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A COUPLED PROBLEM OF HEAT AND MASS TRANSFER APPLIED TO POROUS TEXTILE MEDIA SURROUNDING THE HUMAN FOOT

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Abstract. There have been many attempts in the literature to model human thermoregulation systems in order to predict central temperature and other heat stress indicators (see [3, 9], among many others). These models depend on a wide range of variables, including individual characteristics, surrounding textile and environmental factors. The main objective of this work is to formulate a stable, realistic and versatile 2-D mathematical model describing the heat transfer of the human foot (bare foot and foot surrounded by textile materials). The novelty, but also the difficulty , lies on the theoretical multiphysics that models the textile as a porous media, involving energy transport but also mass transport of liquid water, water vapour and gas, including evaporation phenomena.

The numerical solution to the global problem involves a segregated algorithm and fixed point techniques for the nonlinearities jointly with finite elements spatial discretizations. Implementation has been performed through commercial software COMSOL^{TM} Multiphysics.

1 INTRODUCTION

Most of models to predict human thermoregulation systems are based on empirical properties that simplify human body global geometry, or simply apply theories in onedimensional models (see [4, 9], among others, and their references). There are some works that particularize the studies for some body parts. In the context of thermal modelling of the human foot followed by numerical simulation, it should be mentioned Covill et al. works [2, 3] using the finite element method.

The thermal model of the bare foot is based on the metabolic bio-heat equation, including the blood flow as a heat source and the effects of sweating. Here, this equation is

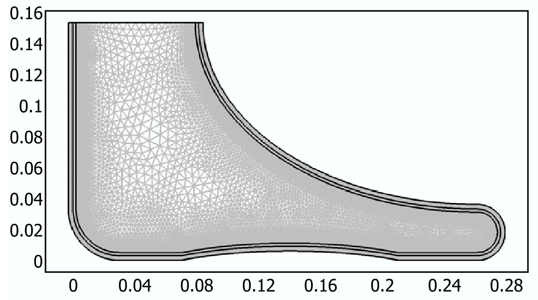

Figure 1: Domains Ω (with internal tissue and skin) and Ω_c textile medium (in meters) and mesh (16216 triangular finite elements of $1mm$ of characteristic size, approx.).

coupled to mass and energy transfer models at the textile that surrounds the foot (sock). Final formulation leads to a highly non-linear partial differential equations system, with adequate boundary conditions and heat transfer conditions between media.

The global model is solved by means of a segregated algorithm, fixed point techniques for the non-linearities along with finite elements spatial discretizations. Implementation is performed by using commercial software COMSOL^{TM}Multiphysics.

Finally, some simulations of several examples try to show that the numerical results are qualitatively acceptable. This 2-D coupled model can be a first step for equations and parameters adjustments, in order to apply it to more realistic three-dimensional geometries but, in any case, allows to carry out certain analysis on geometry design and on textiles composition that could be of great interest for the manufacturing industry.

2 THERMAL MODEL FOR THE FOOT-SOCK COUPLING

Usually, a hard simplification of the foot organs (bones, muscles, tendons, etc) is made in order to reduce the complex internal composition to a single tissue, where the thermal and biological properties are averaged. However, consideration of the foot skin as another tissue does not add new formulation difficulties.

Here, it is assumed that the transverse heat flow is much smaller than the longitudinal and vertical ones. So, the equation should be verified in a two-dimensional domain, Ω, that corresponds to a longitudinal section of an average human foot of a length of $28cm$, approximately (see Fig. 1, where it also shows the mesh used for the finite element method, the foot internal tissue, the $2mm$ thick skin and the $3mm$ thick sock). This equation in steady state form is given by:

$$-\nabla \cdot (k \nabla T) - \rho_b c_b \omega_b (T_b - T) = Q_m, \quad \text{in } \Omega \,, \tag{1}$$

where k the is thermal conductivity ($W\,m^{-1}\,K^{-1}$), ρ_b the blood density ($kg\,m^{-3}$), c_b the blood specific heat ($J\,kg^{-1}\,K^{-1}$), ω_b the blood perfusion (s^{-1}), T_b the blood temperature (K), Q_m the metabolic heat rate ($W\,m^{-3}$) and T is the unknown temperature (K).

