MATHEMATICAL AND NUMERICAL APPROACH FOR A CRASHWORTHY PROBLEM

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Abstract. Vehicle crashworthiness has been improving in recent years with attention mainly directed towards reducing the impact of the crash on the passengers. An optimal way to achieve this target is by exclusive use of specific impact attenuators, such as strategically placed tubular elements. Many of the mechanical devices are designed to absorb impact energy under axial crushing, bending and/or combined loading. An important requirement is that these structural members must be able to dissipate large amount of energy by controlled collapse in the event of a collision. Generally, the total energy dissipated depends on the governing deformation phenomena of all or part of the structural components of simple geometry, such as thin-walled tubes, cones, frames and sections. The energy absorbing capacity differs from one component to the next in a manner which depends on the mode of deformation involved and the material used.

During the last decades the attention given to crash energy management has been centred on composite structures. The use of fibre-reinforced plastic composite materials in automotive structures may result in many potential economic and functional benefits due to their improved properties respect to metal ones, ranging from weight reduction to increased strength and durability features.

Although significant experimental work on the collapse of fibre-reinforced composite shells has been carried out, studies on the theoretical modelling of the crushing process are quite limited since the complex and brittle fracture mechanisms of composite materials. Most of the studies have been directed towards the axial crush analysis, because it represents more or less the most efficient design.

In the present paper, a mathematical approach on the failure mechanisms, pertaining to the stable mode of collapse (Mode I) of thin-walled composite circular tubes subjected to axial loading, was investigated. The analysis was conducted from an energetic point of view; it is therefore necessary to identify the main energy contributions and then equate the total internal energy to the work done by the external load. The average crush load can be obtained minimizing the force contribution, function of several variables, on a domain using a numerical approach. Comparison between theory and experiments concerning crushing loads and total displacements was analysed, showing how the proposed analytical model is efficient for predicting the energy absorption capability of axially collapsing composite shells.

1 INTRODUCTION

The survivability of driver and passengers in an accident is achieved by a combination of the crash resistance of the car and its ability to absorb energy. This has been obtained by providing a survival cell, which is extremely resistant to damage, around which energy absorbing devices are placed at strategic points on the vehicle. From the numerous experimental studies [1-7], it is generally accepted that thin-walled tubes offer the most weight efficient solution for crashworthy aspects. The collapse behaviour of cylindrical and conical shaped shells of round cross sections has received attention in the recent years, because of their possible application to the design for crashworthiness of automotive vehicles. Moreover composite materials are now used in automobiles because they promise to be far more efficient than conventional materials. Carbon fibre reinforced plastic (CFRP) composites, in particular, provide significant functional and economic benefits such as enhanced strength, durability, weight reduction, lower fuel consumption and high level of structural vehicle crashworthiness. The challenge of design is to arrange the column of material such that the destructive zone can progress in a stable manner. On the contrary to the response of metals, progressive crushing of composite collapsible energy absorbers is dominated by extensive micro-fracture instead of plastic deformation. After the initial peak, the force displacement curve during axial crushing is much flatter than the plastically deforming metal tube and the amount of energy absorbed is, therefore, much greater. The mean load of crushing becomes an important factor to estimate while choosing a material and geometry for an impact energy absorbing application.

Recently the experimental campaign have been joined to the numerical analysis in order to predict the final deformation and validate the results obtained [8-10]. Contrary, very few attempts have been made to analyse the collapse mechanism of composite shells from the theoretical point of view [11-18]; this because the difficulty to model analytically the brittle behaviour and heterogeneity of these composite structures. Some additional work is rather on modelling of the plastic folding of round metallic tubes [19-21], always from the energetic point of view.

