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NOVEL KINETIC CONSISTENT 3D MHD ALGORITHM FOR HIGH PERFORMANCE PARALLEL COMPUTING SYSTEMS

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Abstract. The impressive progress of the kinetic consistent schemes in the solution of the gas dynamics problems and the development of the effective parallel algorithms for the modern high performance parallel computing systems lead to the development of advanced methods for the solution of the magnetohydrodynamics problems for plasma physics. The novel feature of the method is the formulation of the complex Boltzmann-like distribution function of the kinetic method with the implementation of the electromagnetic interaction term. The numerical method is based on the explicit schemes, due to the logical simplicity and high efficiency of the algorithm and the easy adaptation to the modern high performance parallel computing systems.

1 INTRODUCTION

The tremendous progress in that the development of high performance computing systems, especially expecting drastically new exascale computing systems gives new opportunities for mathematical modelling of most important physical phenomena in present and future.

A feature of the present is the development of technologies and computer systems are well ahead of the software development. The software problems are primarily associated with the complexity of adaptation of the algorithms to the high performance computing systems architecture. In particular they refer to one of the important requirements as the accuracy in combination with the correctness of initial mathematical models. Another requirement for the methods is their logical simplicity and high efficiency at the same time. Numerical algorithms should be simple and transparent from a logical point of view.

One of the important directions to overcome these problems is the development of non-traditional approach to initial mathematical models and computational algorithms. In the present study for the solution of the multidimensional magnetogasdynamics problems kinetic difference schemes are proposed. They are convenient from the physics point of view, because the gas dynamics and magnetogasdynamics quantities are defined from close relations between the kinetic and gas dynamics description of physics processes [1, 2].

Another aspect is the study of the explicit finite difference schemes, which seem to be preferable for the future high performance parallel computing, especially in term of their simplicity and well adaptation for parallel program realization, including hybrid high performance parallel computing systems. The weakness of explicit schemes is a strictly limited time step that ensures the computational stability. The advanced explicit kinetic finite difference schemes have soft stability condition giving the opportunity to enhance the stability and to use very fine meshes [3].

The mentioned aspects are used for the development of the framework for the study of dynamics of the hot conducting gas media in strong magnetic fields at high performance parallel computing systems.

2 THEORETICAL ISSUES

2.1 Gas Dynamics Processes

The kinetic theory described the gas dynamics by the Boltzmann differential equation through the evolution of the distribution function $f(\boldsymbol{x},\boldsymbol{\xi},t)$ [4]:

$$\frac{\partial}{\partial t}f(\boldsymbol{x},\boldsymbol{\xi},t) + \boldsymbol{\xi}\cdot\nabla f(\boldsymbol{x},\boldsymbol{\xi},t) = C(f) \tag{1}$$

where $C(f)$ is a nonlinear integral operator which describes the collisions between gas molecules.

This evolution equation is following naturally from relations between the kinetic and gas dynamics description of continuous media. The macroscopic observables such as density, momentum, energy flux as a functions of **x** and t are obtained from the moments of the distribution function with respect to the macroscopic velocity. The evolution equations for these hydrodynamics quantities are obtained by integrating Eq.1 over molecular velocities **ξ** with summational invariants $(m, m\xi, 1/2m\xi^2)$. The computational interest in kinetic formulations of the gas dynamics is high due to the linearity of the differential operator on the left side of Eq.1 . Nonlinearity is confined by the collision term, which is generally local in **x** and t.

An important feature is that the collision integral vanishes in the equilibrium state, when the local Boltzmann distribution function f is Maxwellian:

$$f_0(\boldsymbol{x},\boldsymbol{\xi},t) = \frac{\rho(x,t)m^{1/2}}{\left(2\pi kT(x,t)\right)^{3/2}}\exp\left\{-\frac{m}{2kT(x,t)}\left(\xi-\boldsymbol{u}(x,t)\right)^2\right\} \tag{2}$$

This leads to use this model for numerical methods and for possible generalizations in order to provide a natural kinetic description of systems of conservation laws. This approximation is sufficient for the hydrodynamics processes and called kinetic approach [1].

