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A HIGH-ORDER FULLY COUPLED ELECTRO-FLUID-DYNAMICS SOLVER FOR MULTIPHASE FLOW SIMULATIONS

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Abstract. *A high-order discontinuous Galerkin Finite Element solver is developed for solving electro-fluid-dynamics problems. The solver is employed to perform numerical simulations of deformation of a droplet suspended in another immiscible liquid by applying steady and oscillatory electric fields. The level set method is adopted to represent the common interface of the droplet and surrounding medium. Electrostatics equation with a jump in the dielectric property at the interface is solved to find the electric field distribution. The incompressible Navier-Stokes equations including the surface tension force are solved to find the flow field. The Electrostatics and Navier-Stokes equations are coupled through changes in the geometry because of the deformation of the droplet and the dielectrophoretic body force, which is present at the interface.*

1 INTRODUCTION

Numerical simulations of deformation of a droplet in steady and oscillatory electric fields are performed in the present study. The droplet, which is shown in figure 1, is suspended in another immiscible fluid with the same density and viscosity but a different dielectric property (permittivity). The droplet and surrounding fluid are considered as perfect dielectrics. By applying an electric field, the fluids are polarized and because of the jump in the dielectric property, the dielectrophoretic force exerts at the interface of the droplet and surrounding fluid. The droplet continues to deform until a force balance between the electric force, pressure and surface tension force is achieved and the droplet becomes a spheroid, see e.g. Torza et al. [1]. The deformation of the droplet is defined in figure 1.

A two-way coupling exists between the fluid and electric sub-problems. On one-hand, the electric force exerts at the interface of the droplet and surrounding fluid and on the other hand, the deformation of the droplet changes the geometry for the electric field computation. Therefore, an electromechanical approach is required, which includes solving the governing equations of both electric and fluid fields, computing the electric force and capturing the movement of the interface of the droplet and surrounding fluid. Supeene et al. [2] have considered a moving mesh approach to find the movement of the interface, which is suitable for small deformations. Hua et al. [3] have used a front tracking/finite volume method. In the present study, a high-order discontinuous Galerkin Finite Element method (DG) is employed and a one-fluid approach is followed, which enables us to solve one set of the governing equations for the droplet and surrounding fluid.

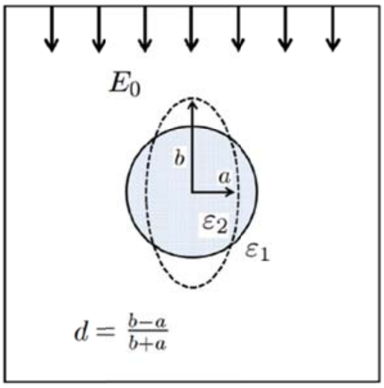

Figure 1: A droplet suspended in another immiscible liquid with a different dielectric property. Deformation of the droplet, d, in response to applying an electric field is shown.

2 METHODOLOGY

The interface is represented as the zero iso-value of a level set function, φ. The governing equations are the electrostatics, continuity, incompressible Navier-Stokes and level- set advection equations:

$$\begin{aligned} &\nabla \cdot \left(\varepsilon(\varphi)\nabla\Phi\right) = 0, \\ &\vec{E} = -\nabla\Phi, \\ &\nabla \cdot \vec{u} = 0, \\ &\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u} = -\nabla p + \frac{1}{Re}\Delta\vec{u} - \frac{1}{We}\delta(\varphi)\kappa\vec{n} - \frac{1}{2}|\vec{E}|^2\,\nabla\varepsilon(\varphi), \\ &\frac{\partial \varphi}{\partial t} + \vec{u}\cdot\nabla\varphi = 0, \end{aligned} \tag{1}$$

where ε represents the dielectric property and Φ is the electric potential. For the case of steady or slowly oscillating electric fields, the electrostatics equations are valid, where the electric field, $\vec{E}$, is defined as the gradient of the electric potential and a Laplace equation is solved to compute the electric potential. A diffuse interface model is used to regularize the jump in the dielectric property at the interface. A signed-distance level set function is used in combination with the diffuse interface model to assure the same thickness of the interface everywhere. The solution algorithm is shown in figure 2. For more information see Emamy [4].

