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MINIMIZATION OF THE PHENOMENON OF VIBRATIONS INDUCED BY VORTICES USING SURROGATE BASED OPTIMIZATION

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Abstract. In this paper we briefly describe the finite element procedure developed to simulate the two dimensional fluid-structure interaction of a rigid circular cylinder, immerse in an incompressible viscous fluid flow which is coupled with optimization techniques in order to minimize the amplitude of the cylinder displacements. Due to the high computational cost associated with the numerical simulations, surrogate models are built using kriging based data fitting scheme. The results obtained when the whole methodology is applied to the minimization of Vortex Induced Vibration (VIV) on Fluid Structure Interaction (FSI) problems are presented and discussed.

1 INTRODUCTION

Several applications, in different engineering fields, are subjected to vibration as a result of flow induced phenomena. Such behavior can compromise the integrity of the structure or make it uncomfortable for human use. The analysis of these problems, involve the study of a coupled fluid-structure interaction model and can be done using computational modeling.

This work uses a stabilized like Petrov-Galerkin “ALE” finite element formulation with Euler time-integration for the fluid-dynamics analysis [1]. This scheme represents an SUPG-like algorithm (Streamline Upwind Petrov-Galerkin [2] with the Fractional Step Method to stabilize the pressure field. For the structural analysis it uses a simple lumped model with three degrees-of-freedom and the Newmark Method [3]. The fluid-structural coupling is solved through interfacing and implemented and tested in a segregated approach, using an algorithm to control errors due to the existing time delay between the fluid and structural analysis [4].

In order to minimize VIV, here we employ two techniques. The first one is an acoustic signal, simulated as an increase of the boundary layer linear momentum, in order to control the flow field parameters, specifically the vortex shedding frequency. The second technique applied is the positioning of a plate behind the cylinder.

In order to minimize the transversal vibrations of the cylinder, the parameters involved in the cases studied were investigated by the application of optimization techniques. As optimization techniques commonly involves several calls of the numerical simulator, which may turn the optimization task into a very time consuming process, surrogate models using kriging based data fitting are employed in substitution to the coupled fluid-structure numerical simulations. The optimization algorithm of choice is the Sequential Quadratic Programming (SQP). This will be embedded here in an interactive procedure, named Sequential Approximate Optimization (SAO) [5]. A trust region based method is used to update the design variable space for each local (sub problem) optimization solution. As will be shown, the optimization process resulted in a reduction of up to 85% of the vibrations amplitude.

2 NUMERICAL FORMULATION

The coupling between fluid and structure fields is characterized by displacements of some of the boundaries of the domain. The regions close to these moving boundaries are more naturally discretized with a Lagrangean approach. The fluid regions away from the moving boundaries, however, are more naturally treated with a conventional Eulerian formulation, with a fixed reference frame. In this work, we use an Arbitrary Lagrangean Eulerian framework to combine these two approaches in a single numerical technique. The differential equations that describe the dynamics of the fluid and the structure therefore must be written in this framework. Hence, there are three different fields that characterize a fluid structure interaction problem: fluid dynamics, structure dynamics and mesh dynamics which will be described below.

2.1. Fluid Dynamics

The incompressible Navier-Stokes Equations, in an ALE description, without thermal effects, in continuous form, can be written as:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{c} \cdot \nabla)\mathbf{u} - \upsilon \Delta \mathbf{u} + \nabla p = \mathbf{f} \quad \text{in } \Omega \times (0,t) \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0 \qquad \text{in } \Omega \times (0,t) \tag{2}$$

where Ω is the spatial fluid domain, t is the time variable, (0,t) is the time interval, $\mathbf{u}$ is the velocity field, υ is the kinematic viscosity, p is the pressure, $\mathbf{f}$ is the external force vector, ∇ is the gradient operator and Δ is the Laplacian operator, and $\mathbf{c}$ is the relative velocity field between fluid and mesh. Considering that the Eqs. (1-2) must be manipulated in order to generate a Poisson equation for pressure field, the physical boundary was divided in two non-overlapping parts Γ_{du} and Γ_{nu} in which the Dirichlet and Neumann boundary conditions are prescribed to each equation, respectively. The

Dirichlet and Neuman boundary conditions are:

$$\mathbf{u}=\bar{\mathbf{u}} \quad \text{in } \Gamma_{du}, \quad p=\bar{p} \quad \text{and} \quad \mathbf{n}\cdot\boldsymbol{\sigma}=\bar{\mathbf{t}} \quad \text{in } \Gamma_{nu} \tag{3}$$

where $\boldsymbol{\sigma}$ is the viscous stress tensor, $\mathbf{n}$ is the unit outward normal vector, and $\bar{t}$ is the surface stress or traction. An upper bar refers to a prescribed value. Finally, initial conditions must be known in the whole domain at the initial time.

