V International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2013
S. Idelsohn, M. Papadrakakis and B. Schrefler (Eds)

DETERMINATION OF 3D WIND INDUCED VIBRATION OF CABLES FOR CABLE–STAYED BRIDGES

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Key words: Cable–stayed bridge, wind–structure interaction, Vibration, Aeroelastic, Resonance.

Abstract. This paper is focused in the analysis of cable stays for Viaducto Zaragoza Bridge located in the city of Puebla, Mexico. Computational fluid dynamics based on finite element method is used to simulate wind forces acting on the cable stays, which are coupled to a structural model of each model analyzed for two wind velocities, associated with serviceability limit state (50 km/hr or 14 m/s) and maximum wind velocity (140 km/hr or 14 m/s). Thus, a stabilized fluid flow formulation is presented to solve an ALE fluid flow while a geometrically non–linear solid elements are used to model the cable stays. Both solutions are coupled using a strong coupling technique to perform an aeroelastic analysis of the cable stays. The results suggest that transversal vibration to wind action can generate undesirable vibrations conditions as the resonance phenomenon in cable stays.

1 INTRODUCTION

Cable–stayed bridges are a highly nonlinear structural system where superstructure deck is supported on several points of its length by cables anchored directly to a support column. Nowadays different cable geometries are commonly used to transmit loads between deck and the pylon (or support column). The most typical cable geometries are shown in Figure 1.

The concept of cable–stayed bridges can be traced to the XVII century [1], the economic viability of this kind of bridges had to wait the development of two aspects: (a) Decks constructed of steel and concrete, and (b) High resistance steel cables.

While the materials used in structural components of these structures are under linear–elastic behaviour for normal operation conditions, the overall performance of the entire structure is often highly nonlinear due to: (i) Nonlinear behaviour of cable stays for axial load–strain relationship caused by its self–weight; (ii) Nonlinear relationship for elements under axial load – bending; and (iii) Large displacements in the structure even for normal operation loads.

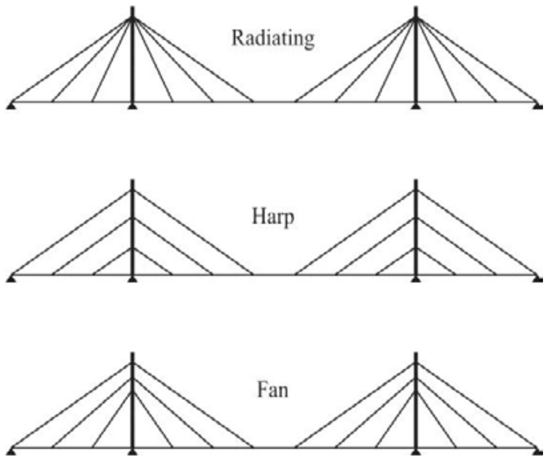


Figure 1 Typical geometries for cable–stayed bridges

1.1 General Bridge Description

The geometry employed in the Viaducto Zaragoza Bridge requires a system of inclined and horizontal cables to support the superstructure deck to the inclined pylon or mast as can be shown in Figure 2. Table 1 shows normal operating conditions for each cable stay of the bridge