The textile medium Ω_c can be considered as an unsaturated porous medium. Therefore, the model should consider (apart from the thermal equation) water and gas transport, in order to take into account sweating effects from the foot to the environment. In general, the thermal model consists of a diffusion equation for the unknown temperature, T_c, with a generation term that includes sweating/evaporative effects. That is:

$$-\nabla \cdot (k_c \nabla T_c) = -S_T, \quad \text{in } \Omega_c \ , \tag{2}$$

where S_T is a heat source\sink due to sweating and is given by:

$$S_T = H_{lat} S_v \ , \tag{3}$$

being H_{lat} the latent heat of vaporisation (Jkg^{-1}) and S_v the evaporation rate which is a priori unkown too. And finally, k_c is the thermal conductivity ($W\, m^{-1}\, K^{-1}$) which will be determined from solid and gas properties of the textile, as follows:

$$k_c = k_t(1-\phi) + k_w \theta_l \phi + k_g (1-\theta_l)\phi \ , \tag{4}$$

where k_t, k_w and k_g are the thermal conductivity of fibers, liquid water and gas mixture in the sock, respectively, ϕ the textile porosity coefficient and θ_l the liquid water concentration in the sock.

To model liquid water transport in the porous medium, Richards equation is used (see [7], among others), which consists of a variant of Darcy's law considering that both the hydraulic conductivity and the matric potential depend on the saturation level of the medium (Darcy's law only considers saturated porous media):

$$-\rho_l \nabla \cdot (K_h(\theta_l) \nabla \psi_h(\theta_l)) = -S_v, \quad \text{in } \Omega_c \ , \tag{5}$$

with water volumetric concentration,θ_l, expressed as per-unit, being the unknown of the problem. Moreover, ρ_l denotes the density of liquid water ($kg\, m^{-3}$), K_h the hydraulic conductivity ($m\, s^{-1}$), ψ_h the hydraulic potential (m) and S_v the vaporisation rate (a priori unknown).

The hydraulic potential can be written as the sum of two potentials (see [7] for details):

$$\psi_h = \psi_m + \psi_z \ , \tag{6}$$

where ψ_m denotes the hydraulic potential and ψ_z denotes gravitational potential that depends on the position of the terrestrial gravitational field:

$$\psi_z = \rho_l \, g \, (z - z_0) \ , \tag{7}$$

with g denoting the gravity acceleration ($m\, s^{-2}$) and z the height, (m), measured in relation to a reference value, z_0.

Following van Genutchen approximation [6], the hydraulic conductivity is given by:

$$K_h(\Theta) = K_s K_r(\Theta) = K_s \Theta^{\frac{1}{2}} \left(1 - \left(1 - \Theta^{\frac{1}{m}}\right)^m\right)^2 \ , \tag{8}$$

where K_s is the constant value of the hydraulic conductivity in saturation $(m\,s^{-1})$ and K_r the relative hydraulic conductivity, that depends on the effective saturation $\Theta = \frac{\theta_l-\theta_r}{\theta_s-\theta_r}$ and, therefore, on the concentration of liquid water in the material, θ_l. This effective saturation depends on θ_l and on the parameters θ_r and θ_s, that denote the residual concentration of moisture and the water concentration in saturation, respectively. Moreover, in (8) it is found the parameter m, that is usually written as $m = 1 - \frac{1}{n}$, and both m and n are usually named as the shape parameters of the retention curve.

Finally, the matric potential, following van Genutchen proposal [6], is written:

$$\psi_m(\Theta) = \frac{-\left(\Theta^{\frac{-1}{m}} - 1\right)^{\frac{1}{n}}}{\alpha}\ , \tag{9}$$

where α is also a shape factor of the retention curve.