In this work, a Fractional Step method based in a LU factorization [1,6-9] was applied. In this method the final system is analogous to the method proposed by Chorin [10], and Temam [11], that applies Helmholtz decomposition. The final discrete system is obtained by using a θ method in time, resulting in a trapezoidal discretization, and a Finite Element Method for the spatial discretization [12, 13]. The stabilization of the advective and the gradient pressure terms are obtained with an orthogonal projection of these terms in a finite element space [7,14].

The system of equations from the discrete variational formulation is solved in a segregated way applying a Gauss-Seidel procedure. At the convergence of the block Gauss-Seidel, all system converges to the monolithic system. We are using an edge based data structure, which is advantageous in terms of CPU time, because, in the adopted procedure, most of the discrete terms do not need to be re-computed in each iteration by looping through the elements.

2.2. Structural Dynamics

In this work we only consider dynamics of rigid bodies. The movement of the body is obtained with a straightforward application of Newmark's Method [3,16]. The formulation used in this work leads to an implicit, second order accurate and unconditionally stable time integration scheme. The unconditional stability of this scheme is important because the time step increment for the structural time evolution is taken as the same as the time increment chosen for the CFD solution. This time increment is determined by the stability requirements of the CFD algorithm, and therefore its time scale is completely unrelated to the dynamic behavior of the structure.

2.3. Mesh Dynamics

In the domains with a ALE formulation, the movement of the interface nodes causes distortions on the original shapes of the elements connected to these nodes.

In order to avoid excessive elements distortion, in this work we are solving a modified Laplace equation for the mesh problem that is solved by Finite Element Method using an edge based data structure. The diffusivity coefficient is based on the volume of the elements, and is designed to smooth the distortions caused by the structure displacements [17,18].

The domain is divided as follow: Ω is the domain of the equation, Γ_m is the moving boundary (structure surface), and Γ_f is the fixed boundary. So the classical formulation of the problem results: given v on the boundaries, find v in the domain, such that:

$$\nabla\cdot\left(\left[1+\tau\right]\nabla\right)\mathbf{v}=0 \tag{4}$$

$$\mathbf{v}=\mathbf{v}_0 \text{ in } \Gamma_m \tag{5}$$

$$\mathbf{v}=0 \text{ in } \Gamma_f \tag{6}$$

where $\Gamma=\Gamma_m\cup\Gamma_f$, $\mathbf{v}$ is the displacement vector, $\mathbf{v}_0$ the moving boundary displacement, and it's required to solve the equation for each direction.

Note that the diffusivity coefficient must be defined in order that the smaller elements, generally defined close to the structure, where the small scale effects are present, suffer minimum deformations. This way, the displacements, are sent through the mesh until the bigger elements, close to the fixed boundaries, can smooth the displacements. Kanchi e Masud, [17], proposed the calculus of the coefficient to avoid excessive element deformations. The element coefficient used by Kanchi and Massud [17], Eq. (7), and adopted in this work is:

$$\tau^e = \frac{1 - V_{\min}/V_{\max}}{V^e/V_{\max}} \tag{7}$$

where $V_{\min} =$ minimum element volume of the mesh, $V_{\max} =$ maximum element volume of the mesh, $V^e =$ element volume e.

There are many practical aspects to a successful computational implementation of the procedures described above. Clearly, facilities for dealing with deformable domains, which involve automatic mesh generation, assessment of mesh quality, and automatic mesh movement are all important aspects. Also is important the choice of the coupling algorithm in order to warranty the kinematic and dynamic compatibility and the Geometric Conservation Law. All these were considered and implemented in this work [16].

