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WATER RETENTION IN UNSATURATED SOILS SUBJECTED TO WETTING AND DRYING CYCLES

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Abstract. The suction is an essential parameter to describe and understand the behavior of unsaturated soils. The ability of unsaturated soils to retain water is quantified by determining the water retention curves (WRC), which express the hydraulic behavior of porous materials such as soil. These curves are determined by subjecting samples to several drying and wetting cycles. The curve during drying path is located above the wetting curve, developing a hysteresis phenomenon [1], and value of content water at a given suction value depends on the path used to reach this point.
The aim of this paper is to present a study on the hydraulic behavior of soil, water retention capacity due to drying and wetting cycles, pointing out the hydro-mechanical behavior of unsaturated soils.
In the first part, the effect of physical and mechanical properties of soil [32] (initial void ratio, particle size, cohesion, density...) on the water retention is presented.
In the second part, a complete numerical model was developed, based on the empirical model of Van Genuchten [18], to model the two boundary curves, and the experimental scanning data were best-fitted using the same theory of Mualem model [13]. This complete model requires 4 parameters.
This model has been validated with experimental data on different type of soils: sand [10], [34], U.S. Silica F-95 sand [30].

1 INTRODUCTION

The description and prediction of hydro-mechanical behavior of unsaturated soils require good knowledge of their hydraulic properties. One of these basic properties is the water retention curve (WRC), which connects the water content or the degree of saturation with suction. This relationship was used to estimate the hydraulic conductivity, volume change, and to estimate the aqueous diffusion functions of unsaturated soils ([1]; [2]; [3]). For a given matrix suction, water content in the drying curve is always higher than that in the wetting curve. In other words, soil follows different WRCs during a drying and a wetting process. This phenomenon is referred to as hysteresis. This hysteresis is typical of porous media consisting of interconnected pores of varying sizes, in which air is trapped. The contact angle between the water and the pore surface is greater in an advancing meniscus than in a receding meniscus. The hysteresis can also be caused by irregularities in the cross-sections of the void passages or the "ink-bottle" effect [4].

The term suction in geotechnical engineering has been defined in [5] as potential energy similar to the hydraulic head in saturated soils, corresponding to "the applied energy to carry free water from infinity to an unsaturated soil".
In engineering practice, soil suction is composed of two components: matrix and osmotic suction [1]. The sum of matrix and osmotic suction is called total suction. Matrix suction comes from the capillarity, texture, and surface adsorptive forces of the soil. Osmotic suction arises from the dissolved salts contained in the soil water.
In Geotechnics, matrix suction consist the main part of total suction, and is usually expressed as a negative water pressure in KPa, the difference between air pressure U_a and water pressure U_w.

$$\psi = U_a - U_w \quad (1)$$

2 WATER RETENTION CURVES OF SOIL

The water retention curves are classified into 4 types: boundary drying curve, boundary wetting curve, wetting scanning curves and drying scanning curves.

