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Estimating error associated with a magnetic field model

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Introduction

A common requirement for users of magnetic field models is to have information about the accuracy of the model. For models used in navigation or orientation any signal not captured by the model is part of the error, for example the unmodelled crustal and external fields. Some models are used long after they have been produced and errors associated with forecasting of the core field also inevitably arise.

Estimates of ground-based errors were derived as part of the recent World Magnetic Model (WMM2015) production. The WMM is similar to the IGRF in that it forecasts the core field for 5 years and is severely truncated. These estimates were derived from vector data from observatories and repeat stations around the world combined with scalar data from marine and airborne surveys, and from inter-model comparisons. We update this analysis here assuming that the field can be modelled using one internal magnetic potential and then two potentials, one internal and the other external.

Declination, the element of the magnetic field of greatest interest to many users, is not linear in spherical harmonic model coefficients but can be propagated from the orthogonal components which are linear. This results in further spatial variations in the errors (inclination and horizontal and total intensities are also affected). We call this the geometrical effect.

The geometrical effect is difficult to validate with ground-based measurements because of the poor spatial coverage. We investigate whether satellite data such as those from the **Swarm mission** can provide such validation.

Assuming one potential

Global estimates of the uncertainties in each of the magnetic field elements output by WMM2015 were derived assuming the presence of one scalar potential $B = -\nabla V$.

These estimates were based upon (1) a statistical analysis of the differences between the WMM2010 and its predecessors (going back to WMM1980), and geomagnetic measurements in as many locations as possible at the Earth's surface and (2) inter-comparing different models.

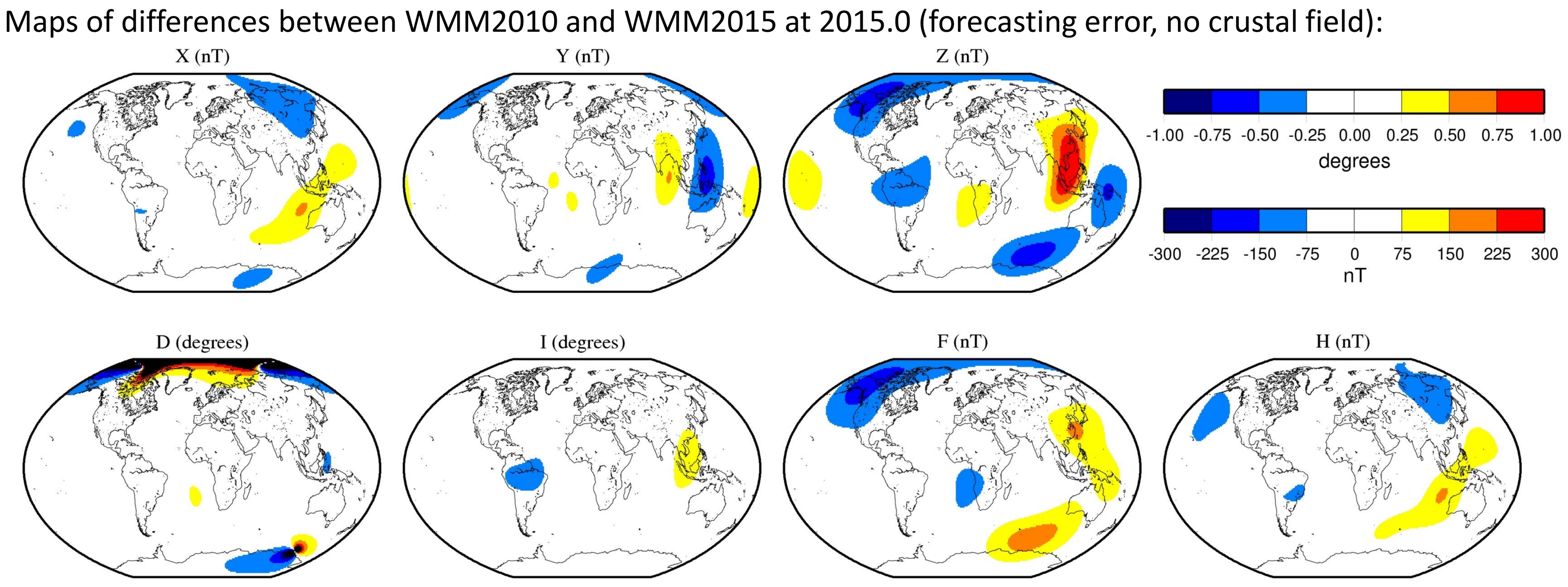
(1) Differences from almost 18000 observations of the vector magnetic field at the surface of the Earth but mostly on land were then re-scaled according to the differences in F from global compilations of marine and aeromagnetic surveys covering both land and sea giving these global RMS differences:

	X (nT)	Y (nT)	Z (nT)	D (°)	I (°)	F (nT)	H (nT)
Repeat stations plus observatory annual means 1980 onwards	187	134	281	0.66	0.32	201	168
Rescaled according to information from EMAG2 and GEODAS	113	81	170	0.40	0.20	121	102

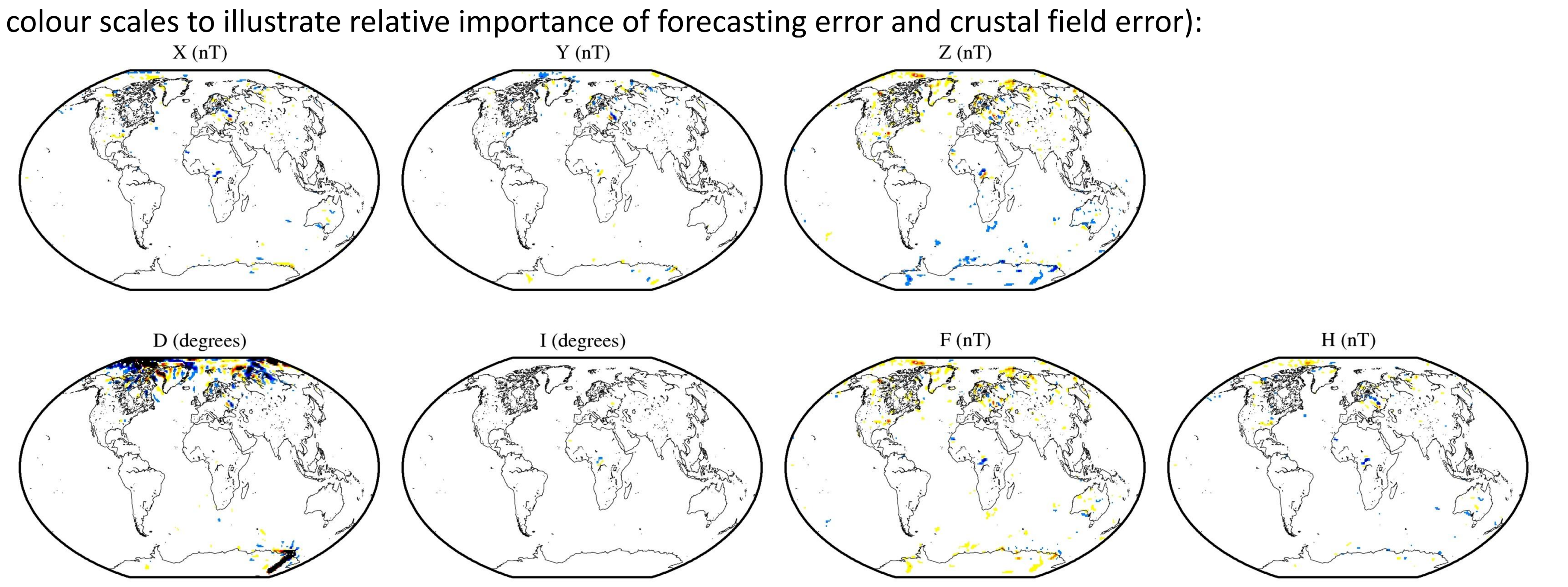
(2) By comparing independent versions of WMM2015 with one another, WMM2010 with WMM2015 and WMM2015 with EMM2015 (a high degree model including more of the crustal field) the following global RMS differences (computed from values on equal lat-long grid and weighted by cosine of latitude) are obtained:

	X (nT)	Y (nT)	Z (nT)	D (°)	I (°)	F (nT)	H (nT)
BGS and NOAA models at 2015.0	3	3	5	0.05	0.01	4	3
BGS and NOAA models at 2020.0	21	17	31	0.16	0.03	29	19
WMM2010 and WMM2015 at 2015.0	46	50	79	0.50	0.10	60	47
WMM2015 and EMM2015 at 2015.0	69	72	100	0.82	0.11	92	69