2.2 Electromagnetic Processes

In [5] was shown that the electromagnetic field does not destroy the validity of the Boltzmann equation and this opened the way to the implementation of the electromagnetic processes term in the Boltzmann distribution function. From the vector nature of the electromagnetic interaction, the distribution function should have also vector behaviour and provide correct kinetic formulation for the evolution of the magnetic field, i.e. the magnetic field should be generally defined as the momentum of Boltzmann-like distribution function.

Few useful attempts to formulate the vector Boltzmann-like distribution function could be found in [7, 8, 9].

We propose an evaluation of the electromagnetic processes in context of complex distribution function including electromagnetic processes. The electromagnetic field is treated as a complex vector field taking to account the axial nature of the magnetic field, following [6]:

$$\boldsymbol{F} = \boldsymbol{E} + i\boldsymbol{B} \tag{3}$$

For the purposes of magneto hydrodynamics, the effect, which a magnetic field exerts on a certain volume, is obtained by integrating the electromagnetic stress tensor over the surface of that volume and the correspondent propagation velocity can be defined as the complex vector of velocity:

$$\boldsymbol{v_{em}} = \boldsymbol{u_{em}} + i\boldsymbol{w_{em}} \tag{4}$$

For first approximation the term defined of electric forces could be neglected and the magnetic term defined through the tension of the magnetic field line showing a similarity to the Alfven wave mechanism:

$$\boldsymbol{w_{em}} = \frac{\boldsymbol{B}}{\sqrt{\boldsymbol{\rho}}} \tag{5}$$

2.3 Proposed Distribution Function for MHD

Using the above definitions we define the local complex Boltzmann-Maxwell distribution function of magnetohydrodynamics:

$$f_M(\boldsymbol{x}, \boldsymbol{\xi}, t) = \frac{\rho(x,t)m^{1/2}}{\left(2\pi kT(x,t)\right)^{3/2}} \exp\left\{-\frac{m}{2kT(x,t)}\left|(\xi - \boldsymbol{u}(x,t)) - i\boldsymbol{w_{em}}\right|^2\right\} \tag{6}$$

The first term on the right-hand side of (6) includes the internal energy of the media and the second term is the electromagnetic field energy. The gas dynamics observables are real scalars and vectors. The complex component includes the dynamics of the macroscopic observables introduced by the evolution of the magnetic field, keeping their specific pseudo-vectorial nature.

The magnetogasdynamics observables are obtained as integrals of the distribution function (6) with the summational invariants $(m, m\xi, 1/2m\xi^2, m\xi^*)$. The integration is performed respect to the molecular velocities ξ in the complex plane.

The proposed complex Boltzmann-Maxwell-like distribution function contains the hydrodynamics terms and the electromagnetic terms. By using the proposed distribution function to calculate mass, momentum and energy fluxes most of the electromagnetic contributions are calculated directly, i.e. one does not have to solve the hydrodynamics and magnetic force components separately.

3 IDEAL MHD SYSTEM OF EQUATIONS

To provide the first step of the formulation of the MHD conservation laws equation, consider the equilibrium gas state with the proposed distribution function. The MHD system of equations is obtained by the integration of (1) with vanishing collision integral with the summational invariants following the definition in (6) :

$$\int_{\gamma} m\frac{\partial f}{\partial t} + \int_{\gamma} m\boldsymbol{\xi}\cdot\nabla f d^3\boldsymbol{\xi} = 0$$
$$\int_{\gamma} m\boldsymbol{\xi}\frac{\partial f}{\partial t} + \int_{\gamma} m\boldsymbol{\xi}\boldsymbol{\xi}\cdot\nabla f d^3\boldsymbol{\xi} = 0$$
$$\int_{\gamma} \frac{1}{2}m\xi^2\boldsymbol{\xi}\frac{\partial f}{\partial t} + \int_{\gamma} \frac{1}{2}m\xi^2\boldsymbol{\xi}\cdot\nabla f d^3\xi = 0$$
$$\int_{\gamma} m\boldsymbol{\xi}^*\boldsymbol{\xi}\frac{\partial f}{\partial t} + \int_{\gamma} m\boldsymbol{\xi}^*\boldsymbol{\xi}\cdot\nabla f d^3\boldsymbol{\xi} = 0 \tag{7}$$

