

RESISTANCE REDUCTION OF A MILITARY SHIP BY VARIABLE-ACCURACY METAMODEL-BASED MULTIDISCIPLINARY ROBUST DESIGN OPTIMIZATION

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Abstract. A method for simulation-based multidisciplinary robust design optimization (MRDO) affected by uncertainty is presented, based on variable-accuracy metamodeling. The approach encompasses a variable level of refinement of the design of experiments (DoE) used for the metamodel training, a variable accuracy for the uncertainty quantification (UQ), and a variable level of coupling between disciplines for the multidisciplinary analysis (MDA). The results of the present method are compared with a standard MRDO, used as a benchmark and solved by fully coupled MDA and fully accurate UQ, without metamodels. The hull-form optimization of the DTMB 5415 subject to stochastic speed is presented. A two-way steady coupled system is considered, based on hydrodynamics and rigid-body equation of motion. The objective function is the expected value of the total resistance, and the design variables pertain to the modification of the hull form. The effectiveness and the efficiency of the present method are evaluated in terms of optimal design performances and number of simulations required to achieve the optimal design.

1 INTRODUCTION

The design of complex engineering system requires simulation-based analysis, addressing the interaction of mutually coupled disciplines. Real world applications are affected by uncertainty and require uncertainty quantification (UQ) and multidisciplinary robust design optimization (MRDO) formulations. Simulation-based design (SBD) for shape optimization has been used in diverse engineering fields, including naval applications [1]. SBD has been widely extended to multidisciplinary design optimization (MDO) problems, including ship design [2]. The assessment of uncertainty in SBD has been presented in [3], whereas a MRDO application addressing operational uncertainties has been shown

in [4]. In order to reduce the MDO and MRDO computational costs, metamodels have been widely applied in several engineering fields. In naval applications, a metamodels-based UQ may be found in [5, 6] and a dynamic radial basis function metamodel for UQ applications in ship hydrodynamics has been presented in [7]. The numerical solution of the MRDO represents a challenge from both the algorithmic and computational viewpoints, especially if computationally expensive simulations are required. Simulation-based multidisciplinary analysis (MDA), UQ, and design optimization need to be effective and efficient, in order to define an optimal solution at a reasonable computational cost.

The objective of the present work is the development and validation of a variable-accuracy method for simulation-based MRDO. Specifically, the optimal solution is identified by variable-accuracy, metamodel-based design optimization. The focus is on two-way steady problems and the method encompasses (a) a variable level of refinement of the design of experiments (DoE) used for the metamodel training, (b) a variable accuracy in the UQ analysis and (c) a variable level of coupling between disciplines in MDA [8].

The SBD application pertains to the hull-form optimization of the DTMB 5415 model, an open-to-public early concept of the DDG-51, a USS Arleigh Burke-class destroyer, widely used for both experimental [9] and numerical investigations [2]. Herein, the SBD optimization is aimed at the reduction of the expected value of the total resistance in calm water, considering stochastic speed. The two-way MDA is defined by the steady hydrodynamics provided by a linear potential flow solver and the rigid body equation of motion. The convergence of MDA is achieved iteratively, for each value of the stochastic speed. Monte Carlo method coupled with Latin Hypercube Sampling (LHS) [8] is used for UQ. The optimization is performed using a single objective deterministic particle swarm optimization (PSO) algorithm [10], using a thin plate spline (TPS) metamodel built on subsequent DoEs, obtained with variable UQ accuracy and MDA coupling. The results are compared to a benchmark solution, obtained by optimization without metamodel and a high level of UQ accuracy and MDA coupling.