Maps of differences between WMM2010 and WMM2015 at 2015.0 (forecasting error, no crustal field):



Maps of differences between WMM2015 and EMM2015 at 2015.0 (crustal field error, no forecasting error. Same colour scales to illustrate relative importance of forecasting error and crustal field error):

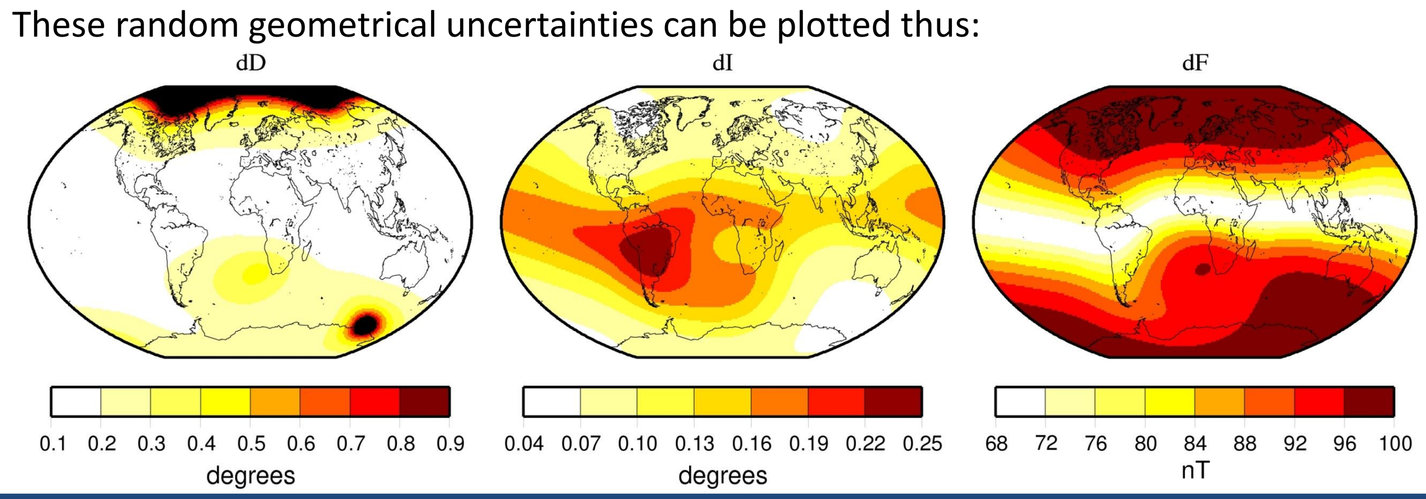


The external field is largely eliminated in both these analyses. Approach (1) considers the modelled and unmodelled parts of the potential but data distributions make mapping the uncertainties difficult. It combines the forecasting error and the crustal field error. Approach (2) considers only the modelled part of the potential but has split the forecasting error from the crustal field error. Both sets of maps are dominated by *systematic* errors arising from forecasting of the core field and missing coefficients for the crustal field.

To investigate whether geometrical effects are present it is assumed the uncertainties in the components which are linear functions of the spherical harmonic coefficients (X, Y and Z) are *globally random and uncorrelated,* and are 69, 72 and 100 nT respectively (= global RMS differences between WMM and EMM models). From the equations for D, I and F in terms of X, Y and Z we get the following *random* errors (H is not considered as it only varies 69-72 nT):

$$\delta D = \frac{\sqrt{(\delta X)^2 \sin^2 D + (\delta Y)^2 \cos^2 D}}{H} \qquad \delta I = \frac{\sqrt{(\delta H)^2 \sin^2 I + (\delta Z)^2 \cos^2 I}}{F} \qquad \delta F = \sqrt{(\delta H)^2 \cos^2 I + (\delta Z)^2 \sin^2 I}$$