3 PROBLEM DEFINITION

3.1 Acoustic Excitation

In the literature (Hiejima et al., [19] and references therein) several experimental and numerical results are reported in which acoustic excitation is applied to an external flow to increase the momentum transfer from the outside flow to the boundary layer and eliminating (or delaying) separation and suppressing (or reducing) vortex induced vibrations in different solid configurations. In this article, our computational system is used to perform an study on the behavior of the fluid-structure problem described by Hiejima et al. [19] in which an idealization of the acoustic excitation is obtained through the application of a periodic velocity excitation on two points at the cylinder surface (see Figure 1). The angle between the stagnation point and the excitation points at ϕ_a. The excitation velocity is given by:

$$V_a = U_a \sin(2\pi f_a t) \tag{8}$$

where, U_a and f_a refer to the periodic velocity excitation amplitude and frequency, respectively. The two excitation velocities are in phase.

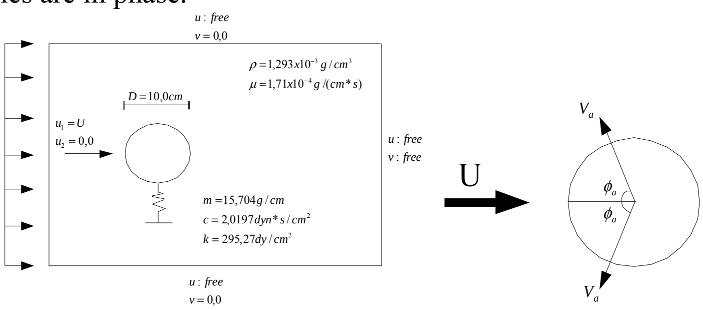

Figure 1: Computational model: data and boundary conditions, and description of the periodic velocity excitation on the surface of the cylinder

Figure 1 also shows the full description of the numerical model, including the: computational domain, boundary conditions, fluid properties and structural parameters of the mass-stiffness-damping system. The free stream velocity is U = 0.0264 m/s and the Reynolds number based on the cylinder diameter is 200. Initially, we performed some numerical simulations considering a fixed cylinder and the vortex shedding frequency obtained at Re = 200 was f_S = 0.043478 Hz. This value is in good agreement with the experimental curve presented by Blevins [20]. The effect of the periodic velocity excitation was initially investigated considering different ratio between the values of the excitation frequency (f_a) and the vortex shedding frequency (f_S), i.e. f_a / f_S = 1.00; 3.51 and 4.45, see Figure 2. Accordingly to Hiejima et al. (1997), the ratio value of 4.45 is close to the experimental value near the transition wave frequency, which is an effective value of frequency for an acoustic excitation to change the flow around a stationary circular cylinder. With such value of excitation they were able to get a considerable increase on the vortex shedding frequency that was quite effective in reducing the vortex induced vibration amplitude, as the experimental results suggests. We picked up two other values around 4.45 in order to study the influence of the excitation frequency on our results. Considering a fixed cylinder and the different ratio (f_a/f_s) mentioned previously, the frequency of the velocity transversal to the flow in a point located inside the vortex shedding region behind the cylinder was studied.

The same analyses were performed considering the cylinder free to vibrate in the direction transversal to the flow. The numerical simulation set up consists of starting with a fixed cylinder and after the vortex shedding becomes periodic we allow the transverse movement, and after the vibration amplitude stabilizes on a constant value we start applying the periodic excitation. In all three cases the cylinder is set free when the time is around 80 seconds and the excitation starts when the time is around 180 seconds. In Figure 2 the displacement histories are plotted for f_a / f_S = 1.00, 3.51 and 4.45, respectively. For f_a / f_S = 4.45, there is reduction on the oscillatory amplitude, and the adopted excitation frequency has an effective effect. The results suggest that an even bigger variation on the vortex shedding frequency might reduce more or even suppress the vibration on the cylinder.

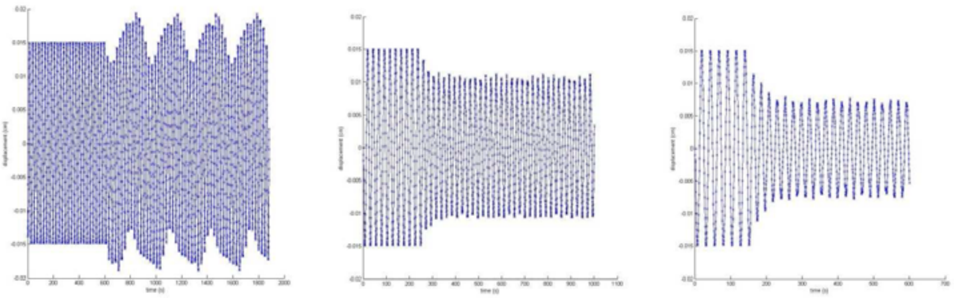

Figure 2: Time history of transversal displacement for f_a/f_s=1.00, 3.51, 4.45, respectively.