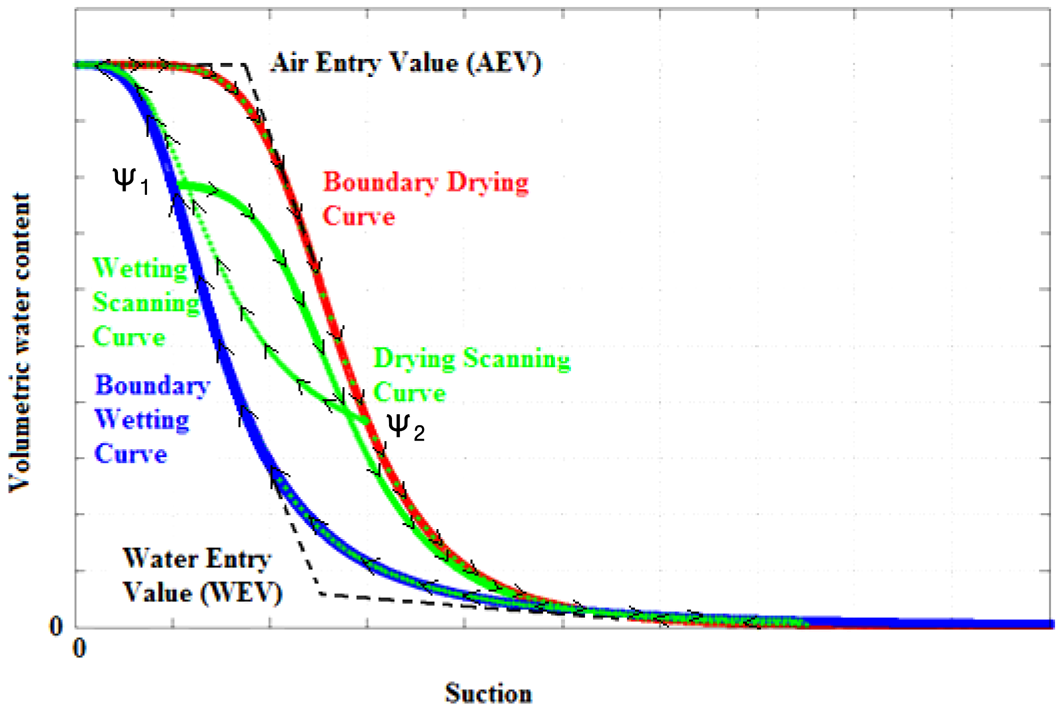

Figure 1: Schematic representation of Water Retention Curve (WRC) with the four curves: the two boundary curves for drying and wetting, and the scanning curves. There are two particular suction values: the air entry value and the water entry value.

On the boundary curves we have:
The air entry value (AEV): it is the higher suction at which the degree of saturation starts to decrease from full saturation at the boundary drying curve.
The Water Entry Value (WEV): it is the lower suction at which the degree of saturation starts to increase from residual water content at the boundary wetting curve.

2.1 Experimental setup for suction

Various experimental tools have been developed to deal with unsaturated soils, or to measure the value of matrix suction (measuring techniques), or to fix it for a test (control techniques) (after [5]). The principal measurement techniques are: Tensiometers, Psychrometers and the filter paper method (for more detail [5]). Most of these instruments have limitations with regard to range of measurement, equilibration times, and cost.

Therefore, there is a need for a method which can cover the practical suction range, be adopted as a basis for routine testing, and inexpensive.

Given the difficulty of measuring the suction, several numerical models have been developed to describe the sigmoid curve of water retention.

2.2 Numerical models

A lot of research has been conducted in the context of WRC; different approaches have been adopted to describe the hysteresis effect in these curves. The various models used to predict hysteretic WRCs can be classified into two categories: conceptual models (domain models) and empirical models.

The first group of conceptual models is based on the theory of independent domains developed by Néel [6] and used by other authors [7]; [8]; [9]. This theory attributes to the water in the soil two areas (group of pores): area without water and water-filled area. The Néel diagram assumes that the soil pore exists in one of two states; full of water or empty. The state of a pore can be characterized by two values of soil suction; namely, (i) drying soil suction, ψ_d ; and (ii) wetting soil suction, ψ_w. When soil suction increases to the drying soil suction of the pore, ψ_d, then the pore is fully drained. When soil suction decreases to the wetting soil suction of the pore, ψ_w, then the pore is filled. The first application of this theory was made by Poulovassilis [10] and Topp [11], both models require all four curves for calibration. Mualem [12];[13];[14] still used the theory of fields, but with two curves for calibration. Other authors have proposed modifications to the theory of fields, to take into account interactions between domains. Several models, requiring more than two branches for calibration have been developed ([15]; [16]). The final model developed by Mualem [17] requires two branches for calibration: it uses a correction factor for variations in water content calculated on the basis of the independent domain theory.

The second group of models is related to empirical models. These models are based on analysis of the shape and properties of the curve.

A number of empirical models have been developed to describe the non-linear (sigmoid) WRC ([18]; [19]; [21]).

All these empirical models are derived from a general equation of the form:

$$a_1\Theta^{b_1} + a_2\, exp(a_3\Theta^{b_1}) = a_4\psi^{b_2} + a_5\, exp(a_6\psi^{b_2}) + a_7 \tag{2}$$

Where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, b_1\ and\ b_2$ are constants, ψ the matrix suction, and Θ the volumetric water content.

