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MODELLING WATER INFILTRATION INTO MACROPOROUS HILL SLOPES USING SPECIAL BOUNDARY CONDITIONS

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Abstract. The formulation of suitable boundary conditions is a very crucial task when modeling water infiltration into macroporous hill slopes. The processes of water infiltration and exfiltration vary in space and time and depend on the flow on the surface as well as in the subsurface. In this contribution we have purposed special system process dependent boundary conditions can be formulated for a two-phase dual-permeability model to simulate infiltration and exfiltration processes. The presented formulation analyses the saturation conditions of the dual-permeability model (e.g. saturation) at the boundary nodes and adopts the boundary conditions depending on the processes at the soil surface such as rainfall intensity. Using a simplified macroporous hill slope and a heavy rainfall event we demonstrate the functionality of our formulation.

1 INTRODUCTION

Fast water infiltration into hill slopes during rainfall is an important issue since it can reduce slope stability and act as trigger for landslides. Modelling the water infiltration is an important key to understand the processes that can lead to a slope failure. The soil of natural slopes may be highly strongly heterogeneous and contains often macropores which strongly affect water flow[1]. For this reason, dual-permeability models are frequently used to simulate the coupled flow processes in such macroporous soils, where the soil is separated into two coupled overlaying domains, a matrix domain containing the small matrix pores and a macropore domain containing larger pores (e.g. earthworm channels, fissures and fractures). Separate balance equations are defined for each domain and mass transfer functions are introduced to describe the fluid exchange depending, for example on the pressure differences between macropore and matrix and the resistance along its interface. Dual-permeability models that simulate the water infiltration into the unsaturated zone are typically based on the Richards equation for both domains as found in Gerke and van Genuchten[2]. A more general dual-permeability model can be obtained when applying the two-phase flow equations instead of the Richards equation. This is necessary when the mobility of the soil air must be taken into account and the air pressure deviates from atmospheric pressure. Typical examples for that are strongly heterogeneous and layered soils where water is ponding and soil air escape is limited[3]. As such a case is investigated here, we decided to use the two-phase (water/air) dual-permeability model developed by Stadler et al.[4] of the multi-scale multi-physics toolbox DuMux[5] for our work.

A special difficulty for modelling the water infiltration processes with a dual permeability model is the definition of reasonable boundary conditions. Most of the rainwater will usually infiltrate via the matrix into the soil until the infiltration capacity of the matrix is exceeded. When surface runoff occurs, water will also infiltrate directly into surface connected macropores. Consequently, the formulation of boundary conditions must be flexible since those for matrix and macropores are coupled and depend on the pressure and saturation conditions which vary in space and time. Such special boundary conditions are also called system-dependent boundary conditions[6]. In this paper we discuss all the different cases which can occur during infiltration and exfiltration and we describe the way they are implemented as boundary conditions in an external module of our two-phase dual-permeability model within DuMu^x.

2 DUAL-PERMEABILITY MODEL

Our dual-permeability model is based on the separation of the soil pores into matrix pores and macropores. Mass balance equations combined with the extended Darcy's law are first defined for both pore systems (domains) separately. They are then linked by a mass transfer equation to describe the fluid exchange between matrix and macropore domain. A detailed review of models and concepts for dual-permeability models can be found in Šimůnek et al. [7].

2.1 Model equations

The balance equations for a two-phase flow dual-permeability system for the wetting phase w and the non-wetting phase n can be written for the matrix domain m and the macropore domain f as:

$$\begin{aligned}
\phi^m \frac{\partial\left(S_w^m \rho_w^m\right)}{\partial t} + \nabla\cdot\left(\rho_w^m \vec{v}_w^m\right) - q_w^m &= \Gamma_w \\
\phi^m \frac{\partial\left(S_n^m \rho_n^m\right)}{\partial t} + \nabla\cdot\left(\rho_n^m \vec{v}_n^m\right) - q_n^m &= \Gamma_n \\
\phi^f \frac{\partial\left(S_w^f \rho_w^f\right)}{\partial t} + \nabla\cdot\left(\rho_w^f \vec{v}_w^f\right) - q_w^f &= -\Gamma_w \\
\phi^f \frac{\partial\left(S_n^f \rho_n^f\right)}{\partial t} + \nabla\cdot\left(\rho_n^f \vec{v}_n^f\right) - q_n^f &= -\Gamma_n .
\end{aligned} \tag{1}$$

