iZAR Model Rocket Workshop

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I. INTRODUCTION

Rockets have always been one of the most challenging and passionate scientific objects in history. Their development has been improved since its first studies in ancient China [1], where they were used as a weapon for firing arrows. It is believed that the Mongols were responsible for bringing rocketry into Europe. Since the 15th century its importance has been balanced between a war weapon and entertainment fireworks. Its importance as a scientific tool began during the 17th century, when Isaac Newton developed the motion laws, including the rocket equation which will be explained later. However, its modern usage as the fundamental way to take scientific instruments out of the atmosphere and into space was studied during the 20th century. After nearly 50 years of study, it was possible to send the first men to the moon and to take a satellite into orbit, and henceforth its importance in space exploration has put its study in one of the most interesting fields in which physics relies.

Its major importance has been the principal motivation of the iZAR group in order to undertake this project. Moreover it provided the opportunity to design and implement what learned from rocket physics and engineering.

II. IZAR DESIGN

The first thing that comes in mind when rocket design is proposed is bullet shape aerodynamics. Some say that the first thought is, usually, the most appropriate, that intuition leads to success, but we needed further investigations.

First of all, there exists the dichotomy between sharpness and bluntness. Doing some research, it can be proven that aerodynamics is not an easy matter. On our project there are different aspects that are involved in the good functioning of the rocket. As it is obvious, drag force is one of the primordial aspects to bear in mind, but stability, point pressure and centre of gravity matter as well. There are shapes that are designed exclusively to reduce the effects of these inconveniences at minimum; some have the form of a water drop, that have a lot of surface to distribute most of the pressure at the nose, and less at the tail. These shapes seem reasonable, but this unbalance of pressure surface sets forward the point of pressure, leading to instability.

One of the principles followed during all the design procedure is that the point of pressure has to be as near as possible to the tail, as the centre of gravity close to the nose. With that in mind, the shape of our rocket should have an increasing surface of pressure as we move from nose to tail. Three fins are implemented with that objective, but the body must help too.

Another important aspect is the velocity, as it has an important role in aerodynamics. That is how we can solve the dichotomy. Some researchers give data that helps with that decision. At low velocities, blunt shapes, like parabolas, show the lowest drag coefficients. If velocity increases and approaches the supersonic limit, sharpness starts to equal any parabola. But these are not the only things to consider. If the final point of the nose is sharp, determine which shape is the best for the rest of the nose is as important. LV-Haack is a profile build on the principle of obtaining the minimum drag with a specified length and volume, this has a sharp point and an ogive-like shape. Another similar profile is called LD-Haack, which searches for the minimum drag with a given length and diameter. This last one is also referred as the Von Kármán ogive, and it is one of the most used in supersonic aviation. Them both are represented by the following function [2] [3].

$$\theta(x) = acos(1 - 2\frac{x}{L})$$

$$Radius(x) = \frac{D}{2\sqrt{\pi}}\sqrt{\theta - \frac{sin(2\theta)}{2} + Csin^3(\theta)}$$

Where D corresponds to the diameter, L to the length of the profile, x is the distance from the tip and C is equal to 0 or 1/3 if it is Von Kármán or LV-Haack.

It is more than proved that there exist multiple factors to consider and options to choose. The iZAR-404 has a profile of LV-Haack with a parabolic tip. In order to improve stability and reduce drag and weight, the shape of the rocket is composed of two noses, facing opposite directions. The one located on the back of the rocket has a major length of definition, and the rear end is cut so as to enable a proper functioning of the engine.

The fins are regulated by the certification requirements. We used curved shapes to approach elliptic contours, which are most aerodynamically efficient, and airfoils profiles to reduce drag, like the wings of planes do.

The rocket is composed of two parts, the nose and the body. The nose houses the payload and because it needs to be ejected, it is not fixed to the body. The payload, composed by the altimeter and the batteries, is concealed by a fabric subjected to the nose by a big ring and adjusted with tape. The body consists of three parts printed separately and fused together.

