VI International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2015
B. Schrefler, E. Oñate and M. Papadrakakis (Eds)

STRUCTURAL DESIGN USING A PARALLEL SEQUENTIAL APPROXIMATE OPTIMIZATION

DONGHUI WANNG*, ZEPING WU*, FAN HU* AND ZHENYU JIANG*

* College of Aerospace Science and Engineering
National University of Defense Technology
109 Deya Road, 410073 Changsha, China
e-mail:lightblue117@gmail.com

Key words: Sequential approximate optimization, Structural design optimization, Parallel sampling strategy.

Abstract. In this paper, a parallel sampling strategy dedicated to SAO is proposed to enhance the competence of exploration and convergence simultaneously in the optimization process. In the parallel sampling procedure, new sampling points are identified in the pareto front of the multi-objective optimization problem, which is solved with the Normal constraint (NC) method. The objectives of the optimization problem are the two indices to represent the competence of exploration and convergence, and the sampled points will be evaluated by the true model in a parallel way. Furthermore, the proposed methodology is evaluated on two benchmark tests. Compared to other optimization algorithms, the PSAO algorithm yields equivalent or better objective values while the number of optimization iterations required to find the same global optima is reduced by multiple orders of magnitude, which substantially reduce the computing costs in the tested structural design optimization tasks, highlighting the applicability of the PSAO structural design optimization problems.

1 INTRODUCTION

Structural design optimization remains an important and challenging topic in the engineering design of lighter, more effective structures [1]. Design optimization aims to determine the optimal shape of a structure by maximizing or minimizing a given criterion, such as stiffness or weight, subject to stress or displacement constraints. Over the past decade, a number of optimization algorithms have been extensively used in structural optimization tasks, such as gradient-based algorithms, evolutionary algorithms (EAs) and approximation-based optimization algorithms [2]. Of course, advantages and disadvantages are associated with any optimization technique.

Several examples of gradient-based optimization applied to structural design problems exist in the literature [3, 4]. However, gradient-based optimization techniques are extremely sensitive to the initial guess and prone to trapping in local optima.

In the past few decades, structural design optimization problems have been increasingly solved by EAs such as the genetic algorithm [5], simulated annealing [6], particle swarm optimization [7] and the artificial bee colony algorithm [8]. EAs present several advantages over gradient-based methods [9]. Drawbacks of these methods are the huge number of function evaluations required.

In approximation-based optimization techniques, objective functions are expressed as low order polynomial approximations to explicit functions. The accuracy of these techniques is acceptable and the computational cost is much reduced. Commonly applied approximation techniques include the response surface method [10, 11], neural network [12, 13], polynomial regression models [14], Kriging methods [15] and the radial basis function (RBF) [16]. However, these techniques introduce error into the meta-model, which reduces their reliability.

The SAO algorithm has been recognized as one of the most attractive approaches for engineering optimization [17]. The success of an SAO algorithm depends chiefly on the approximation technique and the sampling strategy. Therefore, this paper focuses on improving the sampling strategy to the extent that SAO becomes applicable to structural design optimization problems. The paper is structured as follows: Section 2 introduces the general formulation of the SAO approach and proposes a parallel sampling strategy. Finally, the procedure of the PSAO is presented. Section 3 presents two structural optimization case studies, which are used to demonstrate the efficacy of the methodology. Concluding remarks are presented in Section 4.

2 PARALLEL SEQUENTIAL APPROXIMATION OPTIMIZATION

2.1 General framework of the sequential approximation optimization

Let us consider a general structural optimization problem with constraints:

$$\begin{aligned} &\text{find } \ \boldsymbol{X} \\ &\min \ f(\boldsymbol{X}) \\ &s.t. \ \ g_i(\boldsymbol{X}) \le 0 \quad i = 1, 2, \cdots, l \\ &\qquad h_j(\boldsymbol{X}) = 0 \quad j = 1, 2, \cdots, k \\ &\qquad \boldsymbol{X}^L \le \boldsymbol{X} \le \boldsymbol{X}^U \end{aligned} \tag{1}$$

For most structural design problems, the objective function and constraints are implicit functions of design variables, usually obtained by finite element analysis (FEA). Since the computational cost for FEA may be high, the number of analyses carried out during the optimization has the main impact on the efficiency of the algorithm. This has initiated the development of optimization techniques that are suitable for structural design problems [18].

