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CONSOLIDATION PROBLEM SOLUTION WITH A COUPLED HYDRO-MECHANICAL FORMULATION CONSIDERING FLUID COMPRESSIBILITY

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Abstract. There are two principles which may be referred to as essentials to describing soil and rock behavior. The mechanical behavior is associated to the law of conservation of linear momentum, allowing forces balance analysis and the hydraulic behavior is characterized by mass conservation. These phenomena are related: stress-strain state is affected by fluid pressures and vice-versa. Therewith, it is intuitive the understanding of the importance of coupled analyses, which are certainly a more precise manner of describing how mechanical and hydraulic behavior are connected. Given certain difficulties related to the modeling process, porous media numerical model representation is usually simplified. In certain cases, simplifications do not imply on losses in results and behavior prediction. However, some situations require more comprehensive approaches, with development of previously neglected conditions. The main objective of this paper is to present a formulation for fully coupled hydro-mechanical analyses considering fluid and solids compressibility. This formulation, implemented in Finite Element program ALLFINE [1,2,3], was tested for a one-dimensional consolidation case. A sensitivity analyses for the fluid compressibility parameter using modified Cam-clay constitutive model showed that this consideration affects fluid pressure responses significantly, with a delay in fluid pressure dissipation during consolidation process. The simulations results were compared to Terzaghi's analytical solution for the one-dimensional consolidation problem. Also, the comparison of the simulation results to the analytical responses clearly shows the differences between using linear elastic and elastoplastic models. In simulations for different stress levels with the modified Cam-clay model, it is possible to capture a flow induction effect due to high stress levels.

1 INTRODUCTION

The study of geomechanics relates the concepts traditionally employed in geotechnics to reservoir engineering. Hence, it is possible to conduct numerical analyses in order to predict mechanical and hydraulic behaviour of porous media such as oil reservoirs.

Altogether numerical modelling of porous media requires complex relations between the basic variables of the problem. Therefore, coupled analyses should be performed in order to accurately reproduce the effects on porous medium.

This type of analysis has much computational cost and in order to make it more feasible, some simplifying hypotheses are occasionally made. However, some of those simplifications may interfere in model responses, with simulation results not correspondent to the observed behaviour of the studied case.

It is known that the solid matrix and the fluids within the porous medium do not have its compressibility taken into account in traditional models in geotechnics or related areas, such as reservoir geomechanics. This kind of consideration could enhance modelling, contributing to more accurate results in behaviour prediction.

In this paper, simulation results are presented for a one-dimensional consolidation test using modified Cam-clay model. The hydraulic behaviour of a one-meter high soil column is evaluated, with verification of the effect of permeability variation during incompressible and compressible fluid flow. Therewith, it was possible to study the combined effect of compressibility changes in the fluid and permeability of the medium for each type of fluid.

The numerical model here employed for the simulations is a fully coupled hydro-mechanical formulation proposed by Jesus [3], here briefly presented. This formulation was fully described by the author, followed by its implementation, validation and calibration in ALLFINE [1,2,3].

2 PROBLEM FORMULATION

The presented formulation was organized and fully described by Jesus [3]. Here, a brief summary of the equilibrium and the mass conservation equations of the model is shown.

2.1 Spatial solution of the equilibrium equation

The equilibrium equation for a porous medium can be described as:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0 \tag{1}$$

where: σ_{ij} is the stress tensor, b_i is the body forces vector and x_j corresponds to the coordinate system.

This formulation is previewed for a fully saturated system. Hence, the stress-strain relation is based on the definition of the generalized effective stress with influence of solids compressibility on solid matrix stress state, defined as [4]:

$$\{\sigma'\} = \{\sigma\} - \alpha_b \{m\} p \tag{2}$$

where: $\{\sigma'\}$ is the effective stress vector, $\{\sigma\}$ is the total stress vector, α_b is the Biot parameter, $\{m\}^T = \{1\ 1\ 1\ 0\ 0\ 0\}$ for a 3D analysis and p is the fluid pressure (for a saturated analysis).