The equation of the water vapour transport can be written as follows:

$$-\nabla\cdot\left(\frac{\kappa\rho_g}{\mu}\nabla P_v\right) - \nabla\cdot(D_v(T_c)\xi_g(\theta_a)\nabla m_v) = S_v, \quad \text{en } \Omega_c\ , \tag{10}$$

where m_v, a priori unknown, denotes the mass concentration of the vapour in the total textile volume $(kg\,m^{-3})$, κ is the medium's permeability (m^2), ρ_g is the density of the gaseous mixture $(kg\,m^{-3})$, μ is the air viscosity $(kg\,m^{-1}\,s^{-1})$, P_v is the partial pressure of the water vapour $(N\,m^{-2})$, D_v is the effective diffusivity of the vapour in the air $(m^2\,s^{-1})$ and ξ_g is the tortuosity of the vapour transport.

The mass concentration of the vapour in the textile medium can be written as follows:

$$m_v = \rho_v\theta_a \ = \ \rho_v(\phi - \theta_l)\ , \tag{11}$$

being ρ_v the vapour density in the avaliable volume (open pore) $(kg\,m^{-3})$, θ_a the concentration of the non-saturated porous space $(m^3\,m^{-3})$, and ϕ the porosity coefficient of the textile medium.

For calculating the vapour density, ρ_v, a chemical equilibrium hypothesis among the two water phases is imposed. This hypothesis implies a kinetic of phase change infinitely fast since mass interchanges occur instantaneously. In this case, classical Kelvin's equilibrium equation is used (see [1], for instance):

$$\frac{P_v}{P_{v_{sat}}} = exp\left(\frac{(\rho_l g\psi_m)M_l}{(\rho_l RT_c)}\right)\ , \tag{12}$$

where $P_{v_{sat}}$ denotes the partial vapour pressure in saturation conditions (Nm^{-2}), R is the constant of the ideal gases $(JK^{-1}mol^{-1})$ and M_l denotes the molecular weight of the water $(kg\,mol^{-1})$. This expression (12) consists on a dimensionless volumetric relation measuring the saturation of the gas. Moreover, $\frac{P_v}{P_{v_{sat}}}$ is often called relative humidity, h_r, to give it a more physical sense. That is:

$$P_v = P\,h_r\ , \tag{13}$$

where P is the pressure of the gaseous phase ($N\,m^{-2}$), that will be analyzed afterwards.

Now, using the ideal gas law, vapour density ρ_v in (11) is posed in terms of P_v, T_c:

$$\rho_v = \frac{P_v M_l}{RT_c} \ , \tag{14}$$

and, consequently, in terms of P, θ_l and T_c.

Fluxes of the gaseous phase are due to the gradient variations in temperature and pressure and are given by Darcy's law:

$$-\nabla \cdot \left(\frac{\kappa \rho_g}{\mu} \nabla P \right) = S_v, \quad \text{in } \Omega_c \ , \tag{15}$$

where the main unknown is the pressure of the gaseous phase, P, ($N\,m^{-2}$). Moreover, (15) involves the permeability of the medium κ (m^2), the air density, μ ($kg\,m^{-1}\,s^{-1}$), and the gaseous phase density ρ_g ($kg\,m^{-3}$), that can be also written in terms of P and T_c using the ideal gas law:

$$\rho_g = \frac{P M_g}{RT_c} \ , \tag{16}$$

where M_g is the molecular weight of the gaseous mixture.

Remark that equation (12) gives an implicit description of the source term S_v. So, it can be eliminated as problem unknown combining (5) and (10), and using (12) to close a system in terms of T_c, θ_l and P. In short, the coupled mathematical model consists on the one hand on the bio-heating equation (1) in the domain that comprises the foot , being the unknown the temperature (T). On the other hand, in the inner part of the textile medium the equations for energy flow (2), water transport (5) and gas transport (15) are considered. In this case, the unknowns are the temperature (T_c), the volumetric concentration of the liquid water (θ_l), the pressure of the gaseous phase (P) and the vaporisation rate (S_v), that can be calculated in a direct way by coupling the system with the vapour transport equation (10).

