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METHOD FOR ESTIMATING PARAMETERS OF COUPLED PROBLEM OF INTERACTION OF GAS FLOWS LOADED BY SOLID PARTICLES WITH SOLIDS

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Abstract. The present paper outlines a method of processing and analysing experimental data based on the methodology of the solution of inverse heat transfer problems. An algorithm and the results of the computational and experimental study of heat transfer in the vicinity of the critical point of a specimen in a high-enthalpy particles-loaded flow are presented. One of the main difficulties here is how to determine coefficients of the mathematical model, which provide its adequacy to real processes. Direct measurement of most characteristics of heat transfer is usually impossible, and their theoretical estimates are often far from being true and often contradictory. That is why, the unknown parameters of the heat-balance equation at the external moving boundary of the specimen are determined from the inverse problem of heat transfer, which is solved by the method of iterative regularization. The results of experimental data processing for the interaction of particles-loaded flows with plane surfaces of the cylindrical specimen are also presented as well as the optimal experiment design problems for corresponded experiments.

NOMENCLATURE

b	thickness of a specimen
f	experimental measurements
g	increment of unknown function
J	minimized (residual) functional
M	number of temperature measurements
$\overline{P}$	unknown (desired) parameters vector
γ	descent step

δ measurements error
θ sensitivity function
σ deviation of measurements

1 INTRODUCTION

With the development of new materials and processes, identification of system parameters has been the prime goal for understanding and defining these systems. Under development is an approach to a study of high temperature thermal processes, based on the principles of identification of non-linear systems with distributed parameters. One of the main difficulties is how to determine the coefficients of a model, which is adequate to real processes. Methods based on solving boundary inverse heat conduction problems are also widely used in experimental investigations of the thermal interaction between solids and the environment.

Presently, heat and mass transfer in heterogeneous media are being studied intensively [1], [2], [3]. This interest is associated with the important practical applications of the results of these investigations in aerospace technology, nuclear power engineering, turbine manufacture, chemical technology, and other fields.

The present work outlines a method of processing and analyzing experimental data based on the methodology of the solution of inverse heat transfer problems. The experiments are conducted in a gas-dynamic stand specially designed for modeling particles-loaded flows. Solid particles are introduced into the gas flow through a special particle source (Figure 1). The uniformity of the particle distribution over the flow cross-section and the steady flow rate of the particles during the experiment are ensured by a special supply system. The particle velocities are calculated as described in [4]. Experimental investigation of the thermal interaction of the particles-loaded flow with the material is conducted at a special calorimetric module (Figure 2).

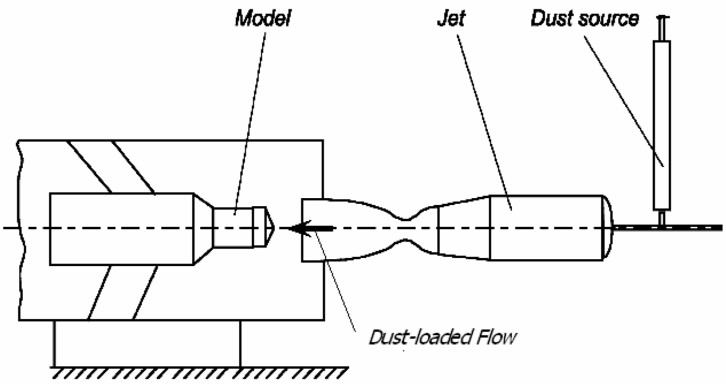

Figure 1: Experimental facilities

The structure of the specimen's models (Figure 3) permits the use of a one-dimensional mathematical model of thermal conduction. The heat transfer in the calorimeter is described by a homogeneous heat-conduction equation

$$C(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) \qquad (1)$$
$$T = (x,\tau), \quad x \in (0, b(\tau)), \quad \tau \in (\tau_{\min}, \tau_{\max}]$$

where

$$b(\tau) = b(\tau_{\min}) - \int_{\tau_{\min}}^{\tau} V_{er}\ d\tau$$

is the coordinate of the specimen external surface heated by a two-phase flow and undergoing erosion, and V_{er} is the linear rate of erosion. $\tau_{\min}, \tau_{\max}$ are the times at which the experiment begins and ends; $C(T)$ is the volume heat capacity; λ is the thermal conductivity. The initial temperature and boundary condition at the internal boundary are known, and take the form

