compared with the linearized method.

Threedimensional model of the whole rock massif with excavated caves is usually very large, i.e. it contains at least hundreds of thousands degrees of freedom. Such large problems are hardly solvable on single processor computers. In fact, there are two possibilities how to solve these problems efficiently. One is based on domain decomposition methods which can be performed on parallel computers [4, 5, 6]. Second possibility is based on adaptive methods where the mesh and the degree of approximation polynomials are changed with respect to error estimates [7].

The paper is organized as follows. Section 2 describes very briefly material models used. Section 3 summarizes variants of the arc-length method. Section 4 deals with numerical results.

2 Material Models

As was mentioned before, this contribution deals with preliminary analysis of rock where the mechanical load cases are accompanied by thermal load. The mechanical analysis is based on the damage theory. The simplest isotropic damage model was not used with respect to different strength in tension and compression. The used orthotropic damage model is described in the next subsection.

2.1 Orthotropic Damage Model

The main drawback of the scalar isotropic damage model is that it uses only one damage parameter for all principle directions regardless of tension or compression. Once the damage parameter caused by exceeding limit strain in one principle direction evolves, it reduces stiffness in all remaining principle directions even though they should not be influenced. This drawback is not significant in the case of the one-dimensional stress state such as pure bending but it becomes more important especially for the three-dimensional stress state.

That led to development of the more advanced damage model which can describe better the 3D problems. In reference [8], the authors proposed general anisotropic model for concrete which contains nine material parameters. The laboratory measurements of the required material parameters has to be performed but it caused difficulties for certain cases. Additionally, the model required a significant number of internal variables that have to be stored. These difficulties led to development of a simplified version of the model which is based on six material parameters - three for tension and another three parameters for compression.

The model is based on the following stress-strain relation

$$\sigma_\alpha = (1 - H(\varepsilon\alpha)D^t_\alpha - H(-\varepsilon_\alpha)D^c_\alpha)[(3K - 2G)\varepsilon_v + 2G\varepsilon_\alpha], \tag{1}$$

where the index α stands for the index of principle components of the given quantity. The model defines two sets of damage parameters D^t_α and D^c_α for tension and compression, respectively. In the equation (1), the symbol $H()$ denotes the Heaviside function, K is the bulk modulus, G is the shear modulus and ε_v stands for volumetric strain.

There are many evolution laws that can be used for D_t and D_c description. In our problems, the two evolution laws for the damage parameters are used similarly to the laws used in the scalar isotropic damage model. The first law gives better results for compression but the determination of the material paremeters is more complicated. It can be written in the form

$$D_\alpha^\beta = \frac{A_\beta(|\varepsilon_\alpha^\beta| - \varepsilon_0^\beta)^{B_\beta}}{1 + A_\beta(|\varepsilon_\alpha^\beta| - \varepsilon_0^\beta)^{B_\beta}}, \tag{2}$$

where β represents indices t or c which are used for tension and compression. A_β, B_β are material parameters controlling the peak value and slope of the softening branch and ε_0^β is the strain threshold. The second law involves correction of the dissipated energy with respect to the size of elements and it describes tension better. It is defined by the non-linear equation (3) which can be solved using the Newton method

$$(1 - D_\alpha^\beta)E|\varepsilon_\alpha^\beta| = f_\beta \exp\left(-\frac{D_\alpha^\beta h|\varepsilon_\alpha^\beta|}{w_0^\beta}\right). \tag{3}$$

In the above equation, f_β represents the tensile or compressive strength and w_0^β controls the initial slope of the softening branches. More details about the implemented models can be found in [9, 10, 11].

2.2 Models for Transport Processes

With respect to limited space, detailed description of material models for transport processes are not included in this paper. They can be found e.g. in references [12, 13, 14]. Efficient computer implementation of transport processes can be found in [15].

3 Arc-length Method

With respect to the softening part of stress–strain diagram, the systems of non-linear algebraic equations are solved by one of arc-length methods. Detailed description of the method can be found in references [16, 17].