The Biot parameter (α_b) is defined by the following expression:

$$\alpha_b = 1 - \frac{\{m\}^T \left[D^{ep}\right]\{m\}}{9k_s} \tag{3}$$

where: $[D^{ep}]$ is the constitutive matrix and k_s is the bulk modulus of solids.

The Principle of Virtual Work is employed to solve the equilibrium equation in space using FEM [5]. The matrixes that represent the solved equilibrium equation are assembled and may be written as:

$$[K]\{\dot{u}\} + [C]\{\dot{p}\} = \{\dot{F}\} \tag{4}$$

where:

$$[K] = \int_\Omega [B]^T [D][B]\, d\Omega$$

$$[C] = \int_\Omega [B]^T \alpha_b \{m\}\left[N^p\right] d\Omega$$

$$\{\dot{F}\} = \int_\Omega [N]^T \{\dot{b}\}\, d\Omega + \int_\Gamma [N]^T \{\dot{\tau}\}\, d\Gamma$$

$[K]$ is the stiffness matrix, $\{u\}$ is the nodal displacements rate vector, $[C]$ is the solids-fluid coupling, $\{\dot{p}\}$ is the nodal fluid pressure rate vector and $\{F\}$ is the external forces rate vector.

2.2 Spatial solution of the mass conservation equation

The mass conservation equation considered in this paper involves both liquid and solid phases. In order to represent the mass conservation of the entire porous medium, volume of solids and fluid within a finite element should be balanced in a total element volume approach [4].

The mass conservation equation for the liquid phase is defined by the following expression:

$$\frac{\partial}{\partial t}\left(\theta\rho^f\right) + \frac{\partial}{\partial x_i}\left(\theta\rho^f \dot{U}_i\right) = 0 \tag{5}$$

where: θ is the volumetric water content, ρ^f is the fluid density and U_i is the real fluid velocity.

The real fluid velocity can be expressed as:

$$\dot{U}_i = \dot{u}_i + \frac{w_i^f}{\theta} \tag{6}$$

where: $\dot{U}_i$ is the real fluid velocity vector, $\dot{u}_i$ is the velocity of the solids vector, w^f_i is the fluid velocity due to percolation vector and θ is the volumetric fluid content of the medium.

The real fluid velocity assumption implies on taking into account the effects of porosity changes in the porous medium. The velocity of the solids u_i is related to the displacements of the medium.

The evolution law for liquid fluids density is expressed as [6,7]:

$$\frac{d\rho^f}{\rho^f} = \frac{1}{k_f} dp \tag{7}$$

where: ρ^f is the fluid density, k_s is the bulk modulus of the fluid and p is the fluid pressure.

The mass conservation equation for the solid phase may be presented as:

$$\frac{\partial}{\partial t}\left[(1-\phi)\rho^s\right] + \frac{\partial}{\partial x_i}\left[(1-\phi)\rho^s \dot{u}_i\right] = 0 \tag{8}$$

where: ϕ is the material porosity, ρ^s is the solids density and $\dot{u}_i$ is the solids velocity vector.

The evolution law for solids density is expressed as [4]:

$$\frac{(1-\phi)}{\rho^s}\frac{D\rho^s}{Dt} = \left[\frac{(\alpha_b-\phi)}{k_s}\frac{Dp^R}{Dt} - (1-\alpha_b)\dot{u}_{i,i}\right] \tag{9}$$

where: ρ^s is the solids density, k_s is the bulk modulus of rock crystals, α_b is the Biot parameter, ϕ is the porosity of the rock and p^R is the pressure of the fluids (for saturated medium p^R=p).

The spatial solution for the mass conservation equation via FEM can be made with Galerkin method, a weighted residual method [5].