In the classical approximation-based optimization procedure summarized in [19], the accuracy of the surrogate model could be degraded by an ill-chosen initial sample, leading to a deceptively positioned optimum. Here we assume that our optimum design is the best result of the true function, not that of the surrogate. Results from the surrogate are therefore evaluated by comparison with the true function evaluations. Additional calls to the true function are used to both validate the surrogate and enhance its accuracy. Thus, as shown in Figure 1, the SAO approach selects the new points at which the true function is called. Applying a series of new infill points based on some infill criteria (also known as a sampling strategy), the objective function is sampled using a constantly changing surrogate model [20].

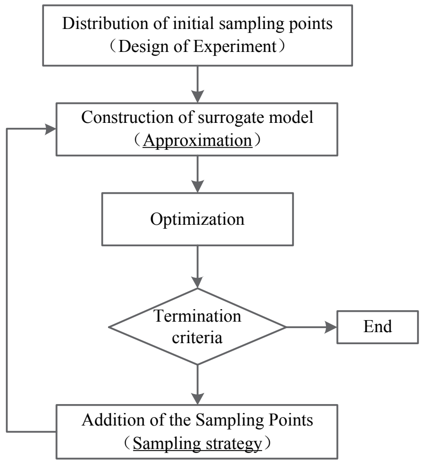

Figure 1: General framework of the sequential approximation optimization

The SAO-based structural optimization can be mathematically expressed as:

$$\begin{aligned}
&\text{for} \quad n = 1, 2, \cdots \\
&\text{find} \quad \boldsymbol{x} \\
&\min \ f^{(n)}(\boldsymbol{x}) \\
&s.t. \quad g_i^{(n)}(\boldsymbol{x}) \le 0 \quad i = 1, 2, \cdots, l \\
&\qquad h_j^{(n)}(\boldsymbol{x}) = 0 \quad j = 1, 2, \cdots, k \\
&\qquad S^{(n)}(\boldsymbol{x}) \le 0 \\
&\qquad \boldsymbol{x}^L \le \boldsymbol{x} \le \boldsymbol{x}^U
\end{aligned} \tag{2}$$

the m-dimensional design variable $\boldsymbol{X}$ is scaled into an m-dimensional unit hypercube $\boldsymbol{x}$ by

$$x_i = \frac{X_i - X_i^L}{X_i^U - X_i^L} \quad i = 1, 2, \cdots, m \tag{3}$$

Here $f^{(n)}(\boldsymbol{x})$ is the nth approximation of the objective function, $g_i^{(n)}(\boldsymbol{x})$ and $h_j^{(n)}(\boldsymbol{x})$ are the nth approximations of the constraints. In addition, $S^{(n)}(\boldsymbol{x})$ is a constraint deduced from the sampling strategy in the nth iteration.

2.2 Parallel sampling strategy

Since its proposal, a number of sampling strategies have been applied to the SAO algorithm [21, 22]. These strategies can be roughly divided into three categories: exploitation, exploration and balanced exploitation/exploration. The sampling strategies are usually performed by infilling the new point sequentially, which imposes restrictions on the capability of exploration and convergence. In this section, a parallel sampling strategy is proposed.