The equilibrium condition of a structure after discretization by the finite element method has the form

$$\boldsymbol{f}_{int}(\boldsymbol{d}) = \boldsymbol{f}_c + \lambda \boldsymbol{f}_p \tag{4}$$

where $\boldsymbol{d}$ denotes the vector of nodal displacements, $\boldsymbol{f}_{int}$ denotes the vector of internal forces, $\boldsymbol{f}_c$ denotes the vector of constant prescribed forces, $\lambda \boldsymbol{f}_p$ denotes the vector of proportionally changing forces and λ denotes the scalar load-level multiplier. The vector of unbalanced forces has the form

$$\boldsymbol{r}(\boldsymbol{d}, \lambda) = \boldsymbol{f}_c + \lambda \boldsymbol{f}_p - \boldsymbol{f}_{int}(\boldsymbol{d}) \tag{5}$$

and it is the residual. The relationship between the forces $\boldsymbol{f}$ and the displacements $\boldsymbol{d}$ has to be obtained by an incremental process where increments of length along the curve $\boldsymbol{f}$—$\boldsymbol{d}$ are used. Within each increment, iteration process is needed in order to attain the equilibrium state. Let the i-th increment be known, i.e. the vector $\boldsymbol{d}_i$ and the parameter λ_i are known and $\boldsymbol{r}(\boldsymbol{d}_i, \lambda_i) = \boldsymbol{0}$. Expansion of the residual has the form

$$\boldsymbol{r}(\boldsymbol{d}_{i+1}, \lambda_{i+1}) = \boldsymbol{r}(\boldsymbol{d}_i, \lambda_i) + \frac{\partial \boldsymbol{r}(\boldsymbol{d}_i, \lambda_i)}{\partial \boldsymbol{d}} \delta \boldsymbol{d}_i + \frac{\partial \boldsymbol{r}(\boldsymbol{d}_i, \lambda_i)}{\partial \lambda} \delta \lambda_i = -\boldsymbol{K}_{i,0} \delta \boldsymbol{d}_{i,1} + \boldsymbol{f}_p \delta \lambda_{i,1} = \boldsymbol{0} \quad (6)$$

Let the vector $\delta \boldsymbol{d}_{i,1}$ be in the form

$$\delta \boldsymbol{d}_{i,1} = \delta \lambda_{i,1} \boldsymbol{v}_{i,1} \quad (7)$$

The length of arc in the first iteration within the increment can be written

$$(\delta \boldsymbol{d}_{i,1})^T \delta \boldsymbol{d}_{i,1} + \psi^2 (\delta \lambda_{i,1})^2 \boldsymbol{f}_p^T \boldsymbol{f}_p = (\delta \lambda_{i,1})^2 \boldsymbol{v}_{i,1}^T \boldsymbol{v}_{i,1} + \psi^2 (\delta \lambda_{i,1})^2 \boldsymbol{f}_p^T \boldsymbol{f}_p = (\Delta l)^2 \quad (8)$$

where the scaling parameter ψ was defined. The increment of the scalar load multiplier has the form

$$\delta \lambda_{i,1} = \pm \frac{\Delta l}{\sqrt{\boldsymbol{v}_{i,1}^T \boldsymbol{v}_{i,1} + \psi^2 \boldsymbol{f}_p^T \boldsymbol{f}_p}} \quad (9)$$

New system of equations has to be solved in second iteration

$$\boldsymbol{f}_c + (\lambda_i + \delta \lambda_{i,1}) \boldsymbol{f}_p - \boldsymbol{f}_{int}(\boldsymbol{d}_i + \delta \boldsymbol{d}_{i,1}) - \boldsymbol{K}_{i,1} \delta \boldsymbol{d}_{i,2} + \boldsymbol{f}_p \delta \lambda_{i,2} = \boldsymbol{0} \quad (10)$$

