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THERMOELECTRIC SIMULATION OF ELECTRIC MACHINES WITH PERMANENT MAGNETS

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Abstract. The objective of this work is to describe some numerical tools developed to perform the thermoelectric simulation of electric machines. From the electromagnetic point of view, we will focus on the computation of nonlinear 2D transient magnetic fields where the data concerning the electric current sources involve potential drops excitations. From the thermal point of view, once the electromagnetic losses are known, we will show an application of a Galerkin lumped parameter method (GLPM) to simulate the thermal behavior of an electric motor. The proposed methods are applied to the simulation of a permanent magnet synchronous electric motor.

1 Introduction

One of the limitations in designing electric machines is that temperature of their different components has to remain below some prescribed thresholds. The temperature of the machine depends on the electromagnetic losses, which are the source term in the energy equation. Thus, to create new high-performance electric machines an accurate numerical simulation of their electromagnetic and thermal behavior is needed.

In order to minimize the electromagnetic losses, the magnetic cores of electrical machines are laminated media consisting of a large number of stacked steel sheets, which are orthogonal to the direction of the currents traversing the coils. This geometry allows us to compute the electromagnetic fields in a plane transversal to the device by assuming that the magnetic flux lies in that plane, and then determining the losses a posteriori (see [4]). This methodology is very interesting because reduces the complexity of dealing with the laminated structure of the machine (see [4]). We will use the axial component of the magnetic vector potential as the main unknown of the mathematical model and we will describe the way of providing different kinds of current sources to the system. In particular, if the sources are given in terms of voltage drops we will develop a numerical method to compute periodic solutions by determining a suitable initial current intensity which avoids large simulations to reach the steady state. The problem is numerically solved by using an implicit time discretization scheme combined with a finite element method for space approximation.

For the thermal modeling of an electric motor, we adopted an alternative to the classical finite element (FE) method, since the numerical simulation of machines composed of a large amount of pieces can be computationally demanding. The so-called lumped parameter (LP) models use a simplification of the original problem requiring the design of a network that properly represents the physical behavior of the problem [5, 7]. With such network, an approximate problem is established where the spatially distributed variables are changed by a set of scalar unknowns.

Inspired in the LP methods, we propose in [3] a new family of methods called Galerkin lumped parameter (GLP) methods. These methods are inpired by the LP methods and the techniques used in the reduced basis methods. They consist of using Galerkin approximations of a weak formulation of the original problem in the small finite-dimensional space spanned by a special basis well adapted to the physics of the problem. GLP methods allow us to solve the problem in two steps: in the first step, a basis adapted to a decomposition of the computational domain is calculated; in the second one, the global solution is calculated by solving a small ODE. Another advantage is that the basis is independent of some magnitudes (sources, for instance), allowing us to solve several different cases with the same basis.

2 Mathematical modelling

In this section we will describe the electromagnetic and thermal models.

2.1 Electromagnetic model

We state a 2D transient magnetic problem which arises in the mathematical modelling of laminated magnetic media in the presence of permanent magnets.

Let us assume that the current sources $\mathbf{J}$ have non-null component only in the z space direction and that this component does not depend on z, i.e., $\mathbf{J} = J_z(x, y, t)\mathbf{e}_z$. We also assume that the laminated core is invariant along the z-direction and that, in the field equations, we neglect the effects of eddy currents in this direction because the steel shells are electrically isolated. In this case, the core can be considered as a homogeneous medium and it is easy to see that the magnetic field $\mathbf{H}$, and then the magnetic induction, $\mathbf{B}$, have only components on the xy-plane and both are independent of z. Thus, for a given current density $\mathbf{J}$, the 2D transient magnetic problem in the xy-plane transversal to the device reads:

$$\mathbf{curl}\,\mathbf{H} = \mathbf{J}, \tag{1}$$

$$\operatorname{div}\mathbf{B} = 0. \tag{2}$$

Notice that, $\mathbf{B}\cdot\mathbf{n}$ and, in absence of surface currents, $\mathbf{H}\times\mathbf{n}$ are continuous through the interface of different media.