The simplest exploitation sampling strategy for SAO is to find the optimum of the surrogate model $s(\boldsymbol{x})$, while exploration maximizes the minimal Euclidean distance between sampling points $d(\boldsymbol{x})$, given by

$$d(\boldsymbol{x}) = \min(\sqrt{(\boldsymbol{x} - \boldsymbol{x}_i^{(n)})^T (\boldsymbol{x} - \boldsymbol{x}_i^{(n)})}) \quad (i = 1, 2, \cdots, N^{(n)}) \tag{4}$$

where $N^{(n)}$ is the number of sampling points before the nth sampling. Here we adopt an adaptive sampling strategy by solving the multi-objective optimization problem

$$\begin{aligned} max: \ & s^{(n)}(\boldsymbol{x}),\ d(\boldsymbol{x}) && \boldsymbol{x}_{\min} \le \boldsymbol{x} \le \boldsymbol{x}_{\max} \\ s.t. \ & g_i^{(n)}(\boldsymbol{x}) \le 0 && i = 1, 2, \cdots, l \\ & h_j^{(n)}(\boldsymbol{x}) = 0 && j = 1, 2, \cdots, k \end{aligned} \tag{5}$$

where l is the number of inequality constraints and m is the number of equality constraints of original optimization problems, $s^{(n)}(\boldsymbol{x})$ is the meta-model constructed before the nth sequential sampling. Then the optimal solution of (1) together with the real response evaluated by the original model will be regarded as new sampling points to update the surrogate model. Solving problem (5), the Pareto front is easy to be obtained by Multi-objective Optimization Evolutionary Algorithms (MOEAs), such as NSGA or NSGA-II.

Given the number of parallel sampling points, denoted by k , the procedure of parallel sampling is shown as follows:

Step 1: solve the multi-objective optimization problem (5) to get the Pareto front;

Step 2: select k points on the Pareto front evenly as the sampling points, which is illustrated in Figure 2;

Step 3: Once these k sampling points are identified, those selected points are evaluated in parallel using the true model.

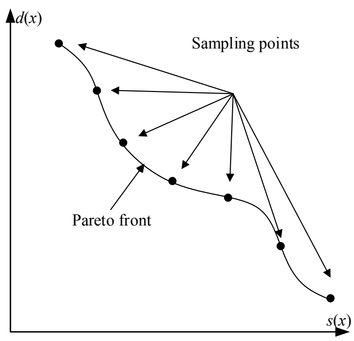

Figure 2: The illustration on the sampling points on the Pareto front

2.3 Procedure of the parallel sequential approximation optimization

Figure 3 presents a detailed flowchart of the PSAO algorithm, which is roughly divided into four blocks: the Initial stage, Approximation stage, Termination criteria and Sampling stage. Each block is elaborated below:

(1) Initial stage

In this stage, the m-dimensional design variable is scaled into an m-dimensional unit hypercube, which is then sampled by the Optimal Latin Hypercube Design (OLHD) method. The number of sampling points N is generally estimated from the following rule:

$$N = \begin{cases} 5m - 10m & m \le 10 \\ 100 & m > 10 \end{cases} \tag{6}$$

Finally, the objective function and the constraints of the sampling points are evaluated from the true model, and an initial sample set is generated.