The governing equations are discretized using the DG method within an in-house CFD code, Kummer [5]. The DG method employs a high-order local polynomial representation of the solution. However, the locality of the solution makes it discontinuous at the cell boundaries. To solve the incompressible Navier-Stokes and continuity equations, a projection scheme is employed. Considering that the inter-cell discontinuity of the solution may be large in case of low resolution (coarse grids/low polynomial degrees), the projection scheme is adapted for the DG method, Emamy [6], to provide a long-term stable and accurate scheme. Moreover, a DG weak-formulation is used to compute derivatives of the flow field variables and electric potential.

The surface tension force is modeled using the continuum surface force model (CSF), Brackbill et al. [7]. To decrease the intrinsic spurious velocities, the surface tension force is computed by using high-order polynomials for computation of the normal vector, $\vec{n}$, and curvature, κ. The normal vector is computed as the gradient of a signed-distance level set function, φ_{SD}. The curvature is the divergence of the normal vector by definition. A reinitialization equation is solved in each time step to find φ_{SD} from the level set function φ, which provides the position of the interface by solving the level set advection equation, Mousavi [8].

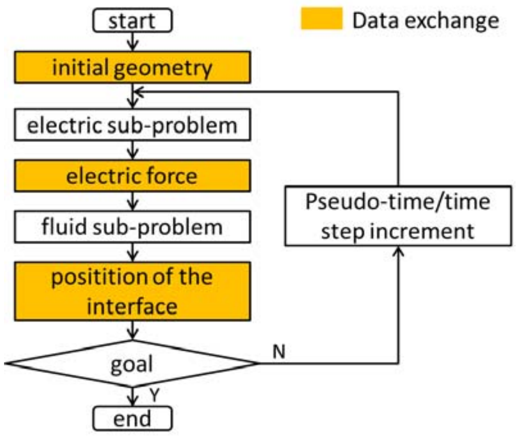

Figure 2: The solution algorithm for the coupled Electro-Fluid-Dynamics problem.

To find the electric potential, the Laplace equation is discretized using an interior penalty method (IP-MCP), which Emamy [4] has provided for the case of a regularized jump in the coefficient (dielectric property). After solving for the electric potential, the electric field and electric force are computed. The electric force is added as a body force to the Navier-Stokes equations, (1).

3 NUMERICAL RESULTS

To perform the numerical simulations, a Reynolds numbers of 1 is considered because the physical problem, which we consider, is a creeping flow. The numerical settings and boundary conditions from Karcher [9] are shown in figure 3. In figure 4, a test case with a negligible surface tension force is considered. In absence of the surface tension force, the droplet continues to deform in response to the electric force and there is no equilibrium state. The droplet preserves the shape of an ellipse while deforming. A convergence study for this test case is performed by grid refinement, which is shown in figure 5. Using polynomial degrees of 5 for the flow field variables, a convergence rate of 3.25 is achieved for the coupled problem. A reduced convergence rate is expected because of the regularization of the jump in the dielectric property at the interface.

Including the surface tension force, in figure 6, effect of the size of the computational domain and boundary conditions on the deformation of the droplet in the equilibrium state, d_∞, is studied. Dirichlet velocity (wall) and Dirichlet pressure boundary conditions are compared. For larger domains, influence of the boundary conditions becomes smaller. For this test case, which includes the surface tension force, we have applied the capillary time step size restriction [7] and modified it for the high-order DG method as

$$\Delta t_C = \sqrt{\frac{We\, h^3}{2\pi\, k^6}}\,, \tag{2}$$

where, We is the Weber number, h is the element size and k is the polynomial degree for the pressure and velocity.

As a final test case, an oscillating electric potential difference with frequency of 1 is applied as the boundary condition. Deformation of the droplet versus time is shown in figure 7, which has a period of 0.5. This means that the droplet oscillates with a frequency that is twice the excitation frequency. This frequency is expected by the presence of $|\vec{E}|^2$ in the formulation of the dielectrophoretic force as a body force in the Navier-Stokes equations.

4 CONCLUSIONS

The coupled electro-fluid-dynamics simulations, predict the expected physical behavior of a perfect dielectric droplet suspended in another perfect dielectric immiscible fluid when steady and oscillatory electric fields are applied. Applying a steady electric field the droplet deforms to an ellipse for 2D simulations. In case of the negligible surface tension force the droplet continues to deform until it bursts. In presence of the surface tension force the droplet reaches an equilibrium state. If the computational domain is large enough the effect of boundary conditions on the deformation in equilibrium state is negligible. Considering the surface tension force, the capillary time step size restriction is modified for the high-order DG method. In case of an oscillatory electric field the droplet oscillates with a frequency, which is twice the excitation frequency.