In order to apply a standard finite element method, let us consider a bounded domain Ω composed by several connected conductors, permanent magnets, a ferromagnetic core and the air around. Let us denote by Ω_i, $i=1,\cdots,N$ the conductors in Ω representing the cross section of the coils and by Ω_i, $i=N+1,\ldots,N+M$ the permanent magnets in Ω. We also denote by Ω_{N+M+1} the complementary domain occupied by the air and the ferromagnetic core, i.e., $\Omega_{N+M+1}=\Omega\setminus\cup_{i=1}^{N+M}\Omega_i$. We will suppose that all of the conductors are stranded conductors, which makes it possible to assume that the current density is uniformly distributed and expressed in terms of the total current across each conductor Ω_i. Actually, for each conductor Ω_i, we will see that the source can be given in terms of either the current or the potential drop per unit length in the z-direction. Then, we must solve the following system of equations:

$$\mathbf{curl}\,\mathbf{H}=\frac{I_i(t)}{\mathrm{meas}(\Omega_i)}\mathbf{e}_z \quad \text{in } \Omega_i, i=1,\cdots,N, \tag{3}$$

$$\mathbf{curl}\,\mathbf{H}=\mathbf{0} \quad \text{in } \Omega_i, i=N+1,\cdots,N+M, \tag{4}$$

$$\mathbf{curl}\,\mathbf{H}=\mathbf{0} \quad \text{in } \Omega_{N+M+1}, \tag{5}$$

$$\mathrm{div}\,\mathbf{B}=0 \quad \text{in } \Omega. \tag{6}$$

This model is completed with the constitutive law relating the magnetic field to the flux density. In particular, we will assume a linear behavior for the air, $\mathbf{B}=\mu\mathbf{H}$, while the coils and the laminated media may have a nonlinear behavior, $\mathbf{B}=\mu(|\mathbf{H}|)\mathbf{H}$. On the other hand, permanent magnets will be modelled by the linear constitutive law: $\mathbf{B}=\mu\mathbf{H}+\mathbf{B}^r$, where $\mathbf{B}^r$ is the so-called remanent flux density which is assumed to have the form $\mathbf{B}^r=B_x^r(x,y,t)\mathbf{e}_x+B_y^r(x,y,t)\mathbf{e}_y$. Notice that $\mathbf{B}^r$ may depend on time due to the orientation of the permanent magnets change with an eventual motion of the machine.

Next, we will introduce a magnetic vector potential to solve the two-dimensional model. Since $\mathbf{B}$ is divergence free, there exists a so-called magnetic vector potential $\mathbf{A}$ such that $\mathbf{B}=\mathbf{curl}\,\mathbf{A}$. Under the assumptions above, we can choose a magnetic vector potential of the form $\mathbf{A}=A_z(x,y,t)\mathbf{e}_z$ (see, for instance, [6]). Thus, in terms of $\mathbf{A}$, the transient magnetic model reads:

$$\mathbf{curl}(\nu_i(|\,\mathbf{curl}\,\mathbf{A}|)\,\mathbf{curl}\,\mathbf{A})=\mathbf{J} \quad \text{in } \Omega_i,\ i=1,\cdots,N, \tag{7}$$

$$\mathbf{curl}(\nu_i\,\mathbf{curl}\,\mathbf{A})=\mathbf{curl}(\nu_i\mathbf{B}_r) \quad \text{in } \Omega_i,\ i=N+1,\cdots,N+M, \tag{8}$$

$$\mathbf{curl}(\nu_{N+M+1}(|\,\mathbf{curl}\,\mathbf{A}|)\,\mathbf{curl}\,\mathbf{A})=\mathbf{0} \quad \text{in } \Omega_{N+M+1}, \tag{9}$$

$$[\nu_i\,\mathbf{curl}\,\mathbf{A}\times\boldsymbol{n}]=\nu_i\mathbf{B}^r\times\boldsymbol{n}, \quad \text{on } \partial\Omega_i, i=N+1,\cdots,N+M. \tag{10}$$

where [.] denotes the jump across $\partial\Omega_i$, $\mathbf{n}_i$ is the outward unit normal vector to $\partial\Omega_i$ and ν_i denotes the magnetic reluctivity of Ω_i. In the air $\nu_{N+M+1}=1/\mu_0$, (where μ_0 denotes the magnetic permeability of the empty space), while in the ferromagnetic material ν_{N+M+1}

is a nonlinear function of $|\mathbf{B}| = |\,\mathbf{curl}\,\mathbf{A}|$. Notice that inside the permanent magnets, the term $\mathbf{curl}(\nu_i \mathbf{B}^r)$ has the same effect as an equivalent current density inside the permanent magnet. In general, both ν_i and $\mathbf{B}^r$ are constant in the magnet so the right-hand side of (8) is null, but, since $\nu_i \mathbf{B}^r$ is only non-null in the magnet, its tangential component has a jump discontinuity across the surface of the magnet similar to a surface current density-like of value $\nu_i \mathbf{B}^r \times \mathbf{n}$ (see (10)). This interface condition is implicitly included in the weak formulation of the problem to be used for finite element approximation.