The present study addresses a mathematical and numerical approach on the failure mechanism, pertaining to the stable mode of collapse (Mode I), of thin-walled FRP composite circular tubes subjected to axial loading in order to predict the mean loads during collapse. The analysis is based on previous research [11-17] with the attempt to eliminate some simplifications dictated by experimental evidence and improve the modeling from the mathematical point of view. After the definition of the theoretical modeling using an energetic approach, different experimental tests were conducted in order to obtain some parametric values and crushing data useful for the model validation. Comparison between theory and experiments concerning loads and crushes was good enough, indicating therefore that the proposed theoretical model may be an efficient strategy for predicting the energy-absorbing capability of the axially collapsing composite shells, despite the complexity of the phenomenon.

2 THEORETICAL MODELING

During the crushing of a composite circular tube under axial impact, after the initial peak, the load oscillates around a mean load *P*. The first sharp drop in the load is due to the formation of a main circumferential intrawall crack of height *h* at the top end parallel to the axis of the shell wall. As the deformation proceeds further, the externally formed fronds curl downwards with the simultaneous development, along the circumference of the shell, of a number of axial splits followed by splaying of the material strips. The post crushing regime is characterized by the formation of two lamina bundles bent inwards and outwards due to the flexural damage; they withstand the applied load and buckle when the load or the length of the lamina bundle reaches a critical value. At this stage, a triangular debris wedge of pulverized material starts to form; its formation may be attributed to the friction between the bent bundles and the platen of the drop mass of the hammer. Due to the constant size of the section, crush zone progresses through successive cycles that are repeated in the same manner. For this reason, the theoretical model investigated takes into account only the first cycle of deformation and the crush zone can be idealized as shown in Figure 1, where *R* is the mean radius, *H* is the height and *T* is the thickness of the shell.

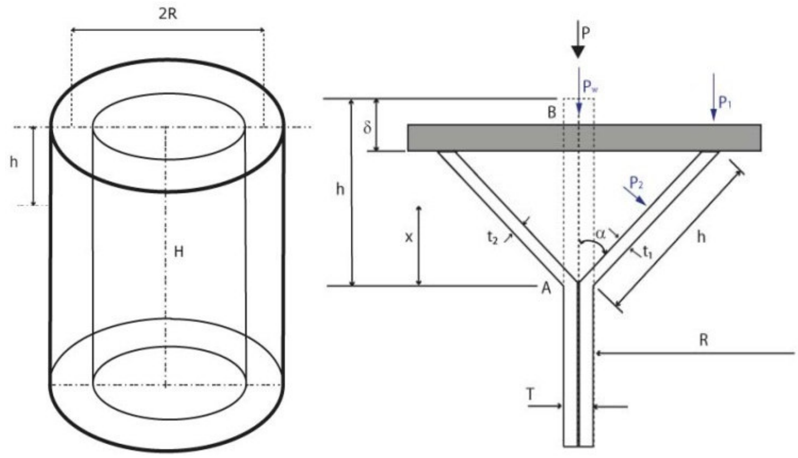

Figure 1: Idealized crush zone model of cylindrical tube wall

In order to simplify the deformation mechanism, the following assumptions have been adopted: the internal and external fronds maintain a constant length equal to *h*, thereby implying the same opening angle α; the transition between straighten and bended zone is sudden, so the central crack is placed in A as shown in Figure 1; the elastic energy associated with the first impact phase was not considered, because very low respect to the other contributions. It has been observed in earlier studies [15] that energy is absorbed in four principal modes during the formation of crush zone in progressive crushing of cylindrical tubes: work required for bending of petals (W_b), work required for petal formation (W_h), work required for circumferential delamination (W_c) and energy dissipated due to friction between the debris wedge and fronds and between fronds and platen (W_f). Follow in detail the expressions used for the various energy contributions.

2.1 Bending energy

As the crushing process initiates, fibers bend both inside and outside the shell radius. Let t_1 and t_2 be the thickness of the fiber layers bending inside and outside the shell radius and α the bending angles. By construction t_2 is equal to T-t_1. Assuming that the fiber layers are perfectly plastic during bending, the work required to bend the fiber inside and outside the shell radius can be expressed as

$$W_b = W_{b1} + W_{b2} = (M_1 + M_2)\alpha = \frac{\pi}{2} R\sigma_0\alpha((T-t_1)^2 + t_1^2) \tag{1}$$

where M_1 and M_2 are the bending moment capacity of the internal and external laminate, respectively, and σ_0 is the ultimate stress in uni-axial tension of the laminate.