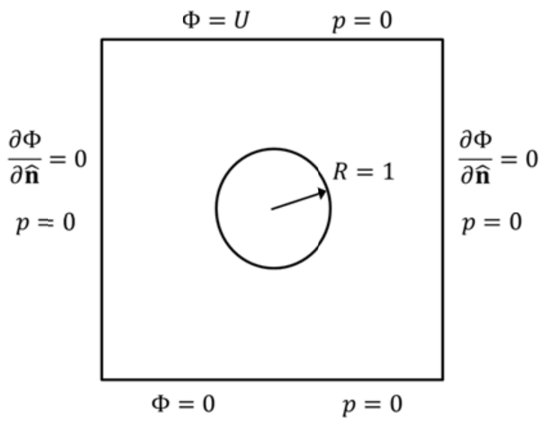

Figure 3: A droplet suspended in another immiscible liquid with a different dielectric property. Boundary conditions for the fluid and electric sub-problems are shown.

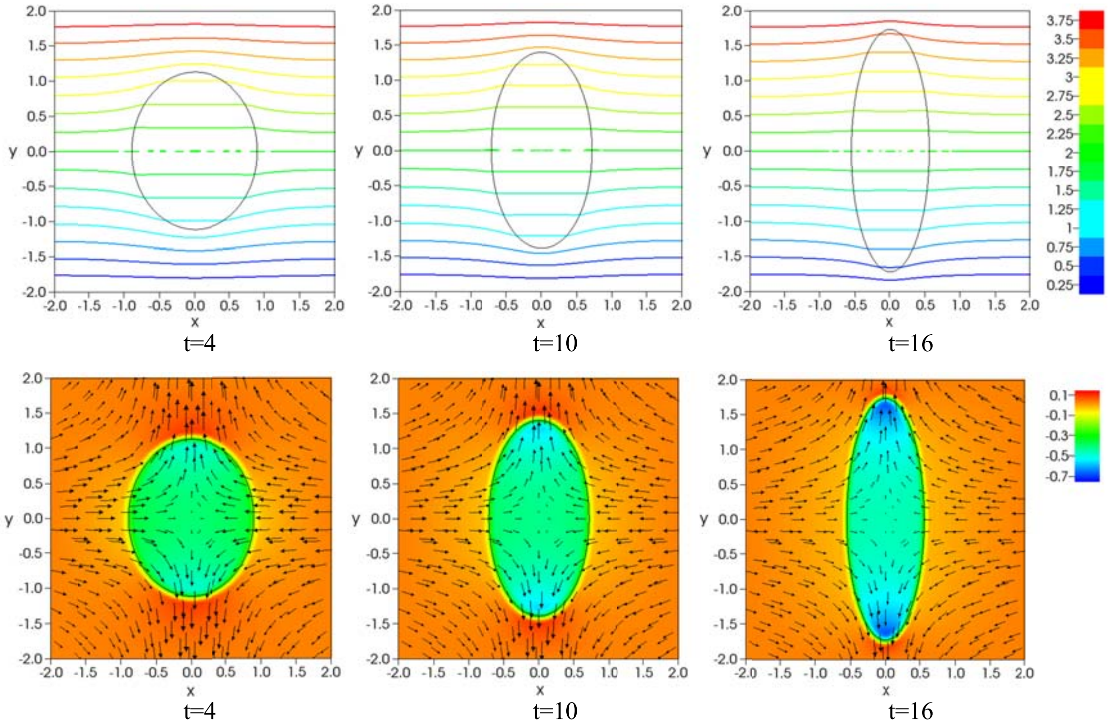

Figure 4: A droplet, which is suspended in another immiscible fluid with a jump ratio of 2 in the dielectric property at the interface, is considered. The surface tension force is neglected. Iso-lines of the electric potential are shown on the first row. Pseudo-colors of the pressure and vectors of the velocity are shown on the second row. An electric potential difference of 4 is applied. The computational domain is a [-2, 2]×[-2, 2] square. The half-thickness of the regularized interface is 1/8. A Cartesian grid of 24×24 cells is employed. Polynomial degree of 8 for the electric potential and 5 for the pressure, velocity and level set function are used. A time step size of 0.01 is used.