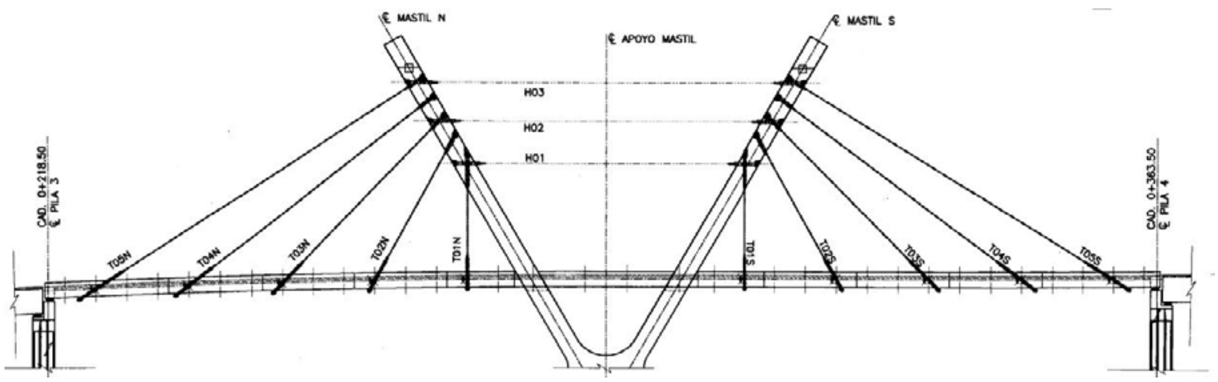


Figure 2 Elevation view of Viaducto Zaragoza Bridge and designation of cable-stays

Wind actions are estimated according local code requirements specified in [2]. According the characteristics of the project, cable stays analyses must be performed for two wind velocities:

1) An average wind velocity (V_{med} = 14 m/s) representing the average operating conditions of the bridge
2) Maximum probable wind velocity ($V_{\max}$ = 40 m/s) representing the maximum wind speed expected for the operations of the bridge.

Table 1 Cable stays characteristics for normal operation conditions

Designation	Length (m)	Strands	Weight (kN)	Tensioning Force (MN)
T01N	17.732	28	12.66	784.5
T01S	17.639	28	12.59	784.5
T02N	23.440	48	28.68	431.5
T02S	23.205	48	28.40	451.1
T03N	32.331	48	39.57	725.6
T03S	31.934	48	39.08	745.3
T04N	42.518	48	52.03	1078.7
T04S	41.854	48	51.22	1304.2
T05N	53.072	34	46.01	1588.6
T05S	52.214	34	45.26	1588.6
H01	39.049	39	77.65	1314.0
H02	45.362	39	90.21	990.4
H03	51.145	39	101.71	2510.3

2 FINITE ELEMENT ANALYSIS OF THE WIND ACTION

An incompressible fluid formulation has been used to simulate wind action due to fluid velocity is lower than 0.3 Mach. Navier–Stokes equation are used to model flow as shown in Eq. (1).

$$\begin{aligned} \mathbf{M}\dot{\mathbf{v}}_{n+1} + \mathbf{K}\mathbf{v}_{n+1} - \mathbf{G}\mathbf{p}_{n+1} &= \mathbf{f}_{n+1}^{\mathrm{ext}} \\ \mathbf{G}^T\mathbf{v}_{n+1} &= 0 \end{aligned} \tag{1}$$

where:

$\mathbf{v}$ = Velocity

$\mathbf{p}$ = Pressure field

$\dot{\mathbf{v}}$ = Acceleration

$\mathbf{M}$ = Mass matrix

$\mathbf{K}$ = Matrix with convective and viscous terms

$\mathbf{G}$ = Matrix to include pressure terms or to consider a compressible flow

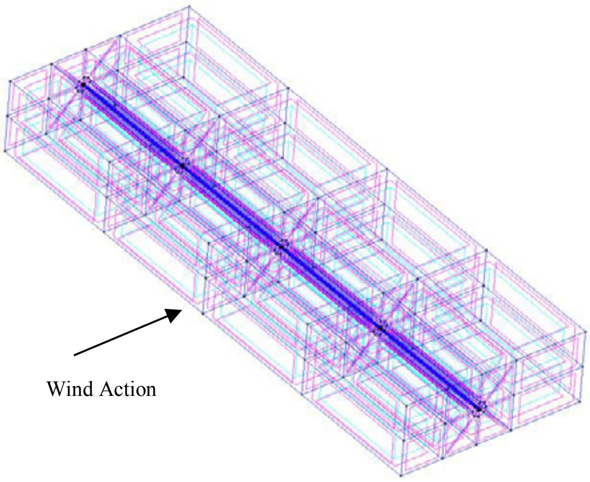
For dynamic fluid flow analysis Eq. (1) is rewritten by Gunzburger [3] as shows in Eq. (2). To perform a faster calculation, Eq. (2) is decoupled using fractional step method proposed by Codina [4] and considering the equation as complete Eulerian formulation, expression is transformed using and ALE formulation, as can be found in Belytschko *et al.* [5] to take in count the structure movement in the domain of analysis.