Cumulative quantities are defined

$$\Delta \boldsymbol{d}_{i,j} = \Delta \boldsymbol{d}_{i,j-1} + \delta \boldsymbol{d}_{i,j} \qquad (\Delta \boldsymbol{d}_{i,1} = \delta \boldsymbol{d}_{i,1}) \quad (11)$$

$$\Delta \lambda_{i,j} = \Delta \lambda_{i,j-1} + \delta \lambda_{i,j} \qquad (\Delta \lambda_{i,1} = \delta \lambda_{i,1}) \quad (12)$$

and they are schematically depicted in Figure 1. Equation (10) can be split into two systems

$$\boldsymbol{K}_{i,1} \boldsymbol{u}_{i,2} = \boldsymbol{f}_c + (\lambda_i + \Delta \lambda_{i,1}) \boldsymbol{f}_p - \boldsymbol{f}_{int}(\boldsymbol{d}_i + \Delta \boldsymbol{d}_{i,1}) \quad (13)$$

$$\boldsymbol{K}_{i,1} \boldsymbol{v}_{i,2} = \boldsymbol{f}_p \quad (14)$$

where the previously defined notation is used. The length of arc has now the form

$$\|\Delta \boldsymbol{d}_{i,1} + \boldsymbol{u}_{i,2} + \delta \lambda_{i,2} \boldsymbol{v}_{i,2}\|^2 + \psi^2 \|\Delta \lambda_{i,1} \boldsymbol{f}_p + \delta \lambda_{i,2} \boldsymbol{f}_p\|^2 = (\Delta l)^2 \quad (15)$$

which is the quadratic equation

$$a_1 (\delta \lambda_{i,2})^2 + a_2 (\delta \lambda_{i,2}) + a_3 = 0 \quad (16)$$

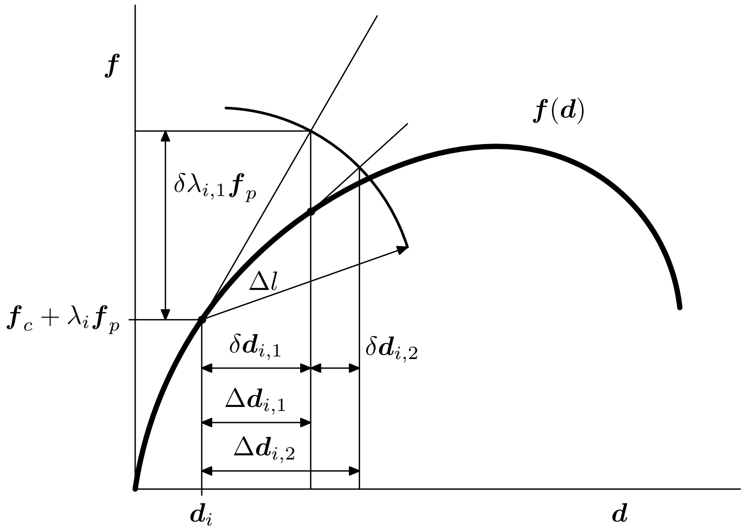

Figure 1: Non-linear force–displacement relationship.

with coefficients

$$a_1 = \boldsymbol{v}_{i,2}^T \boldsymbol{v}_{i,2} + \psi^2 \boldsymbol{f}_p^T \boldsymbol{f}_p \tag{17}$$

$$a_2 = 2\boldsymbol{v}_{i,2}^T (\Delta\boldsymbol{d}_{i,1} + \boldsymbol{u}_{i,2}) + 2\Delta\lambda_{i,1}\psi^2 \boldsymbol{f}_p^T \boldsymbol{f}_p \tag{18}$$

$$a_3 = (\Delta\boldsymbol{d}_{i,1} + \boldsymbol{u}_{i,2})^T (\Delta\boldsymbol{d}_{i,1} + \boldsymbol{u}_{i,2}) + (\Delta\lambda_{i,1})^2 \psi^2 \boldsymbol{f}_p^T \boldsymbol{f}_p - (\Delta l)^2 \tag{19}$$

The increment $\delta\lambda_{i,2}$ is obtained from the quadratic equation (16). New values are again substituted to the residual and equality to the zero vector is checked. The algorithm is summarized in Table 1 and it is called the spherical arc-length method. If the scaling parameter ψ is equal to zero, the method is called the cylindrical arc-length method.