The matrixes that represent the solved mass conservation equation are assembled and written as:

$$[M]\{\dot{p}\} + [L]\{\dot{u}\} + [R]\{p\} = \{Q\} \tag{10}$$

where:

$$[M]=\int_{\Omega}\left[N^{P}\right]^{T}\left[\frac{(\alpha_{b}-\phi)}{k_{s}}+\frac{\phi}{k_{f}}\right]\left[N^{P}\right]d\Omega$$

$$[L]=\int_{\Omega}\left[N^{P}\right]^{T}\alpha_{b}\{m\}^{T}[B]d\Omega$$

$$[R]=\int_{\Omega}\left[N^{P}\right]^{T}\left[\frac{(\alpha_{b}-\phi)}{k_{s}}\{\dot{u}\}+\frac{\phi}{k_{f}}\left(\{\dot{u}\}+\frac{\{w^{f}\}}{\phi}\right)\right]\left[B^{P}\right]d\Omega+\int_{\Omega}\left[B^{P}\right]^{T}\frac{[K]}{\mu}\left[B^{P}\right]d\Omega$$

$$\{Q\}=\left[\int_{\Omega}\left[B^{P}\right]^{T}\frac{[K]}{\mu}\rho_{f}\{g\}(\nabla .y)d\Omega\right]-\int_{\Gamma_{2}}\left[N^{P}\right]^{T}\overline{q}d\Gamma$$

[*M*] is the mass matrix, $\{\dot{p}\}$ is the nodal fluid pressure rate vector, [*L*] is the fluid-solids coupling matrix, $\{\dot{u}\}$ is the nodal displacements rate vector, [*R*] is the flow matrix, {*p*} is the nodal fluid pressure vector and {*Q*} is the external discharges vector.

2.3 Coupling of equations and time solution of the problem

The spatial solution of equilibrium and mass conservation equations may be assembled as a system of equations. This procedure is required for the coupling of the equations. Considering both Equations 4 and 10, we have:

$$\begin{bmatrix}0 & 0\\ 0 & [R]\end{bmatrix}\begin{Bmatrix}\{u\}\\ \{p\}\end{Bmatrix}+\begin{bmatrix}[K] & [C]\\ [L] & [M]\end{bmatrix}\begin{Bmatrix}\{\dot{u}\}\\ \{\dot{p}\}\end{Bmatrix}=\begin{Bmatrix}\{\dot{F}\}\\ \{Q\}\end{Bmatrix} \tag{11}$$

Therefore, the system of equations may be expressed by:

$$[W]\{x\}+[Y]\{\dot{x}\}=\{Z\} \tag{12}$$

Considering the studied phenomenon is transient, this system of equations should be solved in time. Finite Difference (FDM) was the chosen method for the time discretization of the system of equations, already solved in space. This choice was based on the ease of application of FDM for that matter. Evaluating Equation 12 in time stage t+ $\alpha\Delta t$, the solution of the equation system is:

$$\left[\Delta t\alpha[W]_{t+\alpha\Delta t}+[Y]_{t+\alpha\Delta t}\right]\{\Delta x\}=\Delta t\{Z\}_{t+\alpha\Delta t}-\Delta t[W]_{t+\alpha\Delta t}\{x\}_{t} \tag{13}$$

3 METHODOLOGY

3.1 Description of the problem

The studied case consists of a consolidation analysis for a one meter high soil column. This soil sample is assumed to be laterally confined and fully saturated. Fluid flow is restricted in the laterals of the column and free at the top surface, as illustrated in Figure 1. Also, this soil column is discretized in a non-uniform mesh of 10 elements and 44 nodes. 3-D 8-noded elements are shown in Figure 2.

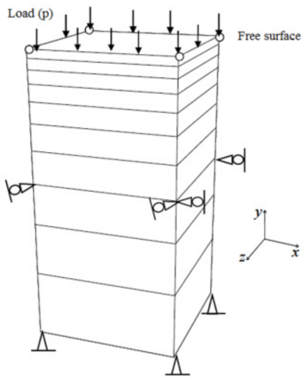

Figure 1. One-dimensional consolidation problem ([2] modified).