The result is the set of Eq.8 which is the ideal magnetohydrodynamics system of equations:

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x_i}\rho u_i = 0$$
$$\frac{\partial}{\partial t}\rho u_i + \frac{\partial}{\partial x_k}\left[\left(p + \frac{B^2}{2}\right)\delta_{ik} + \rho u_i u_k - B_i B_k\right] = 0$$
$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_i}\left[u_i\left(E + p + \frac{B^2}{2}\right) - B_i u_k B_k\right] = 0$$
$$\frac{\partial B_i}{\partial t} + \frac{\partial}{\partial x_k}[u_k B_i - u_i B_k] = 0 \tag{8}$$

4 KINETIC CONSISTENT MHD FINITE DIFFERENCE SCHEME

The model of the kinetic consistent difference schemes is based on the discrete model of evolution of the distribution function and is formulated directly from the Boltzmann kinetic equation.

Let's consider the local volume of the gas with the distribution function at the time t^j. The evolution of the distribution function by first order difference scheme for the kinetic Boltzmann equation can be written as:

$$\frac{f^{j+1} - f^j}{\Delta t} = \xi\frac{f^j_{i+1} - f^j_{i-1}}{2\Delta x} - \frac{|\xi|}{2}\frac{f^j_{i+1} - 2f^j_i + f^j_{i-1}}{\Delta x} + C(f^j) \tag{9}$$

As mentioned before collisions of particles lead to the establishment of the equilibrium state which is adequately described by the single-particle Maxwell distribution function with vanishing of the collision integral in the right part of the balance relations.

Using this feature, let's consider the conditions when the gas is characterized by the equilibrium states, as time or space is comparable with free mean path. The time evolution of the distribution function can be represented as the time evolution of the local Maxwellian distribution function in discrete moments:

- At time t^j, on each cell, the locally constant one-particle Maxwellian distribution function is defined:

$$f_M = \frac{\rho m^{1/2}}{(2\pi kT)^{3/2}} \exp\left\{-\frac{m}{2kT}\left|(\xi-\boldsymbol{u})-i\frac{\boldsymbol{B}}{\sqrt{\rho}}\right|^2\right\} \tag{10}$$

  where the gas dynamics parameters ρ,**u**,T,**B** are not varied on the cell.
- During the time interval $\Delta t = t^{j+1} - t^j$ a collisionless processes of the gas dynamics occurs
- At time t^{j+1} the distribution function is instantaneously maxwellised
- For the time t^{j+2} these processes are repeated

The kinetic consistent differential scheme in this case can be written:

$$\frac{f^{j+1} - f_M^j}{\Delta t} = \xi \frac{f_{i+1,M}^j - f_{i-1,M}^j}{2\Delta x} - \frac{\Delta x|\xi|}{2} \frac{f_{i+1,M}^j - 2f_{i,M}^j + f_{i-1,M}^j}{\Delta x^2} \tag{11}$$

Or in more general form for the multidimensional case equation:

$$\frac{f^{j+1} - f_M^j}{\Delta t} + \frac{1}{\Delta V}\xi_i f_\sigma^j \Delta\sigma_i = \frac{1}{2\Delta V}|\xi_i|\Delta x_i \frac{\partial f^j}{\partial x_i}\Delta\sigma_i \tag{12}$$

where:

$\Delta\sigma_i$ is the surface element $\Delta x_k \Delta x_m$ perpendicular to the direction x_i

f_σ^j is the value of the distribution function at the surface σ between the two volume elements I_i and I_{i+1}.