$$T(x,\tau_{\min}) = T_0(x), \quad x \in [0, b(\tau_{\min})] \qquad (2)$$

$$T(0,\tau) = T_1(\tau), \quad \tau \in (\tau_{\min}, \tau_{\max}] \qquad (3)$$

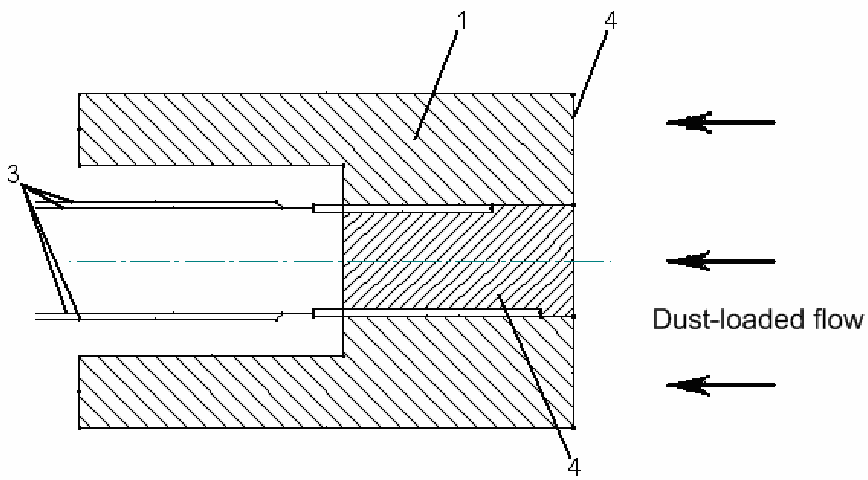

Figure 2: Experimental module: 1 – model, 2 – sensor, 3 – thermocouples, 4 – model surface

At the external boundary, (exposed to the particles-loaded flow) the following conditions are considered

$$-\lambda(T)\left(\frac{\partial T}{\partial x}\right)_{\omega} = q_2(\tau), \quad \tau \in (\tau_{\min}, \tau_{\max}] \tag{4}$$

where

$$\left(\frac{\partial T}{\partial x}\right)_{\omega} = \frac{\partial T(b(\tau), \tau)}{\partial x}$$

$$q_2(\tau) = H \ (parameters\ of\ \ particles - loaded\ \ flow)$$

and at this boundary $(x = b(\tau))$ the condition of heat balance can be considered in the form [5]

$$-\lambda(T)\left(\frac{\partial T}{\partial x}\right)_{\omega} = q_{conv} + q_{turb} + q_r + q_{acc} \tag{5}$$

where q_2 is the heat flux that penetrates to the model in the vicinity of the critical point, q_{conv} is the external convective heat flux, q_{turb} is the heat flux resulting from additional turbulence caused by solid particles, q_r is the additional heat flux resulting from an increase in the surface roughness, as a result of the contact with solid particles, and q_{acc} is the heat flux generated as a result of the accommodation of kinetic energy of solid particles at the specimen surface.

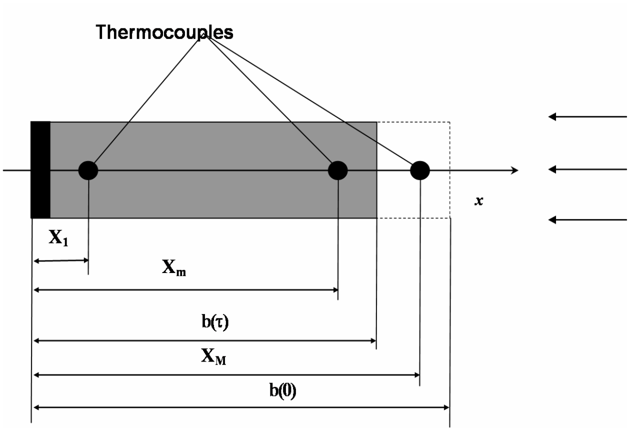

Figure 3: Scheme of sensor.

