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DEVELOPMENT AND IMPLEMENTATION OF AN EULERIAN APPROACH FOR EFFICIENT SIMULATION OF FRICTIONAL HEATING IN SLIDING CONTACTS

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Abstract. Thermal stresses as a result from frictional heating must be considered when designing disc brakes. The rotational symmetry of a disc brake makes it possible to model this system using an Eulerian approach instead of a Lagrangian framework. In this paper such an approach is developed. The sliding object is formulated in an Eulerian frame where the convective terms are defined by the sliding velocity. A node-to-node formulation of the contact interface is utilized. The energy balance of the interface is stated by introducing an interfacial temperature. Both frictional power and contact conductance are included in this energy balance. The contact problem is solved by a non-smooth Newton method. By adopting the augmented Lagrangian approach, this is done by rewriting Signorini's contact conditions to a system of semi-smooth equations. The heat transfer in the sliding body is discretized by a Petrov-Galerkin approach, i.e. the numerical difficulties due to the non-symmetric convective matrix appearing in a pure Galerkin discretization is treated by following the streamline-upwind approach. In such manner a stabilization is obtained by adding artificial conduction along the streamlines. For each time step the thermoelastic contact problem is first solved for the temperature field from the previous time step. Then, the heat transfer problem is solved for the corresponding frictional power. In such manner a temperature history is obtained via the trapezoidal rule. In particular the parameter is set such that both the Crank-Nicolson and the Galerkin methods are utilized. The method seems very promising. The method is demonstrated for two-dimensional benchmarks as well as a real disc brake system in three dimension.

1 INTRODUCTION

In this paper an Eulerian approach for sliding contacts is developed. In the design of machine components like brakes and clutches it is of importance to consider effects from frictional heating. Today, this is mostly done by experiments. The Lagrangian approaches in our commercial softwares usually fail due convergence difficulties in the contact algorithms and too long computational times. An idea to improve these drawbacks is to formulate the problem in an Eulerian frame instead. This is the topic of this paper. The approach is presented for a two-dimensional translating problem. In a forthcoming paper the approach will also be developed for rotating systems as well as for the tree-dimensional case.

Previously, we have studied thermo-mechanical contact problems in the setting of small displacements. In Strömberg [1] thermo-mechanical wear problems were studied for a thermo-elastic body in unilateral contact with a rigid foundation. The development of hot spots was studied by solving the fully coupled equation system using Newton's method. The influence of wear on these hot spots was also investigated numerically. That work was later extended to the case of two thermo-elastic bodies in unilateral contact in Ireman, Klarbring and Strömberg [2]. An earlier work on this topic in the same research group was done by Johansson and Klarbring [3]. In this paper, the thermo-mechanical framework developed in our previous works is now extended to also include large rotations with superimposed small displacements and this is done in an Eulerian framework.

Examples of early works on frictional heating in large displacements are e.g. the papers by Oancea and Laursen [4], and by Agelet de Saracibar [5]. A more recent paper is the one by Rieger and Wriggers [6] where the accuracy of the contact solution, which is most important in order to represent the frictional power sufficiently well, was controlled by adaptive techniques. Another way to improve the contact solution in large displacements is to use the mortar technique. Recently, this was investigated by Hüeber and Wohlmuth [7] for thermo-mechanical friction problems. A nice feature with the presented Eulerian approach in this paper is that the contact region is always well defined and a node-to-node based approach can be adopted, producing very accurate contact solutions. The contact equations are then treated with the celebrated augmented Lagrangian approach where the corresponding equation system is solved by a non-smooth Newton algorithm. The details can be found in Strömberg [8].

One can find several other works where a Lagrangian formulation has been utilized for treating frictional heating, e.g. [9], but it is not easy to find any paper where an Eulerian framework is used. One example is the paper by Pauk and Yevtushenko [10] where a cylinder sliding over a half-space was considered. In this work we present a finite element approach using an Eulerian framework for solving frictional heating in sliding contacts. The fully coupled problem is decoupled in one mechanical part and another thermal problem. These two equation systems are then solved sequentially by using Crank-Nicolson's and Galerkin's settings of the trapezoidal rule in the time discretization. Other possibilities of performing the time discretization are of course also available. For instance, Laursen [11] proposed thermodynamically consistent algorithms for this class of problems. The convective term is stabilized by the streamline-upwind approach. For this task the excellent text-book by Donea and Huerta [12] has been consulted.