3 CONTACT AND BOUNDARY CONDITIONS

Assuming perfect contact between both media (foot and textile medium), transmission conditions for the thermal problems, (1) , (2), are given by the continuity of the temperatures and heat fluxes. For the gas transport equation (15), it is supposed that there are no gas flux from the foot. However, for the water transport equation (5), it is considered that water flux from the foot is given by the evaporative heat losses due to sweating:

$$\mathbf{n} \cdot (-K_h \nabla \psi_h) = -\frac{Q_e}{H_{lat}} \ , \tag{17}$$

being Q_e the sweating rate defined by the same empirical formula that appears in [4] and [9], among others, following the Fanger classical model [5]. This formula links sweating rate with the difference of the foot temperature at the skin surface T (K) and an empirical expression related to the environmental water vapour pressure $P_{v_{atm}}$ (Kpa). That is:

$$Q_e = Q_{eds} + Q_{ews}\,, \tag{18}$$

where Q_{eds} is the heat loss due to implicit sweat secretion when the skin is dry, and Q_{ews} the corresponding heat loss due to evaporation of explicit sweat secretion:

$$Q_{eds} = 0.782\left(T - \left(286.31 + \frac{P_{v_{atm}}}{0.256}\right)\right)\ ;\ Q_{ews} = h_e\, s_h\, 4.275\left(T - \left(286.31 + \frac{P_{v_{atm}}}{0.256}\right)\right)\,, \tag{19}$$

with s_h a nondimensional parameter denoting the skin humidity ($0 \leq s_h \leq 1$, from dry to wet). Moreover, $P_{v_{atm}}$ can be written as follows:

$$P_{v_{atm}} = P_{v_{sat}} H_r\,, \tag{20}$$

being H_r the air relative humidity (on a per unit basis) and $P_{v_{sat}}$ the saturation pressure of the water vapour at the environment temperature (kpa). Using thermodynamical equilibrium, $P_{v_{sat}}$ is classically given by some empirical formula, as the following one (see [8], for example):

$$P_{v_{sat}} = 0.611\, e^{\frac{17.27(T_a-273)}{237,3+(T_a-273)}}\,. \tag{21}$$

In the upper location of the domain, where the foot is connected to the rest of the body, two parts of the boundary need to be distinguished. First, $B1$ the boundary in contact with the rest of the leg, where the same condition of null thermal flux is posed for the temperature T. On the other hand, the boundary between the textile medium and the environment that is denoted by $B1c$ and affects to the temperature T_c, to the water concentration θ_l and to the pressure P. As $B1c$ is extremely small ($3mm$), no heat nor mass transfer is considered through this boundary.

Finally, it is supposed that on the boundary textile-environment, $B2$, there are heat losses due to convection and radiation. Based on this, the boundary condition for the thermal equation (2) can be written as follows:

$$\mathbf{n} \cdot (-k_c \nabla T_c) = h\,(T_c - T_a) + \varepsilon\sigma\left(T_c^4 - T_a^4\right), \quad \text{on } B2, \tag{22}$$

where $\mathbf{n}$ is the normal, unitary and external vector to $B2$, h is the convection coefficient $(W\,m^{-2}\,K^{-1})$, T_c is the temperature on the surface of the textile medium (K), T_a is the environmental temperature (K), ε is the emissivity coefficient (dimensionless) and σ is the Stefan-Boltzmann constant $(W\,m^{-2}\,K^{-4})$.