2.2 Hoop strain energy

The expression for hoop strain energy in a single crush is

$$W_h = W_{h1} + W_{h2} = 2\int_0^h \sigma_0 \varepsilon dV = \sigma_0 \pi h^2 T \sin\alpha \tag{2}$$

where $\varepsilon = x\sin\alpha/\mathrm{R}$ is the shear strain corresponding to the layers bending inside and outside the shell radius. For x it is indicated the distance from A to B, as shown in Figure 1, while dV is the differential volume for the inside (outside) layers.

2.3 Crack energy

The energy required for circumferentially delamination in a single stroke is

$$W_c = 2\pi(R - T/2 + t_1)hG \tag{3}$$

where G is the critical strain energy release rate per unit interlaminar delaminated crack area and $R - T/2 + t_1$ is the tube radius at the crack tip. G is determined experimentally through DCB test as discussed in Section 3.

2.4 Friction energy

After the formation of the internal and external fronds, normal stresses develop on the sides of the debris wedge followed by shear stresses along the same sides due to friction at the interface between the wedge and the fronds. Moreover additional normal and shear stresses develop at the interface between the steel platen and the deforming shell as the formed fronds slide along the interface. The energy dissipated in frictional resistance for a crush distance δ is

$$W_f = 4\pi R(\mu_1 P_1 + \mu_2 P_2)\delta \tag{4}$$

where μ_1 is the coefficient of friction between frond and platen and μ_2 the coefficient of friction between the wedge and the fronds. Due to feasibility $0 < \mu_1 \le \mu_2 \le 1$ and by construction $\delta = h(1-\cos\alpha)$. P_1 is the normal force per unit length applied by the platen to the frond and P_2 is the normal force per unit length applied to the sides of the wedge, as shown in Figure 1. Static equilibrium at interface yields

$$P = 2\pi R(P_w + 2P_1) \tag{5}$$

where P_W is the normal force per unit length applied by the platen to the debris wedge given by

$$P_w = 2(P_2 \sin\alpha + T_2 \cos\alpha) \tag{6}$$

where $T_2 = \mu_2 P_2$ is the frictional force per unit length developed between wedge and fronds. According to [16]

$$P_2 = \sigma_0 h \tag{7}$$

and the horizontal component of the force F_x applied by the wedge to the frond is

$$F_x = P_2(\cos\alpha - \mu_2 \sin\alpha)\ . \tag{8}$$

For feasibility F_x has to be strictly positive, therefore $\alpha < \tan^{-1}(1/\mu_2)$ that implies an angle α not smaller than 45°. The frictional energy (4) can be rewritten as

$$W_f = \mu_1 \delta P + 4\pi R \sigma_0 h \delta(-\mu_1 \sin\alpha - \mu_1\mu_2 \cos\alpha + \mu_2)\ . \tag{9}$$

2.5 External work

The work done by the external load P on the crushing displacement δ in a single progression is

$$W_e = P\delta. \tag{10}$$

Balance of energy yields

$$W_e = W_T = W_b + W_h + W_c + W_f \tag{11}$$

so the mean force P is given by

$$\begin{aligned} P = P(h,t_1,\alpha) &= \frac{1}{\delta(1-\mu_1)}(W_b + W_h + W_c + 4\pi R\sigma_0 h\delta(-\mu_1 \sin\alpha - \mu_1\mu_2 \cos\alpha + \mu_2)) = \\ &= \frac{\pi}{2(1-\mu_1)h(1 - \cos\alpha)} \cdot \\ &\cdot \left\{ 2Gh(2R + 2t_1 - T) + \sigma_0 R\alpha\left(t_1^2 + (T-t_1)^2\right) + 2\sigma_0 h^2\left[T\sin\alpha + 4R(1-\cos\alpha)(\mu_2 - \mu_1\mu_2\cos\alpha - \mu_1\sin\alpha)\right]\right\}. \end{aligned} \tag{12}$$

The expression for determining the crush zone length, the thickness of the plies bending inside the shell radius and the bending angle of the internal plies in a crush cycle is obtained by minimising the average crush load.