$$\begin{aligned} \left(\boldsymbol{v}_h^{n+1}, \boldsymbol{w}_h\right) + c\left(\boldsymbol{v}_h^{n+1}, \boldsymbol{v}_h^{n+1}, \boldsymbol{w}_h\right) - b\left(p^{n+1}, \boldsymbol{w}_h\right) + a\left(\boldsymbol{v}_h^{n+1}, \boldsymbol{w}_h\right) &= \left(\boldsymbol{b}_h^{n+1}, \boldsymbol{w}_h\right) \\ b\left(q_h, \boldsymbol{v}_h^{n+1}\right) &= 0 \end{aligned} \tag{2}$$

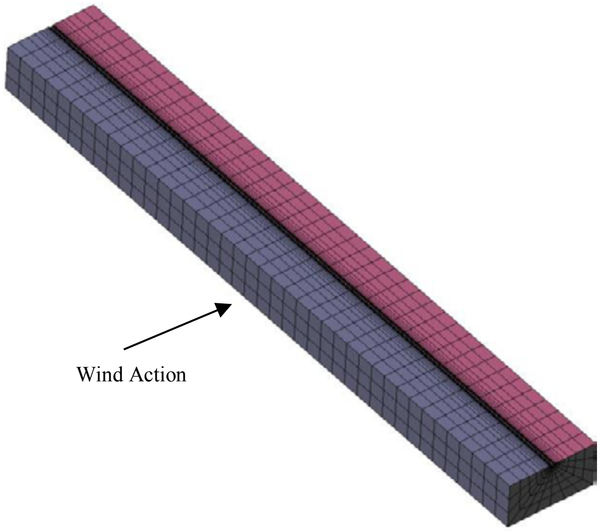
Decoupled and stabilized equations are expressed as can be shown in Eq.(3), which are formulated in four implicit steps for each time increment. The first step is to solve the system at an intermediate velocity, which is a nonlinear formulation. The final pressure is computed in the second step. In the third step the final velocity is calculated and the complete system is stabilized in the fourth step.

$$
\begin{aligned}
\left(\dot{\bar{\boldsymbol{u}}}_h^{n+\alpha_m^{\mathrm{f}}},\boldsymbol{w}_h\right)+c\left(\bar{\boldsymbol{c}}_h^{n+\alpha_f^{\mathrm{f}}},\bar{\boldsymbol{v}}_h^{n+\alpha_f^{\mathrm{f}}},\boldsymbol{w}_h\right)-b\left(p_h^n,\boldsymbol{w}_h\right)+a\left(\bar{\boldsymbol{v}}_h^{n+\alpha_f^{\mathrm{f}}},\boldsymbol{w}_h\right)+\tau\left(\bar{\boldsymbol{c}}_h^{n+\alpha_f^{\mathrm{f}}}\cdot\nabla\bar{\boldsymbol{v}}_h^{n+\alpha_f^{\mathrm{f}}}+\nabla p_h^n-\pi_h^n,\bar{\boldsymbol{c}}_h^{n+\alpha_f^{\mathrm{f}}}\cdot\nabla\boldsymbol{w}_h\right)&=\left(\boldsymbol{b}_h^{n+1},\boldsymbol{w}_h\right)\\
-\frac{\Delta t\,\gamma^{\mathrm{f}}}{\alpha_m^{\mathrm{f}}}\left(\nabla\left[p_h^{n+1}-p_h^n\right],\nabla q_h\right)-\tau\left(\bar{\boldsymbol{c}}_h^{n+\alpha_f^{\mathrm{f}}}\cdot\nabla\bar{\boldsymbol{v}}_h^{n+\alpha_f^{\mathrm{f}}}+\nabla p_h^{n+1}-\pi_h^n,\nabla q_h\right)&=b\left(q_h,\bar{\boldsymbol{v}}_h^{n+1}\right)\\
\frac{\alpha_m^{\mathrm{f}}}{\Delta t\,\gamma^{\mathrm{f}}}\left(v_h^{n+1}-\bar{\boldsymbol{v}}_h^{n+1},w_h\right)-b\left(p_h^{n+1}-p_h^n,\boldsymbol{w}_h\right)&=0\\
\left(\pi_h^{n+1},\eta_h\right)-\left(\bar{\boldsymbol{c}}_h^{n+\alpha_f^{\mathrm{f}}}\cdot\nabla\bar{\boldsymbol{v}}_h^{n+\alpha_f^{\mathrm{f}}}+\nabla p_h^{n+1},\eta_h\right)&=0
\end{aligned}
\tag{3}
$$