Next, we will describe how to impose different kinds of transient sources in the coils. Let σ_i be the electrical conductivity of conductor Ω_i, $i = 1, \cdots, N$. From the assumptions on $\mathbf{J}$ and the Ohm's law, $\mathbf{J} = \sigma_i \mathbf{E}$, we deduce that, in each conductor Ω_i, the electric field $\mathbf{E}$ has to be of the form $\mathbf{E} = E_z(x, y, t)\mathbf{e}_z$. On the other hand, from Faraday's law, a scalar potential V must exist such that

$$\frac{\partial \mathbf{A}}{\partial t} + \mathbf{E} = -\,\mathbf{grad}\, V. \tag{11}$$

Taking into account the form of $\mathbf{A}$ and $\mathbf{E}$, we deduce from this equality that

$$\frac{\partial V}{\partial z} = C_i(t) \quad \text{in } \Omega_i, \; i = 1, \cdots, N,$$

where function $C_i(t)$ represents the potential drop per unit length in direction z, in conductor Ω_i. Hence, from the previous equation and (11) one deduces

$$\sigma_i \frac{\partial A_z}{\partial t} + \sigma_i E_z = -\sigma_i C_i(t) \quad \text{in } \Omega_i, \quad i = 1, \cdots, N. \tag{12}$$

Taking into account the previous discussion, we will assume that, for each conductor Ω_i, either the potential drop $C_i(t)$ or the current $I_i(t)$ is given. In particular, let us suppose there are N_{C} conductors of the first type and $N - N_{\mathrm{C}}$ of the second one.

On the boundary $\partial\Omega$ of Ω, we will consider, for simplicity, a homogeneous Dirichlet boundary condition, $\mathbf{A} = \mathbf{0}$, which means that $\mathbf{B} \cdot \mathbf{n} = 0$ on $\partial\Omega$. This assumption is satisfied in the physical example to be considered below. It also holds, in general, if the computational domain is taken large enough. Another classic boundary condition in magnetostatics is $\mathbf{H} \times \mathbf{n} = 0$. In this case, $1/\mu\, \mathbf{curl}\, \mathbf{A} \times \mathbf{n} = 0$ and further development would be done without any difficulty.

Thus, from Ohm's law, by integrating equation (12) on each Ω_i, the problem to be solved becomes:

Problem 2.1 *Given functions $C_i(t)$, $i = 1, \cdots, N_{\mathrm{C}}$, $I_i(t)$, $i = N_{\mathrm{C}} + 1, \cdots, N$, and initial currents I_i^0, $i = 1, \cdots, N_{\mathrm{C}}$, find a field $\mathbf{A} = A_z(x, y, t)\mathbf{e}_z$ and currents $I_i(t)$, $i =$*

$1, \cdots, N_{\mathrm{C}}$, *such that*

$$\mathbf{curl}(\nu_i(|\,\mathbf{curl}\,\mathbf{A}|)\,\mathbf{curl}\,\mathbf{A}) = \frac{I_i(t)}{\mathrm{meas}(\Omega_i)}\,\mathbf{e}_z \quad \textit{in } \Omega_i,\ i = 1, \cdots, N, \tag{13}$$

$$\mathbf{curl}(\nu_i\,\mathbf{curl}\,\mathbf{A}) = \mathbf{curl}(\nu_i \mathbf{B}^r) \quad \textit{in } \Omega_i,\ i = N+1, \cdots, N+M, \tag{14}$$

$$\mathbf{curl}(\nu_{N+M+1}(|\,\mathbf{curl}\,\mathbf{A}|)\,\mathbf{curl}\,\mathbf{A}) = \mathbf{0} \quad \textit{in } \Omega_{N+M+1}, \tag{15}$$

$$[\nu_i\,\mathbf{curl}\,\mathbf{A} \times \boldsymbol{n}] = \nu_i \mathbf{B}^r \times \boldsymbol{n}, \quad \textit{on } \partial\Omega_i, i = N+1, \cdots, N+M, \tag{16}$$

$$\mathbf{A} = \mathbf{0} \quad \textit{on } \partial\Omega, \tag{17}$$

$$\frac{d}{dt}\int_{\Omega_i} \sigma_i A_z(x,y,t)\ \mathrm{d}x\mathrm{d}y + I_i(t) = -C_i(t)\alpha_i^{-1}, \quad i = 1, \cdots, N_{\mathrm{C}}, \tag{18}$$

$$I_i(0) = I_i^0, \quad i = 1, \cdots, N_{\mathrm{C}}. \tag{19}$$

where α_i denotes the *resistance* of the i-th conductor per unit length in the z direction, that is, $\alpha_i := \left(\int_{\Omega_i} \sigma_i\ \mathrm{d}x\mathrm{d}y\right)^{-1}$. We notice that the case where the currents are given reduces to solve a nonlinear magnetostatics problem at each time in some interval, and hence time appears as a parameter. However, the case with potential drop excitations is more involved because the model becomes a system of degenerate parabolic nonlinear partial differential equations.