2 PROBLEM FORMULATION

The single-objective MDO problem is formulated as

$$\text{minimize } f(\mathbf{x}, \mathbf{a}), \quad \mathbf{x} \in X \subseteq \mathbb{R}^{N_{DV}} \quad (1)$$

whereas the MRDO extension to problems affected by uncertainty reads

$$\text{minimize } \mu(f) = \int_Y f(\mathbf{x}, \mathbf{y}, \mathbf{a})p(\mathbf{y})d\mathbf{y}, \quad \mathbf{x} \in X \subseteq \mathbb{R}^{N_{DV}} \quad (2)$$

where \mathbf{x} collects N_{DV} deterministic design variables, f is the deterministic objective function, μ in Eq. 2 is the expected value of f and $p(\mathbf{y})$ is the probability density function of the stochastic environmental and operating conditions, collected in \mathbf{y} . Box and functional constraints may apply, if required.

The function f depends on several interconnected disciplines. The input of the i -th discipline Δ_i is defined by the set of design variables, $\mathbf{x} = [\mathbf{x}_i^T, \mathbf{x}_S^T]^T$, the set of output

parameters provided by other disciplines Δ_j ($i \neq j$), $\{\mathbf{a}_j\}_{i \neq j}$, and, for Eq. 2, the set of uncertain parameters $\mathbf{y} = [\mathbf{y}_i^T, \mathbf{y}_S^T]^T$. Variables indicated by \mathbf{x}_S are shared by all (or part of) the disciplines, whereas the corresponding \mathbf{x}_i are assumed to be local to the i -th discipline Δ_i . Similarly, the vector \mathbf{y}_S is shared by all (or part of) the disciplines involved and \mathbf{y}_i is local to the i -th discipline Δ_i only (see, e.g., [4]).

In MDO, once the multidisciplinary equilibrium, $\mathbf{a} = [\mathbf{a}_1^T, \dots, \mathbf{a}_{N_\Delta}^T]^T$, is achieved (generally by iterative procedures), the deterministic objective function $f = f(\mathbf{x}, \mathbf{a})$ is evaluated and an optimization algorithm is put on top of MDA.

In MRDO, the multidisciplinary equilibrium is conditional to $\mathbf{y} \in Y$ and UQ is needed on top of MDA (as shown in the top box of Fig. 1). Once the multidisciplinary equilibrium is achieved, the deterministic objective function $f = f(\mathbf{x}, \mathbf{y}, \mathbf{a})$ is evaluated, the stochastic objective function $\mu(f)$ is assessed by UQ, and finally an optimization algorithm is put on top of UQ (see Fig. 1).

3 VARIABLE-ACCURACY METHOD FOR MRDO

A metamodel is interposed between UQ and the optimizer, as shown in Fig. 1. The variable-accuracy metamodel-based MRDO is based on subsequent optimization stages, characterized by: (a) a refinement of the DoE used for the metamodel training, (b) a variable accuracy in the UQ analysis and (c) a variable level of coupling in MDA.

At the first stage, the training points are distributed in the whole design domain and the corresponding objective function values are obtained considering both a low level of accuracy in UQ and a weak coupling between disciplines. After the first optimization stage, a refined subdomain centered in the current optimum is defined. A new training set is used, with the corresponding objective function values obtained increasing both the accuracy of UQ and the coupling in MDA. The procedure is iterated for an appropriate number of stages, achieving fully accurate UQ and fully converged MDA, at the last optimization stage. The pseudo-code of the methodology is presented in Fig. 2, where N_{OS} is the number of optimization stages and N_{TP} is the number of training points per optimization stage.

4 SBD FRAMEWORK FOR MRDO

The SBD optimization framework encompasses three essential and interconnected elements: (a) the analysis tools, (b) the optimization algorithm, (c) the tool for the design modifications. The present toolbox includes a steady potential flow code, coupled with rigid-body equation of motion, a UQ tool based on MC-LHS simulation, a TPS metamodel, a deterministic version of the PSO algorithm, and a tool for geometry modifications based on orthogonal basis functions.