Note that this procedure allows to calculate the energy absorbed in a single crush cycle, as the product of the obtained load P for the vertical displacement δ. The total crush can be seen as the composition of equal single stages that repeat themselves. Therefore in order to evaluate the final deformation of the impact attenuator able to absorb the potential energy Wp

imposed by regulation, it is sufficient to divide *Wp* for the minimum load *P* obtained by previous calculation.

3 FABRICATION AND TESTING OF COMPOSITE SPECIMENS

A single kind of CFRP composite material was used for testing. Despite unidirectional composites being more efficient in energy absorption, fabric reinforced materials tend to be preferred in impact structures in order to ensure a stable crush failure. Therefore prepreg tapes of carbon fibres in epoxy resin with plain reinforcements were used to make the tubes manually. To obtain appropriate value for the σ_0 parameter, standard coupon tests were performed. The tests were performed in tension, according to ASTM Standard D3039, with the warp (weft) direction aligned with the test direction (0° and 90° tests, respoectively) and also with fibres aligned at 45° to the test direction [9]. For the ultimate stress of the laminate was used that obtained from the 45° tensile test (Figure 2), in order to consider the worst condition of resistance given the manual realization of the tubes.

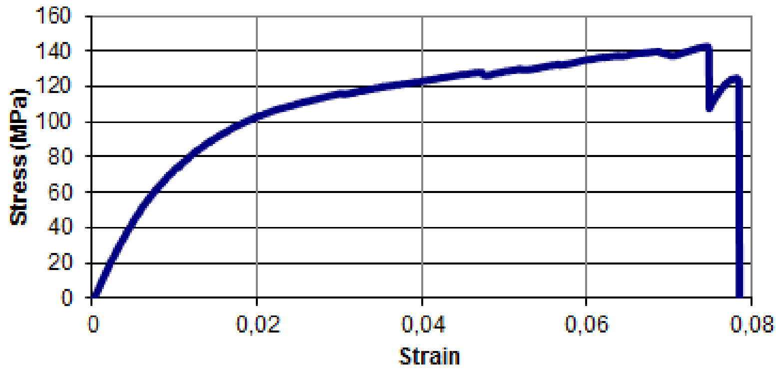

Figure 2: Stress vs strain diagram for prepreg in tensile test

The double cantilever beam (DCB) test was carried out as per ASTM D5528-94, in order to evaluate the mode I interlaminar fracture toughness in the same composite laminate used for the tubes. The test considers a composite beam with an initial delamination crack; the initial delamination is forced to open by applying a force that pull the two beams away from each other (Figure 3). From the test the fracture toughness *G* was calculated using the compliance calibration method, which is the most common in literature.

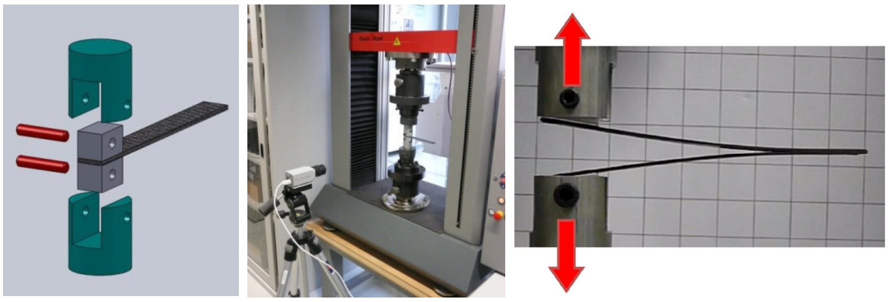

Figure 3: DCB test

In order to obtain experimental evidence about the governing failure mechanism during the axial compression of thin-walled CFRP composite circular tubes, dynamic tests were performed using a drop weight test machine with an impact mass of 294kg and an impact velocity of about 4m/s [9]. During the tests the acceleration of the mass were acquired, sampled and recorded. From the average deceleration, obtained from the beginning of the impact to the instant when the velocity vanishes, is then possible to determine the mean load and the real crushing.

