A structural study of the behavior of bumper cars compared to a system of spatially uncorrelated hard disks

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The emergence of collective patterns from the motion of large groups of humans is widely studied in the literature. Here, we take an uncommon approach to study the structures that appear in bumper cars. We focus on the structural properties instead of the dynamic behavior of the system. After acquiring several clips and obtaining the coordinates of the cars, the radial and angular distribution functions are computed. Additionally, analogous data corresponding to spatially uncorrelated particles is generated to compare its distribution functions to those obtained from the cars' coordinates. Despite the simplicity of the algorithm, these generated distribution functions reproduce the experimental data with high accuracy, suggesting that there is no sensible effect of the individuals in the collective behavior of the system.

I. INTRODUCTION

Emergent phenomena arising from the collective motion of humans have been studied in many specific social settings. Examples of these are the spontaneous formation of certain structures, such as unidirectional lanes or stop-and-go lanes in pedestrian traffic [1], the jamming during escapes from a disaster [2], or the formation of vortex- and gas-like structures during heavy metal concerts [3].

The aim of this paper is to study the behavior of bumper cars. Unlike other works, like those previously referred to, our analysis will not focus on the dynamics of the system; instead, it is studied from a structural point of view. In essence, the goal is to find if, and how, does the inherent consciousness of the individual drivers affect the large-scale behavior of the system.

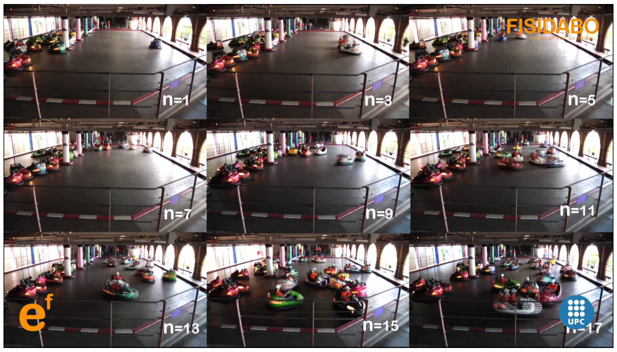
The experiment was carried out in the framework of Fisidabo [4]. Fisidabo is an initiative organized by the degree of Engineering Physics at the Polytechnic University of Catalonia (UPC) with the collaboration of Tibidabo amusement park. Its goal is to involve students in the scientific method by designing, building and carrying out a physics experiment within the facilities of the park. The activity takes place during one academic year.

II. METHODS

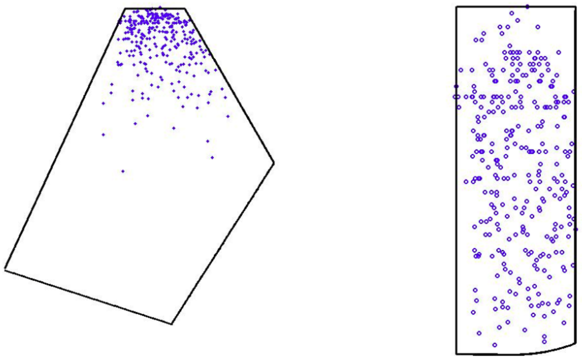
To study the motion of the bumper cars, a static camera was placed in a centered and elevated position with respect to the track. In this way, several videos were recorded, with the variable being the number of cars participating (Fig. 1a). The number of cars in the videos analysed range from 1 to 25, the latter being close to the practical limit at which there are too many cars for them to move continuously.

Static coordinates (i.e., the coordinates of the cars at each frame, but no information about their trajectories) were then obtained from the footage using the "Manual Tracking" plug-in for ImageJ [6].

Due to the perspective of the camera, the first step prior to any data analysis was correcting this perspective in the obtained coordinates (Fig. 1b).



(a) Some frames of the footage used, with different number of cars.



(b) Left: Example of the cars' coordinates as directly obtained from the footage. Right: The same coordinates, once the perspective distortion has been corrected. The images correspond to the coordinates of 4 cars over 75 different frames.