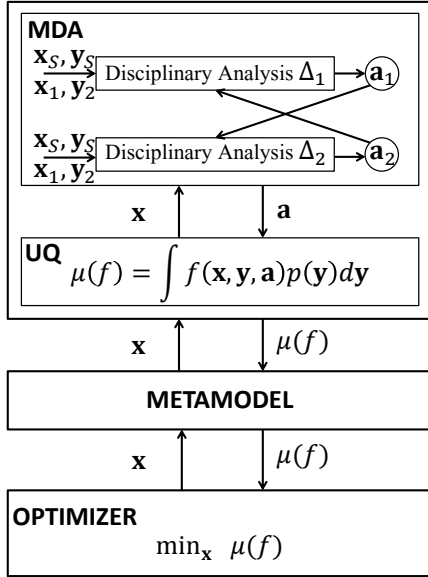


Figure 1: Metamodel-based MRDO method.

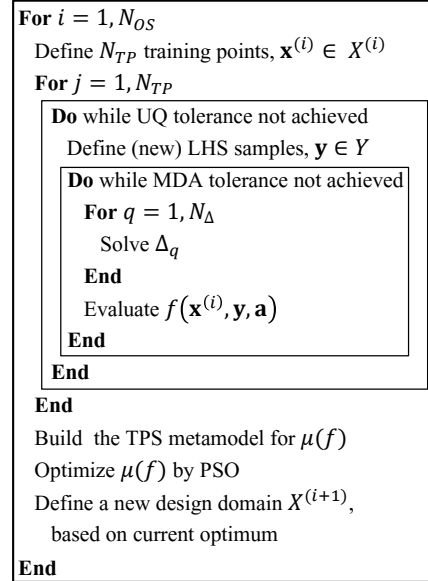


Figure 2: Pseudo-code of the variable-accuracy metamodel-based MRDO method.

4.1 Tools for multidisciplinary analysis

The hydrodynamics is solved using the code WARP (WAVE Resistance Program), developed at CNR-INSEAN. Wave resistance computations are based on linear potential flow theory. For details of equations, numerical implementation and validation see [11]. The wave resistance is evaluated with the transverse wave cut method [12], whereas the frictional resistance is estimated using a flat-plate approximation, based on the local Reynolds number [13]. The inputs (\mathbf{x}_1 and \mathbf{y}_1) of the hydrodynamic solver are the design variables and the speed, respectively, along with sinkage (σ) and trim (τ) values (collected in \mathbf{a}_2). The corresponding outputs (collected in \mathbf{a}_1) are the hydrostatic and hydrodynamic forces and moments.

The steady rigid-body equation of motion is evaluated for the 2DOF (sinkage and trim) problem and includes the equilibrium of the vertical forces F_z , and y -moments M_y . The incremental sinkage ($\Delta\sigma$) at the center of gravity (CG) and trim ($\Delta\tau$) are evaluated by the linearized equations, $\Delta\sigma = F_z / \rho g S_{WL}$ (where ρ is the water density, g is the gravity acceleration and S_{WL} is the waterline surface area), and $\Delta\tau = M_y / \rho g I_{WL}$ (where I_{WL} is the waterline-area moment of inertia about the y -axis, passing at CG). The inputs are the hydrostatic and hydrodynamic forces and moments (\mathbf{a}_1), and the weight force, whereas the outputs (\mathbf{a}_2) are formed by sinkage and trim values.

The MDA achieves the equilibrium solution for \mathbf{a}_1 and \mathbf{a}_2 by an iterative procedure.

4.2 UQ method

The integral in Eq. 2 is approximated as $\mu(f) = 1/N_{\text{UQ}} \sum_{i=1}^{N_{\text{UQ}}} f(\mathbf{x}, \mathbf{y}_i)$, using the MC method. LHS is applied by dividing the uncertain parameter domain with $N_{\text{UQ}} = 2^k + 1$ ($k \in \mathbb{N}$) evenly spaced bins [8].