We notice that, in (13), the currents for $i = N_{\mathrm{C}}+1, \cdots, N$ are given, but those for $i = 1, \cdots, N_{\mathrm{C}}$ are unknown. In order to compute the latter we have added equations (18) and (19) to the system. From the computational point of view, it is better to eliminate the unknowns $I_i(t)$, $i = 1, \cdots, N_{\mathrm{C}}$ from the system. For this purpose, we first obtain $I_i(t)$ from (18) and then replace it in (13) for $i = 1, \cdots, N_{\mathrm{C}}$. Then Problem 2.1 states:

Problem 2.2 *Given functions* $C_i(t)$, $i = 1, \cdots, N_{\mathrm{C}}$, $I_i(t)$, $i = N_{\mathrm{C}}+1, \cdots, N$, *and initial currents* I_i^0, $i = 1, \cdots, N_{\mathrm{C}}$, *find a field* $\mathbf{A} = A_z(x,y,t)\mathbf{e}_z$ *such that*

$$\frac{1}{\mathrm{meas}(\Omega_i)}\frac{d}{dt}\int_{\Omega_i} \sigma_i A_z(x,y,t)\ \mathrm{d}x\mathrm{d}y\ \mathbf{e}_z + \mathbf{curl}(\nu_i(|\,\mathbf{curl}\,\mathbf{A}|)\,\mathbf{curl}\,\mathbf{A})$$
$$= -\frac{C_i(t)\alpha_i^{-1}}{\mathrm{meas}(\Omega_i)}\,\mathbf{e}_z \quad \textit{in } \Omega_i,\ i = 1, \cdots, N_{\mathrm{C}}, \tag{20}$$

$$\mathbf{curl}(\nu_i(|\,\mathbf{curl}\,\mathbf{A}|)\,\mathbf{curl}\,\mathbf{A}) = \frac{I_i(t)}{\mathrm{meas}(\Omega_i)}\,\mathbf{e}_z \quad \textit{in } \Omega_i,\ i = N_{\mathrm{C}}+1, \cdots, N, \tag{21}$$

$$\mathbf{curl}(\nu_i\,\mathbf{curl}\,\mathbf{A}) = \mathbf{curl}(\nu_i \mathbf{B}^r) \quad \textit{in } \Omega_i,\ i = N+1, \cdots, N+M, \tag{22}$$

$$\mathbf{curl}(\nu_{N+M+1}(|\,\mathbf{curl}\,\mathbf{A}|)\,\mathbf{curl}\,\mathbf{A}) = \mathbf{0} \quad \textit{in } \Omega_{N+M+1}, \tag{23}$$

$$[\nu_i\,\mathbf{curl}\,\mathbf{A} \times \boldsymbol{n}] = \nu_i \mathbf{B}^r \times \boldsymbol{n}, \quad \textit{on } \partial\Omega_i, i = N+1, \cdots, N+M, \tag{24}$$

$$\mathbf{A} = \mathbf{0}, \quad \textit{on } \partial\Omega, \tag{25}$$

$$I_i(0) = I_i^0, \quad i = 1, \cdots, N_{\mathrm{C}}. \tag{26}$$

In many applications, there exist two indices i_1 and i_2, $1 \leq i_1, i_2 \leq N_C$, such that $I_{i_1}(t) = -I_{i_2}(t) = I(t)$. In this case, the number of unknown currents in system (20)-(26) is $N_\mathrm{C} - 1$ and, accordingly, we cannot prescribe each potential drop $C_{i_j}(t), j = 1, 2$ arbitrarily, but only the difference of potential drops: $V(t) := C_{i_1}(t) - C_{i_2}(t)$. The previous model can be modified in an easy way to deal with this case (see further details in [1]).

The numerical solution of the electromagnetic Problem 2.2 is done by a backward Euler scheme for time discretization combined with standard continuous piecewise linear finite elements for space discretization. At each time step, we have to solve a nonlinear problem for which we propose an iterative algorithm known as Bermúdez-Moreno algorithm (see [2]).

2.1.1 Computing periodic solutions in the electromagnetic model

If $N_\mathrm{C} = 0$, the nonlinear boundary-value problem (13)–(19) has a periodic solution when the given currents $I_1(t), \cdots, I_N(t)$ are periodic functions of period T. However, the problem of computing periodic solutions is more involved when there are conductors for which we know the potential drops $C_i(t)$ instead of the currents, i.e., if $N_\mathrm{C} \neq 0$. In this case we will assume that the given potential drops are periodic with the same period T and null average, that is,

$$\int_0^T C_i(t)\mathrm{d}t = 0, \quad i = 1, \cdots, N_\mathrm{C}.$$

We will also assume that the given currents $I_i(t),\ i = N_\mathrm{C}+1, \cdots, N$ are periodic functions with common period T. In this case, we have developed a numerical procedure allowing to determine suitable initial currents which avoid large simulations to reach the steady state. We will summarize here the main ideas and refer the reader to [1] for a complete development.