The kinetic consistent scheme of the conservation laws of the macroscopic observables for 3D magnetohydrodynamics processes could be obtained by integrating the balance relation (12) with the summational invariants ($m, m\boldsymbol{\xi}, 1/2m\xi^2, m\boldsymbol{\xi}^*$), using the same integration rules as in Eq.8:

$$\frac{\rho^{j+1} - \rho^j}{\Delta t} + (\rho u_i)_{\hat{x}_i} = \frac{\Delta x_i}{2}\left[\rho u_i Erf(\beta u_i) + \frac{\rho}{\beta\sqrt{\pi}} e^{-\beta^2 u_i^2}\right]_{\bar{x}_i x_i} \tag{13}$$

$$\frac{\rho^{j+1} u_i^{j+1} - \rho^j u_i^j}{\Delta t} + \left[\left(p + \frac{B^2}{2}\right)\delta_{ik} + \rho u_i u_k - B_i B_k\right]_{\hat{x}_k} =$$

$$\frac{\Delta x_k}{2}\left[\frac{\rho u_i}{\sqrt{\pi}\beta} e^{-\beta^2 u_k^2} + \left(\left(p + \frac{B^2}{2}\right)\delta_{ik} + \rho u_i u_k\right) Erf(\beta u_k) - B_i B_k\right]_{\bar{x}_k x_k}$$

$$\frac{E^{j+1} - E^j}{\Delta t} + \left[u_i\left(E + p + \frac{B^2}{2}\right) - B_i u_k B_k\right]_{\hat{x}_i} =$$

$$\frac{\Delta x_i}{2}\left[u_i\left(E + p + \frac{B^2}{2}\right) Erf(\beta u_i) + \frac{E + \frac{1}{2}\left(p + \frac{B^2}{2}\right)}{\beta\sqrt{\pi}} e^{-\beta^2 u_i^2} - B_i u_k B_k\right]_{\overline{x_i} x_i}$$

$$\frac{B_i^{j+1} - B_i^j}{\Delta t} + [u_k B_i - u_i B_k]_{\hat{x}_k} = \frac{\Delta x_k}{2}\left[\frac{B_i}{\sqrt{\pi}\beta} e^{-\beta^2 u_k^2} + B_i u_k Erf(\beta u_k) - u_i B_k\right]_{\bar{x}_k, x_k}$$

Where $\beta = \sqrt{\frac{\rho}{2p+B^2}}, i, k = 1, \dots, 3, x_i = x_k = (x, y, z)$.

5 KINETIC CONSISTENT QUASI MHD EQUATIONS

The kinetic quasi magnetogasdynamics system of equations is closely related to the kinetic consistent scheme and represents a differential form notation for the numerical algorithms.

The balance relation Eq.12 can be rewritten as:

$$\frac{\partial f}{\partial t} + \frac{1}{\Delta V}\int_\sigma \xi_i f_\sigma \, \mathrm{d}\sigma = \frac{1}{2\Delta V}\int_\sigma \frac{|\xi_i|}{|\xi_i|}|\xi_i|\Delta x_i \frac{\partial f}{\partial x_i}\mathrm{d}\sigma = \frac{1}{2\Delta V}\int_\sigma \tau\xi_i^2 \frac{\partial f}{\partial x_i}\mathrm{d}\sigma \tag{14}$$

Using the Gauss-Ostrogradsky formula it is possible to transform Eq.14 to the differential form:

$$\frac{\partial f}{\partial t} + \nabla \cdot \left(\xi f_M^j\right) = \frac{\tau}{2}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_k}\xi_i \, \xi_k f_M^j \tag{15}$$

The quasi magneto gas dynamics system of equations involves explicitly two τ parameters. Gas dynamics processes are introduced by the quantity τ that corresponds to the time of free distance flight of particles, or the characteristic time of particle collisions. By analogy the quantity τ_m is introduced as the characteristic time of propagation of gas dynamics by electromagnetic processes. The characteristic time values τ and τ_m are defined respectively for hydrodynamics and electromagnetic processes equation:

$$\tau = \alpha\frac{\Delta x_i}{\bar{c}_h} \qquad \tau_m = \alpha_m\frac{\Delta x_i}{\bar{c}_m} \tag{16}$$

where:

Δx_i is the size of the computational cell,

$\bar{c}_h, \bar{c}_m$ are the sound and Alfven speeds in the computational cell.

The introduction of the physical meaning of the characteristic time values τ and τ_m provides an important contribution into the understanding of the processes and the simplification of the numerical scheme in Eq.15.