Most of the researchers try to find equations describing the water retention curve using the simplest set of measurable parameters of soil solid phase such as particle size distribution, bulk density.

2.3 For boundary curves

Leong and Rahardjo [20] and Aubertin and Maksoud [21], Fredlund and Houston, and Fredlund and Sillers ([22]; [23]) found that the equation of Van-Genuchten [18] and that of Fredlund and Xing [19] models are most relevant to a variety of soils to model curves limits of water retention. This choice was made based on the Akaike information criterion.

The Van Genuchten equation can be written as:

$$\theta_v = \theta_r - (\theta_s - \theta_r)[1 + \left(\frac{\psi}{a_v}\right)^{n_v}]^{-m_v} \tag{3}$$

a_v is a parameter related to the air entry value (AEV), n_v is a parameter related to the variation of water content in the soil once the suction pressure exceeds the AEV, and m_v is related to residual water content θ_r. The index v in (a_v, n_v, m_v) is equal to w in wetting case and $v = d$ in drying case.

n_v and m_v can be related by: $m_v = 1 - \frac{k_m}{n_v}$ for k_m= 1 we have the VG-Mualem formulation for k_m= 2 we have the VG-Burdine formulation [18]. The Van-Genuchten equation can be expressed in term of degree of saturation:

$$S_{rv} = S_{res} - (S_{rmax} - S_{res})[1 + \left(\frac{\psi}{a_v}\right)^{n_v}]^{-m_v} \tag{4}$$

S_{rv} is the degree of saturation corresponding to suction value ψ, S_{res} is the residual degree of saturation, S_{rmax} is the maximum degree of saturation.

Several authors ([24]; [32]) have correlated the parameters of Van-Genuchten with soil bulk density and with the content of soil organic matter, sand, silt and clay. Different numerical techniques were used to find correlations such as linear regression, nonlinear regression, and multivariate nonlinear optimization.

Other authors have estimated the water retention curve using parameters such as bulk density porosity, pore volume, and texture ([32]; [26]).

The differences between the values of Van-Genuchten parameters for the soils studied in [25] may be governed by differences in their physical and chemical properties. It seems that the particle size has the greatest influence on these parameters. Likewise, this study highlights the influence of compactness.

2.4 For scanning curves

Viane et al. [25] compared six models: (1) Mualem (1974), (2) Mualem (1977), (3) Hogarth et al. (1988), (4) Mualem (1984b), (5) Hanks et al. (1969), and (6) Scott et al. (1983), using the

data set of 7 soils. The authors, after a statistical analysis, found that the models of Mualem give comparable results and provide a good prediction of the scanning curve for models with two branches. Pham and Fredlund [31] have used in their comparison of the two models (Mualem (1974) and Mualem (1984)) are especially relevant to Viane et al. (1994) with three other models.
These studies demonstrated the relevance of the model of Mualem [13] for modeling the scanning curves.
Mualem's model can be drawn from scanning curves from the two limit curves.
The scanning drainage curve, which starts from suction ψ_1 on the limit curve of wetting, is given by:

$$\theta_d(\psi_1,\psi) = \theta_w(\psi) + \frac{[\theta_w(\psi_1) - \theta_w(\psi)]}{[\theta_u - \theta_w(\psi)]}[\theta_d(\psi) - \theta_w(\psi)] \tag{5}$$

Where $\theta_w(\psi)$ is the water content on the boundary wetting curve at suction, ψ ; $\theta_d(\psi)$ is the water content on the boundary drying curve at suction, ψ ; and θ_u is the water content at the meeting point of the two boundary curves at zero suction.
Similarly, the wetting scanning curve, which starts from suction ψ_2 on the drying boundary curve, is given by:

$$\theta_w(\psi_2,\psi) = \theta_w(\psi) + \frac{[\theta_u - \theta_w(\psi)]}{[\theta_u - \theta_w(\psi_2)]}[\theta_d(\psi_2) - \theta_w(\psi_2)] \tag{6}$$

The complete model is then formulated with the two models of Van-Genuchten and Mualem. The same formulation has been adopted for Mualem equations in term of degree of saturation.