The interior of the iZAR-404 has a solid structure build to hold the thin fuselage, the rocket engine, the parachutes and the altimeter. Three structural girders with several circular enforcements are the principal structure. There are 6 small rings joined to that structure of the body where the major parachute is attached, and the big ring fitted in the nose holding the fabric that also fixes the minor parachute to the payload.Two engine adapters were designed to fit them perfectly into our curved interior without losing alignment and stability.

FIG. 1. Rocket final prototype, iZAR-404, Star Not Found, ready to be launched. The UPC and engineering physics logos are the ones printed on the fins.

III. CERTIFICATION REQUIREMENTS

Before rockets can be launched, some requirements for sizes and resilience of the parts of the rocket, as well as some safety requirements and conditions about the construction materials. The list of requirements that both rockets fulfill is as follows

- Fins resist a longitudinal force $F = 2M_{fin}a_{max}$ and a lateral force $F = 0.052S_{fin}v_{max}^2$.
- Applying a transversal force, the maximum fin bending is smaller than 17%.
- Fins have a better alignment with the rocket axis than 5 degrees.
- The maximum bending of the body tube is smaller than the 1% required.
- Engine bracket resist a longitudinal force

$$F = 2T_{fin}$$

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- The respective recovery system are able to withstand the payload and the fuselage and both glide at a vertical velocity according to the specified range, smaller than 5m/s.
- In order to guarantee the recovery of all parts two parachutes are used for each rocket. A hole of 3cm of diameter in the centre of parachutes is done to ensure stability in gliding phase.
- The altimeter is ready to take measures of the altitude as a function of time during the trajectory. A hole has to be done in the nose in order to let the altimeter do the measures.

An important aspect before the launch is to verify the stability of the rocket. In order to start talking about stability the concepts of centre of pressure, centre of mass and margin of stability must be introduced [6]. A rocket will be stable if the margin of stability (the distance between the centre of mass and pressure) is bigger than the maximum diameter of te rocket. The centre of mass is defined as the point of the rocket where is concentrated all the mass of the rocket, whereas the centre of pressure is the point where all the aerodynamic normal forces act.

An easy way to calculate the centre of mass is by hanging the rocket from a string and moving the point of application until the rocket remains totally horizontally. The procedure to calculate the centre of pressure is more analytical. The expression of the centre of pressure is shown in (1)

$$cpA = d_n a_n + d_b a_b + d_f a_f \qquad (1)$$

with a_i are the respective areas in 2D of ech part of the rocket; body, fins and nose, d_i the respective distance of the parts with respect to the base of the rocket,cp the centre of pressure and A the sum of all a_i. It is found that the centre of pressure for the 3D rocket is 20cm far from the bottom of the rocket, using the data from table

1, and the centre of mass 38cm from the bottom,then the difference is bigger than the maximum diameter of the rocket, 6cm.

Part of the rocket	**Area (ai)**	**Distance(di)**
Nose	$30.31cm^2$	45cm
Body 1	$52.01cm^2$	30cm
Body 2	$52.52cm^2$	21cm
Tail	$12.62cm^2$	12cm
Fins	$52,81cm^2$	9cm

With all these requirements and stablity criterion fullfilled the rocket can be launched perfectly safe.