Relations for q_{conv} and q_{acc} have the form

$$q_{conv} = \frac{\alpha}{C_p}(J_e - J_{\omega}) \tag{6}$$

$$q_{acc} = a_{acc} \frac{G_p V_p^2}{2} \tag{7}$$

where α is the heat transfer coefficient; J_e and J_ω are the enthalpy of the flow at the recovery temperature and the temperature of the external surface of the model, respectively; a_{acc} is the accommodation coefficient; G_p is the mass rate of the solid phase; and V_p is the rate of the particles.

The effect of additional turbulence can be taken into account in the form [5]

$$q_{turb} = \frac{\alpha}{C_p}(J_e - J_\omega)a_1\left(\frac{G_p + G_{er}}{G_g}\right)^{n_1} \tag{8}$$

where a_1 and n_1 are non-dimensional coefficients, G_g is the mass rate of the injected gaseous phase, and G_{er} is the mass rate of erosion of the model, defined by

$$G_{er} = \rho V_{er} \tag{9}$$

where ρ is the density of the material of the model and $n_1 = 1$ for a particle concentration of less than 1%.

The increase in the intensity of heat transfer due to the surface roughness is associated with the Reynolds criterion and the relation between value h of the surface roughness and thickness θ of the loss of momentum [6]

$$q_r = \frac{\alpha}{C_p}(J_e - J_\omega) f_r \frac{h}{\theta} \mathrm{Re}_0^{0.5} \tag{10}$$

where f_r is the roughness coefficient. According to [6], the thickness of the loss of momentum is defined by the equation

$$\theta = 0.245\sqrt{\frac{\nu_e}{\beta}}\left(1.4 - 0.4\frac{T_\omega}{T_e}\right) \tag{11}$$

where ν_e is the kinematics viscosity of the gas and

$$\beta = \frac{4V_g}{3\pi R_T}\sqrt{\bar{\rho}(2-\bar{\rho})} \tag{12}$$

is the gradient of the gas in the vicinity of the critical point. Here, V_g is the velocity of the gas flow, R_T is the radius of the specimen, and $\bar{\rho} = \rho_g / \rho_2$ is the ratio of the density of the gas in the flow to its density behind the shock wave. This ratio and the Reynolds number are given by the formulas

$$\overline{\rho} = \frac{(k-1)}{(k+1)} + \frac{2}{(k+1)} \frac{1}{M_\infty^2} \tag{13}$$

$$\mathrm{Re}_0 = \frac{\rho_g V_g R_T}{\mu_g} \tag{14}$$

where k is the adiabatic exponent and μ_g is the dynamic viscosity of the gas. Under these assumptions, the mathematical model of nonsteady-state mass transfer caused by the interaction of materials with two-phase flows is covered by equations (1)-(14).

In general, an inverse problem formulated this way has no unique solution. To provide the uniqueness of the solution, we proposed the simultaneous analysis of the data of several nonstationary experiments under different conditions of thermal interaction of the specimens with particles-loaded flow. The method based on the variations of particle concentration in the forward flow is most appropriate for changing the loading mode.

The characteristics $\alpha/C_p, a_1, f_r, a_{acc}$ are assumed to be constant in the mathematical model described by equations (1)-(14). In this case, the heat-balance equation (4) can be written in the form

$$-\lambda(T)\left(\frac{\partial T}{\partial x}\right)_\omega = H\left(\overline{P}, T_\omega(\tau), \tau\right) \tag{15}$$

where $\overline{P} = \{P_1, P_2, P_3, P_4\}^T$ is the vector of unknown characteristics whose components are defined by the relations $P_1 = \alpha/C_p$, $P_2 = a_1$, $P_3 = f_r$, $P_4 = a_{acc}$ and H is a function of known form. As a result, the inverse problem, which consists in determining the characteristics of heat transfer on the surface of a material, is formulated as follows. It is necessary to determine the vector of unknown parameters satisfying the mathematical model (1)-(15), using the data on additional internal temperature measurements

$$T_n\left(X_{m,n}, \tau\right) = f_{m,n}(\tau), \quad m = 1,\dots,M_n, \quad n = 1,\dots,N \tag{16}$$

where n is the number of the experiment, N is the total number of simultaneously analysed experiments, and M_n is the number of thermocouples in the $n-th$ experiment. The functions $b_n(\tau), C(T), \lambda(T), \tau_{\min}^n, \tau_{\max}^n, T_{0,n}(x)$ and $T_{1,n}(\tau)$, as well as the form of the function H, are known.