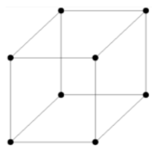

Figure 2. 8-noded 3D element.

An uniform load of a 10000 kPa induces the consolidation process in the sample. The constitutive model employed for these simulations was the modified Cam-clay.

Permeability is defined in this paper as function of porous medium void ratio. Herein, the simulations are performed for three different configurations of this function.

In order to evaluate the combined effect of compressibility and permeability changes, fluid bulk modulus value is also varied, with $k_f = 1\text{x}10^{12}$ kPa for incompressible fluid and $k_f = 1\text{x}10^{5}$ kPa for compressible fluids. The values of fluid bulk modulus and solids bulk modulus used in these simulations do not necessarily correspondent to real values of these parameters. They have been established to facilitate the visualization of the effects of compressibility in the model results, as a sensitivity analysis of the evaluated parameters. It has been shown [3] that depending on the stress state the porous media is subjected to, it is possible to visualize the same effects here presented with real values of these parameters.

The evaluation of the simulation results is made with the values of fluid pressure, which are monitored over time during the consolidation process. The time factor here presented for each time stage is the same predicted by Tergazhi in its analytical solution for the one-dimensional consolidation problem.

3.2 Permeability function

The pore volume reduction during consolidation influences permeability. Therewith, these features are associated and a permeability function may be expressed in order to relate them.

In this study, a permeability function is suggested:

$$k = A\frac{\exp(Be)}{\exp(BC)} \tag{14}$$

where: k is the permeability and e is the void ratio.

Parameter A corresponds to the initial permeability (k_0) and parameter C is equivalent to a reference void ratio value, in this case, the initial void ratio (e_0) for the simulations.

Parameter B was used to calibrate the void ratio variation range of the function. With its variation, it was possible to define three different formats for the permeability function. The function reaches constant values of permeability (function k_1), values 10 times lower than the initial (function k_2) or values 100 times lower than the initial (function k_3) for the final void ratio value. The values for each parameter are presented in Table 1 and its corresponding permeability functions are shown in Figure 3.

Table 1. Calibration parameters for the permeability function.

A		$1{,}0\text{x}10^{-6}$
B	For function k_1	0,0
	For function k_2	95,8
	For function k_3	191,9
C		0,90

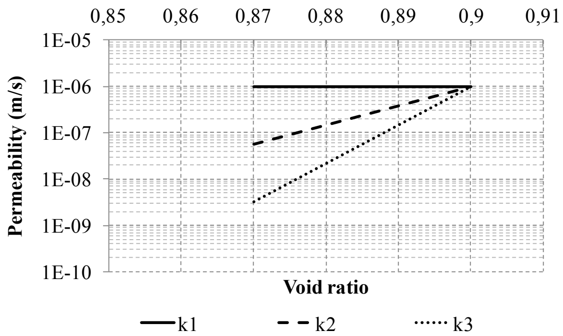

Figure 3. Permeability functions.

4 RESULTS AND DISCUSSION

The performed analyses use the parameters shown in Table 2. The values of permeability vary according to the permeability functions already presented.

Table 2. Parameters for the consolidation simulation.

M	1
p_0	5000 KPa
Overconsolidation ratio (OCR)	0,20
λ	0,0030

κ	0,0003
Poisson coefficient (ν)	0,31
Initial void ratio (e_0)	0,90
Density of the solids (ρ^s)	2,65 kg/m³
Solids bulk modulus (k_s)	1,0x10^{15} kPa
Density of the fluid (ρ^f)	1,00 kg/m³

The values of final void ratio for the modified Cam-clay model analyses are presented in Table 3. There were no differences between final void ratio values for incompressible and compressible fluids.

Table 3. Final void ratio values for modified Cam-clay model simulations.