Where ϕ^i [L^{3}L^{-3}] is the porosity of a domain i (matrix/macropore), S_α^i [L^{3}L^{-3}] the fluid saturation of a phase α, ρ_α^i [kg m^{-3}] the density, $\vec{v}_\alpha^i$ [m s^{-1}] the vector of the Darcy velocity, q_α^i [kg s^{-1}] is a source/sink term and Γ_α [kg s^{-1}] a mass transfer term that describes the exchange between matrix and macropore domain. In a two-phase water/air system the water will be the wetting phase and the air the non wetting phase. The pressure difference between both phases in each domain (i) is equivalent to the capillary pressure p_c^i [Pa] and can be described as function of the effective saturation S_e^i.

$$p_c^i(S_e^i) = p_n^i - p_w^i \tag{2}$$

$$S_e^i = (S_w^i - S_{wr}^i)/(1 - S_{wr}^i - S_{nr}^i)\,. \tag{3}$$

$S_{\alpha r}^i$ is the residual saturation of a fluid. In the presented work we use the formulation after van Genuchten[8] to compute the capillary pressure relationship:

$$p_c^i = \frac{1}{\alpha}\left[(S_e^i)^{-1/m} - 1\right]^{1/n}. \tag{4}$$

The van Genuchten parameters α, m, n are parameters that describe the shape of the relationship. The parameters depend on the soil properties and are different for each domain of the dual-permeability model. The fluids in each domain fill the full pore space of the domain so that the sum of both saturations is in each domain is equal to one:

$$S_w^i + S_n^i = 1\,. \tag{5}$$

The Darcy velocity of a phase in a domain can be computed with the extended Darcy law:

$$\vec{v}_\alpha^i = -\underline{\underline{K}}\frac{k_{r\alpha}^i}{\mu_\alpha^i}(grad\, p_\alpha^i - \rho_\alpha^i g)\,, \tag{6}$$

where $\underline{\underline{K}}$ [m^2] is the intrinsic permeability tensor, μ_α^i [kg m^{-1} s^{-1}] the dynamic viscosity and k_α^i [-] is the relative permeability which can be calculated with the van Genuchten relationship in combination with the model of Mualem[9]:

$$k_{rw}^i = \sqrt{S_e^i}\left[1 - \left(1 - (S_e^i)^{1/m}\right)^m\right]^2 \tag{7}$$

$$k_{rn}^i = \left(1 - S_e^i\right)^{1/3}\left[1 - (S_e^i)^{1/m}\right]^{2m}. \tag{8}$$

Where m, n are again the van Genuchten parameters which depend on the considered domain. The mass transfer between both domains is approximated by a first-order transfer equation[10] that depends on the pressure differences between both domains

$$\Gamma_\alpha = s\rho_\alpha\lambda_\alpha\beta(p_\alpha^m - p_\alpha^f)\,, \tag{9}$$

s [m^{-1}] is a scaling factor between the regarded soil volume [m^3] and the macropore surface. β [m] is a surface resistance parameter and λ_α [kg m^{-1} s^{-2}] the mobility (relative permeability over dynamic viscosity).

2.2 Numerical model

The four balance equations (eq. 1) of the two-phase dual-permeability model together with supplementary and further conditions (eqs. 2-9) yield to a strongly coupled system of four non-linear partial differential equations with mixed parabolic / hyperbolic character. DuMu^x applies a local and global mass conservative box method (mixture of Finite-Element and Finite-Volume Method) for the spatial discretization of the dual permeability model. The time discretization is done with a full implicit Euler scheme[11]. Further, the Newton-Raphson Method is used for the linearization of the system[12].