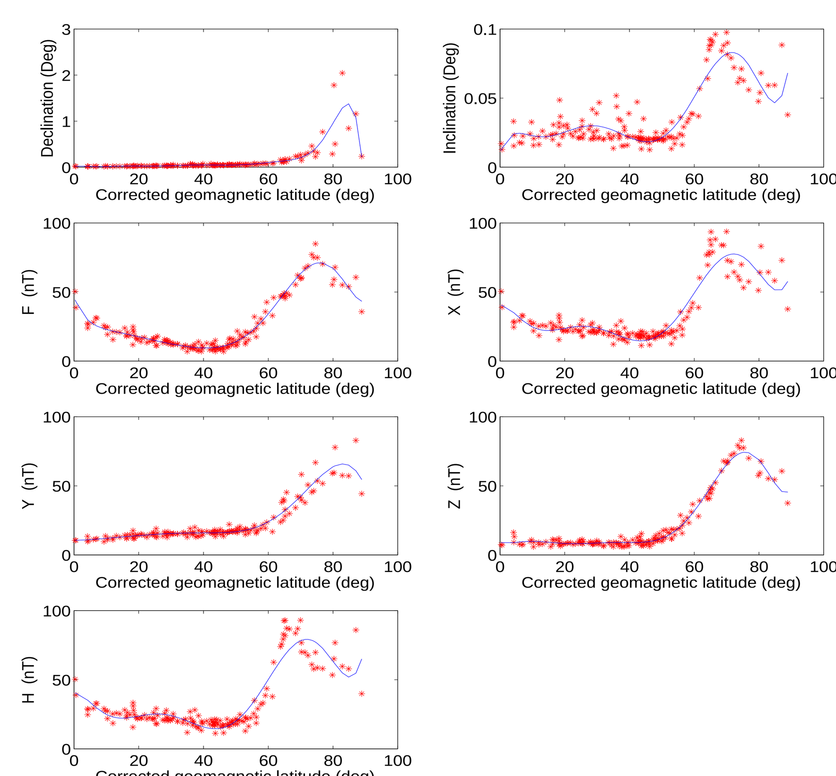
These random geometrical uncertainties can be plotted thus:



Where the horizontal field is small the uncertainties in D and F are large. Likewise, where the total field is small the uncertainty in inclination is large.

Assuming two potentials

In reality the magnetic field observed at the surface of the Earth has two sources – one inside the Earth and one outside – and therefore 2 potentials: $B = B_i + B_e \quad B_{i/e} = -\nabla V_{i/e}$



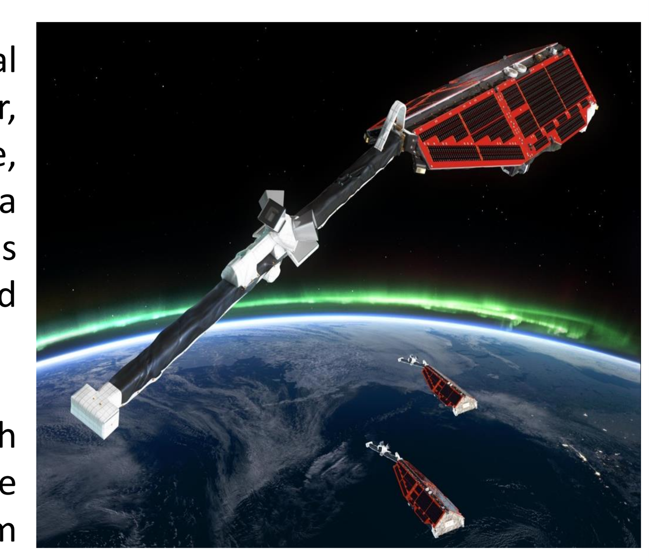
The contribution to the overall error budget from the external potential can be estimated using observatory hourly mean data. On average ~86,000 data points for each component and for each observatory were used, the maximum span being 14 years. The RMS differences between cubic splines with 1 year knot intervals and the hourly mean data are plotted according to absolute corrected geomagnetic latitude.

The effect of the electrojets can be seen clearly in these plots, with peaks at 70-80° latitude associated with the auroral electrojets and at 0-10° associated with the equatorial electrojet. Although a spline is not the same as the WMM it is sufficiently close in terms of capturing the core field signal that it can be said that the uncertainty in the WMM due to the unmodelled external potential is generally of the same order as the uncertainty from prediction. However the uncertainty due to unmodelled crustal field is the greatest.

