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ON THE NECESSITY AND A GENERALIZED CONCEPTUAL MODEL FOR THE CONSIDERATION OF LARGE STRAINS IN ROCK MECHANICS

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Abstract. This contribution presents a generalized conceptual model for the finite element solution of quasi-static isothermal hydro-mechanical processes in (fractured) porous media at large strains. A frequently used averaging procedure, known as Theory of Porous Media, serves as background for the complex multifield approach presented here. Within this context, a consistent representation of the weak formulation of the governing equations (i.e., overall balance equations for mass and momentum) in the reference configuration of the solid skeleton is preferred. The time discretization and the linearization are performed for the individual variables and nonlinear functions representing the integrands of the weak formulation instead of applying these conceptual steps to the overall nonlinear system of weighted residuals. Constitutive equations for the solid phase deformation are based on the multiplicative split of the deformation gradient allowing the adaptation of existing approaches for technical materials and biological tissues to rock materials in order to describe various inelastic effects, growth and remodeling in a thermodynamically consistent manner. The presented models will be a feature of the next version of the scientific open-source finite element code OpenGeoSys developed by an international developer and user group, and coordinated by the authors.

1 INTRODUCTION

Certain rock materials, like rock salt and claystone, play an important role in geotechnical applications (e.g., energy storage), and are characterized by a highly complex material behavior. Irreversible deformations, rate dependent stress-strain effects as well as creep, swelling and shrinking are observed for such materials under realistic load regimes.

Constitutive relations for the mathematical modeling of the mechanical deformation behavior of rock materials or their substitutes (e.g., bentonite) have been developed for decades. Usually, they are phenomenologically, i.e., macroscopically based. Nevertheless, corresponding constitutive models that are known from literature have certain conceptual limitations. Rarely, they consider couplings to thermal and/or hydraulic processes. Additionally, in general such models are formulated as scalar equations to consider specific deformation processes, for instance uniaxial or principal axes states, which makes their transformation to the tensorial description of generalized, three-dimensional stress-strain states difficult. Usually, specific functions for the modeling of particular constitutive effects are defined in a more heuristic manner based on the results of simplified lab experiments. Thus, the compliance with the requirements of the axioms of rational thermodynamics (e.g., thermodynamic consistency, i.e., the a priori compliance with the fundamental theorems of thermodynamics for arbitrary stress-strain states) that are well-approved in the continuum mechanics of materials is not verified. In addition, only a few authors discuss large strain models for rock materials. However, in particular within the context of the above mentioned swelling effects in reality strains of more than 40% are measured for claystone [1], which cannot be numerically simulated in a physically useful manner based on small strain approaches. Consequently, the aforementioned limitations in constitutive modeling of rock materials result in uncertainties of the transformation of most of the existing models from simplified to more complex stress-strain states.

The theoretical framework of large strain mechanics is very well established in the literature (cf. [2, 3] and references given therein). Starting in the 50s of the previous century the development of large strain models for technical materials such as elastomers improved the accuracy of numerical analyses of the mechanical behavior of corresponding components substantially [4]. Based on experimentally observed similarities in the stress response of mechanically loaded biological soft tissues and certain technical materials, in the 80s and 90s large strain models have increasingly been discussed in biomechanics and subsequently extended to biphasic materials such as articular cartilage (cf. [5] and other authors). Currently, large strain models are state-of-the-art in biomechanics. For more realistic results in the numerical prediction of consolidation processes, large strain models have been introduced in soil mechanics starting from the late 90s of the previous century [6]. Currently, large strain models are discussed in a huge amount of publications addressing deformation processes as well as their couplings with other physical effects (e.g., thermal, hydraulic) for technical materials and biological tissues.