We selected the pressures of the non-wetting phase and the saturation of the wetting phase as primary variables. The switch of the boundary conditions presented in the following is determined by an analysis of the values on the actual time level while the primary variables are computed on the new time level.

3 DEFINITION OF SYSTEM DEPENDEND BOUNDARY CONDITIONS

Natural systems like hill slopes are characterized by a strong temporal and spatial variation of subsurface (e.g. saturated/unsaturated) and surface-water flow conditions (e.g. overland flow/dry conditions). It is urgently required to simulate subsurface flow in a natural slope with varying boundary conditions since soil and surface are representing a coupled system. The most common cases where boundary conditions must be adopted are water infiltration during rainfall and water exfiltration during saturated conditions (see Fig. 1).

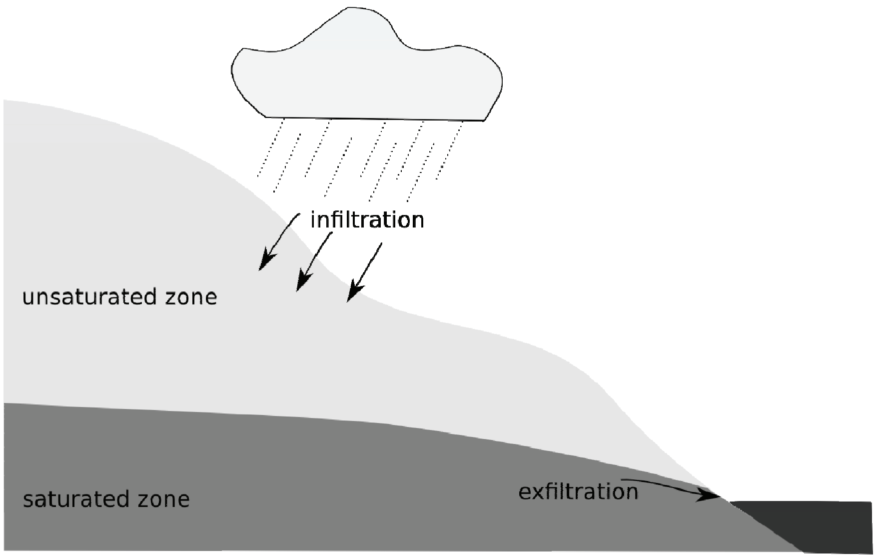

Figure 1: Water infiltration and exfiltration.

Surface runoff can occur in steep hill slopes and when the permeability and / or rainfall intensity are high. However, surface runoff is not taken into account here. The impact of this simplification is reduced when the hill slope gets flatter and when the permeability and rainfall intensity are getting smaller. The presented concept can be easily extended and coupled with surface runoff models.

It is possible to define four different inner states of the system (Fig. 2, left) which depend on the soil conditions in the matrix and macropore domain. In combination with the available water for infiltration, this leads to eight possible cases which must be distinguished for the definition of which will be explained in the following. The available water can be water from a rainfall event, overland flow or ponding surface system-dependent boundary conditions and water.