The proposed method is implemented on 64-bits Windows using Intel Fortran and the sparse Cholesky and LU solvers of Matlab. The pre- and postprocessing are performed on Abaqus/CAE by Python scripts. The implementation seems to be very robust and produce accurate solutions at low computational times. This is demonstrated by presenting numerical examples.

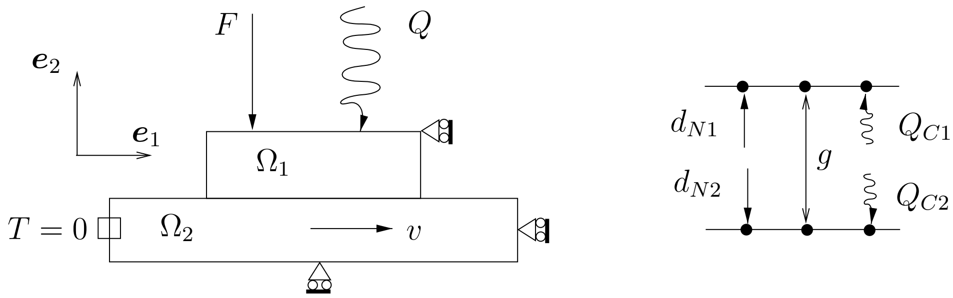

Figure 1: *Two linear thermoelastic bodies in unilateral contact.*

2 GOVERNING EQUATIONS

Let us consider two linear thermo-elastic bodies Ω_m in unilateral contact, see Figure 1. The first body is subjected to external forces $\boldsymbol{F} = \{F_i^A\}$ and heat powers $\boldsymbol{Q} = \{Q^A\}$ at finite element nodes $\boldsymbol{x}_A$ on the top of the first body. The second body is translating with a constant velocity $\boldsymbol{v} = v\boldsymbol{e}_1$ and has superimposed small displacements onto the current rigid body configuration at time t. For each body Ω_m, the nodal displacements are collected in $\boldsymbol{d}_m = \{d_j^A\}$ and the nodal temperature vectors is represented by $\boldsymbol{T}_m = \{T^A\}$, respectively.

At the contact surface of each body, the normal displacements are given by

$$\boldsymbol{d}_{Nm} = \boldsymbol{C}_{Nm}\boldsymbol{d}_m, \tag{1}$$

where the rows of the transformation matrices $\boldsymbol{C}_{Nm}$ contain surface normals in proper positions, i.e.

$$\begin{aligned} \boldsymbol{C}_{N1}^{\text{row}} &= [\mathbf{0}\,[0\,-1]\,\mathbf{0}], \\ \boldsymbol{C}_{N2}^{\text{row}} &= [\mathbf{0}\,[0\,1]\,\mathbf{0}]. \end{aligned} \tag{2}$$

The corresponding normal contact forces $\boldsymbol{F}_{Nm}$ are obtained by

$$\boldsymbol{F}_{Nm} = -\boldsymbol{C}_{Nm}^T\boldsymbol{P}, \tag{3}$$

where $\boldsymbol{P}$ is a vector of Lagrange multipliers which are governed by Signorini's contact conditions:

$$\boldsymbol{P} \geq \mathbf{0}, \quad \boldsymbol{d}_{N1} + \boldsymbol{d}_{N2} \leq \boldsymbol{g}, \quad \boldsymbol{P} \circ (\boldsymbol{d}_{N1} + \boldsymbol{d}_{N2} - \boldsymbol{g}) = \mathbf{0}. \tag{4}$$

Here, $\boldsymbol{g}$ represents a vector of initial gaps g^A between contact nodes in node-to-node contact, see Figure 1, and $\circ$ is the Hadamard product.