3 FACTORS AFFECTING THE WATER RETENTION CURVE

A comprehensive description of the behavior of soils partially saturated in water should take into account:

- the hydraulic behavior (retention properties)
- the mechanical behavior (stress/strain relationship)
- the water flow in porous medium.

All three behaviors are known to be coupled together, and especially, the stress strain behavior is explicitly suction-dependent.
So the researches were conducted to show the different physical and mechanical factors affecting the water retention curve.
In this study, we present the main factor: the effect of void ratio, which is the result of mechanical behavior. There are many other factors; in this article we introduce only the main factor related to coupled hydro-mechanical behavior.

Effect of initial void ratio

Kawai et al. [26] studied the effect of the value of void ratio on the boundary Water Retention Curve (WRC). The pore water tends to migrate with increasing suction and when the value of the suction reaches the air entry value (AEV), the water begins to drain. The AEV reflects the magnitude of the capillary zone of saturation in the soil. The more the pores are large, the smaller AEV become. The AEV is inversely proportional to the logarithm of e (e is the void ratio for the soil). When the soil is wetted, suction decreases and the degree of saturation increases, when the value of sucking decreases beyond a certain value called the Water Entry Value (WEV) (Fig. 1), an increase in the degree of saturation is more remarkable. In this context, Vanapalli et al. [27] published the results of water retention curves of a clay loam under different states of compaction.

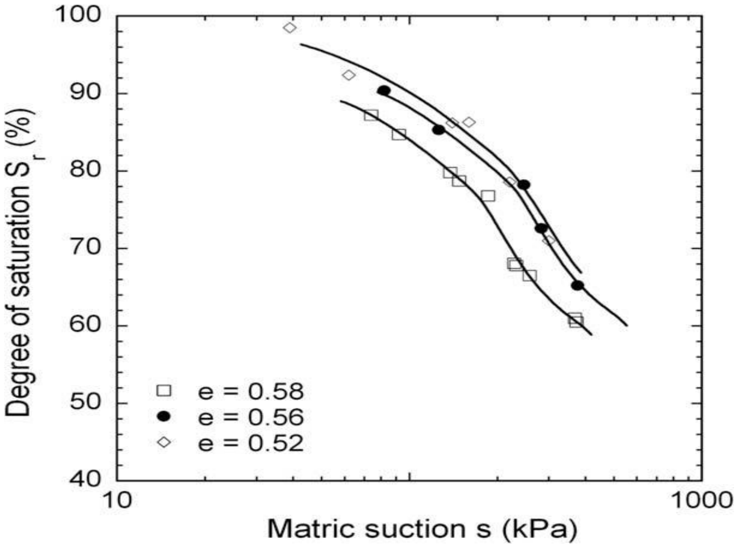

Figure 2: Water retention curves of compacted clay loam under different initial void ratio [27]

To introduce the mechanical coupling, the complete model should take into account the effect of the evolution of soil porosity on the boundary water retention curve.

4 COMPLETE MODEL TAKING INTO ACCOUNT THE VARIATION OF POROSITY

Recent discussions on stress frameworks for unsaturated soils [28]; [29] propose to scale down the analysis to the pore scale to understand the physical implications of saturation. The water retention curve shows that for the same net stress (defined as the difference between total stress and pore air pressure) and same amount of matrix suction, a specimen on a drying path reaches a higher degree of saturation than a sample under a wetting path.

Given that a difference in the degree of saturation induces a different repartition of water in the soil pores, modified proportions of bulk water and meniscus water are encountered according to the suction change following the wetting or the drying path.

It is agreed that suction within the meniscus part of the pore water acts only on the forces at inter-particle contacts whereas suction in bulk water is seen as an overall (isotropic) water pressure deficiency [33]. The skeleton effective stress could thus be identified as a function of external stresses, interstitial fluid pressures or suction, and repartition of pore water or degree of saturation. In other words, whenever a capillary hysteresis is identified in a soil water retention curve, it might significantly affect the state of effective stress within the soil.

In this study, the complete model is based on the Van-Genuchten equation (equation (3)) for boundary curves and on the use of Mualem equations for scanning curves (equations (5), (6)).