4.3 TPS metamodel

A TPS metamodel is used for the design optimization. Given a set of training points $\{\mathbf{x}_i\}, i = 1, \dots, N_{\text{TP}}$, and the corresponding objective function values $f(\mathbf{x}_i)$, the objective function $f(\mathbf{x})$ is approximated, using radial-basis functions (RBF), as per $\hat{f}(\mathbf{x}) = \sum_{i=1}^{N_{\text{TP}}} d_i \varphi[r(\mathbf{x}, \mathbf{x}_i)]$, where $\varphi(r) = r^2 \log r$ is the RBF kernel, with $r = \|\mathbf{x} - \mathbf{x}_i\|$. The coefficients d_i are the solution of $\{\varphi(\mathbf{x}_i, \mathbf{x}_j)\}\{d_j\} = \{f(\mathbf{x}_i)\}$. Herein, the training set is distributed in the design domain using Hammersley sequence sampling (HSS) [8].

4.4 Optimization algorithm

The deterministic version of the PSO algorithm is used for the optimization [10]. The swarm dimension is set to $4 \cdot N_{\text{DV}}$, the swarm is initialized using HSS over the design variables domain and its boundaries. The PSO coefficients are set as $\chi = 0.721$, $c_1 = c_2 = 1.655$, [10]. The number of function evaluations is set to $N_{\text{PSO}} = 512 \cdot N_{\text{DV}}$.

4.5 Shape modification method

Shape modifications are represented in terms of orthogonal basis functions ψ_j ($j = 1, \dots, N_{\text{DV}}$), defined over surface-body patches as

$$\psi_j(\xi, \eta) := \alpha_j \sin\left(\frac{p_j \pi \xi}{A_j} + \phi_j\right) \sin\left(\frac{q_j \pi \eta}{B_j} + \chi_j\right) \mathbf{e}_{k(j)} \quad (\xi, \eta) \in [0; A] \times [0; B] \quad (3)$$

where α_j is the j -th (dimensional) design variable; p_j and q_j define the order of the basis function in ξ and η direction respectively; ϕ_j and χ_j are the corresponding spatial phases; A_j and B_j are the patch extension in ξ and η respectively, and $\mathbf{e}_{k(j)}$ is a unit vector. Modifications may be applied in x , y or z direction ($k(j) = 1, 2, 3$ respectively).

5 OPTIMIZATION PROBLEMS

The objective function for the problem in Eq. 1 is the total resistance ($f = R_T$) of the DTMB 5415 [9] in calm water at 24 [kn], whereas the objective function for the problem in Eq. 2 is the expected value of the total resistance in calm water, evaluated over a stochastic speed y , with $y \in [18; 30]$ [kn] following a uniform probability density function.

Two (normalized) design variables, $x_1 = \alpha_1/2$ and $x_2 = \alpha_2$, are used. The shape modifications are obtained as per Eq. 3 with $j = 1, 2$ and $k = 2$. The associated parameters are $p_1 = 2.0$, $\phi_1 = 0.0$, $q_1 = 1.0$, $\chi_1 = 0.0$, $\alpha_1 \in [-2.0; 2.0]$ and $x_1 \in [-1.0; 1.0]$; $p_2 = 1.0$, $\phi_2 = 0.0$, $q_2 = 2.0$, $\chi_2 = 0.0$, $\alpha_2 \in [-1.0; 1.0]$ and $x_2 \in [-1.0; 1.0]$. Geometric constraints include fixed length between perpendiculars (L_{BP}) and displacement.

Potential flow calculations are performed using a 150x30 panel grid for the hull surface. The computational domain for the free surface is defined within 1 hull length upstream, 3 lengths downstream and 1.5 lengths aside, discretized with 30x44, 90x44 and 30x44 surface panels, respectively. The grid convergence analysis is provided in [14].

6 NUMERICAL RESULTS

Benchmark solutions for both deterministic MDO and stochastic MRDO are presented in the following, and compared to the variable-accuracy metamodel-based results.