It should be observed that the amplitude of the oscillations were small with the cylinder vibrating under the influence of the vortex formation and shedding behind the cylinder, and that the characteristic of the vortex induced vibrations were directly affected by the change on the frequency of such vortex formation and shedding. Also, further investigation considering different ration between the values of the excitation frequency (f_a) and the vortex shedding value (f_S), and also considering different application points and amplitude of excitation were pursued in order to gain a better insight on the behavior of this application to conduct further studies considering optimization techniques.

3.2 Positioning of a plate behind the cylinder

The model described by Zdravkovich [21] and studied by Correia [22] for the suppression (or reduction) of vortex shedding consists in positioning a flat plate behind the cylinder, where the vortex street is developed. The length of the plate is assumed to be equal to the one studied by Correia [21], fixed as 1.14 times the cylinder diameter. The position of the plate relative to the cylinder is the design variable of the problem (***c***).

In order to compare the results with those obtained by the application of the acoustic excitation, the physics and geometric parameters were considered the same for both cases. Figure 3 shows the geometry scheme adopted for this case.

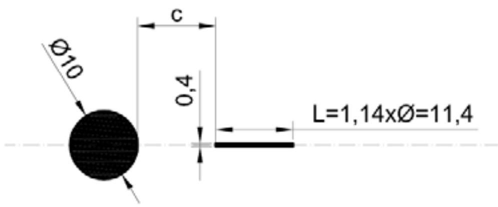

Figure 3: Problem definition to solution by positioning a plate behind the cylinder.

3.3 Formulation of the Optimization problem

Mathematically, a general optimization problem is formulated as:

$$\begin{aligned} &\text{Minimize } f(\boldsymbol{x}) \\ &\boldsymbol{x} \in \mathbb{R}^{ndv} \\ \text{Subject to: } &h_k(\boldsymbol{x}) = 0, \qquad k = 1,\dots,ne \\ &g_i(\boldsymbol{x}) \le 0, \qquad i = 1,\dots,ni \\ &x_{lj} \le x_j \le x_{uj}, \quad j = 1,\dots,ndv \end{aligned} \tag{9}$$

in which ***x*** are the design variables. The function *f(x)* is the objective of the problem. The functions $g_i(x)$ and $h_k(x)$ represent, respectively, the inequality and equality constraints. The side constraints have inferior limits x_l and superior limits x_u. *ne*, *ni* and *ndv* are, respectively, the numbers of equality constraints, inequality constraints and design variables.

In preparation for the optimization process, for the particular problems here addressed, some procedures are required to be previously performed. Considering the cylinder fixed, a fluid problem at Re=200 is analyzed until is ensured the stabilization of the vortex shedding frequency. This frequency is calculated and registered through the transversal velocity of a point inside the vortex street behind cylinder. Then, the analysis proceeds and the cylinder's displacements are released, turning the simulation into a fluid-structure interaction problem. At this time, the structure's displacements start to increase until stabilization, when finally a vortex shedding suppression technique is ready to be applied.

When considering optimization tools, the objective function is the amplitude of the transversal displacements of the cylinder.

For the particular problem of the acoustic excitation, it should be observed from Figure 4 (a) that simulation’s time after releasing the cylinder can be divided into three intervals. The first one corresponds to the time interval before the application of the acoustic excitation, followed by the second interval, when the acoustic excitation is applied and a modification on amplitude is observed. On the third and last interval, between 300 and 600 seconds, the amplitude of the cylinder’s transversal displacements is considered stabilized and the absolute maximum value of transversal displacement is calculated and sent to optimization algorithm as the objective function value. The parameters to be changed (design variables (dv) candidates) are: frequency, amplitude and the location of the acoustic excitation.