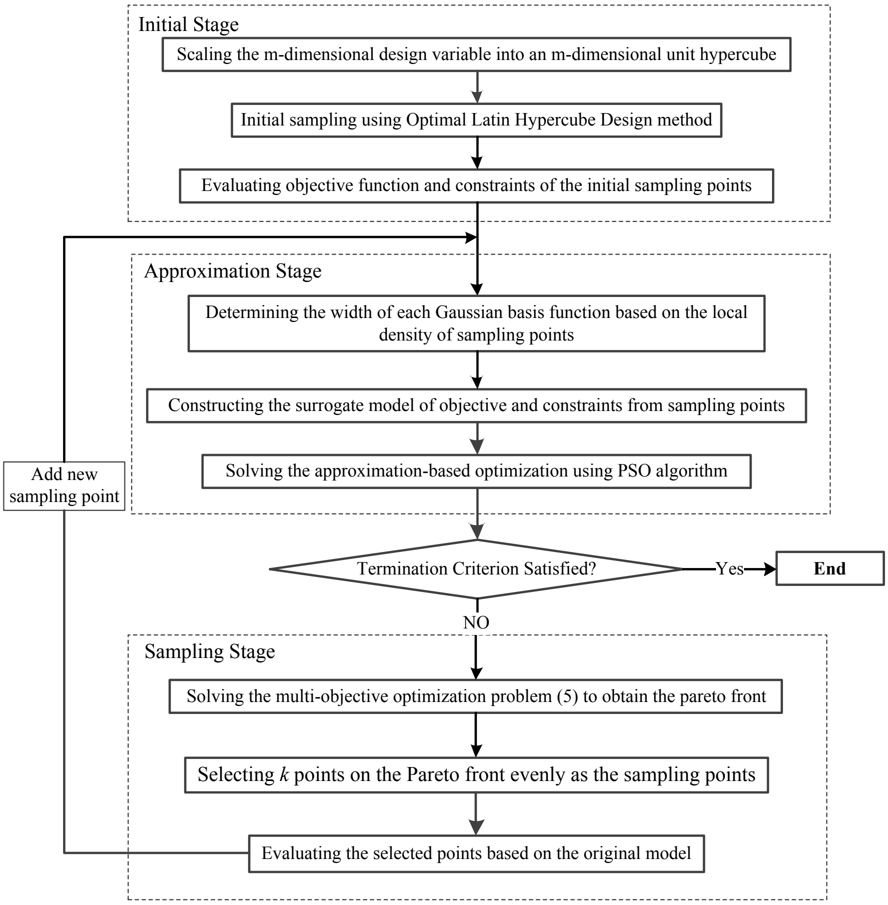

Figure 3: Procedure of the PSAO

(2) Approximation stage

Based on the sample set, a surrogate model of the objective function and its constraints is constructed using the enhanced approximate technique [23], whose kernel widths are obtained based on the local density of sampling points. The proposed point-density based estimate of the width of basis function demonstrates good performance for both uniform and non-uniform sampling points. Finally, the surrogate-based optimization problem is solved by the PSO algorithm [7].

(3) Termination criteria

The PSAO is terminated under the following criteria:

(i) If the relative distance between the optimal solutions of two successive iterations is below 1%, then evaluate criterion (ii). Otherwise, advance the SAO to the Sampling stage;

(ii) If the relative error between the optimal objective functions under the constraints imposed by a penalty is less than 1%, then evaluate criterion (iii). Otherwise, advance the PSAO to the Sampling stage;

(iii) If the relative error between the objective functions of the surrogate model and the true model in the current iteration is less than 1%, then convergence is reached and the PSAO algorithm is terminated. Otherwise, advance the PSAO to the Sampling stage.

(4) Parallel sampling stage

The detail of the parallel sampling strategy has been descripted in section 2.2. When sampling points are selected, the sampling points together with their responses based on the original model are added to the sample set to update the surrogate model in the next iteration.

3 STRUCTURAL DESIGN OPTIMIZATION CASE STUDIES

In this section, two case studies from simple to complex are taken from the engineering practices to investigate the general-purpose application and advantages of the proposed PSAO. The effectiveness and robustness of the proposed PSAO algorithm is validated in comparison with other optimization techniques in this section as well.

3.1 Test case 1

The first test case is a 72-member space truss with numerous design variables and constraints. The geometry and material properties, as well as the node and member numbering system, are shown in Figure 4. The optimization objective is to minimize the structural weight. The design variables are the cross-sectional areas of the truss members, grouped as shown in Figure 4. This grouping reduces the number of design variables to 16 member groups. The area is allowed to vary between 0.1 and 2.5 in2. The structure is subject to two loading conditions, as detailed in Table 2. The maximum allowable stress (tension or compression) is 25.0 ksi per member group, while the maximum allowable planar displacement of each node, in either the x or y direction, is ±0.25 in for both loading cases.

Table 1: 72-bar truss member area groups.