For the liquid water transport equation (5), a similar condition is given. That is, the flux is proportional to the difference of the vapour partial pressure in each media:

$$\mathbf{n} \cdot (-K_h \nabla \psi_h) = \frac{k_{atm} M_l}{R T_c}\left(P_{v_{atm}} - P_v\right), \quad \text{on } B2, \tag{23}$$

where k_{atm} is the mass conductivity of the interface textile-air ($m\,s^{-1}$), M_l is the molecular mass of the water ($kg\,mol^{-1}$), R is the perfect gases constant ($J\,mol^{-1}\,K^{-1}$), $P_{v_{atm}}$ is the vapour pressure in the atmosphere ($N\,m^{-2}$) and P_v is the vapour pressure in the porous medium (Nm^{-2}).

For the gas transport equation (15) it is supposed that existing flux through the boundary is proportional to the pressure differences between atmosphere pressure and gas phase pressure in the porous medium, as proposed in the following equation:

$$\mathbf{n}\cdot\left(-\frac{\kappa\rho_g}{\mu}\nabla P\right)=\frac{k'_{atm}M_g}{RT_c}\left(P_{atm}-P\right),\quad \text{on } B2, \tag{24}$$

where k'_{atm} is the gas mass conductivity of the interface textile-air (supposed to be two orders of magnitude lower than the mass conductivity of the vapour flux) ($m\,s^{-1}$), M_g is the molar mass of the air and P_{atm} is the atmosphere pressure ($N\,m^{-2}$).

4 NUMERICAL SOLUTION AND EXAMPLES

For the numerical solution of this bi-dimensional coupled model, commercial software COMSOL^{TM} Multiphysics is used. This software is based on the finite element method for spatial discretizations and it seems adequate for solving the partial differential equations system of the coupled and strongly non-linear model presented in previous sections.

In order to facilitate the introduction of the model in the computer code, a series of functions, variables and constants have been defined, appearing in tables 1 and 2 of the appendix.

The first simulation example tries to highlight the numerical results obtained from the global model. So, Fig. 2 shows the temperature distribution for the bare foot and for the foot surrounded by a sock with a porosity of 50% and for an environmental temperature of 25^oC and relative humidity of 35%. The temperature inside the foot slightly increases for being surrounded by the sock, also increasing the temperature on its external surface. Specifically, at the point of the tip of the toe with coordinates $(0.28\,m, 0.015\,m)$ it increases from 31.03^oC to 32.49^oC, while at the external point of the sock at the tip toe, with coordinates $(0.283\,m, 0.015\,m)$, it is of 29.01^oC.

Next example shows the variations of the unknowns of the global system inside the sock for an extreme case of environmental temperature of 35^oC. With the aim of visualizing these variations in a domain of such an small thickness, a zoom of the tip of the toe is considered (see Fig. 3) taking a line of nodes from the interior point of the sock in contact with the skin (point coordinates $(0.28\,m, 0.015\,m)$) to the external surface (point coordinates $(0.283\,m, 0.015\,m)$). The decrease of the temperature in this line inside the sock shown in Fig. 3 is due to the thermal gradient generated by the difference of temperature between the foot and the environment. A similar behaviour can be observed in Fig. 4 for the sock pressure though it is not quantitatively very relevant in this case.

Variations of water concentration at the same line are very small as showed the Fig. 5.

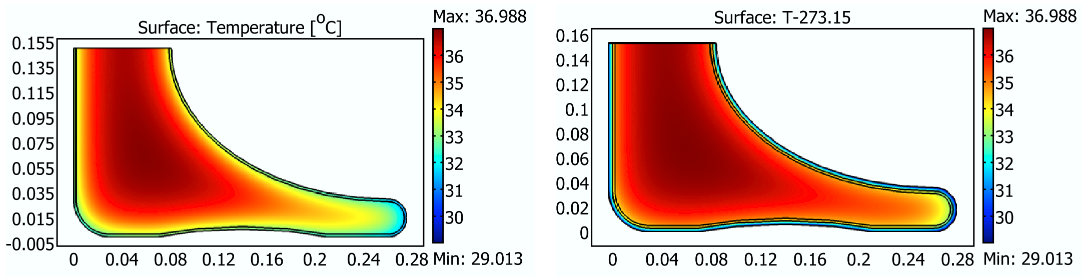

Figure 2: Temperature distribution using the simple model of bare foot (left) and the coupled model for the foot and sock (right).