IV. THE ROCKET EQUATION: THEORY FUNDAMENTALS

The purpose of this part is to explain the trajectory of an object which losses mass along the motion. This is especially important when implementing a rocket trajectory, as a great part of the mass is the fuel used to propel it. The equation that models the motion of a body is the Newton's second law.

$$\vec{F} = m\vec{a}$$

However, in rockets this well-known equation cannot be applied for the reasons aforementioned, and what prevails is the rocket equation, which is as follows [4].

$$M\frac{d\vec{v}}{dt} = \vec{F} + \frac{dM}{dt}\vec{u}$$

A. Rocket Equation

The right term of the previous ocasion is the force. That force is comprised of several parts. Its most important is the term $\vec{T} = \frac{dM}{dt}\vec{u}$ which is the thrust of the rocket, and comprises the acceleration due to the gas propulsion. The quantity $\vec{u}$ relates to the relative ejection velocity with respect to the rocket. It is important to note that it is a vectorial magnitude, as the rocket motion is vertical as well as horizontal, due to the initial launch angle and the weather conditions. In this initial expression, two major effects directly affect the trajectory: Gravity and Drag. Gravity is a vertical force, assumed of the same magnitude through all flight.

$$\vec{F_g} = m\vec{g}$$

Drag is a force component against the motion, quadratically dependent on the velocity of the rocket, as it can be seen in the drag coefficient equation [4].

$$\vec{F_D} = \frac{1}{2}\rho v^2 C_D A$$

In this equation, C_D is the drag coefficient, which is given by the literature; A is the cross-sectional area of the rocket, where it is the largest. RHO represents the air density, which has been taken constant at a value of 0.25 (Kg/m3). It is important to know that the drag force is usually a small fraction of the gravity. Nonetheless, it must be taken into account, so the final rocket equation to simulate is[4]:

$$M\frac{d\vec{v}}{dt} = \vec{T} - \vec{F_g} - \vec{F_D}$$

V. SIMULATIONS

A fundamental part of this project is to predict the rocket estimated trajectory using a simulation software. In this case of study, the software MATLAB was used. The main objective is using the aforementioned equations of the rocket to estimate how the trajectory would be in the different stages of the flight. As they consist on a set of differential equations with no analytical solution, to integrate the velocity and attain the position the integrating method of Runge-Kutta was used [5]. For the simulations, a series of assumptions have been mad and those are applied in all flight stages. Firstly, the air density, as well as gravity, are considered to be constant in all altitudes. Also, no lateral forces are taken into account, thus permitting a 2D simulation of the trajectory. There are 5 principal stages in the rocket flight trajectory:

A. Rocket flight up to burn out

The first stage starts with the ignition and lasts until all the propellant is burned out. Since the thrust increases gradually, when its value overcomes gravity the rocket will take off. In order to simulate that, firstly the thrust curve of the rocket's engine D9-3 is modelled with a series of linear functions that imitate the original non linear one. Using the drag coefficient given in the Model Rocket Workshop manual (approximately 0.15) and taking into account an initial inclination due to the rocket lauch base, the rocket equation is applied. In it it is also taken into account that mass linearly decreases with time, and the thrust is aligned with the rocket's longitudinal axis. In the simulation is studied and plotted the altitud in function of time and in function of the cylindrical coordinate. The code is annexed in XI.

B. Rocket flight from burn out to nose and fuselage separation

The second part starts when all the propellant is consumed. At this moment, the rocket starts a ballistic trajectory, where mass is constant, and the forces acting are only gravity and drag. While gravity acts downward, drag acts as a viscous force against the motion. This has been taken into account, and the plots of this parts have been made exactly as before. This part last the delay time of the rocket, since burn out until nose and fairing are separated.

C. Rocket flight from separation till parachute opening

For the treatment of this part of the flight, nose and fairing have to be treated separately,as each have their own drag coefficient. It is also important to consider the impulse created by the ejection, which will propulse the nose along the propagation direction, whereas the fairing will be propulsed in the opposite direction. The new drag coefficients will be 0.78 for the fuselage and 0.47 for the nose.The momentum change is 13 N with a duration of 0.01s. This part last until the peak altitude is reached, as from that moment the velocity will be negative. It is considered that at this point the parachutes start acting.

D. Parachute gliding

The last part of the flight consist on the parts lowering with the parachutes opened until those parts reach the floor. As it was wanted to ensure a certain landing velocity, the areas of the parachutes had to be determined, and the drag coefficient used with the parachutes was of 1.15, as stated in the guidelines of the Model Rocket Workshop. As in the second part of the flight, the only forces acting are gravity and drag.