Solution of the quadratic equation (16) is straightforward but only one root has to be used for next computation. One of the criteria used has the form

$$\cos\theta = \frac{\Delta\boldsymbol{d}_{i,j+1}^T \Delta\boldsymbol{d}_{i,j}}{(\Delta l)^2} \to \max \tag{20}$$

Substitution of (11) and (12) leads to the form

$$\cos\theta = \frac{1}{(\Delta l)^2} \Delta\boldsymbol{d}_{i,j}^T (\Delta\boldsymbol{d}_{i,j} + \boldsymbol{u}_{i,j+1} + \delta\lambda_{i,j+1}\boldsymbol{v}_{i,j+1}) \tag{21}$$

Both roots of the equation (16) are substituted to the expression (21) and the root leading to the larger value is selected.

Linearized form of the arc-length leads to the expression

$$\delta\lambda_{i,j+1} = \frac{-\frac{1}{2} l_{i,j} - \Delta\boldsymbol{d}_{i,j}^T \boldsymbol{u}_{i,j+1}}{\Delta\boldsymbol{d}_{i,j}^T \boldsymbol{v}_{i,j+1} + \psi^2 \Delta\lambda_{i,j} \boldsymbol{f}_p^T \boldsymbol{f}_p} \tag{22}$$

and no root selection procedure is needed.

Table 1: Algorithm of the Arc-length Method

$\lambda_0 = 0, \boldsymbol{d}_0 = \boldsymbol{0}$
For $i = 0, 1, 2, \ldots$
$\quad \Delta\lambda_{i,0} = 0$, $\Delta\boldsymbol{d}_{i,0} = \boldsymbol{0}$, $\boldsymbol{r}_{i,0} = \boldsymbol{0}$
$\quad$ For $j = 0, 1, 2, \ldots$
$\quad \boldsymbol{u}_{i,j+1} = \boldsymbol{K}_{i,j}^{-1}\boldsymbol{r}_{i,j}$
$\quad \boldsymbol{v}_{i,j+1} = \boldsymbol{K}_{i,j}^{-1}\boldsymbol{f}_p$
$\quad a_1 = \boldsymbol{v}_{i,j+1}^T\boldsymbol{v}_{i,j+1} + \psi^2\boldsymbol{f}_p^T\boldsymbol{f}_p$
$\quad a_2 = 2\boldsymbol{v}_{i,j+1}^T(\Delta\boldsymbol{d}_{i,j} + \boldsymbol{u}_{i,j+1}) + 2\Delta\lambda_{i,j}\psi^2\boldsymbol{f}_p^T\boldsymbol{f}_p$
$\quad a_3 = \Vert\Delta\boldsymbol{d}_{i,j} + \boldsymbol{u}_{i,j+1}\Vert^2 + (\Delta\lambda_{i,j})^2\psi^2\boldsymbol{f}_p^T\boldsymbol{f}_p - (\Delta l)^2$
$\quad a_1(\delta\lambda_{i,j+1})^2 + a_2(\delta\lambda_{i,j+1}) + a_3 = 0 \quad \Rightarrow \quad \delta\lambda_{i,j+1}$
$\quad \delta\boldsymbol{d}_{i,j+1} = \boldsymbol{u}_{i,j+1} + \delta\lambda_{i,j+1}\boldsymbol{v}_{i,j+1}$
$\quad \Delta\boldsymbol{d}_{i,j+1} = \Delta\boldsymbol{d}_{i,j} + \delta\boldsymbol{d}_{i,j+1}$
$\quad \Delta\lambda_{i,j+1} = \Delta\lambda_{i,j} + \delta\lambda_{i,j+1}$
$\quad \boldsymbol{r}_{i,j+1} = \boldsymbol{f}_c + (\lambda_i + \Delta\lambda_{i,j})\boldsymbol{f}_p - \boldsymbol{f}_{int}(\boldsymbol{d}_i + \Delta\boldsymbol{d}_{i,j})$
$\quad$ if $\Vert\boldsymbol{r}_{i,j+1}\Vert < \varepsilon$, stop
$\lambda_{i+1} = \lambda_i + \Delta\lambda_i$
$\boldsymbol{d}_{i+1} = \boldsymbol{d}_i + \Delta\boldsymbol{d}_i$