The Van-Genuchten equation is improved by adding the D_p parameter to take into account the effect of void ratio evolution on the boundary water retention curves. And D_p can be made a function of the void ratio, according to:

$$D_p = \psi_{ae0} * \lambda(e_0 - e)/e_0 \tag{7}$$

λ is a material coefficient, and $\psi_{ae0}$is the Air entry value at the referential void ratio e_0, in the model this value is defined as the intersection of two lines:

$$\theta_d = \theta_s \tag{8}$$

And the tangent line on boundary drying curve at inflection point defined by:

$$\theta_d = (\theta_s + \theta_r)/2 \tag{9}$$

Hence, Eq. (3) will be modified to:

$$\theta_v = \theta_r - (\theta_s - \theta_r)[1 + \left(\frac{(\psi - D_p)}{a_v}\right)^{n_v}]^{-m_v} \tag{10}$$

In summary, two reference boundary curves (for an initial reference void ratio e_0) are expressed, where θ_w and θ_d are recalculated using the modified Van-Genuchten equation (10) $v = w$ in wetting case and $v = d$ in drying case. The parameter (a_v, n_v) of each curve are determined based on experimental data for a given soil, then the scanning curves are interpolated between the two boundary curves using Mualem equations (equations (5); (6)) depending on the evolution of suction value: the drying case is obtained for positive suction increment whereas the wetting case is obtained for negative suction increment.

Recent studies [28],[29] have adopted the same method of shifting for boundary curve with a linear relation between the air entry value and the void ratio.

This model has four parameters (a_w,n_w;a_d,n_d). By a simple method, these parameters can be determined for each type of soil.

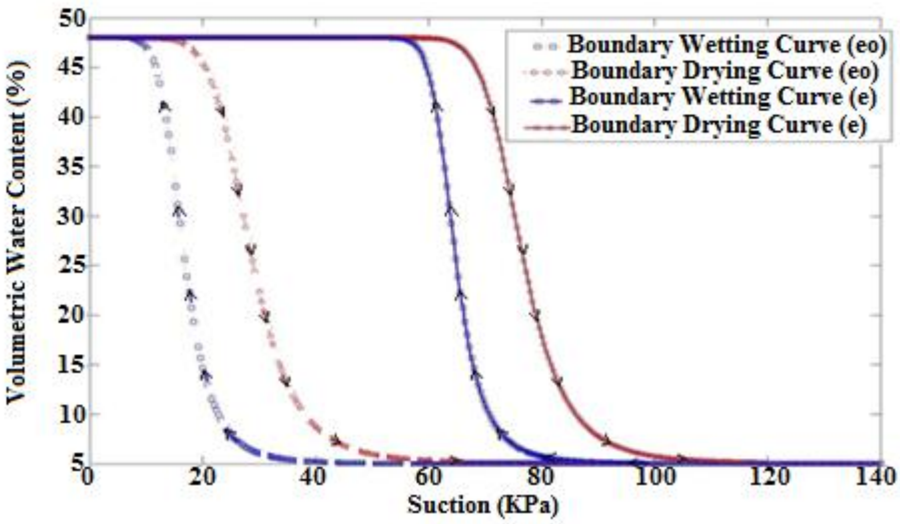

Figure 3: Shape of boundary retention curve with the variation of void ratio: change of void ratio induces a shift of the retention curve (the smaller void ratio, the higher air entry value)

5 VALIDATION OF NUMERICAL MODEL

A comparison of model predictions with experimental data is provided in order to further illustrate the capabilities of the model in simulating real unsaturated soil behavior.
The first experimental data considered are those presented by Lins et al. [34], from test on Hostun sand for which the parameters used in equation (4) are presented in Table 1, and $S_{rmax} = 1, S_{res} = 0$.
The prediction is shown in Fig. 4, from which it can be concluded that the model is reasonably accurate, although the lowest part of the wetting path was not quite exactly predicted. This is due to the number of simplifications adopted, particularly assuming that:

$$m_v = 1 - 1/n_v \tag{11}$$

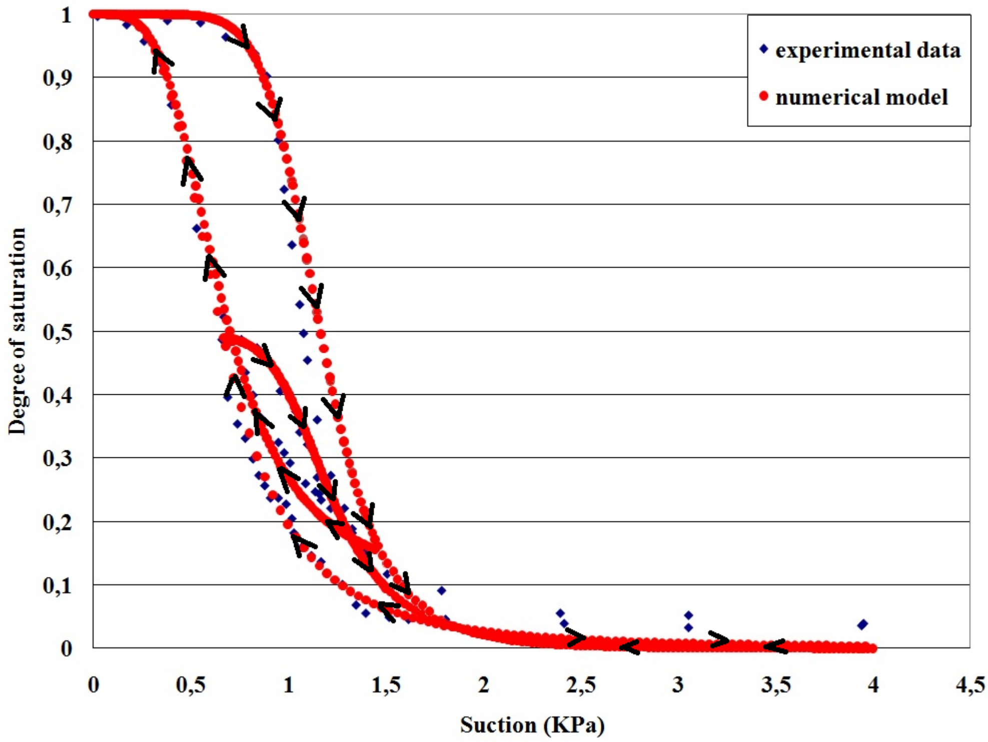

Figure 4: Validation of the proposed WRC model against experimental data from tests on Hostun sand (data after [35]).

The second set of experimental results considered are those presented by Muraleetharan et al. [30] and correspond to a fine poorly-graded sand named US Silica F-95 (Berkeley Springs, West Virginia). The results provided in Muraleetharan et al. [30] are expressed in terms of suction ψ against volumetric water content θ, defined as the volume of fluid divided by the total volume of a representative elementary volume (REV). This volumetric water content θ will be equal to the porosity n when the suction becomes zero. Fig. 5 shows the experimental data points and the model predictions of boundary drying and wetting WRCs and scanning curves, from which it can be seen that the computed results are a reasonably accurate representation of the experimental data.

Table 1: Van-Genuchten parameters for Hostun sand(data after [34]).