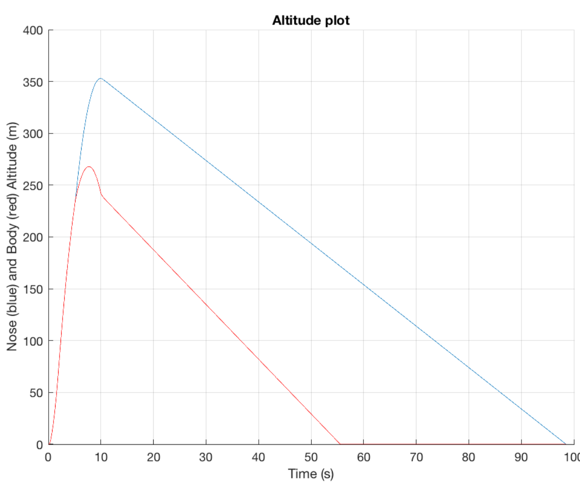

FIG. 2. Graphic corresponding to the altitude reached by the nose (blue) and the body (red), according to the simulation code, with a take-off angle of 5 degrees.

VI. RESULTS

This part of the article refers to the results obtained with the altimeter, which stores the flight altitude through all flight, in order to study the correlation between the simulations and the reality. Comparing one versus the other, one could study how accurate the assumptions made to make the simulations were, or in which stages the simulation fails and how. Unfortunately, no data could be retrieved from the flights. Although the both the design and the simulation prevision were both checked by the project manager, the lauching site was chosen too near the sea, which resulted in our rocket landing on the water Although it was possible to retrieve the body parts and the altimeter, the water damage made it impossible to obtain the anaylisis data.

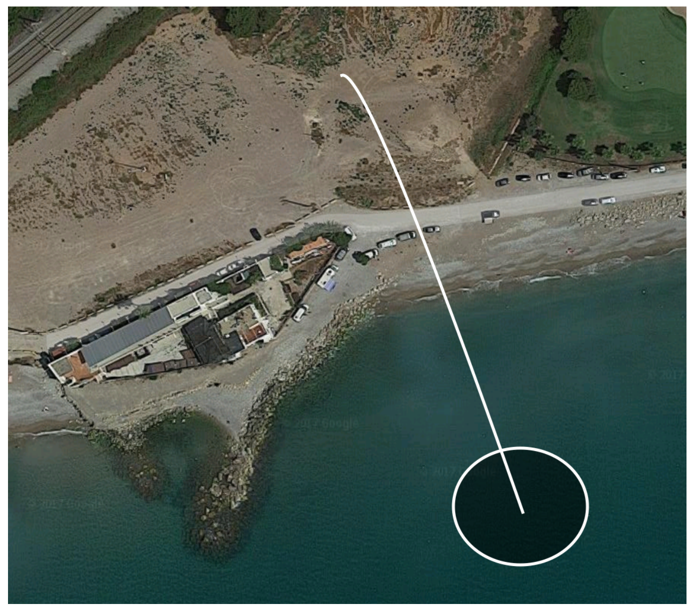

FIG. 3. Panoramic view of the launch site and the rocket trajectory described by iZAR 404. Tha launch took place in June 13th 2017, in Sitges.

VII. CONCLUSIONS

Going in depth into a scientific matter is a process that needs time and effort to be fullfiled. Model rocket engineering has been a no different experience: computer design, code programming, testing procedures, certification requirements or launching protocol are some of the most superficial aspects of a process that has given us a different and deeper view of an engineering project, involving many factors that we can find in superior, more technical environments.

We have chosen a different design, a more aerodynamic shape full of detail and effort that has resulted in an engineering piece that we are proud of.

VIII. REFERENCES

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