6.1 Benchmark solution for MDO

Convergence of iterative MDA is conducted for both design variables and stochastic parameter at domain center ($x_1 = x_2 = 0.0$, $y = 24$ [kn]), corresponding to $Fr = y/\sqrt{gL_{BP}} = 0.330$). Fig. 3 shows the solution change for total resistance, pitch and vertical force coefficients, sinkage and trim (respectively $C_t = 2F_x/\rho y^2 S_{WL}$, $C_{My} = 2M_y^h/\rho y^2 S_{WL}$, $C_{Fz} = 2F_z^h/\rho y^2 S_{WL}$, σ and τ , where F_x is the total resistance, M_y^h is the hydrodynamic moment, F_z^h is the hydrodynamic force) versus MDA iterations.

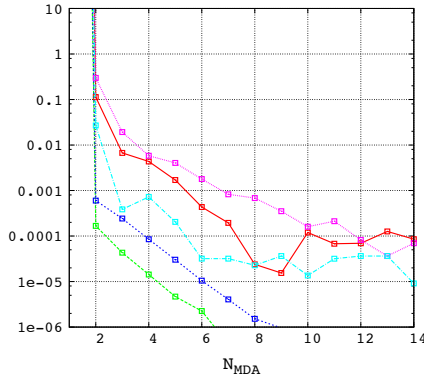


Figure 3: MDA convergence ($x_1 = x_2 = 0$, $y = 0.330$).

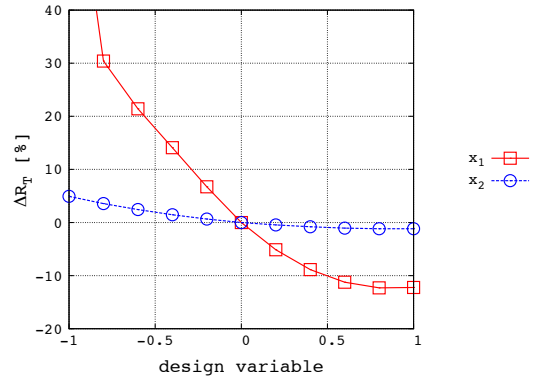


Figure 4: Sensitivity analysis for calm-water total resistance at $Fr = 0.330$.

A benchmark tolerance of 10^{-4} is set for all parameters and the maximum number of MDA iteration $N_{MDA}^{(max)}$ is set equal to 14. The corresponding maximum number of discipline evaluations is defined as $N_S^{(max)} = N_{PSO} \cdot N_{MDA}^{(max)}$ and summarized in Tab. 1.

The sensitivity analysis for the resulting R_T is shown in Fig. 4, versus x_1 and x_2 . The benchmark MDO results are included in Tab. 2, in terms of optimal design variables, objective function value and number of discipline evaluations.

Table 1: MDO and MRDO maximum number of simulations.

Problem	ID	Evaluations	Problem	ID	Evaluations
MDO	N_{PSO}	$512 \cdot N_{\text{DV}}$	MRDO	N_{PSO}	$512 \cdot N_{\text{DV}}$
	$N_{\text{MDA}}^{(\max)}$	14		$N_{\text{UQ}}^{(\max)}$	33
	$N_{\text{S}}^{(\max)}$	$7,168 \cdot N_{\text{DV}}$		$N_{\text{MDA}}^{(\max)}$	14
				$N_{\text{S}}^{(\max)}$	$236,554 \cdot N_{\text{DV}}$

Table 2: MDO and MRDO results.

Problem	Design var. 1	Design var. 2	Obj. [kN]	$\frac{\text{Obj.}^{(\text{B})} - \text{Obj.}}{\text{Obj.}^{(\text{B})}}$	%	$N_{\text{S}}/N_{\text{DV}}$
MDO (benchmark)	0.99	0.99	573.64	-		5,024
Variable-coupling MDO	0.96	0.99	571.30	0.41		674
MRDO (benchmark)	1.00	0.53	712.34	-		167,587
Variable-accuracy MRDO	0.98	0.36	712.82	0.07		18,876

6.2 Variable-accuracy MDO

Three subsequent optimization stages are considered, the MDA coupling is increased at each stage, whereas the number of training points N_{TP} is fixed for each stage. The tolerance for the solution change of C_t , C_{My} , C_{Fz} , σ and τ is set to 10^{-2} , 10^{-3} and 10^{-4} for first, second and third optimization stage, respectively.