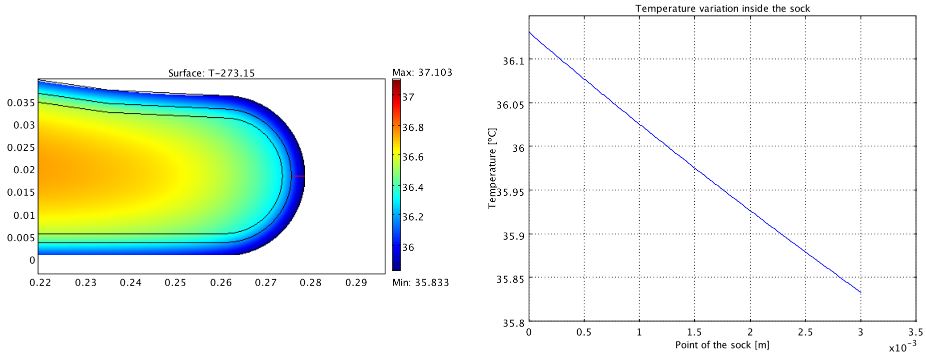

Figure 3: Temperature variation inside the sock (right) at a red line located at the tip of the toe zone (left).

Finally, varying the values of the porosity coefficient of the textile medium, it is intended to validate the qualitative properties of the proposed coupled model. In fact, if the sock porosity tends to be very big, the skin temperature obtained from a bare foot model and the corresponding one using the coupled model with the textile medium should be very similar. And, it can be noted in Fig. 6 that for a porosity of 95% and an environmental temperature of 35^oC, the temperature at the surface of the foot (point $(0.28\,m, 0.015\,m)$) is 36.05^oC and at the external surface of the sock (point $(0.283\,m, 0.015\,m)$) is 35.89^oC, while with the bare foot model the temperature obtained at the same point of the foot surface is 35.74^oC. This validation test is very important since it sets the trend of the temperature respect to extreme porosities.

5 CONCLUSIONS

Literature sources does not show such a complete model as the one here proposed for the thermal coupling of a human foot to a textile medium (a sock or a pant). This model is configured from the classic equation of the bio-heating energy in the foot and the theoretical multi-physics on porous media that affects the textile medium, where mass transport is involved (water, vapour and evaporating phenomena).

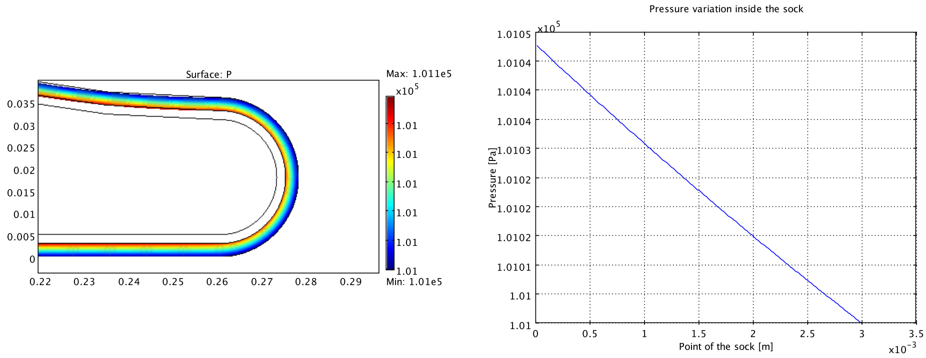

Figure 4: Pressure variation inside the sock (right) at a red line located at the tip of the toe zone (left).