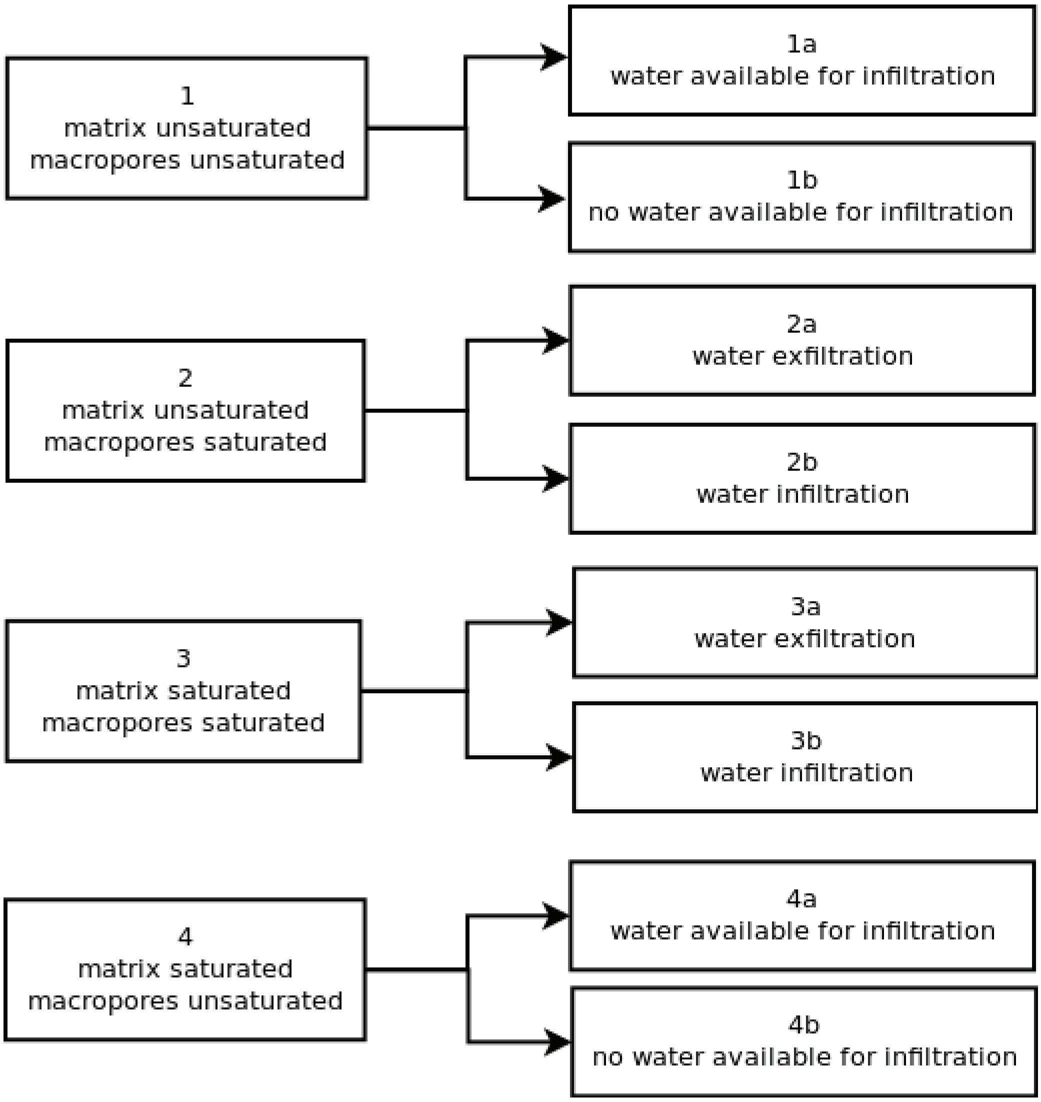

Figure 2: Possible states matrix and macropore domain (left) in combination with possible states at the soil surface (right).

In a first step it is necessary to determine the system state. Therefore, the saturations at boundary nodes are regarded. If a cell is saturated, the mass fluxes (Fig. 3) are additionally computed to analyze whether water is infiltrating or exfiltrating over the boundary surface. The corresponding cell fluxes over the cell surfaces (inner boundaries) are $F_{surface}$ and the fluxes between matrix and macropore domain are mass transfers $F_{transfer}$. If the sum of both fluxes is positive, outflow over the boundary will occur when the element is saturated. A negative sign indicates that water will infiltrate over the surface boundary.