4 RESULTS AND DISCUSSION

As discussed in Section 2, the central issue of the procedure is determining the minimum of the mean load *P*. In the present section, first the existence of a local minimum of *P* inside the domain *D* is proved, than since the roots of the gradient of *P* cannot be analytically determined, the minimum is estimated using numerical methods.

The mean load *P* is a function of the three variables *h*, t_1 and α belonging to the domain $D =]0, H] \times [0, T] \times [\pi/4, \pi/2]$. Consider a minimizing sequence for P, $\{(h_n, t_{1_n}, \alpha_n)\}_{n \in \mathbb{N}}$, in the set D. Equation (12) implies that P tends to infinity when h vanishes therefore h_n is far from zero and by Weierstrass theorem there exists at least a minimum of *P* in D. Figure 4 displays the direction of the gradient of *P* on the boundary of the domain.

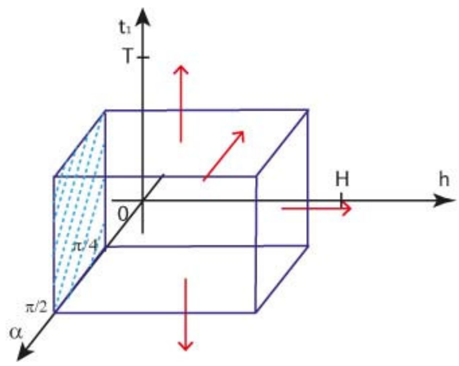

Figure 4: Direction of ∇P on ∂D

In details, choosing $H > \sqrt{\pi T R / \sqrt{2}}$ $\partial_h P|_{h=H} > 0$; setting *R* and *T* such that $RT > 8GH/(\pi\sigma_0)$ $\partial_{t_1} P|_{t_1=0} < 0$; $\partial_{t_1} P|_{t_1=T} > 0$ and $\partial_\alpha P|_{\alpha=\pi/4} < 0$. In order to state that the minimum is inside the domain D, it remains to consider the edge face with $\alpha = \pi/2$. If $\alpha = \pi/2$ the function $P(h, t_1, \pi/2)$ has a minimum P^* in $(h^*, t_1^*, \pi/2)$ with

$$h^* = \frac{\pi\sigma_0 RT}{2\sqrt{2}\sqrt{4\pi R^2 \sigma_0^2(\mu_2 - \mu_1) + \pi\sigma_0^2 R - 2G^2}} \ , \ t_1^* = \frac{1}{2}\left(T - \frac{\sqrt{2}GT}{\sqrt{4\pi R^2 \sigma_0^2(\mu_2 - \mu_1) + \pi\sigma_0^2 R - 2G^2}} \right).$$

Since the one variable function $P(h^*, t_1^*, \alpha)$ is strictly increasing in $\alpha = \pi/2$, there exists an angle $\bar{\alpha} < \pi/2$ in which $P(h^*, t_1^*, \bar{\alpha}) < P^*$, therefore the minimum of *P* is in the interior of *D*.

It is not easy to find the roots of the gradient of P so the minimum is estimated using some nonlinear optimization algorithms implemented with the software Mathematica [23] as a preliminary study. Both the Nelder-Mead "simplex" algorithm, the Random search method [24] and the Differential evolution procedure [25] give the same minimum point of P for every specific cases analyzed; so it is reasonable to assume that the mean load P has an unique minimum internally to the domain.

Table 1 reports the geometrical and material parameters of the cylindrical shells taken into account.