It is assumed that sliding is always developed and that the corresponding frictional forces are given by

$$\boldsymbol{F}_{Tm} = \mu \boldsymbol{C}_{Tm}^T \boldsymbol{P}, \tag{5}$$

where

$$\begin{aligned} \boldsymbol{C}_{T1}^{\mathrm{row}} &= [\mathbf{0}\,[1\,0]\,\mathbf{0}], \\ \boldsymbol{C}_{T2}^{\mathrm{row}} &= [\mathbf{0}\,[-1\,0]\,\mathbf{0}]. \end{aligned} \tag{6}$$

These assumptions are in agreement with Coulomb's law of friction when sliding is developed.

By introducing finite element shape functions $N^A = N^A(\boldsymbol{x})$ and performing a finite element discretization, the following equilibrium equations in forces can be derived:

$$\begin{aligned} \boldsymbol{K}_1 \boldsymbol{d}_1 - \hat{\boldsymbol{K}}_1 \boldsymbol{T}_1 &= \boldsymbol{F} + \boldsymbol{F}_{N1} + \boldsymbol{F}_{T1}, \\ \boldsymbol{K}_2 \boldsymbol{d}_2 - \hat{\boldsymbol{K}}_2 \boldsymbol{T}_2 &= \boldsymbol{F}_{N2} + \boldsymbol{F}_{T2}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} \boldsymbol{K}_m = [K_{ik}^{BA}], &\qquad K_{ik}^{BA} = \int_{\Omega_m} E_{ijkl} \frac{\partial N^A}{\partial x_l} \frac{\partial N^B}{\partial x_j}\,\mathrm{d}V, \\ \hat{\boldsymbol{K}}_m = [\hat{K}_i^{BA}], &\quad \hat{K}_i^{BA} = \int_{\Omega_m} \alpha(3\lambda + 2G) N^A \frac{\partial N^B}{\partial x_i}\,\mathrm{d}V, \end{aligned} \tag{8}$$

$E_{ijkl} = \lambda \delta_{ij} \delta_{kl} + G(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$, λ and G are Lame's coefficients, respectively, and α represents the thermal expansion coefficient.

The energy balance for the first body reads

$$\boldsymbol{M}_1 \dot{\boldsymbol{T}}_1 + \boldsymbol{O}_1 \boldsymbol{T}_1 = \boldsymbol{Q} + \boldsymbol{Q}_{C1}, \tag{9}$$

where

$$\begin{aligned} \boldsymbol{M}_m = [M^{BA}], &\quad M^{BA} = \int_{\Omega_m} \rho c N^A N^B\,\mathrm{d}V, \\ \boldsymbol{O}_m = [O^{BA}], &\quad O^{BA} = \int_{\Omega_m} k \frac{\partial N^A}{\partial x_i} \frac{\partial N^B}{\partial x_i}\,\mathrm{d}V, \end{aligned} \tag{10}$$

ρ is the mass density, c is the heat capacity and k is the thermal conductivity. Furthermore, by introducing the contact conductance ϑ and the frictional dissipation at each contact pair as

$$\mu \boldsymbol{P} v, \tag{11}$$

we can define the heat power transferred at the first contact surface as

$$\boldsymbol{Q}_{C1} = \frac{\vartheta}{2} \boldsymbol{P} \circ (\boldsymbol{S}_2 \boldsymbol{T}_2 - \boldsymbol{S}_1 \boldsymbol{T}_1) + \frac{1}{2} \mu \boldsymbol{P} v. \tag{12}$$

Here, we have also introduced $\boldsymbol{S}_i$, where $\boldsymbol{S}_i^{\text{row}} = [\mathbf{0}\,[1]\,\mathbf{0}]$, in order to obtain the nodal temperatures at the contact surfaces. In a similar way, we can define the heat power transferred at the second contact surface as

$$\boldsymbol{Q}_{C2} = \frac{\vartheta}{2}\boldsymbol{P} \circ (\boldsymbol{S}_1\boldsymbol{T}_1 - \boldsymbol{S}_2\boldsymbol{T}_2) + \frac{1}{2}\mu\boldsymbol{P}v. \tag{13}$$

In the energy balance for the second body convective terms appear due to the speed v. These are represented by $\boldsymbol{N}\boldsymbol{T}_2$, where

$$\boldsymbol{N} = [N^{BA}], \quad N^{BA} = \int_{\Omega_2} \rho c v \frac{\partial N^A}{\partial x_1} N^B \,\mathrm{d}V, \tag{14}$$