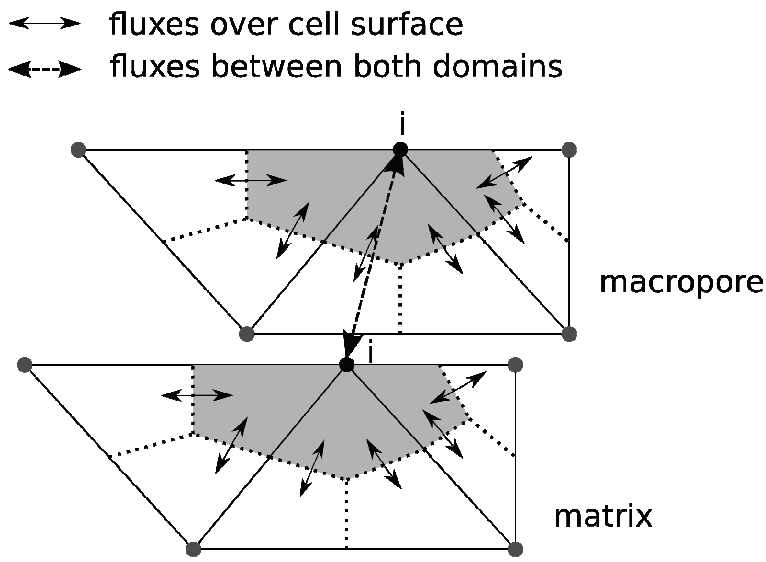

Figure 3: Fluxes at boundary node (i) for matrix and macropore domain and mass transfer fluxes for fluid exchange between both domains.

In the following we discuss the four possible cases and the corresponding sub-cases which can occur. Based on the presented concept Dirichlet and Neumann boundary conditions can be prescribed for both domains. The pressure of the gas phase is chosen as primary variable in both domains and is set to atmospheric pressure as long no water table stands above the soil surface. The saturation of the water phase is variable and Dirichlet or Neumann boundary conditions are set depending on the system state.

3.1 Case 1 – unsaturated matrix and unsaturated macropore domain

No ponding of water occurs as long as both domains are unsaturated. Thus, atmospheric pressure is set as Dirichlet boundary condition for the gas phase of both domains (**1a, 1b**). A Neumann no-flow boundary condition is set for the water phase in both domains (**1b**) if no water is available for infiltration. If water is available, it is checked whether the actual infiltration capacity $q^m_{w,\max}$ of the matrix is exceeded. If yes, the remaining water infiltrates via the macropores ($q^m_w = q^m_{w,\max}$ and $q^f_w = q^f_w - q^m_{w,\max}$). If not, the whole water infiltrates via the matrix ($q^m_w = q_i$ and $q^f_w = 0$). Due to the high macropore conductivity there is usually no limitation for the water infiltration into the macropores until they are saturated.

3.2 Case 2 - unsaturated matrix and saturated macropore domain

Macropore flow will usually only occur when the matrix is saturated. However, in some cases the water can bypass an unsaturated matrix and flow through the macropore domain. This is a special case which may occur at the toe of a slope where water can flow out through saturated macropores. Regardless of whether water infiltrates or exfiltrates (**2a, 2b**), atmospheric pressure is set for the gas phases in both domains (Dirichlet boundary condition).

The definition of boundary conditions is very complex for case 2 since a high non-equilibrium between matrix and macropore domain exists. For single domain concepts water will only exfiltrate when the soil is fully saturated. This can be also prescribed for dual-permeability models when a Neumann no-flow boundary condition for both domains is set. However, the water can bypass an unsaturated matrix and escape to the surface. Thus, a Neumann boundary condition is set for the water phase of the matrix and a Dirichlet boundary condition is set for the macropore domain (**2a**). The water can also infiltrate from the surface into saturated macropores during infiltration (**2b**). If more water than available infiltrates via the macropores the Dirichlet boundary condition for the water phase is switched to a Neumann boundary condition.