Area group	Truss members	Area group	Truss members
A1	1, 2, 3, 4	A9	37, 38, 39, 40
A2	5, 6, 7, 8, 9, 10, 11, 12	A10	41, 42, 43, 44, 45, 46, 47, 48
A3	13, 14, 15, 16	A11	49, 50, 51, 52
A4	17, 18	A12	53, 54
A5	19, 20, 21, 22	A13	55, 56, 57, 58
A6	23, 24, 25, 26, 27, 28, 29, 30	A14	59, 60, 61, 62, 63, 64, 65, 66
A7	31, 32, 33, 34	A15	67, 68, 69, 70
A8	35, 36	A16	71, 72

Table 2: 72-bar truss loading cases

Load case	Node	Fx [kips]	Fy [kips]	Fz [kips]
1	1	5.0	5.0	-5.0

2	1	0.0	0.0	-5.0
	2	0.0	0.0	-5.0 -5.0
	3	0.0	0.0	-5.0 -5.0
	4	0.0	0.0	-5.0 -5.0

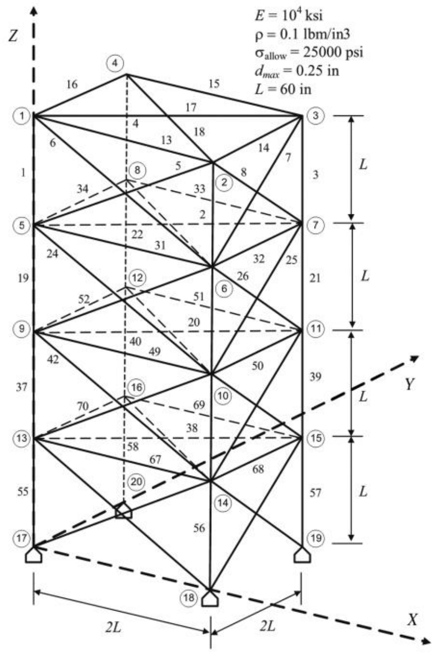

Figure 4: 72-Bar truss geometry

The optimization problem is solved by the proposed PSAO algorithm for 100 initial sampling points. In each iteration, 3 sampling points are selected in parallel. Figure 5 illustrates the relative distance and the relative error during the iteration history. The termination criterion is satisfied after 102 iterations and the evolution of the objective function is shown in Figure 6. The PSAO results are compared against those of recent publications; namely, the original SAO [9], penalty based PSO [24], and ant colony algorithms [25]. Table 3 summarizes the previously published results for the 72-bar truss problem using the different optimizers. Due to the parallel sampling strategy, the capacity of exploration and convergence of the original SAO is greatly enhanced. Hence, the number of iteration is reduced. It is noteworthy that the number of function evaluations for PSAO (406) is more than that for the original SAO (252). However, in PSAO, the sampling points are evaluated by the true model in parallel in each iteration, the actual computation time is equal to 202 function evaluations. Moreover, the SAO algorithm (PSAO and original SAO) shows a much better performance than EAs, and the number of function evaluations required to find the optimal solution is reduced from order 10^4 to order 10^2, indicating a substantial reduction in computing costs.