FIG. 1: From the gathering of the data to its processing prior to the analysis.

The data analysis includes the computation of the radial and angular distribution functions. These are averaged over the frames of each clip, in which the number of cars remains constant.

The radial distribution function (RDF) describes how the density varies as a function of the distance from a given reference particle. The RDF was calculated in a standard way, computing the distances between all particle pairs in each frame and binning them into a histogram, with the corresponding normalization. This normalization accounts for the fact that, even with no correlation between particles (like in an ideal gas) the likelihood to find a particle at larger distances is greater, simply because we are taking into account more volume (or surface area, in our case). However, it is important to note that, unlike in most molecular dynamics contexts in which this function is computed, the system has a finite size, i.e. there are not any periodic boundary conditions on the system. This causes the appearance of edge effects on the RDF, which is discussed later on.

The angular distribution function (ADF) is defined here as a function describing the probability, given a reference particle, for the two closest particles to be at a certain angle. Similarly to the RDF, the ADF was obtained by computing, for each car, the angle formed by the two closest ones; the results are binned into a histogram as well.

Furthermore, we compared the results with the ones obtained using generated data. The algorithm developed to obtain said data creates a given number of randomly distributed points within the boundaries, which correspond to the track the cars are on. These points are not allowed to be within a given distance of one another or the boundaries. Therefore, the algorithm effectively generates a spatially uncorrelated set of hard disk-like particles.

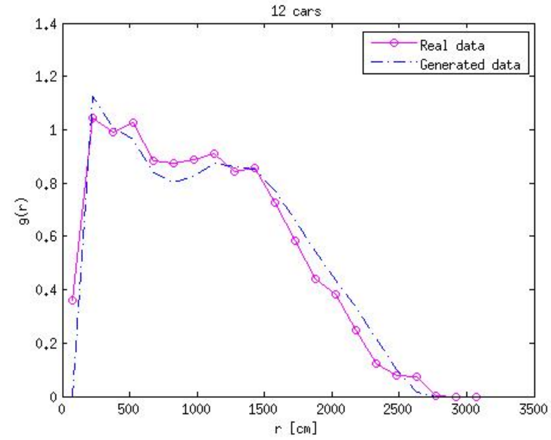
All the computations have been carried out using Matlab.

III. RESULTS AND DISCUSSION

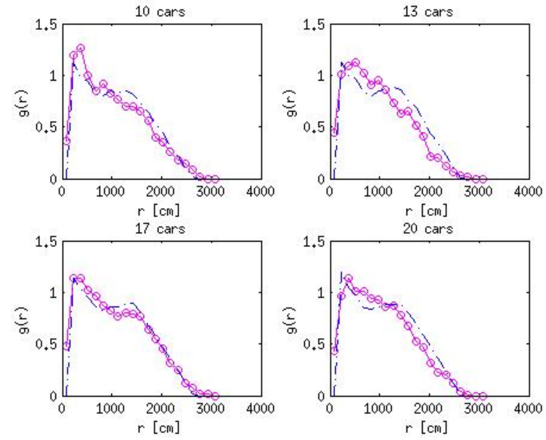
Radial Distribution Function

Figure 2 shows examples of obtained RDFs.

The most noticeable effect (in both real and generated curves) is the decay of the RDF at long distances. The RDF actually goes to zero at a distance of about 30 m. This is not the usual result: an RDF is meant to tend to 1 as the distance tends to infinity, since particles that are separated by a large enough distance are uncorrelated. The fact that our result is different is a direct consequence of the finite size of our experiment: as the distance increases, there are fewer cars that contribute to the RDF (since the only cars that can contribute to values significantly larger than the dimensions of the track are those near the corners); but these cars have other cars at small distances as well, meaning that there will always be many more "counts" at smaller distances. Indeed, the dimensions of the track are 10 m by 29 m, meaning the maximum distance two cars can be apart from each other is about 30 m.