3.3 Case 3 – saturated matrix and saturated macropore domain

For this case it is necessary to check the mass fluxes at the boundary node to control whether water is infiltrating or exfiltrating. As mentioned above, surface runoff can be neglected for macroporous slopes and the pressure in both domains is set as Dirichlet boundary condition, assuming atmospheric pressure during ex-filtration (**3a**). The influence of the pressure increase du the water level will be negligible for small water depths. However, when water stands above the surface (e.g. river), the pressure must be adopted. For infiltration (**3b**) it is checked whether if the infiltration capacity exceeds the available water and in case a switch to a Neumann boundary condition for the water phases in one or both domains (depending on the infiltration rates) is carried out.

3.4 Case 4 – saturated matrix and unsaturated macropores

The last case generally occurs during infiltration (**4a**) if the infiltration capacity $q^m_{w,\max}$ of the matrix is lower than the available water for infiltration. Then the rest of the available water will infiltrate via the macropores ($q_i - F_{w,transfer} - F_{w,matrix} = q^f_w$). This case is implemented by a Neumann boundary condition for the water phase of the macropore domain and Dirichlet boundary conditions for the water phase in the matrix domain (fully saturated). The pressure of the gas phase is set again to atmospheric pressure. If no water is available for infiltration, outflow may occur via the matrix pores (**4b**). However, the water will directly infiltrate into the macropores. A Neumann no-flow boundary condition is set for the water phase in the matrix domain. This leads to an increasing saturation in the macropore domain and avoids water exfiltration until the macropores are saturated.

4 EXAMPLE

Common examples where system-dependent boundary conditions can be demonstrated are small slopes where the water infiltration leads to an increasing groundwater table during infiltration. The model domain (Figure 4) for our example is a simplified macroporous hill slope similar as shown in Figure 1. The soil parameters for the study are given in Table 1. A rainfall event with a intensity of 40mm/h and a duration of two hours is investigated to study water infiltration. The groundwater table at the initial state (Figure 3a) is influenced by the water body at the right side where a water level (e.g. lake) is imposed using Dirichlet boundary conditions for the matrix and macropore domain. The level of the water body is assumed to be constant during the whole simulation time. The initial saturation in the matrix and macropore domain is very low (~0.3). The rainfall intensity will exceed the infiltration capacity of the matrix so that macropore flow will occur directly. Soil parameters and initial conditions are chosen to test most of the possible cases during this extreme situation.

Starting with a low saturated zone (Figure 3a), the water table increases mainly in the right part of the hill slope during water infiltration, the nodes on the right boundary above the water table get saturated and switch to Dirichlet conditions (Figure 3b, right). Most of the water infiltrates via the macropores and bypasses the matrix because of the low permeability of the matrix domain. After the rainfall event has finished, the saturation of the boundary nodes at the right side is reduced and they switch back to Neumann no-flow boundary conditions (Figure 3c, right).