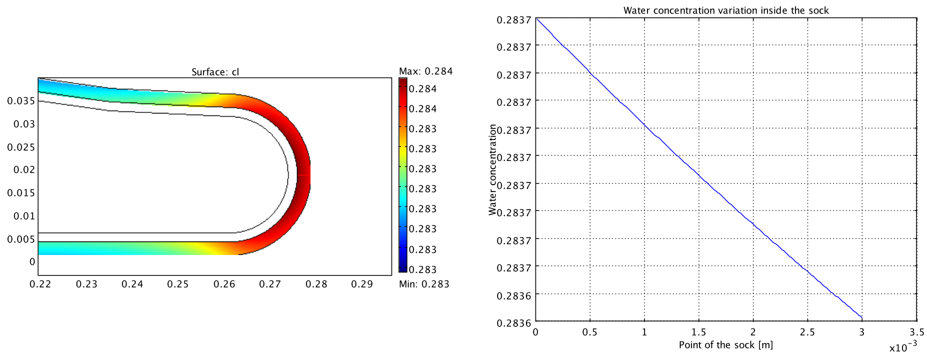

Figure 5: Variation of water concentration inside the sock (right) at a red line situated at the tip of the toe zone (left).

Numerical simulations here presented show a results qualitatively acceptable. Thus, it seems that both the model and the finite elements discretizations selected for the numerical solution and the software used fulfil the objectives of this study.

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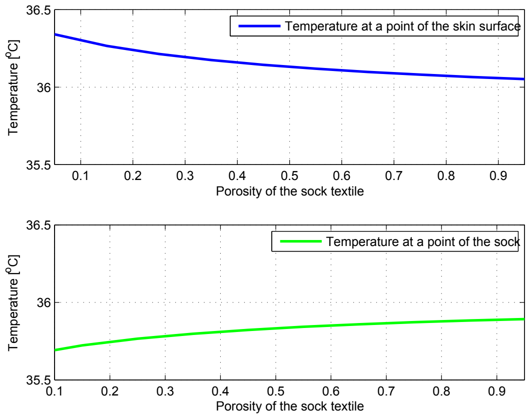

Figure 6: Temperature variation at the point of the skin surface with coordinates $(0.28\,m, 0.015\,m)$ (top) and at the point of the surface of the sock with coordinates $(0.283\,m, 0.015\,m)$ (bottom), for different porosities of the sock textile, and environmental temperature $T_a = 35^o$.

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Appendix: Data of the global model

Var.	Expression	Units	Description
k	k_f *or* k_s	$W/(mK)$	Foot thermal conductivity
S_T	$H_{lat}S_v$	$\frac{J}{s\,m^3}$	Heat Source\Sink due to sweating
k_c	$k_t(1-\phi)+k_w\theta_l\phi+k_g(1-\theta_l)\phi$	$W/(mK)$	Sock thermal conductivity
k_g	$k_{da}+k_v$	$W/(mK)$	Gas thermal conductivity
k_{da}	$0.024+7.73\,10^{-5}T-2.6\,10^{-8}T^2$	$W/(mK)$	Dry air thermal conductivity
Θ	$\frac{\theta_l-\theta_r}{\theta_s-\theta_r}$		Effective water concentration
P_v	$P\,h_r$	kPa	Partial vapour pressure
h_r	$exp\left(\frac{(\rho_l g\psi_m)M_l}{\rho_l RT_c}\right)$		Air vapour fraction
K_h	$K_sK_r(\Theta)$	m/s	Hydraulic conductivity
K_r	$\Theta^{0.5}(1-(1-\Theta^{1/m})^m)^2$	m/s	Residual hydraulic conductivity
m	$1-1/n$		Shape parameter of water retention curve
ψ_h	$\psi_m+\psi_z$	m	Hydraulic potential
ψ_z	$(z-z_0)$	m	Gravitational potential
ψ_m	$\frac{-(\Theta^{-1/m}-1)^{1/n}}{\alpha}$	m	Matric potential
D_v	$D_{v_0}(P_0/P_{atm})(T_c/T_a)^{1.75}$	$\frac{m^2}{s}$	Vapour diffusivity
ξ_g	$0.66\,\theta_a$		Tortuosity of the sock vapour transport
m_v	$\rho_v\theta_a$	$kg/(m^3)$	Vapour mass concentration
ρ_v	$\frac{P_vM_l}{RT_c}$	$\frac{kg}{m^3}$	Water vapour density
θ_a	$\phi-\theta_l$		Non-saturated porosity
ρ_g	$\frac{PM_g}{RT_c}$	$\frac{kg}{m^3}$	Gas density
Q_e	$Q_{eds}+Q_{ews}$	$\frac{W}{m^2}$	Evaporation heat losses
Q_{eds}	$0.782\left(T-\left(286.31+\frac{P_{v_{atm}}}{0.256}\right)\right)$	$\frac{W}{m^2}$	Heating losses due to implicit sweat secretion
Q_{ews}	$h_e\,s_h\,4.275\left(\frac{Q_{eds}}{0.782}\right)$	$\frac{W}{m^2}$	Heating losses due to explicit sweat secretion
$P_{v_{atm}}$	$P_{v_{sat}}\,H_r$	kPa	Environmental water vapour pressure
$P_{v_{sat}}$	$0.611\,exp\left(\frac{17.27(T_a-273)}{273{,}3+(T_a-273)}\right)$	kPa	Environmental water vapour saturation pressure