Thus, the convection matrix $\boldsymbol{N}$ is non-symmetric. When this matrix dominates over the symmetric conduction matrix $\boldsymbol{O}_2$, then the thermal solution might be unstable. This might be stabilized by adding artificial conduction along the streamlines by $\boldsymbol{R}\boldsymbol{T}_2$, where

$$\boldsymbol{R} = [R^{BA}], \quad R^{BA} = \bar{k}v \int_{\Omega_2} \frac{\partial N^A}{\partial x_1}\frac{\partial N^B}{\partial x_1} \,\mathrm{d}V, \tag{15}$$

and $\bar{k}$ is an artificial conduction coefficient. By using (14) and (15), we obtain the following energy balance for the second body:

$$\boldsymbol{M}_2\dot{\boldsymbol{T}}_2 + (\boldsymbol{N} + \boldsymbol{R} + \boldsymbol{O}_2)\boldsymbol{T}_2 = \boldsymbol{Q}_{C2}. \tag{16}$$

3 NUMERICAL TREATMENT

The equations presented in the previous section are treated sequentially for each time step by decoupling the mechanical and thermal equations. That is, for a given temperature distribution the thermo-mechanical contact problem is first solved, then for the obtained contact force distribution the energy balance is solved. Details are presented in this section.

The contact problem is treated by the augmented Lagrangian approach. The key idea is to rewrite (4) as

$$\boldsymbol{P} = (\boldsymbol{P} + r(\boldsymbol{d}_{N1} + \boldsymbol{d}_{N2} - \boldsymbol{g}))_+\,, \tag{17}$$

where $r > 0$ is a penalty coefficient and $(x)_+ = (x + |x|)/2$. (7) and (17) are then put together to form an equation system as

$$\boldsymbol{h} = \boldsymbol{h}(\boldsymbol{d}_1, \boldsymbol{d}_2, \boldsymbol{P}, \boldsymbol{T}_1, \boldsymbol{T}_2) = \mathbf{0}. \tag{18}$$

This is a semi-smooth equation system which is efficiently solved by a Newton algorithm with an inexact line-search procedure for given temperatures $\boldsymbol{T}_m$.