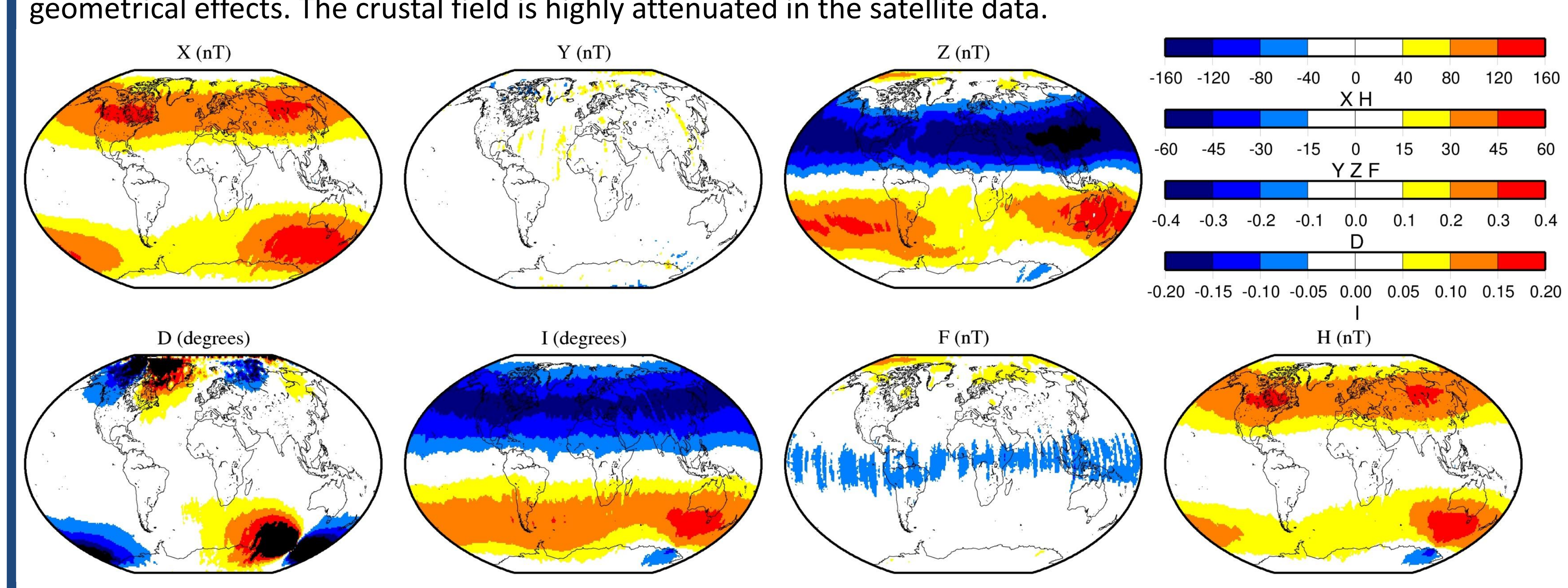
Bringing it all together?

One difficulty in bringing all these error sources together is that they have each been estimated using different, imperfectly distributed, data sources. The data from Swarm are uniformly distributed, albeit not on the surface of the Earth, and may help provide insight into the relative importance of the different error sources, in particular the geometrical effect on D, I and F.

Calibrated data from Swarm are selected on the same basis as a typical selection for magnetic field modelling, namely Kp ≤ 2-, |dDst/dt| ≤ 5 nT/hr, IMF $B_z \geq 0$ nT, 22:30 ≤ local time ≤ 5:00 for data ≤ 50° geomagnetic latitude, and some de-selection for obviously outlying data. By using only these data the error from unmodelled sources, primarily from the external potential as defined above, is reduced. The differences between these observations and WMM2015 are plotted.



The main patterns in these maps are from the external potential, which results in globally systematic errors for X and H. Apart from D there is little similarity between the D, I and F plots below and those above arising from geometrical effects. The crustal field is highly attenuated in the satellite data.



This analysis demonstrates in a simple way the importance of the external potential, in both producing models from satellite data and in assessing errors of widely used models such as the WMM. The geometrical effect is visible in D but less so in I and F where other signals (despite careful selection) mask the expected signal.

Conclusions

The assessment of errors in magnetic field models should take account of the requirements of the users. We assess the errors for a widely used model where the user is requiring values at the surface of the Earth with uncertainties which reflect the difference between the modelled field and the observed field. The analyses here demonstrate the importance of the unmodelled sources, namely the crustal field and the external field. Geometrical effects should also be taken account of, particularly for declination.

Acknowledgements and References

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