Although various experiments for rock materials show comparable large strain effects, hardly any corresponding model is known from the rock mechanics literature. However, well-discussed phenomena such as creep of rock salt and swelling of claystone indicate the necessity of considering large strain approaches in order to avoid physically inappropriate results of numerical simulations. In this study, we present a generalized numerical model for the finite element solution of quasi-static coupled processes in (fractured) porous media at large strains. Without loss of generality, for simplicity of the representation the presented model is restricted to isothermal hydro-mechanical (HM) processes in fully saturated biphasic materials neglecting mass production of the constituents as well as mass transfer between them. The consideration of non-isothermal, multiphase-multicomponent and/or partly saturated effects follows straightforwardly from the presented procedure.

In the following, tensors will be denoted by bold-faced characters in direct notation. Their juxtaposition implies the scalar product of two vectors (e. g., $\boldsymbol{a}\,\boldsymbol{b} = a_i\, b^i$), or a single contraction of adjacent indices of two tensors, while double dots indicate a double contraction of adjacent indices of tensors of rank two and higher (e. g., $\boldsymbol{a} \cdot\cdot\, \boldsymbol{b} = a_i{}^j\, b_j{}^i$). A superposed dot indicates the material rate of a tensor, a superscript $(.)^{\mathrm{T}}$ the transposed tensor. Tensors belonging to the reference configuration of the solid skeleton are denoted by capital letters (additionally labeled by the subscript $(.)_S$), tensors in the current configuration by small letters. The subscripts $(.)_S$ and $(.)_F$ indicate variables corresponding to the solid skeleton and the pore fluid, respectively.

2 CONCEPTUAL MODELING

The model, which is discussed in this paper, is mainly based on the so-called *Theory of Porous Media* (TPM). In brief, the TPM is a combination of the physically based mixture theory (see [7]) with the concept of volume fractions (cf. [8–10] and others). Within the context of this enhanced approach of the mixture theory all kinematical and physical quantities can be interpreted on the macro scale as local statistical averages of the corresponding values of the underlying microstructure.

A comprehensive overview of the history and the current state of the TPM is given, for instance, by [11]. The development of material-independent basic principles (kinematics of transport and deformation, balance relations) to model the behavior of fully and partially saturated porous continua within the context of the TPM, and the formulation of appropriate numerical schemes based on standard Galerkin procedures are discussed in detail by [11–13] and [14] (see also the huge number of references therein).

For the first time, [15] presented a model for large elastoplastic solid skeleton deformations within the context of hydro-mechanical porous media behavior based on the multiplicative split of the deformation gradient. More recently mixed large strain formulations for porous media mechanics are discussed by [6] (elastic solid skeleton), [16] and [17] (elastoplasticity in case of partially saturated models), [18] (dynamic hyperelastic model) and [19] (dynamic elastoplastic approach). While these papers are mainly dedicated to applications in soil mechanics, [5, 20–22] and many other authors present various large strain porous media models adopted to biomechanical problems.

The conceptual basics, numerical aspects and examples of application of the TPM under large strain conditions have been studied by many authors (only a very short overview could be given here), but few of the previous works analyzed a consistent representation of the weak formulation of the governing equations in the reference configuration of the solid skeleton. This description is preferred here, and serves as the foundation of a generalized material approach, the details of which are discussed in [23].

2.1 Preliminary Remarks

Within the context of the TPM, all constituents of the porous medium are understood as *smeared* substitute continua with reduced mass density. Consequently, the porous medium is considered as a substitute continuum model, which is constituted by overlapping homogenized partial continua, and which is able to characterize physical processes in heterogeneously structured materials using the well-known assumptions and thermodynamically based approaches of continuum mechanics.

As usual in the context of the concept of volume fractions, the pore structure as well as the pore distribution are described in a statistically averaged sense using scalar variables representing the fraction of the partial volume of the constituent with respect to the overall volume $d\Omega_0$ in the reference state of a representative elementary volume of the control space. In case of biphasic porous media the volume fractions ϕ_{S0} for the solid skeleton and ϕ_{F0} for the pore fluid (i.e., the porosity) at time $t = t_0$ are defined as follows:

$$\phi_{S0} = d\Omega_{S0} \,/\, d\Omega_0 \,, \qquad \phi_{F0} = d\Omega_{F0} \,/\, d\Omega_0 \tag{1}$$

with the partial volume $d\Omega_{S0}$ of the solid skeleton and the partial volume $d\Omega_{F0}$ of the pore fluid. The saturation condition

$$d\Omega_0 = d\Omega_{S0} + d\Omega_{F0} \qquad \Rightarrow \qquad \phi_{S0} + \phi_{F0} = 1 \tag{2}$$

which is assumed to be fulfilled at each time t, represents a constraint condition.