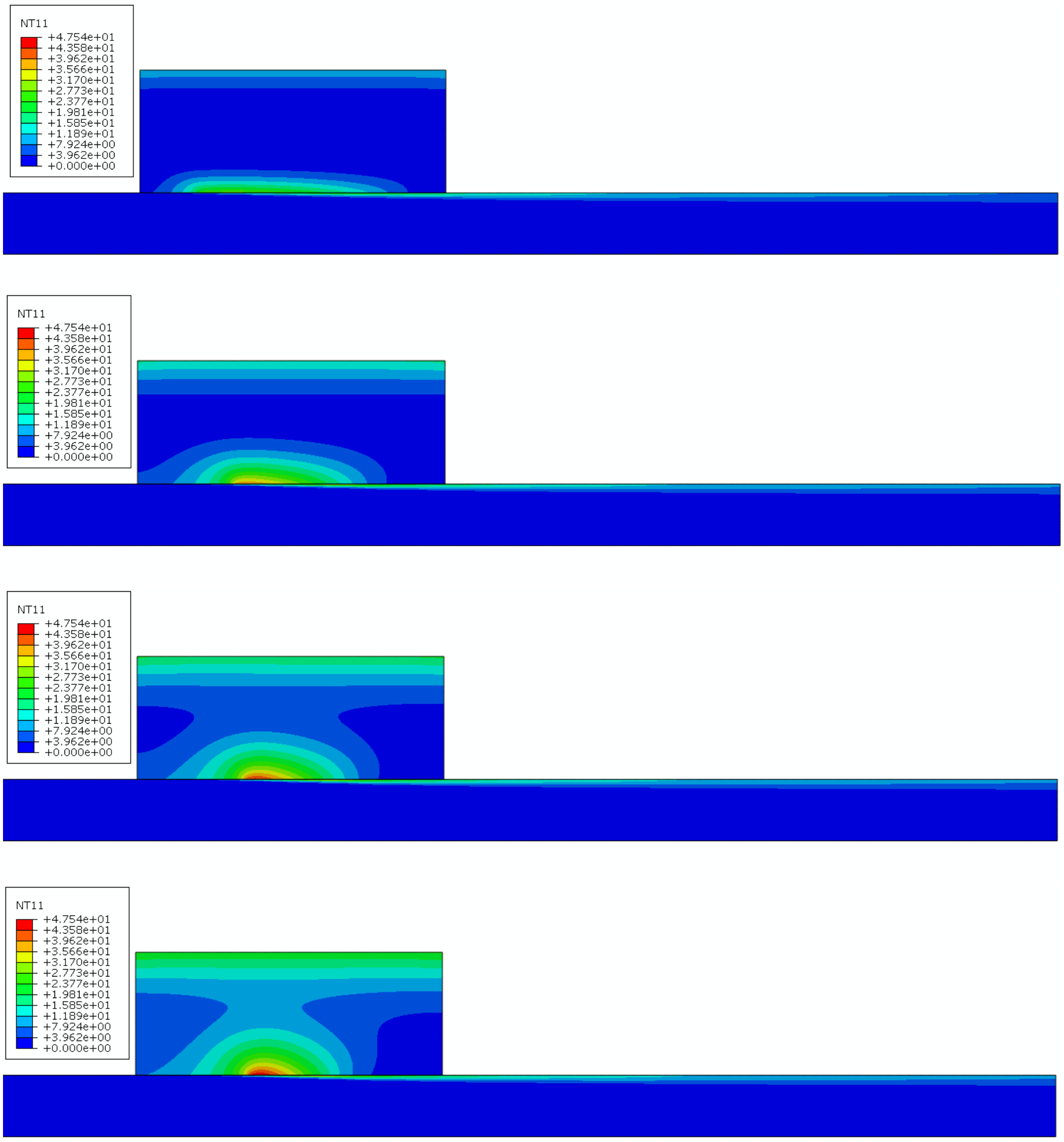

Figure 2: *The nodal temperatures plotted at different times: after 20, 40, 60 and 80 increments, respectively.*

The time rates appearing in the energy balances are discretized by the trapezoidal rule. Let $\boldsymbol{T}_m^n = \boldsymbol{T}_m(t_n)$ at time t_n, then the temperatures at the next time step t_{n+1} are updated according to

$$\boldsymbol{T}_m^{n+1} = \boldsymbol{T}_m^n + \Delta t \left((1-\xi)\dot{\boldsymbol{T}}_m^n + \xi \dot{\boldsymbol{T}}_m^{n+1} \right), \tag{19}$$

where $\Delta t = t_{n+1} - t_n$ and $\xi = 1/2$ (Crank-Nicolson) or $\xi = 2/3$ (Galerkin). (19) inserted in (9) and (16) yields

$$\begin{aligned} \left(\boldsymbol{O}_1 + \frac{1}{\xi \Delta t} \boldsymbol{M}_1 \right) \boldsymbol{T}_1^{n+1} &= \boldsymbol{Q}_1^{\text{eff}} + \boldsymbol{Q}_{C1}, \\ \left(\boldsymbol{N} + \boldsymbol{R} + \boldsymbol{O}_2 + \frac{1}{\xi \Delta t} \boldsymbol{M}_2 \right) \boldsymbol{T}_2^{n+1} &= \boldsymbol{Q}_2^{\text{eff}} + \boldsymbol{Q}_{C2}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} \boldsymbol{Q}_1^{\text{eff}} &= \boldsymbol{Q} + \frac{(1-\xi)}{\xi} \boldsymbol{M}_1 \dot{\boldsymbol{T}}_1^n + \frac{1}{\xi \Delta t} \boldsymbol{M}_1 \boldsymbol{T}_1^n, \\ \boldsymbol{Q}_2^{\text{eff}} &= \frac{(1-\xi)}{\xi} \boldsymbol{M}_2 \dot{\boldsymbol{T}}_2^n + \frac{1}{\xi \Delta t} \boldsymbol{M}_2 \boldsymbol{T}_2^n. \end{aligned} \tag{21}$$