The evolution equations of the gas dynamics parameters and the magnetic field are obtained from Eq.15 by integration with the summation invariants $\left(m, m\xi, 1/2m\xi^2, m\boldsymbol{\xi}^*\right)$, over the molecular velocities under the assumption:

$$\int f^{j+1}\phi(\boldsymbol{\xi})d\boldsymbol{\xi} = \int f_M^{j+1}\phi(\boldsymbol{\xi})d\boldsymbol{\xi} \tag{17}$$

The integration is performed as in Eq.8 and Eq.13. The hydrodynamics and magnetic field quantities are obtained respectively as the real and imaginary parts of the path integral in the complex plane.

The resulting 3D kinetic consistent quasi MHD equations are:

$$\frac{\rho^{j+1}-\rho^j}{\Delta t} + \frac{\partial}{\partial x_i}\rho u_i = \frac{\tau}{2}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_k}\left[\left(p+\frac{B^2}{2}\right)\delta_{ik} + \rho u_i u_k - B_i B_k\right] \tag{18}$$

$$\frac{\rho^{j+1}u_i^{j+1}-\rho^j u_i^j}{\Delta t} + \frac{\partial}{\partial x_k}\left[\left(p+\frac{B^2}{2}\right)\delta_{ik} + \rho u_i u_k - B_i B_k\right] =$$

$$\frac{\tau}{2}\frac{\partial}{\partial x_k}\frac{\partial}{\partial x_m}\left[u_k\left(p+\frac{B^2}{2}\right)\delta_{im}+u_m\left(p+\frac{B^2}{2}\right)\delta_{ik}+u_i\left(p+\frac{B^2}{2}\right)\delta_{km}+\right.$$

$$\left.\rho u_i u_k u_m - u_i B_k B_m - u_m B_i B_k - u_k B_i B_m\right]$$

$$\frac{E^{j+1}-E^j}{\Delta t}+\frac{\partial}{\partial x_i}\left[u_i\left(E+p+\frac{B^2}{2}\right)-B_i u_k B_k\right]=$$

$$\frac{\tau}{2}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_k}\left[\frac{5}{2}\frac{1}{\rho}\left(p+\frac{B^2}{2}\right)\delta_{ik}+u_i u_k E-\frac{B_i B_k}{\rho}E+2u_i u_k\left(p+\frac{B^2}{2}\right)-2\frac{B_i B_k}{\rho}\left(p+\frac{B^2}{2}\right)+\right.$$

$$\left.\frac{1}{\rho}\frac{\rho u^2}{2}\left(p+\frac{B^2}{2}\right)\delta_{ik}-\frac{B^2}{2\rho}\left(p+\frac{B^2}{2}\right)\delta_{ik}-(B_i u_k+B_k u_i)B_m u_m\right]$$

$$\frac{B_i^{j+1}-B_i^j}{\Delta t}+\frac{\partial}{\partial x_k}[u_k B_i-u_i B_k]=$$

$$\frac{\tau_m}{2}\frac{\partial}{\partial x_k}\frac{\partial}{\partial x_m}\left[\frac{B_i}{\rho}\left(p+\frac{B^2}{2}\right)\delta_{km}-\frac{B_k}{\rho}\left(p+\frac{B^2}{2}\right)\delta_{im}-\frac{B_m}{\rho}\left(p+\frac{B^2}{2}\right)\delta_{ik}+\right.$$

$$\left.-u_i u_k B_m-u_i u_m B_k+u_k u_m B_i-\frac{B_i B_k B_m}{\rho}\right]$$

The dissipative terms appear because the construction of the quasi magnetogasdynamics system is based on the assumption that the distribution function slightly changes over the distance between neighbour cells, which is related to the characteristic times τ and τ_m. It was shown in [2] that the dissipative terms of the quasi gas dynamics system, on the right side, are small in comparison with the convective terms. With the condition of cell size equivalent of the free path they converge to the viscous terms of the corresponding Navier-Stokes equations, i.e. the correspondent dissipative terms are associated with real physics processes. An important remark is that in this case the gas dynamics parameters as viscosity and heat conductivity converge from the kinetic theory.