Considering porous media constituents, two different definitions of their mass density are given. For the *effective* (aka *realistic*) density the differential elements of mass dm_{S0} and dm_{F0} of the constituents are related to the partial elementary volumes.

$$\varrho_{SR0} = dm_{S0} \,/\, d\Omega_{S0} \,, \qquad \varrho_{FR0} = dm_{F0} \,/\, d\Omega_{F0} \tag{3}$$

In contrast, the *partial* (aka *global*) mass density of the constituents is related to the elementary volume of the overall continuum.

$$\varrho_{S0} = dm_{S0} \,/\, d\Omega_0 = \phi_{S0}\, \varrho_{SR0} \,, \qquad \varrho_{F0} = dm_{F0} \,/\, d\Omega_0 = \phi_{F0}\, \varrho_{FR0} \tag{4}$$

The averaged density of the (homogenized) overall porous structure is defined as

$$\varrho_0 = \varrho_{S0} + \varrho_{F0} \tag{5}$$

2.2 Kinematics of Transport and Deformation

Below, the description of the kinematics of a multiphase medium is based on two fundamental assumptions

1. At the current time t, each particle located at the position $\boldsymbol{x}$ of the mapping of the real body into the physical space simultaneously consists of material points of all of the partial constituents, and
2. all constituents are characterized by an individual, independent motion process (i.e., transport, deformation) of their material points.

The reference configuration of the porous body is identical to the reference configuration of the solid skeleton, and represents a set $\Omega_0 \subset \mathbb{R}^3$ of material points with the boundary Γ_0 (i. e., an area within the three-dimensional Euclidian space $\mathbb{E}^3$). The material points of the solid skeleton are uniquely defined by their position vectors $\boldsymbol{X}_S \in \Omega_0$. Regarding the individual motion of the considered constituents, material points of the solid skeleton and the pore fluid, both belonging to $\boldsymbol{x}$ at the current time t, were located at different positions in the reference configuration. With the individual laws of motion

$$\boldsymbol{x} \;=\; \boldsymbol{\varphi}_S(\boldsymbol{X}_S, t)\,, \qquad \boldsymbol{x} \;=\; \boldsymbol{\varphi}_F(\boldsymbol{X}_F, t) \tag{6}$$

unique relations between the current position of material points of the constituents in $\mathbb{E}^3$ at any time t, and their assignment to the reference state are given.

Within the context of TPM applications, usually a Lagrangian description is used for the kinematics of the solid skeleton. For physical correctness, the fluid flow as motion relative to the motion of the solid skeleton is originally referred to the current configuration, which is actually a description of Eulerian nature. In order to smooth out some shortcomings of an inconsistent formulation of the motion of individual constituents, [24–26] and others proposed a so-called *generalized material description* of the balance relations of the TPM considering the reference configuration of the solid skeleton as reference configuration of the overall continuum.

The displacement vector for material points of the solid skeleton is the primary kinematical variable of the TPM. Using the motion law (6_1), the displacement vector can be represented as a function of the coordinates of the reference configuration and the time.

$$\boldsymbol{u}_S = \boldsymbol{u}_S(\boldsymbol{x}, t) = \boldsymbol{u}_S(\boldsymbol{\varphi}_S(\boldsymbol{X}_S, t), t) = \boldsymbol{U}_S(\boldsymbol{X}_S, t) = \boldsymbol{x}(\boldsymbol{X}_S, t) \;-\; \boldsymbol{X}_S \tag{7}$$