Figure 3 shows a mesh used to model wind for the analysis of cable H01. Figure 3(b) shows that finite elements near to the contour are smaller to model the boundary layer.



(a) General View



(b) Finite Elements detail

Figure 3 Isometric view of mesh to model wind

3 FINITE ELEMENT ANALYSIS OF THE CABLE STAYS

To estimate the behavior of the cable stays a geometrically nonlinear model of solid is used. The properties of solids for each cable stay is estimated from the real properties showed in Table 1, considering the accessories elements for strands protection (as shown if Figure 4) only provide mass, thus, the cable stays are modeled with solids with the same equivalent properties to all elements.

The expression that describes the behavior of cable stays are obtained from the equation of lineal momentum, and discretized using FEM. The computations consist of solve the following expression

$$
\mathbf{f}^{\mathrm{int}}\left(\mathbf{u}_{n+1}\right)+\mathbf{M}\ddot{\mathbf{u}}_{n+1}=\mathbf{f}^{\mathrm{ext}}\left(\mathbf{u}_{n+1}\right)
\tag{4}
$$

where:

$\mathbf{f}^{\mathrm{int}}$ = Internal forces

$\mathbf{f}^{ext}$ = External forces
$\mathbf{M}$ = Mass matrix
$\mathbf{u}$ = Displacements
$\ddot{\mathbf{u}}$ = Accelerations

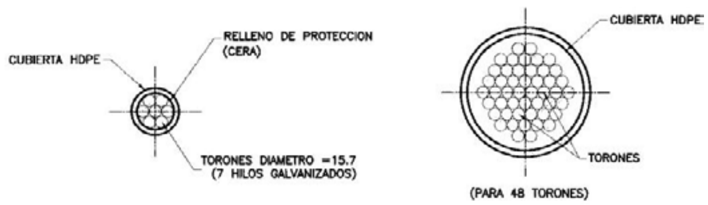


Figure 4 Transversal detail of cable stays

To solve the dynamics of cable stays in time *Generalized*$-\alpha$ method is used to integrate Eq. (4) due to other traditional methods like β*–Newmark* or θ*–Wilson* produce inconsistent results with nonlinear finite element formulation proposed.

One important issue of this work is the modeling of the cable stays tension for the elements. Considering that the spatial position and tension force for each cable stay are known, the one–dimensional stress is transformed in a tensional stress in a 3D solid according to the reference system used for analysis. Thereafter, internal forces due to tension stress are added to the ordinary 3D stress tensor.

Furthermore, to reduce the memory required to solve the equation system, meshes for each cable stay is always horizontal or vertical to coincide with the global reference system. This consideration helps to the development of mesh analysis but leads the problem of cable self–weight. To solve this problem gravity is considered as a unit vector that can be oriented in any direction making the cables have the correct deformation profile according its real position in the bridge.