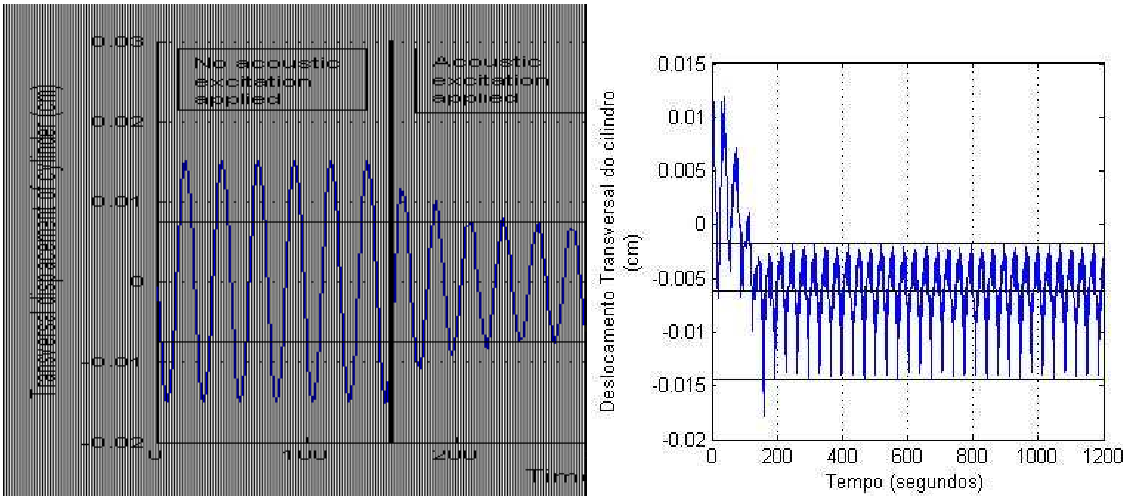

Figure 4: Time history of transversal displacement of cylinder for (a) Acoustic excitation problem and (b) Flat Plate Problem

The objective function of the plate problem is evaluated by an analogous way to that of the acoustic excitation, but at the same time that cylinder is released, the plate slowly starts to move from its initial position to the position defined by the optimization algorithm. The movement of the plate to its final position lasts up to 200 seconds of simulation. For this case, the stabilization of the cylinder vibrations is considered only on the interval defined by the final 300 seconds of simulation. During the preliminary tests, it was observed that after the release of the cylinder, an initial and larger displacement is observed and the cylinder assumes a new position of vibration. This effect is shown on Figure 4 (b). Once we are here interested in the reduction of vibrations, this displacement was not considered for evaluation of the objective function. So, the amplitude of vibration considered is relative to its final position of vibration.

4 SURROGATE MODEL

The challenge in the process of surrogate model construction is to provide a substitute model with sufficient accuracy. Several strategies can be used to build the approximated model [23]. Here, kriging (ordinary) based data fitting approach is considered. The main idea of this model is to assume that errors are not independent but rather assume that the correlations between errors is related to the distance between corresponding points modeled by a Gaussian process around each sample point. The main advantages of this scheme are to easily accommodate irregularly distributed sample data, and the ability to model multimodal functions with many peaks and valleys. Kriging models provide exact interpolation at the sample points. Details of such procedure can be seen elsewhere [23].

5 OPTIMIZATION STRATEGY

In order to obtain the optimum design of the problem tackled in this work the SAO methodology will be employed. As surrogate models have a limited range of accuracy, the design space of the approximate optimization problem is restricted to the sub region called trust region whose dimensions are adaptively managed by the SAO strategy depending on surrogate accuracy [5]. Mathematically, each sub problem k is defined as:

$$
\begin{aligned}
&\text{Minimize } \hat{f}(x) \\
&\text{Subject to} \\
&\hat{g}^k(x) \le 0, i = 1,...,m \;\; \hat{g}^k(x) \le 0, i = 1,...,m \\
&x_l \le x_l^k \le x \le_u^k \le x_u ,\; k = 0,1,2, ...,\mathrm{k}_{\max} \\
&\text{Where} \\
&x_l^k = x_c^k - \frac{\Delta^k}{2} \text{;}\; x_u^k = x_c^k + \frac{\Delta^k}{2}
\end{aligned}
\qquad (10)
$$

In the above equations, $\hat{f}(x)$ and $\hat{g}(x)$ are, respectively, the surrogate objective and constraints functions, x_c^k is the center point of the trust region, Δ^k is the width of the trust region and x_l^k , x_u^k are, respectively, the lower and upper bounds of the design variables at the k^{th} SAO iteration [5].

5.1 Reuse of samplings

When considering the original algorithm of the SAO methodology, the high fidelity model simulations are conducted on a set number of points in the trust region for each iteration. The selected output computed at the samplings are used to build the surrogate model in the sub region of the design space. In the next iteration, other points (same quantity) are selected to build another surrogate model.