For $t \in [0, T]$, let us denote by $\mathbf{F}_t = (F_{t,1}, \cdots, F_{t,N_\mathrm{C}})$ the mapping from $\mathbb{R}^{N_\mathrm{C}}$ into itself such that, to the vector of currents $\vec{I} = (I_1, \cdots, I_{N_\mathrm{C}}) \in \mathbb{R}^{N_\mathrm{C}}$ associates the numbers

$$F_{t,i}(\vec{I}) = \alpha_i \int_{\Omega_i} \sigma_i A_z(x, y, t)\ \mathrm{d}x\mathrm{d}y,\ i = 1, \cdots, N_\mathrm{C}.$$

We notice that computing $\mathbf{F}_t(\vec{I})$ requires to solve a nonlinear magnetostatics problem at each time t, in order to determine field $A_z(x, y, t)$. By using this mapping, equations (18) can be rewritten as

$$\sum_{j=1}^{N_\mathrm{C}} (D\mathbf{F}_t(\vec{I}))_{ij} \frac{dI_j(t)}{dt} + \alpha_i I_i(t) = -C_i(t), \quad i = 1, \ldots, N_\mathrm{C}, \tag{27}$$

where $(D\mathbf{F}_t(\vec{I}))_{ij}$ denotes the ij-th element of the Jacobian matrix $D\mathbf{F}_t(\vec{I})$ of $\mathbf{F}_t$ at point $\vec{I}$. Let us assume the following hypothesis:

$$\frac{\alpha_i T}{\min\limits_{t,\vec{I}}(|D\mathbf{F}_t(\vec{I})|)_{ii}} \ll 1, \ i = 1, \cdots, N_{\mathrm{C}}.$$

In this case, the term involving α_i can be neglected in (27). Thus, by using algebraic operations (see [1]) we deduce that to compute an initial condition leading to a periodic solution from the initial time, we can solve the system of equations:

$$\mathbf{F}_0(\vec{I}_0) = \frac{1}{T}\Big(\int_0^T \mathbf{F}_t(\mathbf{0})\mathrm{d}t + \int_0^T (T-s)\vec{C}(s)\mathrm{d}s\Big). \tag{28}$$

Notice that, in order to solve (28), it is first necessary to compute the term $\mathbf{F}_t(\mathbf{0})$ by solving a magnetostatics problem for each value of $t \in [0, T]$. Once this term has been computed, the nonlinear system (28) has the important feature that only involves the magnetostatics problem for time $t = 0$.

2.2 Thermal model

Given the volumetric heating, calculated with the electromagntetic model explained in the previous section, we can tackle the calculation of the temperature in the electric motor. We suppose that domain Ω is divide in several subdomains, connected through surfaces called *ports*. Following the GLP method explained in [3], we pose the heat equation for the temperature θ:

$$\rho\, c\, \frac{\partial\theta}{\partial t} - \mathrm{div}(k\, \mathbf{grad}\, \theta) = f \quad \text{in } \Omega \times [0, T], \tag{29}$$

$$\theta(x,t) = \theta_l^P(x,t) \text{ on } \Gamma_l^P, \ l = 1, \ldots, n^P, \tag{30}$$

$$-k\frac{\partial\theta}{\partial\mathbf{n}}(x,t) = \alpha_l(\theta(x,t) - \theta_l^C(x,t)) \text{ on } \Gamma_l^C, \ l = 1, \ldots, n^C, \tag{31}$$

$$k\frac{\partial\theta}{\partial\mathbf{n}} = 0 \text{ on } \Gamma^A, \tag{32}$$

$$\theta(x,0) = \theta_0(x) \quad \text{in } \Omega, \tag{33}$$

where ρ is the density, c is the specific heat, k is the thermal conductivity, f is the volumetric heating, α_l are the convective heat transfer coefficients, θ_l^C are the convective temperatures and θ_0 is the initial temperature.

Here, the boundary Γ of the domain is divided into three parts: $\Gamma^P = \cup_{l=1}^{n^P}\Gamma_l^P$ are the *ports*, i.e., the surfaces of the domain connecting two or more sub-domains. We assume that each Γ_l^P is a connected component of Γ^P. $\Gamma^C = \cup_{l=1}^{n^C}\Gamma_l^C$ are the *convective boundaries*, where convective heat transfer conditions are applied. Finally, Γ^A, are the thermally isolated (or adiabatic) surfaces.