4 WIND–CABLE STAY INTERACTION ANALYSIS

The cable stay and the fluid are solved in a domain containing both models, as can be shown in Figure 3, solving both problems in a coupled way, as occur in real world.

To solve both systems, a partitioned approach is employed, *i.e.*, for each time step behavior of cable stay and fluid are computed independently using Aitken schemes (Wüncher, [6]) to ensure convergence of both coupled systems. This methodology has been employed by Valdés [7] given excellent results for aeroelastic problems of several structures, such as those studied by Valdés *et al.* [8] and Hernández and Valdés [9].

The procedure for the solution of wind–cable stay interaction is as follows:

1. Solve the cable stay to predict the displacements according the acting external forces. Initial step consider tension force and gravity only.
2. Obtained displacements of cable–stays are passed to the fluid mesh, adjusting the mesh to match with deformed profile of the cable.
3. Wind dynamics is solved to estimate acting forces on cable–stay surface.
4. Updating fluid forces acting on cable–stay surface.

This procedure is repeated until convergence criteria is reached for each time step. Detailed information for above procedure can be found in [7].

5 OBTAINED RESULTS

Figure 5 shows pressure distribution for certain time step al half-length of cable stay T03N. Both figures shown suction due to vortices generation that induces transversal vibration on cable stays respect to wind action.

(a) Average velocity (14 m/s) (b) Maximum velocity (40 m/s)

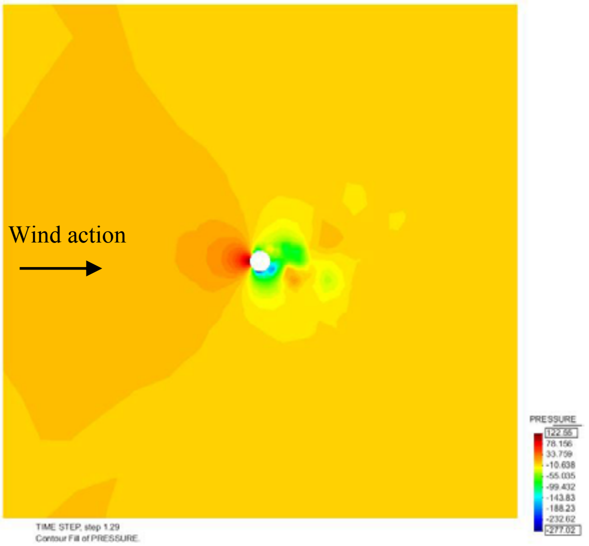


Figure 5 Transversal views for pressure distribution at half length of cable stay T03N

Vortices are not statically and "move" through the cable length, as can be seen in Figure 6 which shows pressure distribution in a cross section along the entire length of cable stay T03N for both analysis conditions.

(a) Average velocity (14 m/s) (b) Maximum velocity (40 m/s)