Furthermore, (12) and (13) can also be written as

$$\begin{aligned} \boldsymbol{Q}_{C1} &= \boldsymbol{S}_{P2} \boldsymbol{T}_2 - \boldsymbol{S}_{P1} \boldsymbol{T}_1 + \boldsymbol{Q}_\mu, \\ \boldsymbol{Q}_{C2} &= \boldsymbol{S}_{P1} \boldsymbol{T}_1 - \boldsymbol{S}_{P2} \boldsymbol{T}_2 + \boldsymbol{Q}_\mu, \end{aligned} \tag{22}$$

where $\boldsymbol{S}_{Pi}$ and $\boldsymbol{Q}_\mu$ all depend on $\boldsymbol{P}$. By putting together (20) and (22), one obtaines an equation system on the following form:

$$\boldsymbol{A}(\boldsymbol{P}) \left\{ \begin{array}{c} \boldsymbol{T}_1 \\ \boldsymbol{T}_2 \end{array} \right\} = \boldsymbol{Q}(\boldsymbol{P}), \tag{23}$$

which of course becomes a linear system for given multipliers $\boldsymbol{P}$.

In conclusion, let $\boldsymbol{d}_m^n$, $\boldsymbol{T}_m^n$, $\boldsymbol{P}^n$ be given at time t_n, then $\boldsymbol{d}_m^{n+1}$, $\boldsymbol{T}_m^{n+1}$, $\boldsymbol{P}^{n+1}$ are obtained by the following steps:

Step 1:

$$\boldsymbol{h}(\boldsymbol{d}_1^{n+1}, \boldsymbol{d}_2^{n+1}, \boldsymbol{P}^{n+1}, \boldsymbol{T}_1^n, \boldsymbol{T}_2^n) = \boldsymbol{0}$$

is solved by Newton's method, details can be found in [8].

Step 2:

$$\left\{ \begin{array}{c} \boldsymbol{T}_1^{n+1} \\ \boldsymbol{T}_2^{n+1} \end{array} \right\} = \boldsymbol{A}(\boldsymbol{P}^{n+1})^{-1} \boldsymbol{Q}(\boldsymbol{P}^{n+1}).$$

Step 3:

$$\dot{\boldsymbol{T}}_m^{n+1} = -\frac{1-\xi}{\xi} \dot{\boldsymbol{T}}_m^n + \frac{\boldsymbol{T}_m^{n+1} - \boldsymbol{T}_m^n}{\xi \Delta t}.$$

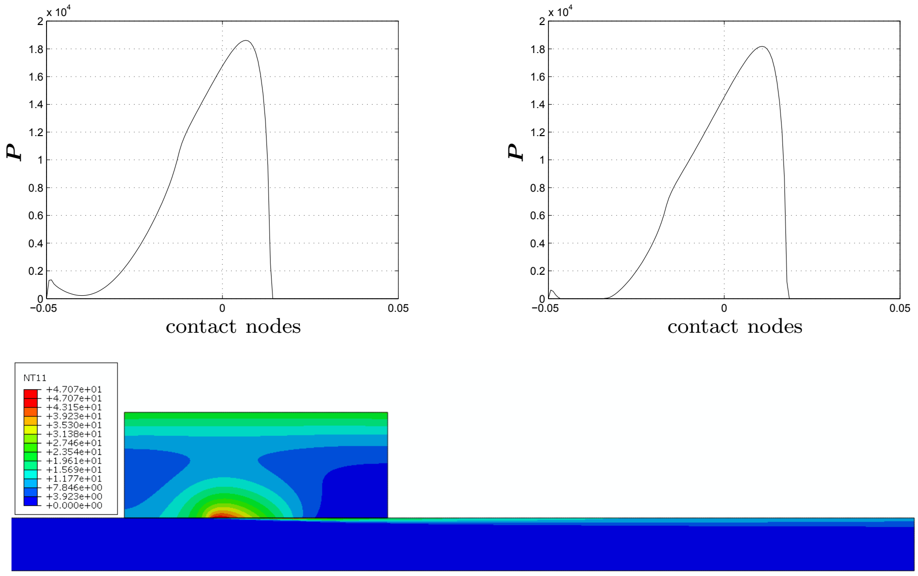

Figure 3: *A comparison when the friction force is neglected in the equilibrium equations.*