In this work, the algorithm used for application of SAO strategy is modified. The modification consists in save the samples of the high fidelity model in a data base during the SAO process. Then, in each SAO iteration, this data base is consulted in order to check the possibility to reuse the samples for the new surrogate model construction. With this approach, the information obtained by a high computational cost can be reused during the process. The details of such scheme can be found in [24].

6 EXAMPLES

6.1 Optimization of the acoustic excitation problem

An optimization problem considering as design variables the frequency and the amplitude is solved by two versions of SAO procedure. Initially, the SAO algorithm already used on the optimization of single design variables is considered. Next, some modifications were applied to the algorithm, resulting in a potential gain of performance and robustness. The modification encompasses the reuse of samplings from previous SAO iterations that are located in the current sub problem trust region. Those points together with the generated samples of current sub region make a richer model to build the surrogate to be used in the current SAO iteration. This higher level of information allows surrogate models to represent a better approximation to the high fidelity model without computational cost addition, once those samples were already evaluated which in the previous SAO version were being discarded. A preliminary study [24] considering an analytic function shows the potential gain of performance with the modifications of the algorithm.

For the problem here analyzed, using both SAO strategies, the angle was fixed as 80º and the initial design values for frequency and amplitude were considered 4.00 and 65 cm/s respectively, in design spaces limits between 0 to 10 and 50 to 90 cm/s.

In the initial SAO version, whose results are presented on Figure 5 (a) to (c), the parameters that lead to the maximum reduction of vibrations of the cylinder were found in nine SAO iterations. Relative to the initial design, the optimum value found represents a reduction of approximately 50%.

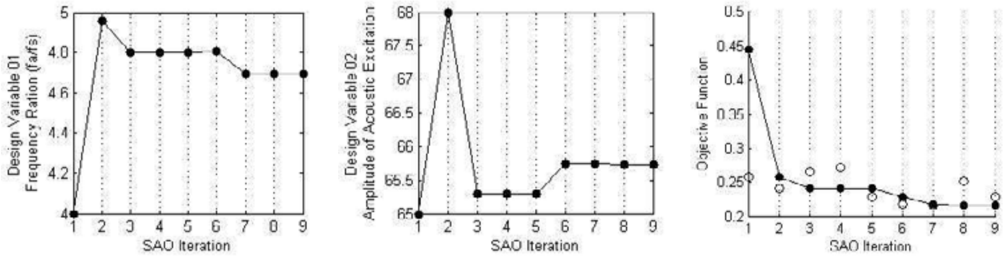

Figure 5: 2 dv problem (traditional SAO) – iteration histories: (a) Frequency, (b) Amplitude and (c) Objective function.

Figure 6 (a) to (c) present the results of SAO procedure with re-use of samples from previous iterations.

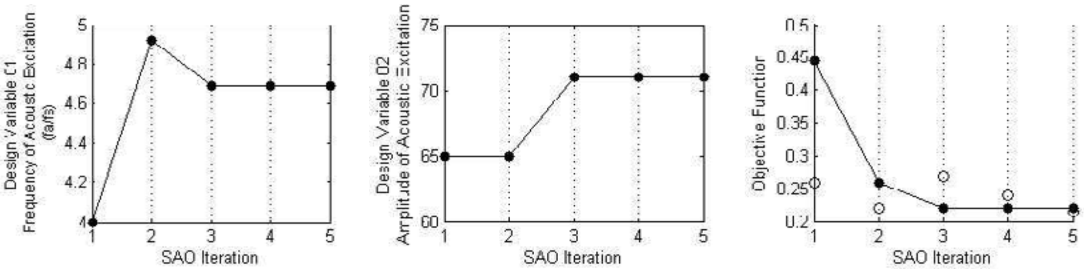

Figure 6: 2 dv problem (SAO with re-use sampling) – iteration histories (a) Frequency, (b) Amplitude and (c) Objective function.

It can be observed that with the later SAO alternative a reduction on the number of iterations when compared with the traditional procedure, which represents an economy of computational cost of almost a half. Moreover, the optimum values found with both procedures were similar.

Table 1 and 2 present in summary the numerical results obtained using the SAO strategy for the optimization of the acoustic excitation problem. The shaded cells of Table 1 refer to the fixed variables on the optimization process. In both tables the following cases are presented:

- SAO 01: Two design variables optimization: frequency and amplitude of acoustic excitation (traditional SAO scheme);
- SAO 02: Two design variables optimization: frequency and amplitude of acoustic excitation (SAO with re-use sampling scheme);

Table 1: Summary of SAO procedures – Design Variables.