We consider a domain decomposition in order to construct an adapted basis in each sub-domain: the domain Ω splits into several sub-domains Ω_i, $i = 1, \ldots, N$, connected among them through boundaries called *ports*. Thus, in the boundary of each sub-domain Ω_i, called Γ_i, we distinguish three parts: the *ports*, $\Gamma_i^P = \bigcup_{j=1}^{n_i^P} \Gamma_{ij}^P$, consisting of all the boundaries between sub-domains; *the convective boundaries*, $\Gamma_i^C = \bigcup_{j=1}^{n_i^C} \Gamma_{ij}^C$ and the *the isolated boundary*, Γ_i^A.

The basis for the i-th sub-domain consists of $n_i^P + n_i^C$ elements, to be called φ_{ij}^P : $j = 1, \ldots, n_i^P$, and φ_{ij}^C : $j = 1, \cdots, n_i^C$ which are defined as the unique solutions to the following stationary boundary-value problems:
For $i = 1, \ldots, N$ and $j = 1, \ldots n_i^P$ find $\varphi_{ij}^P \in H^1(\Omega_i)$ satisfying,

$$-\operatorname{div}(k\, \mathbf{grad}\, \varphi_{ij}^P) = 0 \quad \text{in } \Omega_i, \tag{34}$$

$$\varphi_{ij}^P(x) = \delta_{jl} \text{ on } \Gamma_{il}^P,\ l = 1, \ldots, n_i^P, \tag{35}$$

$$k\, \frac{\partial \varphi_{ij}^P}{\partial \mathbf{n}} + \alpha_l \varphi_{ij}^P = 0 \text{ on } \Gamma_{il}^C,\ l = 1, \ldots, n_i^C, \tag{36}$$

$$k\, \frac{\partial \varphi_{ij}^P}{\partial \mathbf{n}} = 0 \text{ on } \Gamma_i^A. \tag{37}$$

For $i = 1, \ldots, N$ and $j = 1, \ldots n_i^C$ find $\varphi_{ij}^C \in H^1(\Omega_i)$ satisfying,

$$-\operatorname{div}(k\, \mathbf{grad}\, \varphi_{ij}^C) = 0 \quad \text{in } \Omega_i, \tag{38}$$

$$\varphi_{ij}^C(x) = 0 \text{ on } \Gamma_{il}^P,\ l = 1, \ldots, n_i^P, \tag{39}$$

$$k\, \frac{\partial \varphi_{ij}^C}{\partial \mathbf{n}} + \alpha_l (\varphi_{ij}^C - \delta_{jl}) = 0 \text{ on } \Gamma_{il}^C,\ l = 1, \ldots, n_i^C, \tag{40}$$

$$k\, \frac{\partial \varphi_{ij}^C}{\partial \mathbf{n}} = 0 \text{ on } \Gamma_i^A. \tag{41}$$

We notice that each problem is established in one single sub-domain Ω_i, it is independent on time, and the stiffness matrix is the same for all the problems so it can be assembled and factorized only once. Now, the global basis can be constructed using the solutions of them. It consists of two types of functions:

1. the elements w_l^P that coincide with φ_{ij}^P in Ω_i when the l-th global port is the j-the local port of Ω_i, and that are zero otherwise;

2. the elements w_l^C that coincide with φ_{ij}^C in Ω_i when the l-th global convective boundary is the j-th local convective boundary of Ω_i, and that are zero otherwise.

Let us call $\mathcal{V} \subset H^1(\Omega)$ the linear space spanned by the above set of $n^P + n^C$ functions. The lumped parameter model is defined as the Galerkin approximation of the weak formulation of problem (29)–(33) corresponding to this basis:

For $t \in [0,T]$, find $\tilde{\theta}(.,t) \in \mathcal{V}$ satisfying

$$\int_\Omega \rho\, c \frac{\partial \tilde{\theta}}{\partial t} \tilde{\psi}\, dx + \int_\Omega k\, \mathbf{grad}\, \tilde{\theta} \cdot \mathbf{grad}\, \tilde{\psi}\, dx + \sum_{l=1}^{n^C} \int_{\Gamma_l^C} \alpha_l\, \tilde{\theta}\, \tilde{\psi}\, d\Gamma$$

$$= \int_\Omega f \tilde{\psi}\, dx + \sum_{l=1}^{n^C} \int_{\Gamma_l^C} \alpha_l\, \tilde{\theta}_i^C\, \tilde{\psi}\, d\Gamma\ \forall \tilde{\psi} \in \mathcal{V} \tag{42}$$

$$\tilde{\theta}(x,0) = \tilde{\theta}_0(x) \ \text{ in } \Omega, \tag{43}$$

where $\tilde{\theta}_0$ denotes a projection of the initial condition θ_0 on the space $\mathcal{V}$.