In order to define N_{TP} : (1) a target number of total evaluations $N_{\text{S}}^{(\text{T})} = 0.1 N_{\text{S}}^{(\max)}$ is assumed; (2) at the j -th stage, the number of evaluations is calculated as $N_{\text{S}}^{(j)} = N_{\text{TP}} \cdot N_{\text{MDA}}^{(j)}$; (3) a target number of MDA iterations is defined according to Fig. 3, providing $N_{\text{MDA}}^{(\text{T})} = 4, 7, 14$ for first, second and third optimization stage. Accordingly, N_{TP} equals 29.

Table 3: Metamodel-based MDO and MRDO evaluations.

Problem	ID	Stage 1	Stage 2	Stage 3	$N_{\text{S}}/N_{\text{DV}}$
Variable-coupling MDO	N_{TP}	29	29	29	
	$N_{\text{S}}^{(j)}$	87	210	377	674
Variable-accuracy MRDO	N_{TP}	39	39	39	
	$N_{\text{S}}^{(j)}$	1,209	4,680	12,987	18,876

Variable-coupling metamodel-based MDO results are included in Tab. 2 and are found in close agreement with the benchmark values. The number of evaluations for each optimization stage is summarized in Tab. 3. The number of evaluations associated to the variable coupling metamodel-based MDO is about 13.4% of the corresponding benchmark value in Tab. 2. Figure 5 shows the subsequent metamodel-based optimization stages, with the corresponding optima.

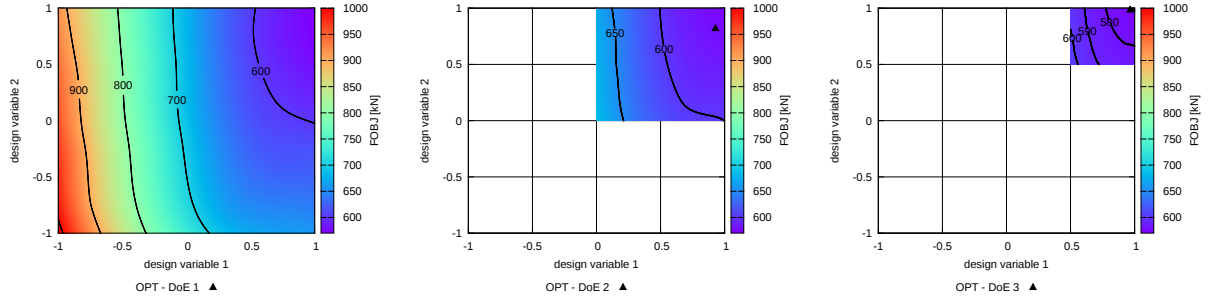


Figure 5: Metamodel-based MDO with subsequent refinement of design variable domain.

6.3 Benchmark solution for MRDO

Convergence of UQ is studied versus the number of samples $N_{UQ} = 2^k + 1$, $k \in \mathbb{N}$, for $x_1 = x_2 = 0.0$ and a benchmark tolerance for MDA equal to 10^{-4} . Fig. 6 shows the solution change of the expected value of the total resistance ($\Delta\mu/\mu$) versus N_{UQ} .

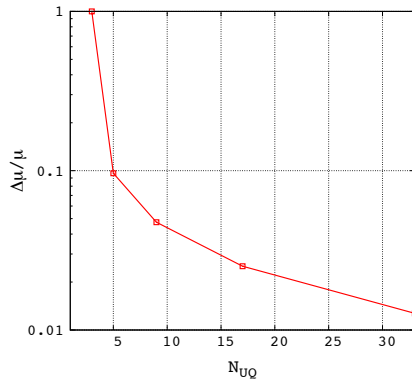


Figure 6: UQ convergence ($x_1 = x_2 = 0$, $y \in [0.250, 0.410]$).