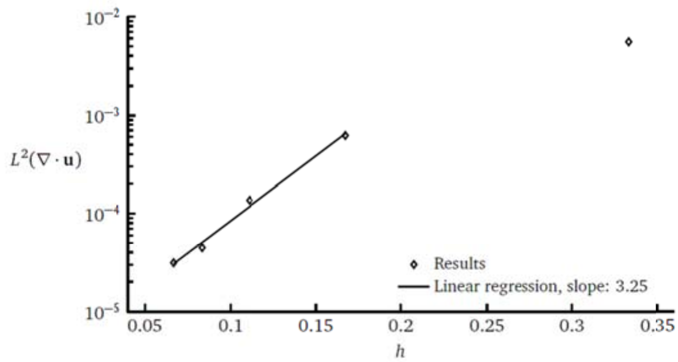

Figure 5: Convergence study is performed for the test case of a droplet suspended in another immiscible fluid with a jump ratio of 2 in the dielectric property at the interface. L2-norm of divergence of the velocity is shown, as a measure for the error, vs. the element size, h, after the first time step. The computational domain is a [0, 2]×[-2, 2] quadrilateral, where a symmetry boundary is used at x=0 axis. The half-thickness of the regularized interface is 1/7. Polynomial degree of 8 for the electric potential and 5 for the pressure, velocity and the level set function are used. A time step size of 0.01 is applied.

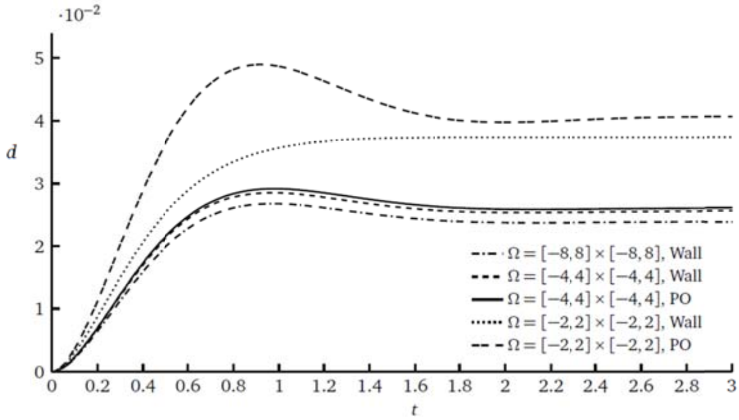

Figure 6: Deformation, d, vs. time, t, is shown for a test case with a jump ratio of 10 in the dielectric property at the interface and a Weber number of 0.1. Different domain sizes and boundary conditions are tested. Wall stands for a no-slip boundary condition for the velocity and pressure-outlet (PO) stands for a Dirichlet boundary condition for the pressure. The half-thickness of the regularized interface and the element size of the Cartesian grids are 1/8. Polynomial degrees of 8 for the electric potential and 5 for the pressure, velocity and the level set function are used. A time step size of 5×10-4 is used by considering the capillary time step size restriction.

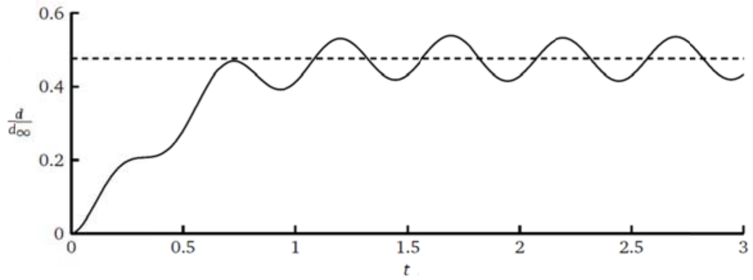

Figure 7: Relative deformation, d/d∞, is shown vs. time, t, for a test case of an oscillating droplet. An oscillating potential difference of 4cos(2πt) is applied. A jump ratio of 10 in the dielectric property at the interface and a Weber number of 0.1 are considered. The dashed line shows the relative mean value of 0.48. The computational domain is a [-2, 2]×[-2, 2] square with the wall boundary conditions. The half-thickness of the regularized interface and the element size of the Cartesian grid are 1/8. Polynomial degrees of 8 for the electric potential, 5 for the pressure and velocity and 4 for the level set function are used. A time step size of 5×10-4 is used by considering the capillary time step size restriction.

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