Now, we seek a solution that, at each time t, belongs to the space spanned by the reduced basis:

$$\tilde{\theta}(x,t) = \sum_{l=1}^{n^P} \theta_l^P(t) w_l^P(x) + \sum_{l=1}^{n^C} \theta_l^C(t) w_l^C(x),$$

Here, coefficients θ_l^P and θ_l^C denote the temperature at the ports and the convective boundaries, respectively. In order to determine those coefficients, we replace $\tilde{\theta}(x,t)$ in equations (42)–(43), obtaining an ordinary differential system of dimension $n^P + n^C$. We note that the dimension of this system is much smaller than the classical finite element method.

3 Numerical results

In the first part of this section we will show some numerical results obtained with a Fortran code implementing the numerical methods described for the electromagnetic problem.

Figure 1-left shows the cross-section of a permanent magnet synchronous electric motor having 16 magnet poles and three phase windings, each of them composed of 16 coils with 31 turns. This motor has buried permanent magnets, entirely enclosed in the solid rotor structure. We consider that both rotor and stator have nonlinear magnetic cores, laminated in the direction of the current, which allows us to solve a 2D problem in the cross section of the device. Furthermore, we notice that we can solve the model in an eighth of the geometry by imposing an evenly periodic boundary condition due to the configuration of coils and magnets.

The motor is driven by a uniformly distributed sinusoidal three phase current. The permanent magnets are assumed to have a remanent flux density of 1.26 T oriented parallel to the radial direction at each magnet center (i.e., parallel to its edges). On the other hand, rotor and stator are composed by a nonlinear material described by the curve $\mathbf{B}(\mathbf{H})$ depicted in Figure 1-right, while all the other materials are considered to be linear. We notice that the high relative permeability of ferromagnetic core with respect to the air surrounding the device would ensure that most of the flux will remain inside the stator,

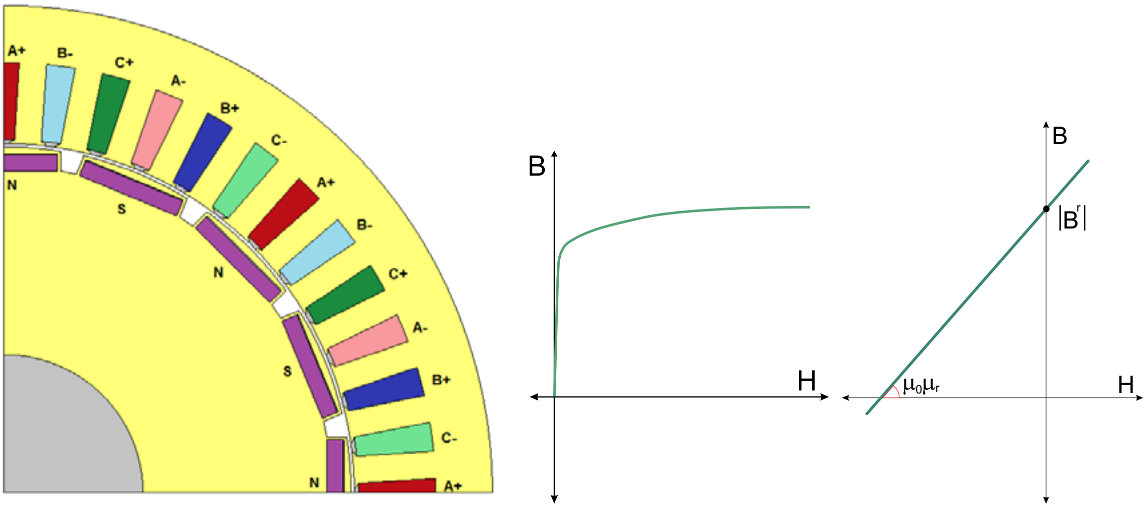

Figure 1: Sketch of the motor geometry with current phases and permanent magnet orientation (left). $\mathbf{B}(\mathbf{H})$ curve for rotor and stator and permanent magnet (right).

and therefore flux lines are constrained to follow the stator boundary. This leads us to impose a Dirichlet homogeneous condition on the exterior boundary of the device.

Figure 2-left shows the modulus of the computed flux density distribution and Figure 2-right shows the corresponding vector field. From the values of the magnetic flux density at each point, we have computed the losses in each material by using a posteriori estimation techniques (see, for instance, [4]). These losses have been introduced as sources in the thermal model by assuming that are invariant in the z-direction.

For the heat equation, the GLP method has been implemented in a computer program by using Matlab. The numerical algorithm consists of two parts, that can be executed independently:

1. in the first one, functions φ_{ij}^P and φ_{ij}^C are calculated as solution of systems (34)–(37) and (38)–(41);

2. in the second one, function $\tilde{\theta}$ is calculated as solution of an ordinary differential system of equations equivalent to (42)–(43).