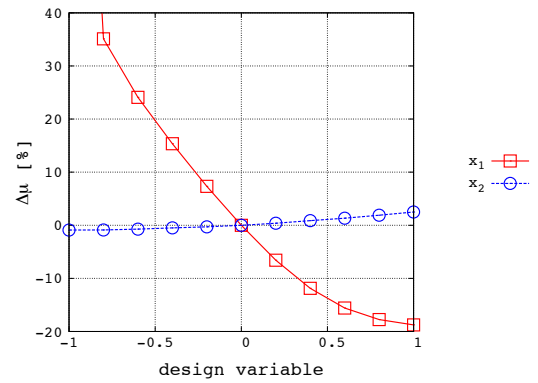


Figure 7: Sensitivity analysis for calm water expected value of total resistance.

A benchmark tolerance for solution change is set to $2 \cdot 10^{-2}$ and a number of $N_{UQ}^{(\max)} = 33$ is set. The maximum number of evaluations is defined as $N_S^{(\max)} = N_{\text{PSO}} \cdot N_{UQ}^{(\max)} \cdot N_{\text{MDA}}^{(\max)}$

and summarized in Tab. 1.

The sensitivity analysis for each design variable is shown in Fig. 7, whereas the benchmark MRDO results are included in Tab. 2.

6.4 Variable-accuracy MRDO

Three subsequent optimizations stages are considered, the UQ accuracy, together with the MDA coupling, is increased at each stage, whereas the number of training points N_{TP} is fixed for each optimization stage. The tolerance for the solution change of the expected value of the total resistance is set to $5 \cdot 10^{-2}$, $3 \cdot 10^{-2}$ and $2 \cdot 10^{-2}$, for the first, second and third optimization stage, respectively. The MDA tolerances are the same as for the MDO problem.

In order to define N_{TP} : (1) a target number of total evaluations $N_S^{(T)} = 0.1 N_S^{(max)}$ is assumed; (2) the number of evaluations for the j -th optimization stage is expressed as $N_S^{(j)} = N_{TP} \cdot N_{UQ}^{(j)} \cdot N_{MDA}^{(j)}$; (3) a target number of UQ samples is selected according to Fig. 6), which provides $N_{UQ}^{(T)} = 9, 17, 33$ for the first, second and third optimization stage; (4) the target number of MDA iterations is the same as the MDO problem. Accordingly, N_{TP} is found equal 39.

Metamodel-based MRDO results are included in Tab. 2 and are found in close agreement with benchmark values. The number of evaluations for each optimization stage are summarized in Tab. 3. The number of simulations associated to the variable-accuracy metamodel-based MRDO is about 11.3% of the corresponding benchmark values in Tab. 2. Figure 8 shows the subsequent metamodel-based optimization stages with the corresponding optima.

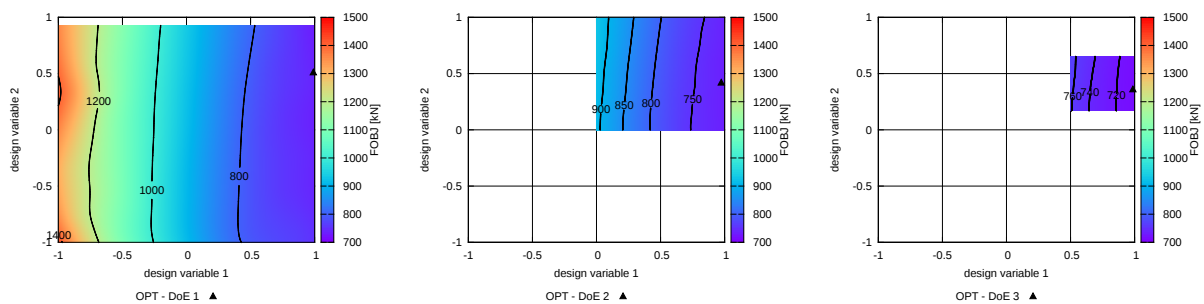


Figure 8: Metamodel-based MRDO with subsequent refinement of design variable domain.