4 NUMERICAL EXAMPLES

The problem in Figure 1 is here considered as a numerical benchmark. The dimensions of the two bodies are taken to be 0.1x0.04 [m^2] and 0.5x0.02 [m^2], respectively. The plain strain assumption is adopted with a thickness of 1 [m]. The first body is meshed using 8151 elements and for the second body 20735 elements are used. Young's modulus is 2.1E11 [Pa], Poisson's ratio is 0.3, the expansion coefficient is 1.2E-5 [1/K], the density is 7800 [kg/m^3], the heat capacity is 460 [J/kgK], the conductivity is 46 [W/mK] and the conduct conductance is taken to be φ =1 [W/NK].

A total heat power of Q =5760 [W] is applied on the top of body Ω_1 as well as a total force of F =72E4 [N] (corresponding to a pressure of 7.2 [MPa]). Both the total heat power and the total force are equally distributed over all contact nodes on the top surface. The heat power is applied at time zero and the force is ramped up using a log-sigmoid function for 20 time increments. The problem is solved for 80 time increments with a constant time step Δt =0.125 [s]. The speed of the second body is v =1 [m/s]. Thus, the total sliding distance is 10 [m]. The evolution in temperatures for this problem when $\mu = 0.1$ are plotted in Figure 2.

The bottle-neck of the algorithm is to solve the linear system appearing in the Newton algorithm. Typically 4-6 such linear systems have to be solved for getting convergence in each time step. The system is also non-symmetric du to the friction force. One approach to speed up these calculation is to assume that the friction force has a little influence on the thermal solution. If the friction force is neglected in the mechanical problem, then only