For the second case, we consider an electric motor designed by the University of Mondragon and the Orona company. In this example we consider a motor at room temperature that starts at $t = 0$ and works for 2 hours. In order to check the program, electric losses were measured by the University of Mondragon and they are given as data to the program. The program also admits electric losses calculated with an external code. The motor is decomposed into 34 pieces, each of them represents a sub-domain in the GLP method. When the temperature is constant at ports, the total number of basis functions in GLP method is only 70 and the relative error in $L^2(0,T);\Omega)$ respecto to the FE solution is close to 2% (see [3]).

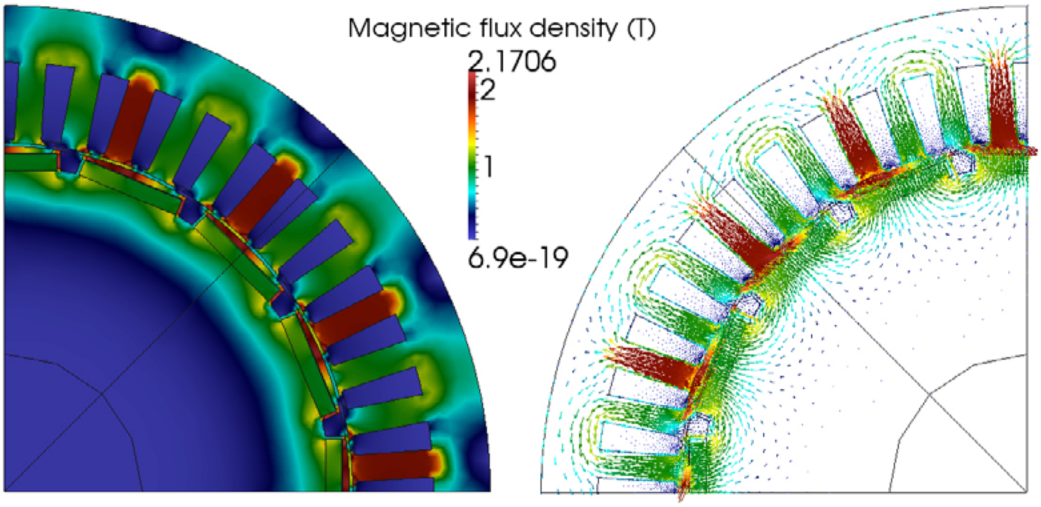

Figure 2: Modulus (left) and vector field (right) of the magnetic flux density in the motor.

In order to reduce the error derived from the fact that temperature is taken to be constant at the ports, a "nodal" version GLP method was programmed. In this case, a *port* is each single node between two pieces. Now, the meshes of the different sub-domains must be conforming on the common interfaces and the number of basis functions is related to the number of nodes on the ports, not to the number of ports or convective surfaces. In Figure 3 a cross-section of the motor can be observed. Temperature was calculated with the nodal version.

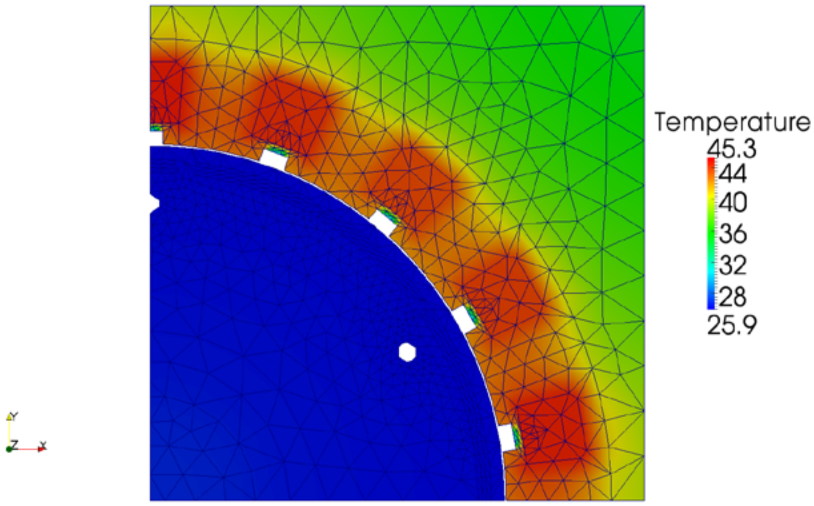

Figure 3: Temperature calculated with the nodal GLP method.

In this example, the nodal version presents a relative error with respect to the standard

FE method ten times smaller than the GLP method. Still, step 2 of the algorithm is up to 2.5 times faster than the standard FE method.

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