Kinematical reflections regarding the balance relations and constitutive models at large strains are usually based on the deformation gradient

$$\boldsymbol{F}_S \;=\; (\mathrm{Grad}_S\, \boldsymbol{x})^{\mathrm{T}} \;=\; (\mathrm{Grad}_S\, \boldsymbol{U}_S)^{\mathrm{T}} \;+\; \boldsymbol{I} \tag{8}$$

providing the mapping of material line elements of the solid skeleton from the reference into the current configurations. The determinant J_S of the deformation gradient

$$J_S \;=\; \det \boldsymbol{F}_S \;=\; d\Omega \,/\, d\Omega_0 \tag{9}$$

represents the volume ratio of the solid skeleton smeared over the current configuration with respect to the reference configuration. Based on the deformation gradient, different strain measures can be defined. Within the context of the generalized material description the right Cauchy-Green tensor $\boldsymbol{C}_S = \boldsymbol{F}_S^{\mathrm{T}} \boldsymbol{F}_S$ and Green's strain tensor $2\boldsymbol{E}_S = \boldsymbol{C}_S - \boldsymbol{I}$ are of particular interest.

2.3 Effective Stress Concept

Based on experimental observations on saturated soils [27] introduced the concept of effective stresses, in order to calculate the overall stress state in porous media. This heuristic principle implies the decomposition of the stress state at any spatial point of the current configuration into partial stresses. Characterizing the interaction between the moving pore fluid and the deforming solid skeleton, one of the partial stresses acts equally in material points of the pore fluid and the solid skeleton that occupy the same location in the current configuration. This partial stress can be represented using a second order isotropic tensor (hydrostatic state), whose coefficient is known as pore pressure p. Consequently, the overall stress state has to be determined by additional partial stresses, which are caused by the history of the fluid transport as well as the deformation of the solid skeleton themselves. These partial stresses are called effective stresses.

The generalized material description of the partial stresses for the solid skeleton and the pore fluid of a saturated biphasic porous medium in terms of 2nd Piola-Kirchhoff partial stress tensors $\boldsymbol{T}_S$ and $\boldsymbol{T}_F$ follows from the corresponding spatial description performing usual pull-back operations.

$$\boldsymbol{T}_S = -J_S\,\phi_S\,p\,\boldsymbol{C}_S^{-1} + \boldsymbol{T}_S^E \tag{10a}$$

$$\boldsymbol{T}_F = -J_S\,\phi_F\,p\,\boldsymbol{C}_S^{-1} + \boldsymbol{T}_F^E \tag{10b}$$

Here, $\boldsymbol{T}_S^E$ and $\boldsymbol{T}_F^E$ denote the effective stress tensors for the solid skeleton and the pore fluid, respectively. Neglecting internal friction forces of the pore fluid, and considering the saturation condition (2), the 2nd Piola-Kirchhoff overall stress tensor $\boldsymbol{T}$ is defined as follows:

$$\boldsymbol{T} = \boldsymbol{T}_S^E - p\,\boldsymbol{S}_\mathrm{v} = \boldsymbol{T}_S^E - \boldsymbol{T}_\mathrm{v} \quad \text{with} \quad \boldsymbol{S}_\mathrm{v} \overset{\text{def}}{=} J_S\boldsymbol{C}_S^{-1} \tag{11}$$

2.4 Constitutive Models for the Solid Skeleton

The effective stress tensor $\boldsymbol{T}_S^E$ is characterized by the deformation of the solid skeleton as well as non-mechanical processes (e. g., thermal and/or (electro-)chemical effects in geo- and biomechanics). Preferring phenomenological, macroscopic constitutive concepts, this causes certain additive decompositions of the effective stress tensor. As known from respective approaches in solid mechanics, appropriate evolutional relations for partial stresses can be thermodynamically consistently formulated considering the classical axioms of material theory (cf. [2, 28]).