2 IDENTIFICATION OF THE MATHEMATICAL MODEL

The algorithm used to solve the inverse problem is constructed on the basis of the gradient method of minimization of the mean square functional of discrepancy (see Alifanov, Artyukhin et al. (1995)). In the case under consideration, the discrepancy functional is composed for the entire body of simultaneously analysed experiments and has the form

$$J(\overline{P}) = \sum_{n=1}^{N} \sum_{m=1}^{M_n} \int_{\tau_{\min}^n}^{\tau_{\max}^n} \left(T_n(X_{m,n}, \tau) - f_{m,n}(\tau)\right)^2 d\tau \tag{17}$$

where $T_n(x,\tau)$ is the solution of the boundary-value problem equations (1)-(15) for the $n-th$ experiment.

The iteration process of successive approximations of the unknown vector $\overline{P}$ is constructed in accordance with the following procedure:

1. The initial approximation of the vector of required parameters is preset:

$$\overline{P}^0 = \left\{P_1^0, P_2^0, P_3^0, P_4^0\right\}^T .$$

2. The value of the required vector at the next iteration is calculated according to the formula

$$\overline{P}^{s+1} = \overline{P}^s + \Delta\overline{P}^s, \quad s = 0,1,... \tag{18}$$

where s is the iteration number. The increment $\Delta\overline{P}^s$ is determined from the condition

$$\frac{\partial J\left(\overline{P}^s + \Delta\overline{P}^s\right)}{\partial \Delta\overline{P}^s} = 0 \tag{19}$$

3. The condition $J\{\overline{P}\} = \delta^2$ of stopping of the iterative process is verified, where δ^2 is the integral error of temperature measurements:

$$\delta^2 = \sum_{n=1}^{N} \sum_{m=1}^{M_n} \int_{\tau_{\min}^n}^{\tau_{\max}^n} \sigma_{m,n}(\tau)^2 d\tau \tag{20}$$

where $\sigma_{m,n}(\tau)$ is the deviation of $\{m,n\}$-th measurement. If this condition is satisfied, then the iterative process is terminated. In the opposite case, the procedure of successive approximations is continued.

Following the approach suggested to calculate increments of the desired vector $\Delta\overline{P}^s$, the minimized functional (17) at the $(s+1)$-th iteration can be presented as

$$J(\overline{P}^{s+1}) = \sum_{n=1}^{N} \sum_{m=1}^{M_n} \int_{\tau_{\min}^n}^{\tau_{\max}^n} \left(T_n^s(X_{m,n}, \tau) + \Delta T_n^s(X_{m,n}, \tau) + o(\Delta P^2) - f_{m,n}(\tau)\right)^2 d\tau \tag{21}$$

where

$$\Delta T_n^s(X_{m,n}, \tau) = \sum_{k=1}^{4} \Delta T_{n,k}^s(X_{m,n}, \tau) = \sum_{k=1}^{4} \frac{\partial T_n^s(X_{m,n}, \tau)}{\partial P_k} \Delta P_k = \sum_{k=1}^{4} \theta_{n,k}(X_{m,n}, \tau) \Delta P_k \tag{22}$$

Here the temperature $T_n^s\left(X_{m,n},\tau\right)$ can be determined from the solution of the direct problem (1)-(15) at $\overline{P}=\overline{P}^s$, and $\theta_{n,k}(x,\tau),\ k=1,2,3,4$ satisfy the following set of equations

$$C(T)\frac{\partial\theta_{n,k}}{\partial\tau}=\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial\theta_{n,k}}{\partial x}\right)+\frac{\partial T}{\partial x}\frac{d\lambda}{dT}\frac{\partial\theta_{n,k}}{\partial x}+$$
$$+\left(\frac{\partial^2 T}{\partial x^2}\frac{d\lambda}{dT}+\frac{d^2\lambda}{dT^2}\left(\frac{\partial T}{\partial x}\right)^2-\frac{\partial T}{\partial\tau}\frac{dC}{dT}\right)\theta_{n,k} \tag{23}$$
$$\theta_{n,k}=\theta_{n,k}(x,\tau),\quad x\in\left(0,b^n(\tau)\right),\quad n=1,2,...,N,\quad \tau\in\left(\tau_{\min}^n,\tau_{\max}^n\right]$$