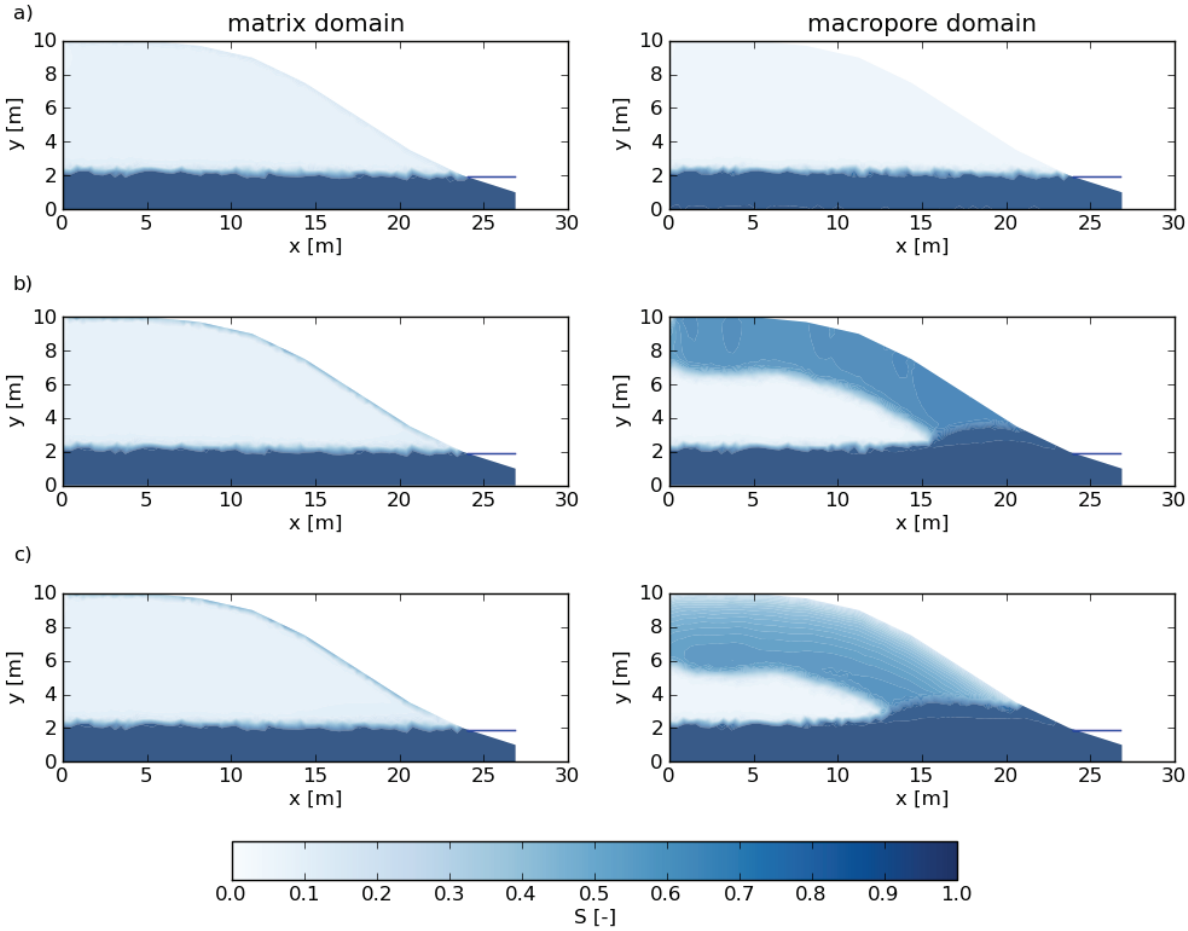

Figure 4: Water saturation of the matrix (left) and macropore domain (right) for various times (a = 0h, b = 2h and c = 2.5h) for an infiltration rainfall rate of 40mm/h and a period of two hours.

Table 1: Soil parameters

	matrix domain	macropore domain
S_{wr} [-]	0.14	0.05
S_{nr} [-]	0.05	0.05
K [m^2]	1.0E-13	1.0E-11
van Genuchten n [-]	2.2	4.2
van Genuchten α [Pa^{-1}]	5.0E-4	8.0E-3
Φ [-]	0.4	0.08
β [m]	2.0E-13	2.0E-13
s [m^{-1}]	1	1

5 CONCLUSIONS

In this paper we have proposed a concept to simulate simplified interactions between surface and subsurface flow on macroporous hill slopes where the overland flow is not taken into account yet. The two-phase dual-permeability model of the numerical simulator DuMux was extended by special system-dependent boundary conditions to simulate infiltration and exfiltration processes on macroporous hill slopes. An idealized system with a low matrix infiltration capacity was investigated to test the model capabilities during a high rainfall event. Due to the low permeability of the soil matrix the rainwater bypassed the soil matrix and water infiltration and exfiltration mainly occurred via the macropores. In future work the proposed concept will be further extended and coupled with a surface runoff model.

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