In particular, the formulation of thermodynamically consistent constitutive relations is based on the conceptual analysis of the combination of the first and second laws of thermodynamics, which is frequently called the *Clausius-Duhem inequality*. Based on corresponding definitions of the energy and entropy balances for the individual constituents, the Clausius-Duhem inequality for the considered porous media problem can be represented in generalized material description as follows:

$$\frac{1}{2}\boldsymbol{T}_S^E \cdot\cdot\, \dot{\boldsymbol{C}}_S - \varrho_{S0}\,\dot{\tilde{\psi}}_S \geq 0 \tag{12}$$

Defining relation (12) certain assumptions on the relationships between partial balance laws of the constituents and their counterparts for the overall continuum, and on the variable characterizing the momentum exchange due to the interaction between the inviscid pore fluid and the solid skeleton have been taken into account. Furthermore, the volume balance of the overall continuum, the decomposition of the partial stress tensors (10a), (10b), and the saturation condition (2) have been used (for details see [10, 23]).

Dependent on the material properties of the solid skeleton, its partial free energy $\widetilde{\psi}_S$ represents a function of an elastic strain measure and, occasionally, on different internal variables. Consequently, analyzing relation (12) a thermodynamically consistent constitutive relation for $\boldsymbol{T}_S^E$ can be defined as well as potential evolutional equations for stress-type quantities, which are work-conjugated to the internal variables. Within this context, the definition of specific constitutive relations is based on the multiplicative split of the deformation gradient into an elastic and various inelastic parts $\boldsymbol{F}_S = \boldsymbol{F}_S^{\mathrm{e}}\,\boldsymbol{F}_S^{\mathrm{i}}$, dependent on the relevant physical effects. Obviously, the following approaches that are well-approved for technical materials and/or biological tissues are of interest for applications in rock mechanics (in brackets only one early relevant reference is given, respectively, although, currently a huge number of publications exist on this topic): elastoplastic models ($\boldsymbol{F}_S^{\mathrm{i}} = \boldsymbol{F}_S^{\mathrm{p}}$; [29]), viscoelastic models ($\boldsymbol{F}_S^{\mathrm{i}} = \boldsymbol{F}_S^{\mathrm{v}}$; [30]), growth models ($\boldsymbol{F}_S^{\mathrm{i}} = \boldsymbol{F}_S^{\mathrm{g}}$; [31]).

3 GOVERNING EQUATIONS

In this section, the balance laws for mass and momentum related to the overall continuum are presented to provide a complete set of governing equations, which enables the solution of coupled quasi-static initial-boundary value HM problems for saturated biphasic porous media in terms of a generalized total Lagrangian finite element approach. The proposed conceptual steps can be summarized as follows:

1. Development of the weak formulation from the strong form of the problem
2. Time discretization of rate terms of the integrands of the weak formulation
3. Consistent linearization of individual nonlinear functions of the integrands of the weak formulation using Taylor series representations
4. Spatial discretization of the linearized weak formulations using standard Galerkin procedures

3.1 Governing Balance Relations of the Overall Continuum

For the important case of quasi-static loading, inertial forces are neglected, and the local balance of (linear) momentum of the overall continuum reduces to the classical form of the equilibrium conditions given in generalized material description (cf. [14] and others)

$$\mathrm{Div}_S\left(\boldsymbol{T}\boldsymbol{F}_S^T\right) + \varrho_0\boldsymbol{B} = \boldsymbol{0} \tag{13}$$

where $\varrho_0\boldsymbol{B}$ represents the barycentric overall volume force.

Starting from the material time derivative of the saturation condition (2) with respect to the motion of a material point of the solid skeleton using the relations between the effective and partial density (4) of the constituents as well as the individual mass balances neglecting mass exchange between the constituents, and assuming intrinsic incompressibility of the constituents, the volume balance of the overall biphasic saturated porous medium is originally obtained in spatial description (cf. [14] and others). Its generalized material formulation is given as:

$$J_S\, \boldsymbol{C}_S^{-1} \cdot\cdot\, \dot{\boldsymbol{E}}_S \,+\, \mathrm{Div}_S\, \widetilde{\boldsymbol{W}}_F \,=\, 0 \tag{14}$$

Frequently, the term $\widetilde{\boldsymbol{W}}_F \overset{\text{def}}{=} \phi_{F0} \boldsymbol{W}_F$ is called *filter velocity* representing the relative velocity of the pore fluid against the deforming solid skeleton. In the mathematical sense, $\varrho_{FR0} \widetilde{\boldsymbol{W}}_F$ can be understood as the mapping of the spatial mass flux vector per unit time and surface area into the reference configuration of the solid skeleton.