$$\theta_{n,k}\left(x,\tau_{\min}^n\right)=0,\quad x\in\left[0,b^n\left(\tau_{\min}^n\right)\right],\quad n=1,2,...,N \tag{24}$$

$$\theta_{n,k}(0,\tau)=0,\quad \tau\in\left(\tau_{\min}^n,\tau_{\max}^n\right] \tag{25}$$

$$\lambda(T)\frac{\partial\theta_{n,k}(b(\tau),\tau)}{\partial x}-\frac{\partial T}{\partial x}\frac{d\lambda}{dT}\theta_{n,k}(b(\tau),\tau)-\frac{\partial H}{\partial T}\theta_{n,k}(b(\tau),\tau)-\frac{\partial H}{\partial P_k}=0, \tag{26}$$
$$\tau\in\left(\tau_{\min}^n,\tau_{\max}^n\right]$$

Then, using equation (22), a system of linear algebraic equations can be obtained

$$\sum_{l=1}^{4}\Delta P_l\sum_{n=1}^{N}\int_{\tau_{\min}^n}^{\tau_{\max}^n}\theta_{n,l}\left(X_{m,n},\tau\right)\theta_{n,k}\left(X_{m,n},\tau\right)d\tau=$$
$$=-\sum_{n=1}^{N}\sum_{m=1}^{M_n}\int_{\tau_{\min}^n}^{\tau_{\max}^n}\theta_{n,k}\left(X_{m,n},\tau\right)\left(T_n\left(X_{m,n},\tau\right)-f_{m,n}(\tau)\right)d\tau,\quad k=1,2,3,4 \tag{27}$$

from which the increments $\overline{\Delta P}$ can be determined.

3 EXPERIMENTAL APPROVING

In implementing the algorithm described, the value problem is solved by a finite difference method on an implicit four-point scheme. In the numerical solution the direct nonlinear problem is treated by iteration in the coefficients. The approximation of all three boundary value problems is carried out on one and same difference grid, making it possible to achieve error matching. Below we provide results of handling the experimental data obtained during the four-point process of an experimental device. The tests differed from each other by the mass discharge values of solid phase. The particle diameter of the solid phase was 250 μm. The rate of the solid phase particles was V_p =1083 m/sec, the gas rate was V_g =1797 m/sec, and the mass discharge of gas was G_g =1490 kg/(m^2/sec). Variation of the length of the specimen $b(\tau)$ as a result of erosional destruction in the course of the experiment is shown in Figure 4. The remaining characteristics are shown in Table 1.

Cylindrical specimen of radius 40mm, prepared from copper, had different initial thickness $b(0)$ (Figure 4), as well as an amount and coordinates of thermocouple devices in the transducers, while one of the thermocouples was located on the rear surface, and the remaining ones – at the internal points of the transducers, located in the vicinity of the critical point of the model. The results of thermocouple measurements in all specimens are shown in Figure 3. In solving the inverse problem the experimental data obtained in the six tests were analyzed simultaneously. The temperature value calculated in this case at the points of the thermocouple devices are also given in Figure 5. The results of processing the experimental data are presented in Table 2.

Table 1: Experimental Data

No, of tests	G_p, kg/(m^2/sec)	T_0, K	V_{er}, kg/(m^2/sec)
1	5.1	1621	2.51
2	6.9	1593	3.52
3	0	1581	0.0
4	7.2	1612	3.91
5	7.8	1579	4.14
6	0	1637	0.0