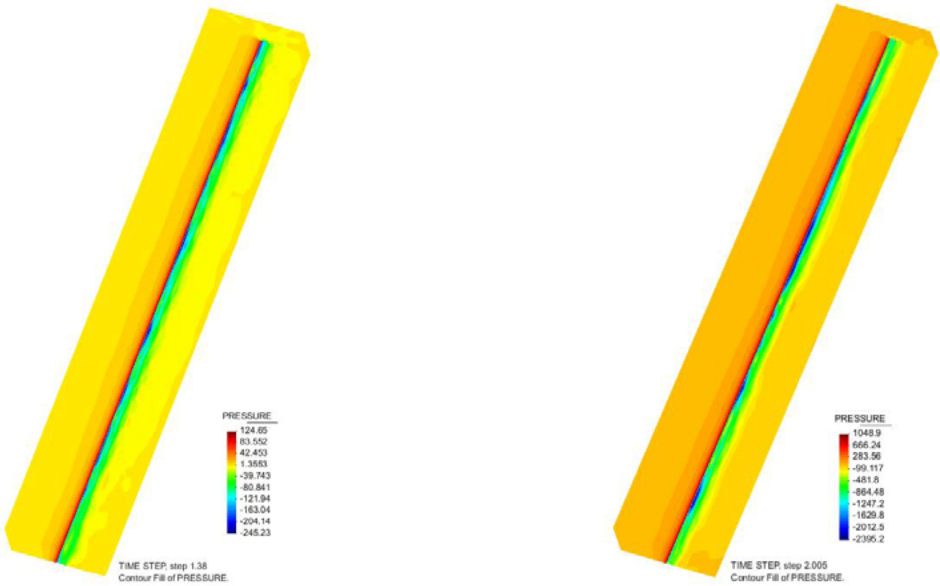


Figure 6 Longitudinal views of pressure distribution for cable stay T03N

Variation of the acting forces on the cable length in conjunction with operational conditions for each case cause the vibration of cable stay with certain profile deformation and vibrational frequency.

As can be noted in Figure 7 the displacements in transversal direction to wind action dominate the total displacement at this position, making evident that the cable has a typical resonant behavior. Furthermore, the displacements shown in Figure 8 suggest that transverse displacements do not dominate the total displacements, however, is evident that transversal displacements increases with time, but their influence on the total displacements is minimal. It is noteworthy that in the presented case, there is a greater potential that resonance phenomena occurs at wind velocity less than the maximum analyzed, which is more probably to occur in bridge life.

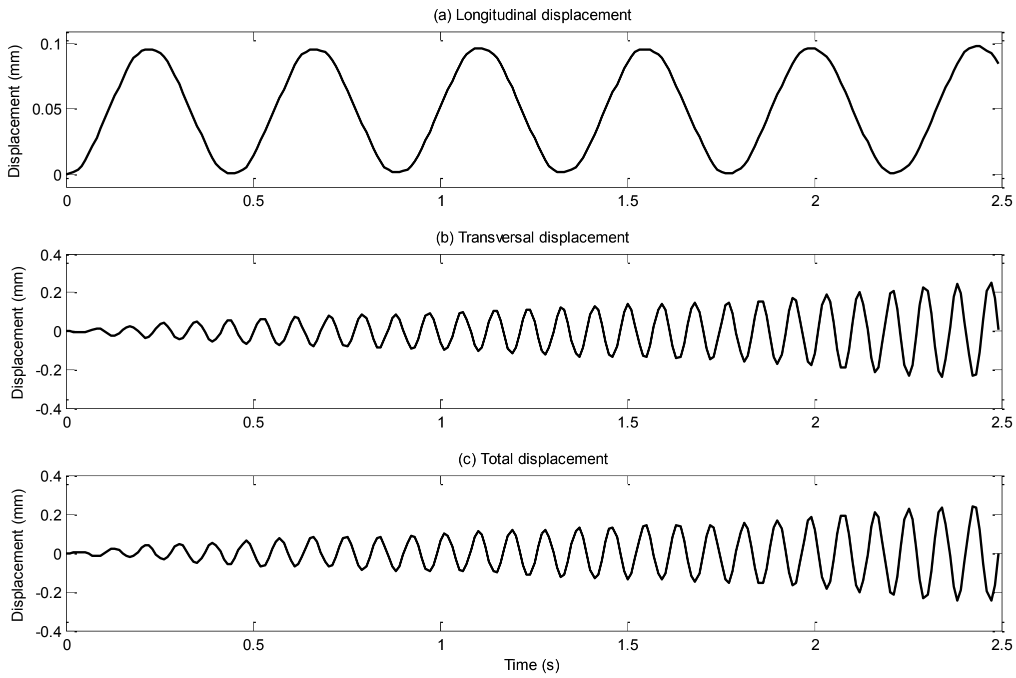


Figure 7 Displacements history at half length of T03N cable for average velocity (14 m/s)

Obtained results suggest that the most critical conditions for cable stays are transversal vibration respect to wind action, and is not necessary have a high wind speeds to induce resonance in cable stays, which may lead in fatigue on the strands or their anchorage systems.