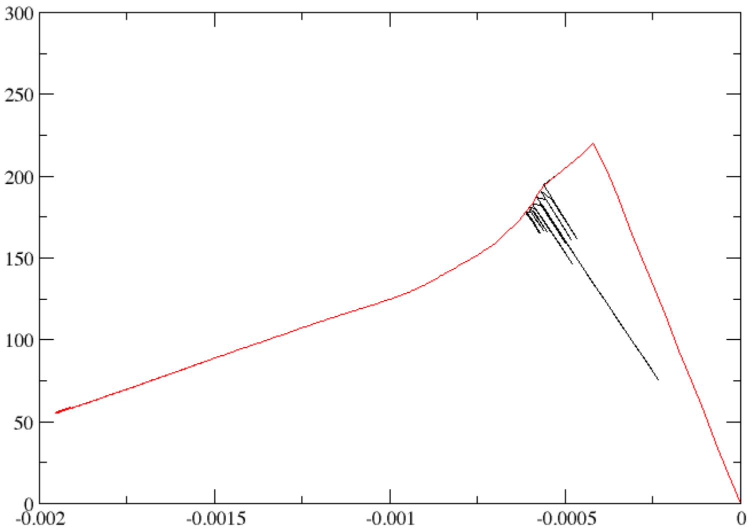

Figure 2: Difficulties with non-linear solvers in softening branch.

4 Numerical results

Two problems were numerically modelled in this preliminary study. First problem was represented by a compression of cylinder which was used for material parameter identification. Cylindrical rock samples were tested in laboratory [1] and the Young modulus, stress-strain relationships and the compressive strength were obtained. Difficulties were observed during the numerical analysis in softening branch of stress-strain relationship. The spherical arc-length method converges very slowly while the linearized arc-length method exhibits incorrect unloading–loading cycles. The situation is depicted in Figure 2 where the red curve is obtained by the spherical method and the black curve by the linearized method.

Second problem was represented by a rock massif with excavated caves. Two different caves were taken into account. Distribution of the damage parameter is depicted for both cases in Figure 3.

5 Conclusions

Analysis of rock massif based on the orthotropic damage material model was performed. Load cases were represented by the dead load and thermal load. Identification of material parameters was done with the help of laboratory tests. Difficulties with solution of non-linear systems of equations were observed.

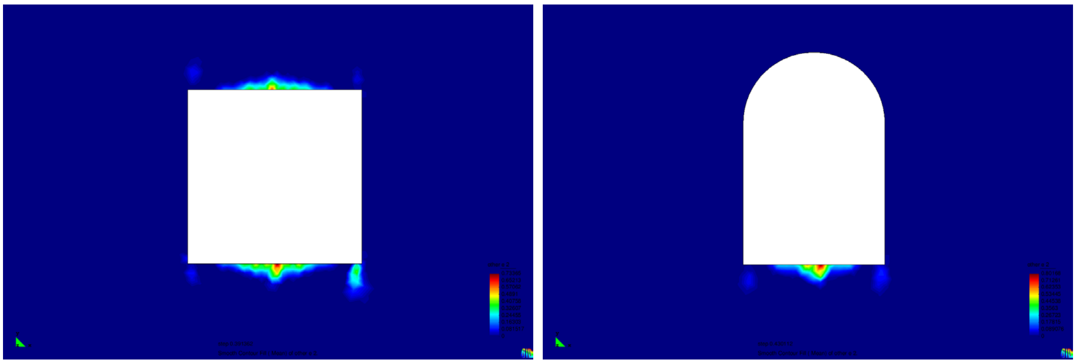

Figure 3: Distribution of the damage parameter in the vicinity of caves.

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