Permeability	**Final void ratio**
k_1 function	0,876
k_2 function	0,877
k_3 function	0,877

As noticed, volume change is small, but the effects of permeability variation can be noticed in fluid pressure results due to the chosen permeability functions. The evolution of the fluid pressure during the simulation of the consolidation process is monitored for specific time factors (T=0; 0,2; 0,5 e 0,8). The results for incompressible fluid analysis are presented in Figures 4 to 7.

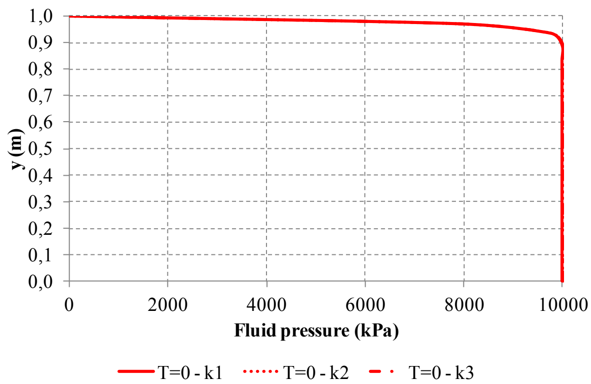

Figure 4. Results of fluid pressure for incompressible fluid simulation (T=0).

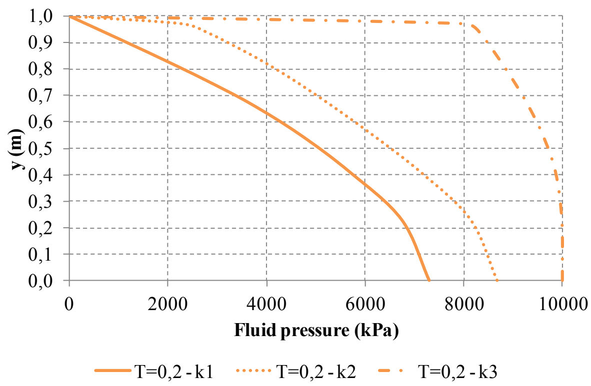

Figure 5. Results of fluid pressure for incompressible fluid simulation (T=0,2).

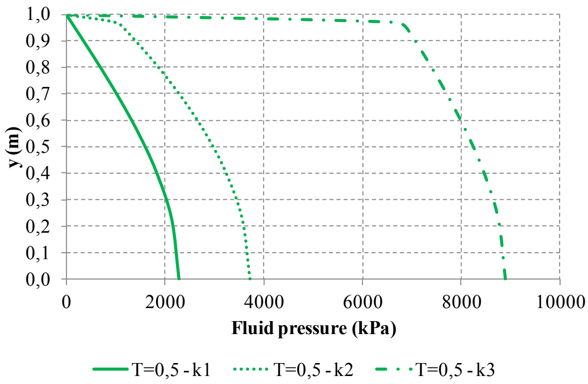

Figure 6. Results of fluid pressure for incompressible fluid simulation (T=0,5).

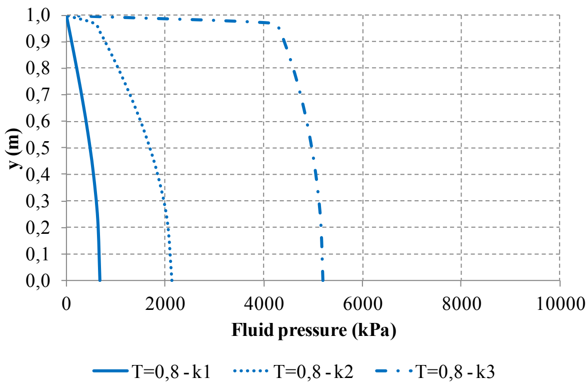

Figure 7. Results of fluid pressure for incompressible fluid simulation (T=0,8).