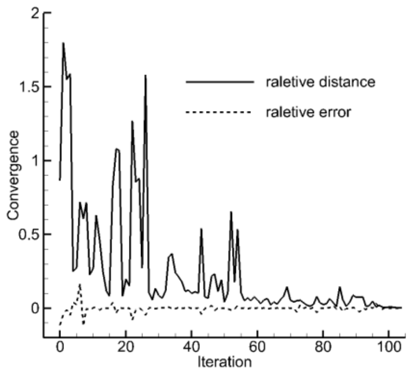

Figure 5: The best solution convergence history

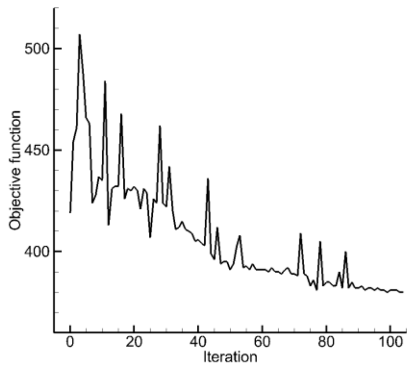

Figure 6: The objective function history

Table 3: Optimization results for the 72-bar truss

Design Variables	PSAO	SAO [9]	Perez and Behdinan(PSO) [24]	Camp and Bichon (ACO)[25]
A1 [in]	**0.157**	0.157	0.162	0.156
A2 [in]	**0.546**	0.549	0.509	0.550
A3 [in]	**0.405**	0.406	0.497	0.390
A4 [in]	**0.566**	0.555	0.562	0.592
A5 [in]	**0.520**	0.513	0.514	0.561
A6 [in]	**0.518**	0.529	0.546	0.492
A7 [in]	**0.100**	0.100	0.100	0.100
A8 [in]	**0.100**	0.100	0.110	0.107
A9 [in]	**1.258**	1.252	1.308	1.303
A10 [in]	**0.513**	0.524	0.519	0.511
A11 [in]	**0.100**	0.100	0.100	0.101
A12 [in]	**0.100**	0.100	0.100	0.100
A13 [in]	**1.898**	1.832	1.743	1.948
A14 [in]	**0.513**	0.512	0.519	0.508
A15 [in]	**0.100**	0.100	0.100	0.101
A16 [in]	**0.100**	0.100	0.100	0.102
Max. stress [psi]	**24999.67**	24943.87	24485.67	24939.59
Max. disp.[in]	**0.2500**	0.24992	0.2497	0.2500
Weight [lb]	**379.61**	379.90	381.91	380.24
Function	**202***	252	>20000	18500

3.2 Test case 2

This section presents the design of a moderate-dimensional case study of a bracket structure based on the proposed structural design framework. As depicted in Figure 7 eleven parameters that significantly affect the performance of the bracket are selected as design variables. Table 4 presents the feasible range of the design variables. The CAE model of the bracket is shown in Figure 8. Young's modulus and Poisson's ratio are settled as 200 GPa and 0.3. The bracket is subjected to a stretching force and a bending moment induced by the forces P1=4.7 kN and P2 = 4.2 kN loaded at the center of such screws as L1, L2, L3 and L4. In addition, the bracket is fixed at the screws including R1, R2, R3 and R4. The optimization objective is to minimize the volume while the maximum stress is constrained to be less than 200 MPa and the total displacement less than 2cm.

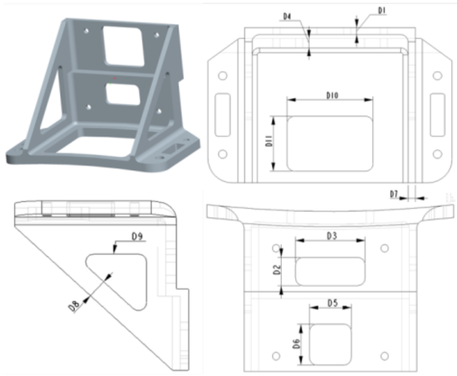

Figure 7: Description of the selected design variables

Table 4: Design space

Design variable (cm)	Lower bound	Upper bound
D1	1.0	3.5
D2	12.0	40.0
D3	10.0	25.0
D4	2.0	6.0
D5	12.0	40.0
D6	10.0	20.0
D7	1.5	4.0
D8	5.0	13.0
D9	25.0	35.0
D10	35.0	55.0
D11	8.0	50.0