3.2 Weak Formulation

Multiplying Eq. (13) with an arbitrary test function $\boldsymbol{V}_S = \boldsymbol{V}_S(\boldsymbol{X}_S)$ ($\boldsymbol{V}_S = \boldsymbol{0}$ at the Dirichlet part Γ_{0DU} of the boundary of the domain under consideration) and integrating the result over the volume of the undeformed domain Ω_0, the weighted form of the equilibrium conditions for the overall continuum, i. e., the weak formulation follows. With

$$\mathrm{Div}_S\big(\boldsymbol{T}\, \boldsymbol{F}_S^{\mathrm{T}}\big)\, \boldsymbol{V}_S = \mathrm{Div}_S\big(\boldsymbol{T}\, \boldsymbol{F}_S^{\mathrm{T}} \boldsymbol{V}_S\big) - \boldsymbol{T}\, \boldsymbol{F}_S^{\mathrm{T}} \cdot\cdot\, (\mathrm{Grad}_S \boldsymbol{V}_S)^{\mathrm{T}} \tag{15}$$

neglecting volume forces, using Eq. (8) as well as the decomposition (11) of the overall stress tensor, based on Gauss-Ostrogradski's integral theorem and considering the symmetry of the 2nd Piola-Kirchhoff stress tensor, the weak formulation of the equilibrium conditions of the overall continuum becomes:

$$\int\limits_{\Omega_0} \boldsymbol{T}_S^E \cdot\cdot\, \boldsymbol{E}_S\,(\boldsymbol{U}_S; \boldsymbol{V}_S)\; d\Omega_0 - \int\limits_{\Omega_0} p\,(\boldsymbol{S}_{\mathrm{v}} \cdot\cdot\, \boldsymbol{E}_S\,(\boldsymbol{U}_S; \boldsymbol{V}_S))\; d\Omega_0 = \int\limits_{\Gamma_{0NU}} \bar{\boldsymbol{R}}_U \boldsymbol{V}_S\, d\Gamma_0 \tag{16}$$

Here, $\bar{\boldsymbol{R}}_U$ is the given external loading related to the corresponding Neumann surface Γ_{0NU}, and the kinematic variable

$$2\boldsymbol{E}_S(\boldsymbol{U}_S;\boldsymbol{V}_S) \overset{\text{def}}{=} (\mathrm{Grad}_S \boldsymbol{V}_S)^{\mathrm{T}} + \mathrm{Grad}_S \boldsymbol{V}_S + \mathrm{Grad}_S \boldsymbol{U}_S (\mathrm{Grad}_S \boldsymbol{V}_S)^{\mathrm{T}} + \mathrm{Grad}_S \boldsymbol{V}_S (\mathrm{Grad}_S \boldsymbol{U}_S)^{\mathrm{T}} \tag{17}$$

has been defined in order to simplify further representations.

Considering the stress decomposition (11), multiplying Eq. (14) with an arbitrary test function $q = q(\boldsymbol{X}_S)$ ($q = 0$ at the Dirichlet part Γ_{0Dp} of the boundary of the domain under consideration) and integrating the result over Ω_0, the weighted form of the volume balance relation for the overall continuum, i. e., the weak formulation follows. Adopting Gauss-Ostrogradski's integral theorem to the corresponding flux term in an analogous manner as it was done for the equilibrium conditions, a simplified representation can be derived

$$\int\limits_{\Omega_0} \Big(\boldsymbol{S}_{\mathrm{v}} \cdot\cdot\, \boldsymbol{E}_S \left(\boldsymbol{U}_S; \dot{\boldsymbol{U}}_S\right)\Big)\; q\, d\Omega_0 - \int\limits_{\Omega_0} (\mathrm{Grad}_S\, q)\, \widetilde{\boldsymbol{W}}_F\, d\Omega_0 = - \int\limits_{\Gamma_{0Np}} \bar{R}_p\, q\, d\Gamma_0 \tag{18}$$

where $\bar{R}_p$ defines the prescribed fluid flux related to the part Γ_{0Np} of the surface affected by an external impact. In order to integrate the pore pressure as primary variable into