The Navier-Stokes thermal flux vector is identified within the dissipative terms of the energy equation:

$$\Pi_{ik}^{NS}=\frac{\tau}{2}\left[p\frac{\partial u_i}{\partial x_k}+p\frac{\partial u_k}{\partial x_i}-\frac{2}{3}p\frac{\partial u_m}{\partial x_m}\delta_{ik}\right]=\mu\left[\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}-\frac{2}{3}\frac{\partial u_m}{\partial x_m}\delta_{ik}\right] \tag{19}$$

The gas viscosity μ is related to the gas pressure p and the characteristic time τ as $\mu = \tau/2\, p$.

The Navier-Stokes thermal flux vector is identified within dissipative terms of the energy equation:

$$q_i=\frac{\tau}{2}\left[\frac{5}{2}p\frac{\partial}{\partial x_i}\frac{p}{\rho}\right]=k\frac{\partial T}{\partial x_i} \tag{20}$$

with thermal coefficient expressed as $k=\frac{5}{2}\mathrm{Pr}^{-1}R\frac{\tau}{2}p$, with Pr Prandl number.

The similar analysis of dissipative terms of the electromagnetic processes gives the estimation of the smallness of values of dissipative terms of the electromagnetic processes and with the correct condition for the size of cells the equation converges to the correct representation of the magnetic viscosity. The resistivity is identified within the dissipative term of the magnetic field evolution equation:

$$\Pi_{ik}^{B} = \frac{\tau_m}{2}\left[\left(p + \frac{B^2}{2}\right)\left(\frac{\partial B_i}{\partial x_k} - \frac{\partial B_k}{\partial x_i}\right)\right] = \eta\left[\left(\frac{\partial B_i}{\partial x_k} - \frac{\partial B_k}{\partial x_i}\right)\right] \tag{21}$$

6 COMPUTATIONAL ALGORITHM

The numerical algorithm uses a Cartesian, staggered, divergence free mesh configuration detailed description presented in [10]. The hydrodynamics observables: mass density, momentum and energy density are defined at the cell centre. The components of the magnetic field are defined at the face centres of cells. A duality is established between the fluxes and electric field at the edges. The electric field is then utilized to make an update of the magnetic fields that preserves the solenoidal nature of the magnetic fields and ensures that the magnetic fields in an magneto hydrodynamics modelling remain strictly solenoidal up to discretization errors.

Generally we are using the explicit numerical scheme, believing that it is perspective for the modern high performance computing systems due to the logical simplicity and efficiency of the algorithms. The finite volume method is used to update the conserved observables: mass, momentum and energy by calculating the fluxes of these observables across the cell face. The update of the magnetic field is more complicated procedure and performed via electric field integration along the edge of the cells. For the calculation of the observables is used the proposed distribution function method described above. For the time evolution is used explicit scheme of the integration of the quasi magnetohydrodynamics system of equations. The code uses a variable time steps, the time step in an explicit scheme is controlled by a Courant type condition on the time step estimation [2].

7 RESULTS OF NUMERICAL MODELING

The simulation framework is created on the base of Fortran 90 and c++, parallel implementation on MPI. The demonstration of the performance of the present method is performed on the base of the solution of the spherical expansion problem of ionised gas and the solution of the expansion problem of ionised gas in strong magnetic field. The simulations are performed for the Cartesian rectangular mesh $100 \times 100 \times 100$ in the physics domain [0,1] .

The first solution is the three dimensional thermal expansion of the ionised gas in the physical domain. The initial conditions are represented as the unit cell in the centre of the physical region with pressure of 100 in comparison to the overall area with pressure 1. Fig. 1,2,3 present the state of the simulation in time 0.03. On the 3D picture the arrows represent the velocities of the ionised gas and the colour represent the density of gas.

The second solution is a three dimensional thermal expansion of ionised gas in a strong uniform magnetic field along the z coordinate. The initial conditions are the same, in addition the magnetic field is $5/\sqrt{\pi}$. On the Fig. 4,5,6 the view is shown of the process of expansion of the hot ionized gas in a strong uniform magnetic field. The picture shows the time when the conductive gas is confined in the cylindrical area of space along z.