Table 1: Variables defined to eased model understanding (in order of appearance in the paper)

Param.	Value	Units	Description
k_f	0.5	$W/(m\,K)$	Thermal conductivity of the internal tissue of the foot
k_s	0.2	$W/(m\,K)$	Skin thermal conductivity
ρ_b	1060	kg/m^3	Blood density
c_b	3850	$J/(kg\,K)$	Blood specific heat
ω_b	$0.54\,10^{-3}$	s^{-1}	Blood perfusion volumetric rate
T_b	310.15	K	Blood temperature
Q_m	300	W/m^3	Metabolic heat of the foot tissues
H_{lat}	$2.25\,10^6$	$\frac{J}{kg}$	Water condensation conductivity
k_t	0.06	$W/(mK)$	Fibers thermal conductivity
k_w	$2.7\,10^{-1}$	$W/(mK)$	Liquid water thermal conductivity
k_v	$2.5\,10^{-2}$	$W/(mK)$	Water vapour thermal conductivity
ϕ	0.5	m^3/m^3	Sock porosity
ρ_l	1000	$\frac{kg}{m^3}$	Liquid water density
θ_r	0.095	$\frac{m^3}{m^3}$	Sock residual saturation
θ_s	0.41	$\frac{m^3}{m^3}$	Maximum sock saturation
n	1.31		Shape parameter of water retention curve
K_s	$7.22\,10^{-7}$	m/s	Saturated hydraulic conductivity
g	9.82	m/s^2	Gravity acceleration
z_0	0	m	Reference height for gravitational potential
α	0.019	m^{-1}	Shape parameter of water retention curve
κ	10^{-14}	m^2	Air absolute permeability
μ	$1.7\,10^{-5}$	$kg/(m\,s)$	Air viscosity
D_{v_0}	$2.92\,10^{-5}$	m^2/s	Vapour diffusivity (at 0^oC and 1 bar)
P_0	101	kPa	Reference pressure
P_{atm}	101	kPa	Atmospheric pressure
R	8.31	$\frac{J}{mol\,K}$	Ideal Gases constant
h_e	10	$W/(m^2\,K)$	Convection coefficient
s_h	$s_h \in [0,1]$		Skin humidity
T_a	298	K	Environmental temperature
ε	0.97		Skin emissivity
σ	$5.67\,10^{-8}$	$W/(m^2\,K^4)$	Stefan-Boltzman constant
H_r	0.35		Environmental relative humidity (on a per unit basis)
M_l	18	g/mol	Water molecular mass
M_g	18	g/mol	Gas molecular mass
k_{atm}	0.137	m/s	Mass conductivity at the interface sock-air
k'_{atm}	0.001	m/s	Gas mass conductivity at the interface sock-air

Table 2: Constants defined in the model (in order of appearance in the paper).