Optimization Procedure	Initial design			Optimum design		
	Frequency	Amplitude	Angle	Frequency	Amplitude	Angle
SAO 03	4.0000	65.0000	80.0000	4.6943	65.7285	80.0000
SAO 04	4.0000	65.0000	80.0000	4.6894	70.9660	80.0000

Table 2: Summary of SAO procedures - Objective Function and number of iterations.

Optimization Procedure	SAO Iterations	Objective Function		Objective Function Reduction
		Initial Point	Optimum Point	
SAO 04	9	0.4446	0.2170	51,2%
SAO 05	5	0.4446	0.2210	50,3%

The re-use scheme highlights its capability in terms of SAO iterations. Other optimizations processes considering one and three design variables were also conducted and can be found in [24].

6.2 Optimization of the flat plate problem

As described previously, only one design variable was considered for the problem of positioning a flat plate behind the cylinder. This design variable is the distance between the plate and cylinder, and is normalized by the cylinder diameter.

The initial design variable value considered is 3.50, based on the parametric study presented in previous section. The design space was defined between 1.45 and 4.34, referred by the studies of Correia [22]. The SAO algorithm that reuses samples is used for this problem. After five SAO iterations, the strategy converged to a minimum of 0.1362. This value represents a reduction on cylinder vibrations of almost 87% related to the original configuration (without plate). Taking the initial objective function value as reference, the reduction of the objective function on the optimum solution is equal to 3%. However, it is important to highlight that the initial point chosen was expected to be near the optimum, based on the parametric study previously conducted.

Figure 7 (a) and (b) present, respectively, the histories of the design variable and the objective function over the SAO iterations of this problem.

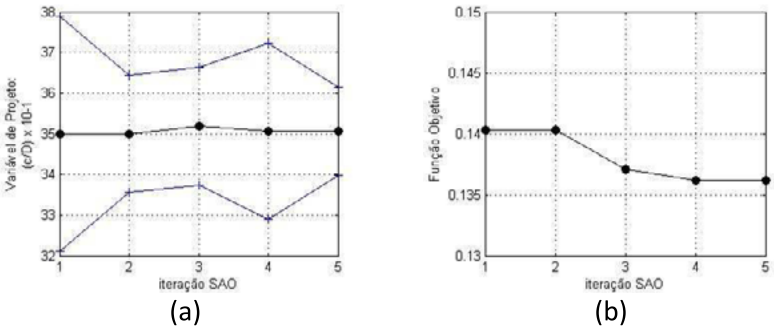

Figure 7: Flat plate problem – SAO histories: (a) design variable and (b) objective function.

7 CONCLUSIONS

In the present work an integrated tool to solve fluid-structure interaction problems. The VIV problem was investigated obtaining very good results. The overall conclusions of present study are:

- The use of surrogate models was fundamental to conduct the optimization of the fluid structure interaction problems.
- Reuse of samplings at SAO algorithm provided computational savings and increased its robustness.
- The optimization considering acoustic excitation represented 80% vibration reduction. So far the best literature reported improvement is 60%.

- For the flat plate behind the cylinder problem, the application of the developed optimization tool reduced the cylinder vibrations in 86%.
- The techniques employed were satisfactory for its aim and should be applied to other examples.