Figure 9 (a) and (b) show the MDO and MRDO optimal shapes, compared to the original. It is worth noting that the optimal MRDO and MDO configurations fall in different points of the design variable domain. Nevertheless, the corresponding shapes are similar, as also found in earlier research for a catamaran configuration [5].

Finally, a parametric analysis of the total resistance versus Fr is presented in Fig. 9 (c), for the optimal and original configurations. The performances of benchmark and variable-

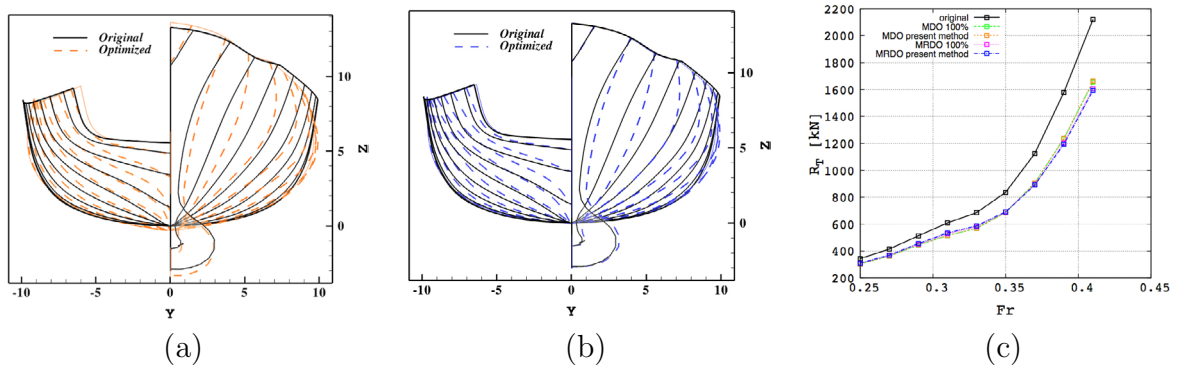


Figure 9: (a) Optimal MDO hull shape *vs* original, (b) Optimal MRDO hull shape *vs* original, (c) Parametric analysis of the total resistance *vs* Fr number.

accuracy solutions are found in close agreement for both MDO and MRDO, and present an overall improvement in the whole speed range. Moreover, MDO and MRDO solutions perform similarly; however the MRDO solution presents an overall best performance, whereas the performance of the MDO solution is found slightly better for deterministic $Fr = 0.330$.

7 CONCLUSIONS

A methodology for simulation-based multidisciplinary design optimization for problems affected by uncertainty has been presented. The approach encompasses a variable-accuracy metamodel-based design optimization, with (a) a variable level of refinement of the DoE used for the metamodel training, (b) a variable accuracy in the UQ analysis, and (c) a variable level of coupling in MDA.

The methodology has been applied to the resistance optimization of the the DTMB 5415 model, within a stochastic speed range. A potential flow solver has been coupled with the rigid body equation of motion (steady), the UQ has been performed by MC-LHS, a TPS metamodel has been used for the design optimization, and a PSO algorithm has been used for the optimization. Two design variables have been used, for the hull form modification.

Both deterministic MDO and stochastic MRDO have been solved. MDO and MRDO variable-accuracy metamodel-based solutions have been found in very close agreement with the corresponding benchmark solutions (obtained by fully convergent UQ and MDA, without metamodels). The present method allowed for a reduction of the computational cost by 13.4% and 11.3% for MDO and MRDO, respectively (Tab. 3).

The parametric analysis, conducted for the optimal designs over the speed range, revealed that the MRDO solution presents an overall best performance, whereas the MDO solution shows the best deterministic performance. As in earlier research, MDO and MRDO solutions have been found similar.

Future work will focus on the use of different surrogate techniques. Specifically, the

current *a priori* definition of the training points for the metamodel will be replaced by dynamic metamodeling techniques (e.g. [7]). The sensitivity of the results to diverse metamodels will be also investigated. The extensions to higher-dimensional problems (with more than two design variables) will be also addressed.

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