(a) Example of the obtained RDF. This specific instance corresponds to 12 cars.



(b) Some more examples of the obtained RDFs (10, 13, 17 and 20 cars).

FIG. 2: Radial distribution function. Dash-and-dotted lines correspond to the generated data. They represent uncorrelated particles with a radius $r = 75$ cm, which is approximately the effective radius of the cars.

Aside from that, the important result is the fact that the shape of the RDFs obtained from the real and the generated data are very similar. This holds when the number of cars varies, with no significant changes on the shape of the curves.

Angular Distribution Function

The analysis of the results (Fig. 3) shows that, for a low number of cars, there is a clear majority of small angles. An intuitive explanation of this involves imagining only three particles (which is the minimum number to calculate an angular distribution). In this situation, a large angle can only appear with the three particles fairly

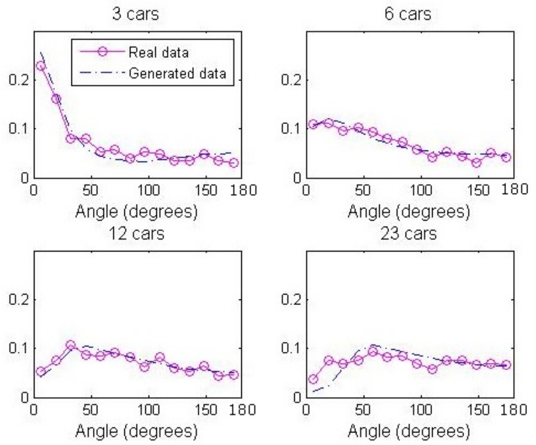


FIG. 3: Angular distribution functions for 3, 6, 12 and 23 cars. Dash-and-dotted lines correspond to generated data ($r = 75$ cm).

aligned; however, this configuration implies two "counts" of a near-zero angle per each "count" of a large (close to 180°) angle. Therefore, one can expect the observed preponderance of small angles.

On the other hand, when the number of cars becomes large enough, there is an "exclusion", or absence, of small angles. The video clips show that, for such large densities, most cars are in contact with each other. This explains the absence of small angles: when particles that occupy a given space are packed together, there is a minimum possible value for the angle that can be formed between them, simply because they have to keep at a minimum distance. In fact, the angular distribution functions for the largest numbers of cars show a distinct peak at 60°, consistent with an hexagonal packing.

As with the case for the RDF, the real data and the generated show very similar behaviors, despite the simplicity of the way the generated data are obtained.

IV. CONCLUSIONS

Generated and experimental data produce almost the same radial and angular distribution functions. The divergence between experimental and generated functions is likely to be mostly due to the lack of repetitions of the experiment, that is, we think that a larger time of observation would produce an experimental function closer to the generated one.

The generation of data has been performed without taking into account any dynamic interactions, but only geometrical conditions of the system. The resemblance between the two kinds of functions suggests that, in complex systems like this one, the macroscopic properties depend on general restrictions rather than in the details of local interactions. Furthermore, the fact that cars are driven by conscious human beings does not appear to have any physical consequences: from the structure point of view, the system behaves exactly as if the movement was random.

This method of data generation is an excellent way to study static properties. It allows for a very accurate idea of the behaviour of the system with a lower computational cost than full simulations involving forces and interaction potentials.

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[1] Mehdi Moussaïda, Dirk Helbingb, and Guy Theraulaza, *How simple rules determine pedestrian behavior and crowd disasters*, Proc. Natl. Acad. Sci. U.S.A. **108**, 6884 (2011).
[2] D. Helbing, A. Johansson, and H. Z. Al-Abideen, *Dynamics of crowd disasters: An empirical study*, Phys. Rev. E **75**, 046109 (2007).
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[4] http://fisidabo.org/portfolio/particules-intel-ligents/
[5] http://enginyeriafisica.etsetb.upc.edu/
[6] http://rsbweb.nih.gov/ij/plugins/track/track.html