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REFERENCES

1- Antunes, A. R. E., 2008, "A computational system for the analysis of 3D incompressible fluid flow using a Fractional Step Formulation with Edge based FEM and Parallel Processing", D.Sc. Thesis, Recife, Brazil (*in Portuguese*).
2- Brooks, A. N. & Hughes, T. J. R., "Streamline Upwind/Petrov-Galerkin Formulations for Convection Dominated Flows with Particular Emphasis on the Incompressible Navier-Stokes Equations", Computer Methods in Applied Mechanics and Engineering, vol. 32, pp. 199-259, 1982.
3- Hughes, T. J. R., 1987, "The Finite Element Method", 1ª Edition, United States of America, Prentice-Hall, Inc., 803p
4- Bloom, J. F. and Leyland, P., 1998, "Consistency Analysis of Fluid-Structure Interaction Algorithms", European Cong. on Comp. Methods in App. Science and Eng. (ECCOMAS), Barcelona, Spain, 15 p. (in CD-ROM).
5- Alexandrov, N., Dennis Jr, J.E., Lewisand, R.M. and Torezon V., A Trust Region Framework for Managing the Use of Approximation Models in Optimization. NASA/CR-201745; ICASE Report No. 97-50, 1997.
6- Antunes, A. R. E., Lyra, P R. M., Da Silva, R. S., Wilmmersdorf, R. B., 2009, "An Implicit Monolithic Finite Element to Solve 3D Incompressible Navier-Stokes Equations using Fractional Step Method and Parallel Edge-Based Implementation", 20th International Congress of Mechanical Engineering, Gramado, RS, Brazil, 10p.
7- Codina, R., 2002. "Stabilized finite element approximation of transient incompressible flows using orthogonal subscales", Comp. Meth. Applied mechanics and Engineering, vol 191, p. 4295-4321.
8- Chang, W., Giraldo, F., Perot, B., 2002, Analysis of an exact fractional step method, J. Computational Physics, vol. 180, p. 183-199.
9- Henriksen, M. O., Holmen, J., 2002. Algebraic splitting for incompressible Navier-Stokes equations, J. Computational Physics, vol. 175, p. 438-453.
10- Chorin, A. J., 1967, "A Numerical Method for Solving Incompressible Viscous Problem", J. Comput. Phys., vol. 2.
11- Temam, R., 1969, Sur l'approximation de la Solutin des Équaciones de Navier-Stokes par la Méthode dês pás Fractionaires (I), Arch. Ration. Mech. Anal., vol. 32 (*in French*).
12- Gresho, P. M., Sani, R. L., 1998. Incompressible flow and the finite element method, John Wiley & Sons, vol. 2.
13- Zienkiewicz, O. C., Taylor, R. L., 1988, "The finite element method: basic formulation and linear problems", vol 1, MacGraw-Hill, 4ª edition.

14- Soto, O., Löhner, R. and Cebral, J., 2001, "An implicit monolithic time accurate finite element scheme for incompressible flow problems", In: 15th AIAA Computational Fluid Dynamics Conference, Anaheim, EUA, June.
15- Soto, O., Löhner. R., Cebral, J. and Camelli, F. 2004. "A stabilized edge-based implicit incompressible flow formulation", Comp. Meth. Applied Mechanics and Engineering, vol. 193, p. 2139-2154.
16- Antunes, A. R. E., Lyra, P. R. M., Willmersdorf, R. B., 2005, "A Methodology and Computational System for the Simulation of Fluid-Structure Interaction Problem". J. of the Braz. Soc. Of mech. Sci. & Eng., Vol. XXVII, No.3, p. 255-265.
17- Kanchi, H., & Masud, A., 2007, "A 3D adaptive mesh moving scheme", Int. Journal for Num. Meth. In Fluids, vol. 54, pp 923-944.
18- Liberato, B. E., Antunes, A. R. E., Lyra, P. R. M., 2010, " Using Laplace equation and Finite Element Method for the solution of Mesh Movement problem in Fluid-Structure applications", VI National Congress of Mechanical Engineering, Campina Grande, PB, Brazil, 11p, (*in Portuguese*).
19- Hiejima, S., Nomura, T., Kimura, K. & Fujino, Y., 1997. Numerical study on the supression of the vortex-induced vibration of a circular cylinder by acoustic excitation. *Journal of Wind Enginnering and industrial aerodynamics*, pp. 325-335.
20- Blevins, R. D., 1986, "Flow-Induced Vibration", R. E. Krieger Publishing, Inc, Malabar/Florida, USA, 363 p.
21- Zdravkovich, M., 1981. Review and classification of various aerodynamic and hydrodynamic means for suppressing vortex shedding. *Eng. & Ind. Aero.*, Issue 7, pp. 145-189.
22- Correia, A. C., 2001. "Computational simulations of vortices formations on a circular cylinder with emphasis on Vortex Induced Vibrations". M. Sc. Thesis. Recife, Brazil (*in Portuguese*)
23- Forrester, A., Sobester, A., Keane, A., Engineering Design via Surrogate Modeling: A Practical Guide. Wiley, 2008
24- Farias Filho, U. P., 2012. "Minimization of the phenomenon of Vibrations Induced by Vortices using Sequential Approximation Optimization", M.Sc. Thesis, Recife, Brazil (*in Portuguese*).