Eq. (18), the filter velocity is eliminated using Darcy's (filter) law in generalized material description neglecting volume forces.

$$\widetilde{\boldsymbol{W}}_F = -\boldsymbol{K}\,\mathrm{Grad}_S\,p \tag{19}$$

3.3 Numerical Scheme

The coupled problem (16), (18) shows a nonlinear dependency on the primary variable $\boldsymbol{U}_S$, and linear dependency on the primary variable p, on the rate variable $\dot{\boldsymbol{U}}_S$ and on the test functions. In order to eliminate rate-dependent terms of primary variables by discretizing them in time, a generalized trapezoidal single-step scheme is applied. Within this context, regarding the material time derivative of the solid skeleton displacements

$$(\boldsymbol{U}_S)_{n+1} = (\boldsymbol{U}_S)_n + \left[\alpha\,(\dot{\boldsymbol{U}}_S)_{n+1} + (1-\alpha)\,(\dot{\boldsymbol{U}}_S)_n\right]\Delta\,t \tag{20}$$

folloows, where the time increment $\Delta\,t$ is given by $t_{n+1} - t_n$, the subscript $(.)_n$ denotes variables at time t_n, which are known from the solution of the previous time step within the context of an incremental numerical scheme, and the subscript $(.)_{n+1}$ denotes variables, which belong to the unknown current solution at time t_{n+1}.

Linearization of the system (16), (18) is required in order to solve it numerically within the context of a Newton-Raphson scheme. It is performed based on Taylor series representations applied to functions of the primary variables $\boldsymbol{U}_S$ and p instead of adopting Gateaux derivatives of functionals. The idea is to find the solution

$$(\boldsymbol{U}_S + \Delta\boldsymbol{U}_S, p + \Delta p) := (\boldsymbol{U}^{i+1}_{S(t+\Delta t)}, p^{i+1}_{(t+\Delta t)}) \tag{21}$$

of the coupled two-field problem for the current $(i+1)$st Newton iteration at time t_{n+1} based on the given solution $(\boldsymbol{U}_S, p) := (\boldsymbol{U}^{i}_{S(t+\Delta t)}, p^{i}_{(t+\Delta t)})$ for the ith Newton iteration at time t_{n+1}, where $(\boldsymbol{U}^{0}_{S(t+\Delta t)}, p^{0}_{(t+\Delta t)}) = ((\boldsymbol{U}_S)_n, p_n)$ serves as initial solution known from the previous time step at time t_n.

Considering the backward Euler case ($\alpha = 1$) in terms of the generalized time discretization scheme (20), based on the proposed linearization procedure, and neglecting terms that are at least quadratic with respect to the increments of the primary variables $\Delta\boldsymbol{U}_S$ and Δp as well as with respect to products of these increments, the solution of the mixed initial-boundary value problem for saturated biphasic media within the TPM at large strains results in the incremental-iterative solution of the linear system