Table 1: Geometrical and material characteristics of the shells

#	R (mm)	T (mm)	H (mm)	σ_0 (MPa)	G (N/mm)	μ_1	μ_2
1	41.25	2.5	200	140	0.5	0.35	0.45
2	41	2	200	140	0.5	0.35	0.45
3	40.75	1.5	200	140	0.5	0.35	0.45
4	26.25	2.5	200	140	0.5	0.35	0.45
5	26	2	200	140	0.5	0.35	0.45
6	25.75	1.5	200	140	0.5	0.35	0.45
7	51.25	2.5	300	140	0.5	0.35	0.45

Table 2 reports the mean load and the final crushing in the minimum configuration for the cases taken into account. In the table there are also the experimental data taken from real tests [9] and the committed percentage error. Despite the simplification adopted the proposed method is able to predict within ±20% the mean crushing load and the total crushing. As it is evident by table, the error tends to increase as the wall thickness decreases; this can be justified by the real deformation which, for thinner thickness, changes from axial splitting in splaying mode to fragmentation mode (Figure 5). This second mode of failure have a lower energy absorption capacity with also lower force values.

Table 2: Mean crush load and final crushing

#	P (kN)			s (mm)		
	Model	Test	Error %	Model	Test	Error %
1	56,1	59,4	5	37,0	35	6
2	43,7	46,4	6	46,6	44	6
3	31,9	26,8	19	65,1	78	16
4	37,7	38,9	3	55,6	54	3
5	29,1	26,1	11	71,8	80	10
6	21,0	17,7	19	97,1	115	15
7	68,3	75,9	10	48,9	44	11

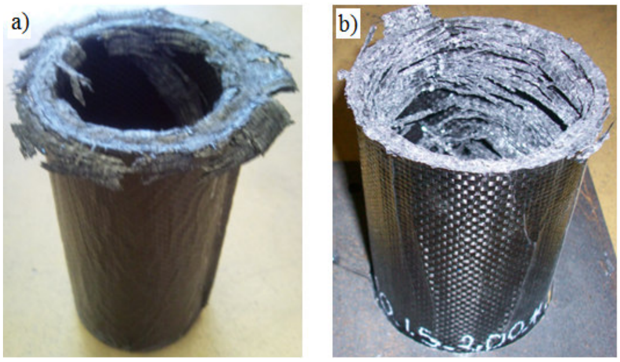

Figure 5: Different crushing mode: a) splaying with axial splitting, b) fragmentation

Figure 6 graphically represents the trend of the mean load for a specific configuration (case 1) in function of the bending angle α and of the thickness of the inside ply t_1, set h equal to the minimum value 2.73 mm; the black point is the minimum of P. As it is observed from experimental evidence, the delamination in all the cylindrical tubes analyzed occurs at about half of the wall thickness.

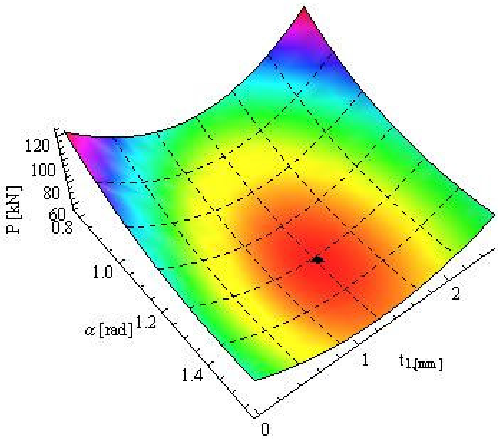

Figure 6: Mean load as function of t_1 and α and minimum point (Case 1)

5 CONCLUSIONS

A crashworthy problem was investigated from a mathematical and numerical point of view. In particular the energy absorption of composite cylindrical tubes subjected to axial loading was analyzed, defining analytically the external load as a function of three variables and identifying the minimum through a numerical approach. Despite some simplifications, the method adopted is able to predict the mean load and the final crushing of tubular specimens made of CFRP composite, once known parameters of the used material. This methodology can therefore be used as the first approach to follow during the design of specific CFRP impact attenuator. Forthcoming analysis will eliminated the assumption of a constant crush length both internally and externally to the axial splits and will consider different numerical methods.

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