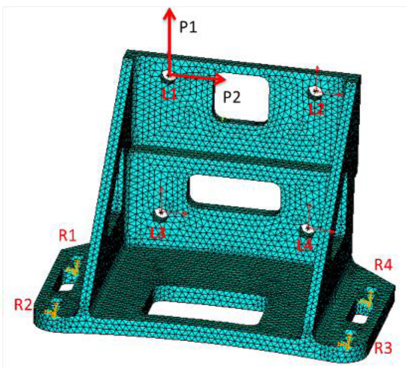

Figure 8: Details of the CAE model

The proposed PASO algorithm is used to solve this optimization problem and the number of initial sampling points is settled as 100. In every iteration, 4 sampling points are selected in parallel. In this test case, the proposed approach is also benchmarked against other optimization techniques as shown in Table 5. The best result from the zero-order optimization method locates on the point of 34675.67 cm^3 after 186 function evaluations. Pure GA with a population of 50 individuals and PSO with a swarm of 40 individuals are also used to solve the optimization problem, and the optimal volumes of 31493.33 cm^3 and 31510.26 cm^3 are found after 120 iterations (6000 FEA evaluations) and 140 iterations (5600 FEA evaluations), respectively. The original SAO requires 196 iterations to find the same level of optimal result, i.e. 296 FEA evaluations in sum [23]. Comparatively, the proposed PASO runs 152 iterations to get the same result as shown in Figure 9 and Figure 10. The total number of FEA iterations is 708. Because of the parallel mechanism, the equivalent number of function evaluations is 252. Based on the proposed approach, the computing costs of the structural optimization are immensely reduced.

Table 5: Comparison of the design optimization results

Optimization method	Volume (cm^3)	Displacement (cm)	Stress (MPa)	FEA evaluations
CAE built-in optimization	34675.67	1.86	196.5	186
GA	31493.33	1.94	198.3	6000
PSO	31510.26	1.96	195.4	5600
SAO	31481.48	1.96	197.6	296
PSAO	**31502.22**	**1.96**	**196.5**	**252***

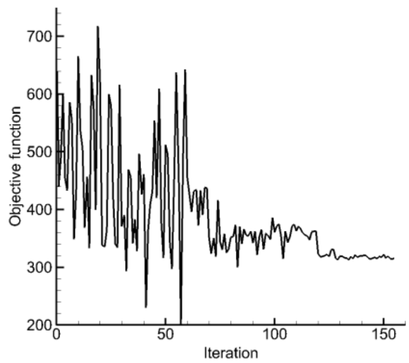

Figure 9: The convergence history

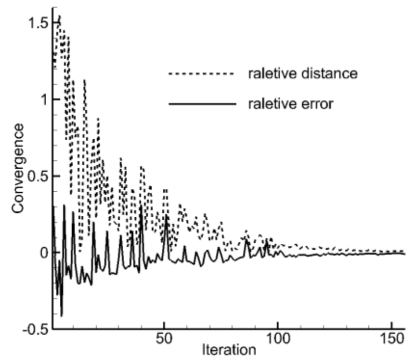

Figure 10: The iteration history of the objective function

4 CONCLUSIONS

This research has presented a parallel sequential approximate optimization (PSAO) algorithm that is suitable for structural design optimization tasks. This approach intends to reduce the computational costs normally associated with structural design problems.

We introduce a parallel sampling strategy that balances exploration and exploitation, allowing high-efficiency searching of the global optimum during the optimization process. The parallel sampling strategy substantially reduces the computation costs required to find the optimal solutions. The feasibility, convenience and efficacy of the proposed structural design optimization algorithm have been investigated through two cased studies. Furthermore, the effectiveness and computational efficiency of the PSAO methodology are benchmarked against other optimization approaches as well. The time consumed to find the optimum is reduced from the order of 10^3 to 10^2 by the PSAO. Accordingly, the proposed PSAO proves to be an adequate strategy for effectively and efficiently handling structural design optimization problems.

REFERENCES

[1] Yildiz, A.R. Comparison of evolutionary-based optimization algorithms for structural design optimization. *Eng. Appl. Artif. Intell* (2013) **26**: 327-333.

[2] Bonte, M.H.A., Boogaard, A.H., and Huétink J. An optimization strategy for industrial metal forming processes. S*truct. Multidiscip. Optim* (2008) **35**: 571-586.