6 CONCLUSIONS

To determinate displacements and forces acting on a cable–stayed bridge, traditional analysis methods are not recommended to use due to highly geometrically nonlinear behavior of this kind of structures. In order to improve the functionality and security for cases like above mentioned, techniques that can predict the behavior more accurately are needed. The presented FEM application permits to visualize vibrations characteristics for cable stays and determinate if they are susceptible to undesirable frequencies, as those associated with resonance phenomena.

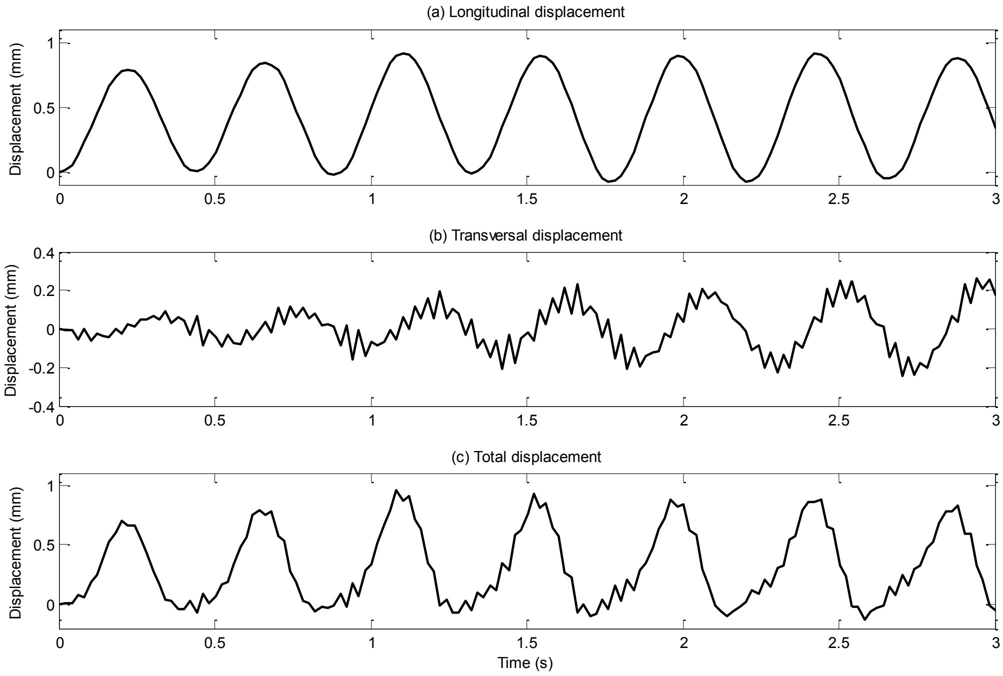


Figure 8 Displacements history at half length of T03N cable for maximum wind velocity (40 m/s)

Obtained results suggest that transversal vibration respect to wind action is very important to determinate safety operation condition of the cable stays, currently most of methodologies which allow the determination vibration periods only can predict frequencies in the direction of wind action, due to the models are considered in 2D (Au *et al.* [10], Starossek [11] among others) making impossible to determinate the vibrations characteristics in transverse direction to wind action.

For presented case of Viaducto Zaragoza Bridge, resonance phenomena is identified for some cable stays, viewing the need to change their operations conditions by adding frictional dampers at deck anchorages. These devices modify the natural vibration frequencies of the cables due to added mas of the device, and reduce the displacements by the damper, decreasing the probability of occurrence of resonance.

7 ACKNOWLEDGMENTS

The authors acknowledge the support the University of Guanajuato for the work development presented in this paper.

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