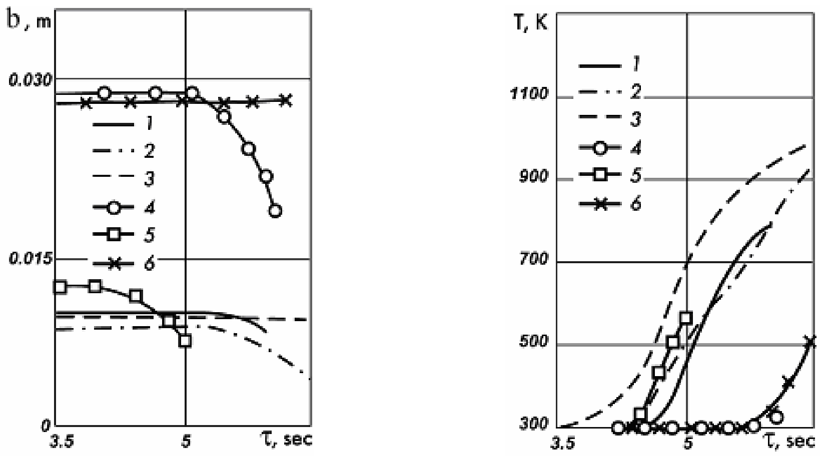

Figure 4: Erosional destruction of specimens and temperature at the left boundaries (6 tests).

Table 2 :Estimated Parameters

Parameters	Estimates
$\frac{\alpha}{C_P}$	$2.42\,kg/(m^2\sec)$
a_{acc}	0.71
a_1	45.53
f_r	0.0371

The result of experimental data processing reveals a reasonably good agreement between the measured and calculated temperature values (Figure 5 and Table 3). Note that the following constrains were imposed on the required parameters while solving the inverse problem $\dfrac{\alpha}{C_P} > 0,\ a_1 \geq 0,\ \ f_r \geq 0,$ and $0 \leq a_{acc} \leq 1$.

Table 3: Estimating errors

No, of tests	J , K^2	$\Delta T_{\max}$, K
1	$0.28*10^5$	22.51
2	$0.9*10^4$	33.52
3	$0.16*10^4$	27.03
4	$0.12*10^5$	23.91
5	$0.2*10^5$	34.14
6	$0.35*10^5$	35.31

Determining the function q_2 in the boundary condition in (15) from the results of temperature measurement at several internal points of the specimen $X_m, m = 1,...,M$, is a well-known boundary inverse heat conduction problem [5]. This problem is solved for each experiment using the method presented in [7] and [8]. The heat fluxes q_2 and temperature of the external calorimeter surface T_ω (Figure 6) indicate that, with increase in mass concentration of the incoming particles to 1.1% the heat flux reaching the material at the "cold" wall is twice that in the case of a flow with no particles.

CONCLUSION

In this paper, a sensitivity method formulation is presented for the solving of the inverse problem of interaction materials with particles-loaded high-enthalpy flows. The objective was to investigate the influence of different factors on the intensity of heat transfer. The implementation of the optimization problem of the inverse formulation is carried out by Newton's method, while the direct and sensitivity problems in each iteration are solved by the finite difference method. The obtained results should be considered to be another step toward the construction of adequate mathematical models that describe the interaction of materials with particles-loaded flows. Further parameter studies and tests on more complicated cases remain to be done in the future to examine more complex mathematical models of heat transfer.