[3] Edke, M.S. and Chang, K.H. Shape optimization of heavy load carrying components for structural performance and manufacturing cost. *Struct. Multidiscip. Optim* (2006) **31**: 344-354.

[4] Dai, L., Guan, Z.Q., Chen, B.S. and Zhang, H.W. An open platform of shape design optimization for shell structure. *Struct. Multidiscip. Optim* (2008) **35**: 609-622.

[5] Goldberg, D. *Genetic algorithms in search, optimization, and machine learning*. New York, NY: Addison-Wesley, (1989).

[6] Kirkpatrick, S., Gelatt, C., and Vecchi, M. Optimization by simulated annealing. *Science* (1983) **220**: 671-680

[7] Kennedy, J and Eberhart, R. *Particle swarm optimization*. Piscataway: IEEE international conference on neural networks, (1995)..

[8] Karaboga, D and Basturk, B. A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm. *J. Global. Optim* (2003) **39**: 459-471.

[9] Wang, D., Wu, Z., Fei, Y. and Zhang, W. Structural design employing a sequential approximation optimization approach. *Comput. Struct* (2014) **134**: 75-87.

[10] Barthelemy, J.F.M. and Haftka, R.T. Approximation concepts for optimum structural design-a review. *Struct. Optim* (1993) **5**: 129-144.

[11] Haftka, R.T and Scott, E.P. Optimization and Experiments-A Survey. *Theor. Appl. Mech* (1996) **25**: 303-321.

[12] Nikolaidis, E., Long, L. and Ling, Q. *Neural Networks and Response surface polynomials for Design of Vehicle Joints*. AIAA-98-1777, (1998).

[13] Berke, L. and Hajela, P. Application of artificial neural nets in structural mechanics, *Struct. Optim* (1992) **4**: 90-98.

[14] Giunta, A., Watson, L.T. and Koehler, J. *A comparison of approximation modeling techniques: polynomial versus interpolating models*. St.Louis: 7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis & Optimization, (1998).

[15] Simpson, T.W., Mauery, T.M., Korte, J.J. and Mistree, F. *Comparison of response surface and Kriging models for multidisciplinary design optimization.* St. Louis: 7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis & Optimization, (1998).

[16] Broomhead, D.S. and Loewe, D. Multivariate functional interpolation and adaptive networks. *Adv. Complex. Syst* (1988) **2**: 321-355.

[17] Bonte, M.H.A. and Fourmentm, L. Optimization of forging processes using finite element simulations - a comparison of sequential approximate optimization and other algorithm. *Struct. Multidiscip. Optim* (2010) **42**: 797-810.
[18] Cheng, G., Xu, L. and Jiang. L. A sequential approximate programming strategy for reliability-based structural optimization. *Comput. Struct* (2006) **84**: 1353-1367.
[19] Kitayama, S., Arakawa, M. and Yamazaki, K. Sequential approximate optimization using radial basis function network for engineering optimization. *Optim. Eng* (2011) **12**: 535-557.
[20] Forrester, A.I.J and Keane, A.J. Recent advances in surrogate-based optimization. *Prog. Aerosp. Sci* (2009) **45**: 50-79.
[21] Hastie, T., Tibshirani, R. and Friedman, J. *The elements of statistical learning*. Springer, Vol. I, (2001).
[22] Jones, D.R. A taxonomy of global optimization methods based on response surfaces. *J. Glob. Optim* (2001) **21**: 345-383.
[23] Wang, D., Hu, F., Ma, Z., Wu, Z. and Zhang, W. A CAD/CAE integrated framework for structural design optimization using sequential approximation optimization. *Adv. Eng. Softw* (2014) **44**:7656-7668.
[24] Perez, R. and Behdinan, K. Particle swarm approach for structural design. *Comput. Struct* (2007) **85**:1579-1588.
[25] Camp, C. and Bichon, J. Design of space trusses using ant colony optimization. *J. Struct. Eng* (2004) **130**: 741-751.