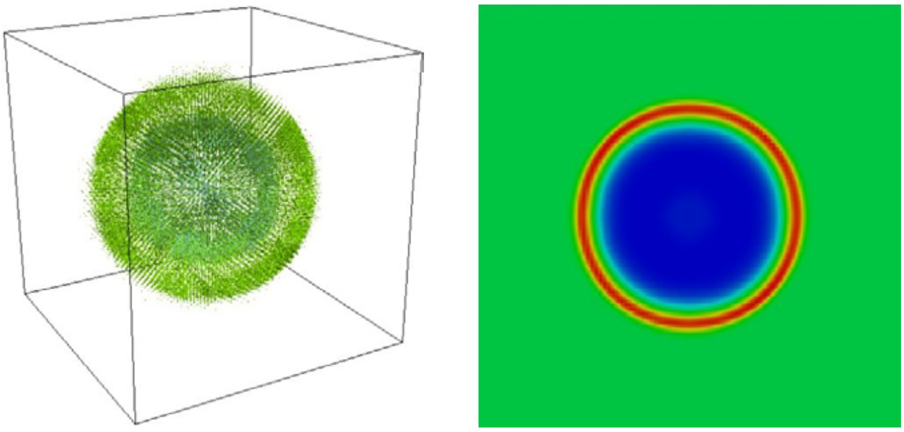

Figure 1: 3D gas expansion view and 2D gas density projection

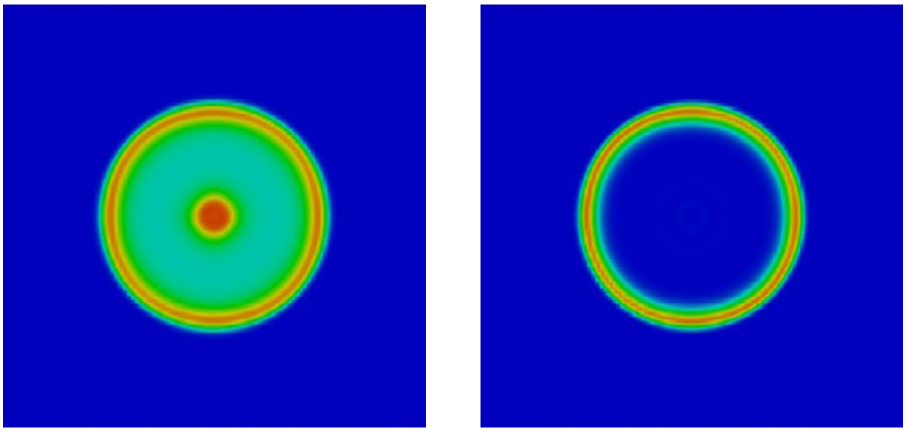

Figure 2: 2D pressure and 2D kinetic energy projection

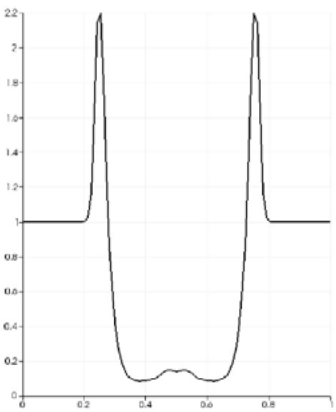

Figure 3: 1D density profile

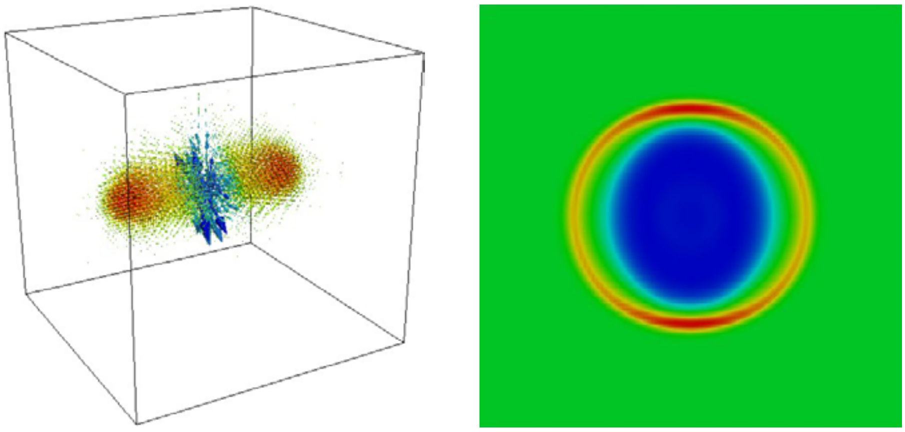

Figure 4: 3D gas expansion in magnetic field and 2D gas density projection

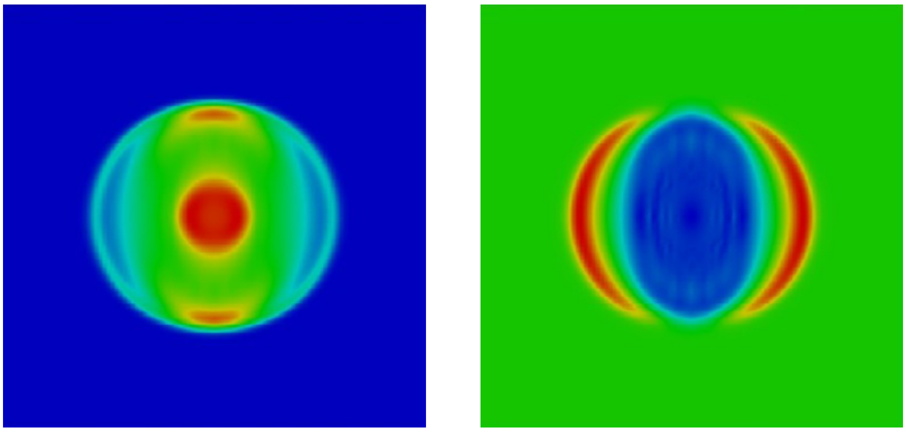

Figure 5: 2D gas pressure and 2D magnetic pressure projections

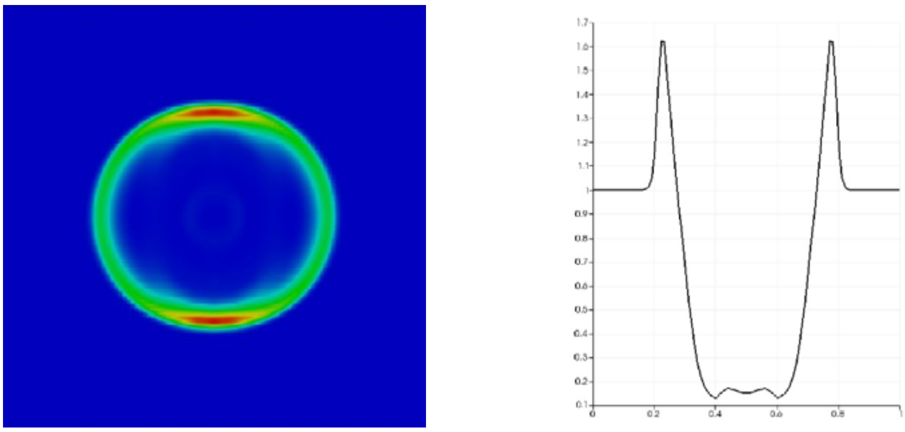

Figure 6: 2D kinetic energy projection and 1D gas density profile

8 CONCLUSIONS

A new 3D kinetic consistent algorithm has been developed for the solution of magnetohydrodynamics problems. The novel feature of the method is that the local complex Boltzmann-like distribution function incorporated most of the electromagnetic processes terms. The fluxes of mass, momentum and energy accross cell interface as well as the magnetic field are calculated by integrating a local complex Boltzmann-like distribution function over the velocity space.

Results of the numerical simulations demonstrate that the proposed method can achieve high numerical accuracy and resolves strong shock waves of the magneto gas dynamics problems.

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