At the first time stage (T=0), values of fluid pressure are equal for all permeability functions simulations. Then, in the following time stages, fluid pressure dissipates over time faster for constant permeability (k_1 function) and slower for the function with which soil reaches a permeability value 100 times lower than the initial value (k_3 function).

It can be inferred from these results that fluid pressure values are influenced by permeability even for low void ratio changes. The straining the porous medium suffers is sufficient to make the permeability function influence noticeable.

Analyzing the shape of the curves, it can be noticed that for permeability function k_3, fluid pressure is high for the lower layers of the sample and it decreases abruptly for the superficial layers (surface at 1 m).

During the process, the superficial layers consolidate first, having their pores closed, forming a low-permeability zone. This causes fluid flow decrease, making the values of fluid pressure higher in other layers of the sample.

It was possible to reproduce this pore closing effect due to the use of modified Cam-clay model. This model adequately represented straining of the sample during the consolidation test. Thus, soil void ratio was appropriately calculated and updated during the simulations.

In Figure 8, two curves are plotted, one for void ratio and other for permeability. These curves complement the understanding of the pore closing process during consolidation when performing simulations with modified Cam-clay model. It can be observed how void ratio gradually decreases until it reaches initial stress (5000 kPa), when it starts to decrease abruptly. As expected, being the permeability a function of void ratio, it follows the same tendency.

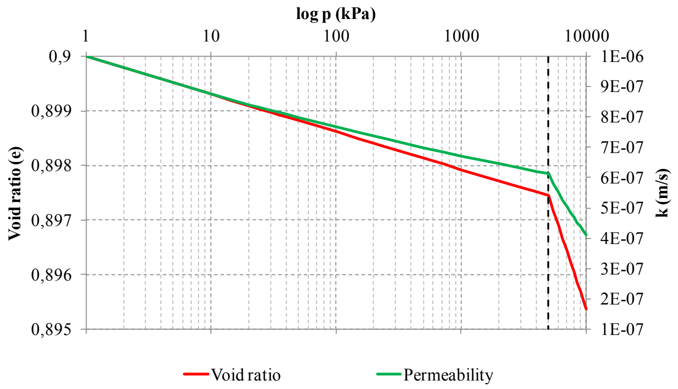

Figure 8. Void ratio and permeability changes with stress increase.

One can deduce the need for an appropriate permeability function, with adequate parameters to represent the pore volume changes over time. Porous medium permeability influences directly fluid pressure and flow, representing an important aspect of numerical modeling.

The following simulation was performed for a compressible fluid. The analysis results are presented in Figures 9 to 12.

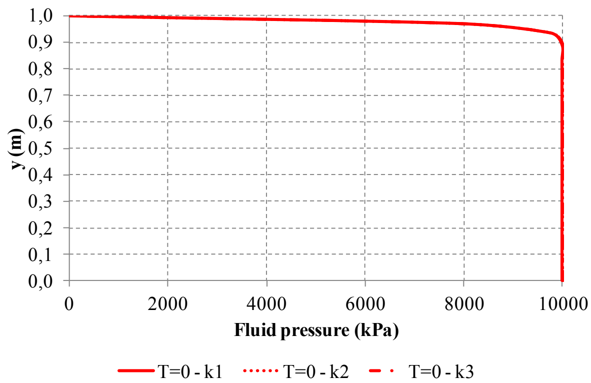

Figure 9. Results of fluid pressure for compressible fluid simulation (T=0).

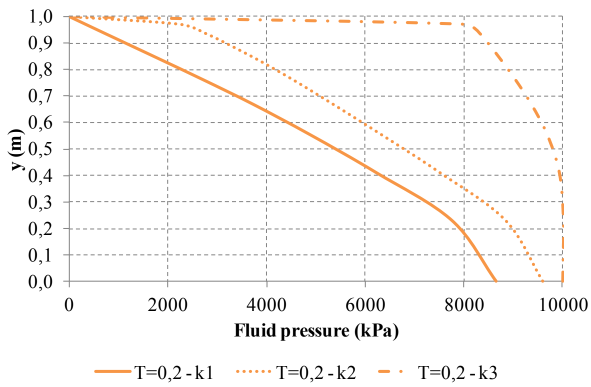

Figure 10. Results of fluid pressure for compressible fluid simulation (T=0,2).