Drying		Wetting	
a_d	n_d	a_w	n_w
1.1376	7.825	0.5963	3.987

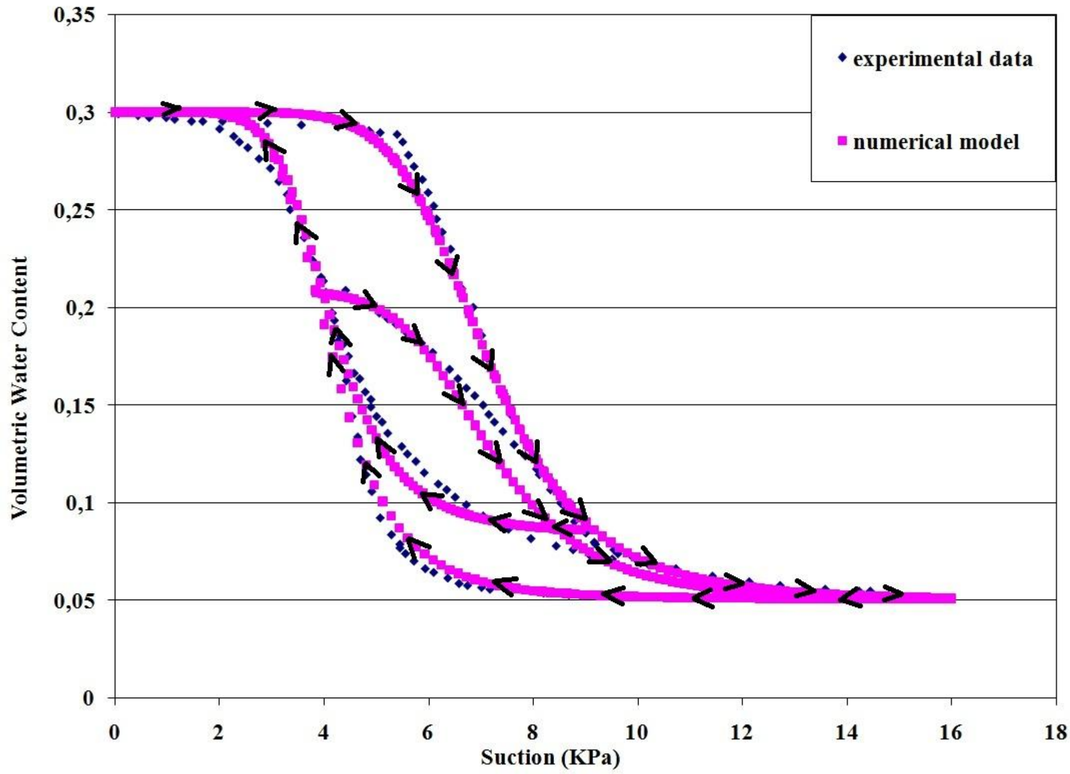

Figure 5: Validation of the proposed WRC model against experimental data from tests on US Silica F-95 fine sand (data after [31])

In order to further illustrate the versatility of the proposed model, for this second validation, the axes will be directly selected as x = ψ and y = θ, as were utilized in the coupled model of Muraleetharan et al. [30]. Now, $\theta_s = 0.3$ and $\theta_r = 0.05$ for the US Silica F-95 sand. The parameters for equation (3) are summarized in Table 2.

Table 2: Van-Genuchten parameters for US Silica F-95 sand (based upon test data after [30])

Drying		Wetting	
a_d	n_d	a_w	n_w
6.7613	8.168	3.9215	7.695

However, in the literature, to the knowledge of the author of this article, there's no a complete experimental data, for water retention curves with the coupled hydro mechanical effect on those curves, in other terms, there's no experimental data of hydraulic loading cycles (the four water retention curves) taking into account the effect of the void ratio. A new experimental setup has been developed, in our laboratory, for measuring the water retention curve and controlling the porosity of the sample subjected to suction cycles.

6 CONCLUSION

A complete representation of the WRC with hysteresis behavior of unsaturated soils is presented, involving a simple series of equations. The great advantage of this technique is its easy application to experimental saturation–suction data for a given soil. The proposed model for water retention curve introduces two reference boundary curves corresponding to a reference void ratio e_0.

The coupled model has eight parameters: four hydraulic calibration parameters: $(a_w;\ n_w;\ a_d;\ n_d)$, one mechanical parameter (λ) to introduce the effect of void ratio evolution on water retention curve, and three physical parameters $(\theta_r;\ \theta_s;\ e_0)$. The first five parameters are calibrated by wetting and drying experimental curves and the last three are directly measured by characterization experiments.

The model presented here deals with the hydro-mechanical behavior and takes into account the deformation characteristics of soils, such as the influence of either the initial or current void ratios. The implementation of the resulting equations into fully hydro-mechanical coupled models, with the stress/strain relation for numerical analyses, is straightforward though, using an effective stress definition, such as Bishop equation.

The model is efficient with two different data sets [31],[35] introducing only the retention behavior. Experiments are in progress to be able to catch the void ratio variation.

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