$$\begin{aligned}
&\int\limits_{\Omega_0} \boldsymbol{E}_S\,(\boldsymbol{U}_S;\boldsymbol{V}_S) \cdot\cdot \frac{\partial \boldsymbol{T}\,(\boldsymbol{E}_S(\boldsymbol{U}_S), p)}{\partial \boldsymbol{E}_S} \cdot\cdot \boldsymbol{E}_S\,(\boldsymbol{U}_S;\Delta\boldsymbol{U}_S)\,d\Omega_0 \\
&\qquad + \int\limits_{\Omega_0} \boldsymbol{T}(\boldsymbol{E}_S(\boldsymbol{U}_S), p) \cdot\cdot \mathrm{Grad}_S\Delta\boldsymbol{U}_S(\mathrm{Grad}_S\boldsymbol{V}_S)^T\,d\Omega_0 - \int\limits_{\Omega_0} \Delta p\,(\boldsymbol{S}_{\mathrm{v}}(\boldsymbol{U}_S) \cdot\cdot \boldsymbol{E}_S\,(\boldsymbol{U}_S;\boldsymbol{V}_S))\,d\Omega_0 \\
&\qquad\qquad = \int\limits_{\Gamma_{0NU}} \bar{\boldsymbol{R}}_U \boldsymbol{V}_S\,d\Gamma_0 - \int\limits_{\Omega_0} \boldsymbol{T}\,(\boldsymbol{U}_S) \cdot\cdot \boldsymbol{E}_S\,(\boldsymbol{U}_S;\boldsymbol{V}_S)\,d\Omega_0
\end{aligned} \tag{22a}$$

$$\int\limits_{\Omega_0} [\boldsymbol{S}_{\mathrm{v}}(\boldsymbol{U}_S) \cdot\cdot\, \boldsymbol{E}_S\,(\boldsymbol{U}_S; \Delta\boldsymbol{U}_S)]\; q\, d\Omega_0 + \Delta t \int\limits_{\Omega_0} (\mathrm{Grad}_S\, q)\; \boldsymbol{K}\; (\mathrm{Grad}_S\, \Delta p)\; d\Omega_0$$
$$= -\Delta t \int\limits_{\Gamma_{0Np}} \bar{R}_p\, q\, d\Gamma_0 \;-\; \Delta t \int\limits_{\Omega_0} (\mathrm{Grad}_S\, q)\, \boldsymbol{K}\; (\mathrm{Grad}_S\, p)\, d\Omega_0$$
$$- \int\limits_{\Omega_0} [\boldsymbol{S}_{\mathrm{v}}(\boldsymbol{U}_S) \cdot\cdot\, \boldsymbol{E}_S\,(\boldsymbol{U}_S; \boldsymbol{U}_S - (\boldsymbol{U}_S)_n)]\; q\, d\Omega_0 \tag{22b}$$

in terms of the increments $\Delta\boldsymbol{U}_S$ and Δp of the primary variables, valid for all functions $\boldsymbol{V}_S \in (H_0^1(\Omega_0))^3$ as well as $q \in L_2(\Omega_0)$.

For the numerical solution of the system (22a), (22b) in appropriate partial spaces $\Delta\boldsymbol{U}_S$, $\boldsymbol{V}_S \in \mathbb{V}_h^3 \subset \mathbb{V}^3$ and Δp, $q \in \mathbb{X}_h \subset \mathbb{X}$ of ansatz functions within the context of a mixed finite element formulation, the usual spatial discretization procedures are applied.

4 CONCLUSIONS

This paper has been contributed to the conceptual modeling and the numerical realization of a mixed finite element formulation for hydro-mechanical processes in saturated biphasic porous media at large strains based on the Theory of Porous Media. Within this context, the work was focused on isothermal, quasi-static models neglecting mass exchange between the phases. Defining all field variables consistently on the reference configuration of the solid skeleton, a generalized material approach has been presented resulting in the realization of a generalized total Lagrangian finite element approach.

The presented finite element procedure has been successfully applied to study several examples from biomechanics and soil mechanics illustrating the capabilities of the algorithms [23]. In comparison to small strain solutions, the presented results show the importance to consider large strain models developed in a consistent way. Consequently, the discussed numerical concept represents a kind of blueprint for the numerical modeling of porous media mechanics considering multiphase-multicomponent flow, reactive transport aspects and nonisothermal effects as typical phenomena observed for geotechnical applications in rock mechanics.

The discussed numerical scheme is prepared to be realized in an object oriented scientific software tool, OpenGeoSys, developed under the coordination of the authors, and applied to various hydrological and geotechnical applications (for details see [32]).

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