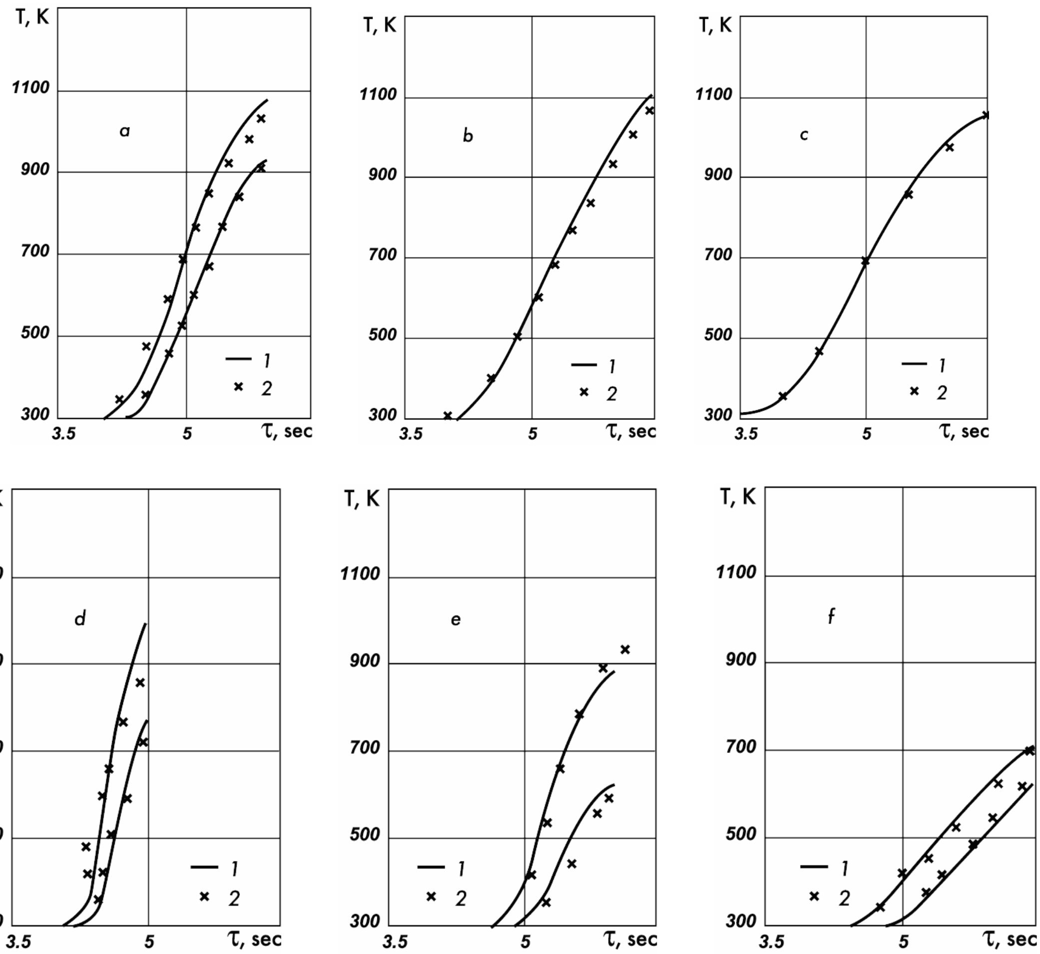

Figure 5: Experimental (1) and calculated (2) temperatures.
a - test 1, b – test 2, c – test 3, d – test 4, e – test 5, f – test 6.

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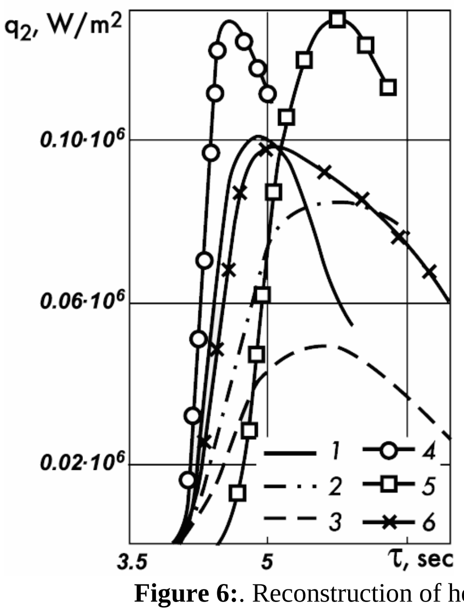

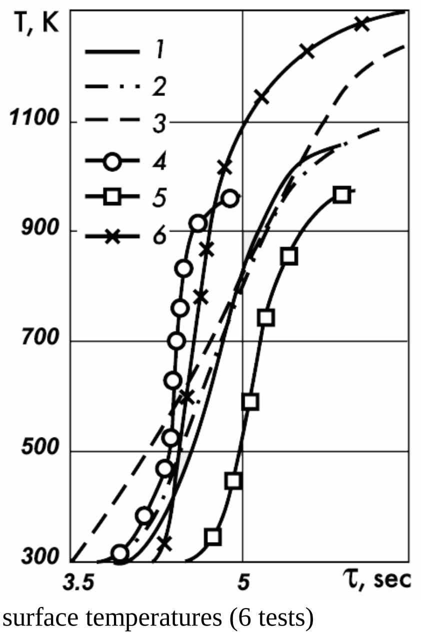

Figure 6:. Reconstruction of heat fluxes and surface temperatures (6 tests)