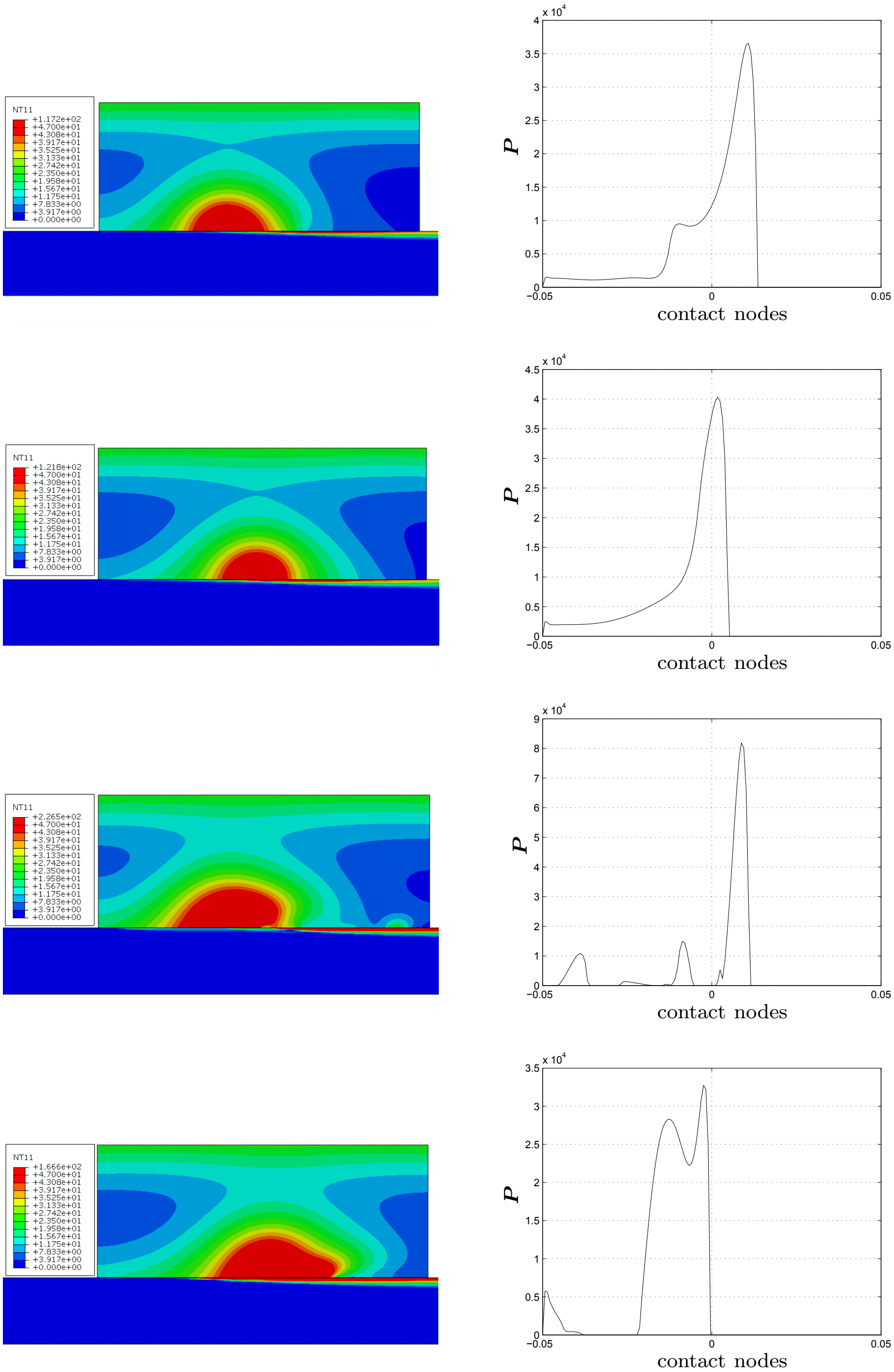

Figure 4: *Comparison of temperatures and contact forces for* $\mu = 0.2$ *and 0.3 when friction forces are included as well as neglected.*

2-3 Newton steps are needed and the linear system also becomes symmetric. In Figure 3 we have utilized this approach and compare the final solution to the original one presented in Figure 2. The resemblance of the two solutions are very close. A similar comparison is performed when the friction coefficient is taken to be 0.2 and 0.3, respectively. The results are presented in Figure 4. Also here a close resemblance between the results is obtained for the two different approaches, with and without frictional forces. The difference shown in the plots for $\mu = 0.3$ depends mostly on a time shift. This will be explained in more detail at the conference.

Another approach for speeding up the calculations but still consider the frictional force is to first solve the frictionless problem and then letting the friction force be defined by the obtained frictionless contact pressure, and solving the friction problem for this constant friction force. That is, at each iteration, (18) is first solved for $\boldsymbol{F}_{Tm} = \boldsymbol{0}$. Let $\hat{\boldsymbol{P}}$ denote the solution and then solve (18) again but now with $\boldsymbol{F}_{Tm} = \mu \boldsymbol{C}_{Tm}^T \hat{\boldsymbol{P}}$. In general, the number of iterations will be twice the number of iterations for the frictionless case. Of course, for this case, we will also have a symmetric Jacobian which is most beneficial for large size problems.

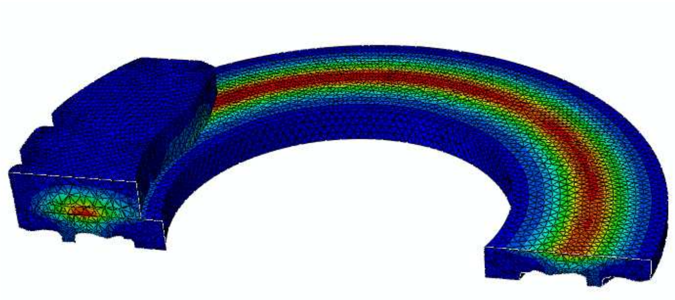

Figure 5: *A heat band developed in a disc brake.*

5 CONCLUSIONS

In this work a method for simulating frictional heating in sliding contacts is developed and implemented. A key idea of the approach is to use an Eulerian frame for the sliding object. The convective term appearing in this approach is stabilized by the streamline-upwind technique. The method seems promising. This is shown by solving a two-dimensional benchmark with a translating object for different coefficients of friction. The next step in the development will be to consider rotating objects. The ultimate goal is to solve frictional heating in disc brakes efficiently. A preliminary result is presented in Figure 5.

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