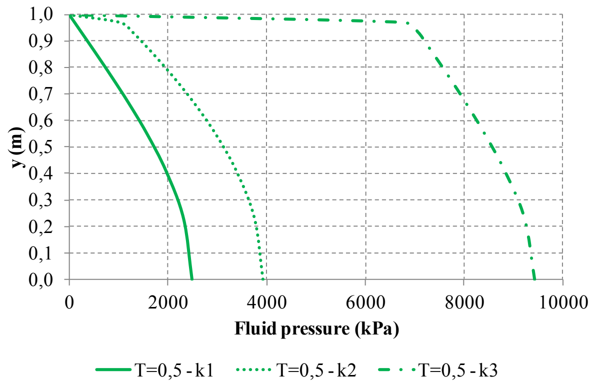

Figure 11. Results of fluid pressure for compressible fluid simulation (T=0,5).

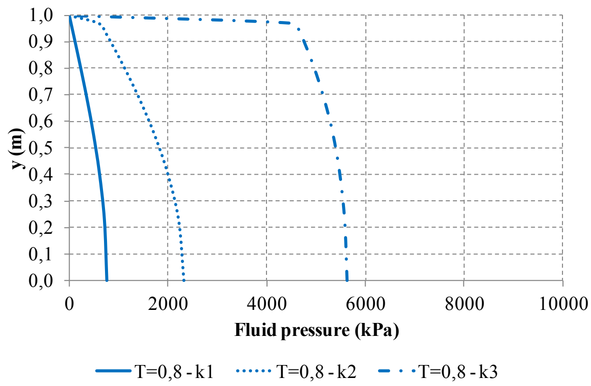

Figure 12. Results of fluid pressure for compressible fluid simulation (T=0,8).

The results for simulations with the compressible fluid follow the same behavior tendency of the incompressible fluid. The compressibility of the fluid causes an effect of delay in fluid pressure dissipation, given by volume changes the fluid suffers. The effect of fluid compressibility in a consolidation process in porous media is detailed in Jesus [3].

It is fair to state that the only difference among the values of pressure for both types of fluids is related to its compressibility, with no influence from the permeability functions.

Thus, by analyzing the format of the fluid pressure curves over time, it can be inferred that fluid flow tendency is the same (the shape of the curves is maintained) regardless the fluid compressibility. Soil consolidation induces pore closing first on the superficial layers of the sample, forming a low-permeability zone at its surface. Fluid pressure within this zone increases, as observed for the results of all time stages for permeability function k_3 curves.

5 CONCLUSIONS

In this paper, the effects of permeability variation in fluid pressure dissipation during a consolidation process were investigated. Pore volume changes influence directly fluid percolation through the porous medium, evidencing that the different permeability functions may alter significantly the responses achieved with the proposed numerical model.

In addition, the combined effect of permeability changes and fluid compressibility is analyzed. One can deduce that fluid compressibility does not alter percolation through the porous medium. Fluid flow takes place with the same tendency observed in incompressible fluid simulations. However, an effect of fluid pressure dissipation delay is registered due to fluid volume changes.

Interesting results were achieved with modified Cam-clay model. This model captures more accurately the effect of pore closing during the consolidation process. The applied load triggered significant changes in void ratio for the superficial layers of the sample. Thereby, a low-permeability zone was formed at the surface of the sample and it prevented fluid pressure dissipation for lower layers (fluid flow out of the sample). This could be easily observed in the results for permeability function k_3.

These analyses highlighted the importance of studying permeability functions for porous medium, in order to better represent its hydraulic behaviour and how an interesting mechanical behaviour of porous medium could be captured with the use of modified Cam-clay model.

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