# Use of geostatistical Bayesian updating to integrate airborne radiometrics and soil geochemistry to improve mapping for mineral exploration

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### Synopsis

Mineral exploration programmes around the world use data from remote sensing, geophysics, and direct sampling. On a regional scale, the combination of airborne geophysics and ground-based geochemical sampling can aid geological mapping and mineral exploration. Since airborne geophysical and traditional soil-sampling data are generated at different spatial resolutions, they are not immediately comparable due to their different sampling density. Several geostatistical techniques, including indicator cokriging and collocated cokriging, can be used to integrate different types of data into a geostatistical model. However, with increasing numbers of variables the inference of the cross-covariance model required for cokriging can be demanding in terms of effort and computational time. In this paper a Gaussian-based Bayesian updating approach is applied to integrate airborne radiometric data and ground-sampled geochemical soil data to maximize information generated from the soil survey, enabling more accurate geological interpretation for the exploration and development of natural resources. The Bayesian updating technique decomposes the collocated estimate into two models: prior and likelihood models. The prior model is built from primary information and the likelihood model is built from secondary information. The prior model is then updated with the likelihood model to build the final model. The approach allows multiple secondary variables to be simultaneously integrated into the mapping of the primary variable. The Bayesian updating approach is demonstrated using a case study from Northern Ireland. The geostatistical technique was used to improve the resolution of soil geochemistry, at a density of one sample per 2 km$^2$, by integrating more closely measured airborne geophysical data from the GSNI Tellus Survey, measured over a footprint of 65 x 200 m. The directly measured geochemistry data were considered as primary data and the airborne radiometric data were used as secondary data. The approach produced more detailed updated maps and in particular enhanced information on the mapped distributions of zinc, copper, and lead. The enhanced delineation of an elongated northwest/southeast trending zone in the updated maps strengthened the potential for discovering stratabound base metal deposits..

Keywords
geostatistics, Bayesian updating, airborne geophysics, geochemistry, mineralization.

## Introduction

Mineral and oil exploration programmes around the world utilize data from remote sensing, geophysics, and direct sampling. These techniques measure different attributes of the Earth, at a range of scales, the integration of which may provide more information than is immediately apparent, if correctly analysed. On a regional scale, the combination of geophysics (airborne magnetic and radiometrics) and geochemical sampling can aid geological mapping and mineral exploration (Smith *et al.*, 1997, Morris *et al.*, 2003). Integration of soil geochemistry, soil gas helium, and *in situ* radiometry (where the detector was placed directly over the soil) was used to delineate the subsurface extent of uranium zones in the Cuddapah Basin, India (Menon *et al.*, 2009). Since airborne geophysical and traditional soil-sampling geochemical data are generated at widely different spatial resolutions, they are not immediately comparable due to different sampling densities. Previous work by Rawlins *et al.* (2007) on the integration of high-resolution radiometrics and detailed soil surveying from eastern England used multivariate geostatistical methods comprising coregionalization and residual likelihood (REML) modelling. In the present study, geostatistical techniques were applied to integrate regional multi-source geophysical and geochemical data. The application of a Bayesian updating approach provided the opportunity to investigate the relationship between the different data types. The threefold objectives were to:

- ➤ Investigate the use of a Bayesian updating approach to integrate different types of spatial geophysical and geochemical information
- ➤ Examine the controls (geological and parent material) on soil geochemistry
- ➤ Interpret the findings in terms of the development of the area's natural resources.

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## Study region and survey data acquisition

### *Geological setting*

Northern Ireland, with an area of only about 14 000 km², is home to a remarkable diversity of geology (GSNI, 1997; Mitchell, 2004a) (Figure 1). The range of rocks present forms a stratigraphic record that commences in the Mesoproterozoic, comprising deformed and metamorphosed sedimentary and volcanic rocks formed between 900 and 600 Ma (Cooper and Johnston, 2004a). With the exception of the Cambrian, all geological systems are represented up to and including the Palaeogene, which comprises basalt lavas and associated rocks that formed between approximately 62 and 55 Ma (Cooper, 2004; Cooper and Johnston, 2004c). The basement rocks of Northern Ireland can be partitioned into three major terranes (Anderson *et al.*, 2004). North of the Highland Boundary Fault occurs the Central Highland or Grampian terrane, which is host to the oldest rocks present in Northern Ireland and includes the Dalradian Supergroup (Cooper and Johnston, 2004a). An extension of the Midland Valley terrane of Scotland, south of the Highland Boundary Fault, hosts limited basement at outcrop, but includes the Tyrone Igneous Complex and small inliers of fossiliferous Ordovican and Silurian sedimentary rocks (Cooper and Mitchell, 2004; Cooper *et al.*, 2011). Devonian and Carboniferous continental red-bed and marine Carboniferous sediments (Mitchell 2004b, 2004c) overlie the basement in the southwest of this terrane and are the focus of this study. The Southern Uplands-Down-Longford terrane lies to the south of the Southern Uplands Fault and is composed of Ordovician and Silurian rocks (Anderson, 2004) intruded by the Lower Devonian Newry Igneous Complex (Cooper and Johnston, 2004b) and Palaeogene Slieve Gullion and Mournes complexes (Cooper and Johnston, 2004c). In the northeast, straddling all three basement terranes, cover rocks of the Antrim Plateau are composed of Palaeogene basaltic lava with underlying Carboniferous, Permian, Triassic, Jurassic, and Cretaceous rocks exposed at the margins. Quaternary superficial deposits cover some 80% of the Northern Ireland and were mainly laid down over the last 120 000 years (Bazley, 2004). They include subglacial tills in the form of rib moraines and swarms of aligned drumlins, and proglacial deposits of clay, silt, sand, and gravel related to meltwaters. Post-glacial alluvium and peat continue to accumulate and obscure bedrock. Remote sensing using airborne geophysics and ground geochemistry are therefore particularly relevant to prospecting in this region.

### *Prospecting history*

Northern Ireland has a varied mineral prospecting history dating from the 18th century (Arthurs and Earls, 2004). In the southwestern region, prospecting for base metals has largely been concentrated within the counties Fermanagh and Tyrone (Figure 1). County Fermanagh and the southern part of County Tyrone comprise shallow-water Carboniferous limestone sediments with interbedded sandstones (Mitchell, 2004c). The Carboniferous includes the non-fossiliferous Ballyness Formation and the Clogher Valley Formation. The Ballyness Formation comprises about 300 m of red and purplish-red sandstones and conglomerates with clasts of mainly white quartz pebbles. The Clogher Valley Formation (400 m) is divided into fenestral micrite, siltstones, sandstones, evaporate beds, and grey mudstone with miospores indicating an early Chadian age (Mitchell, 2004c).

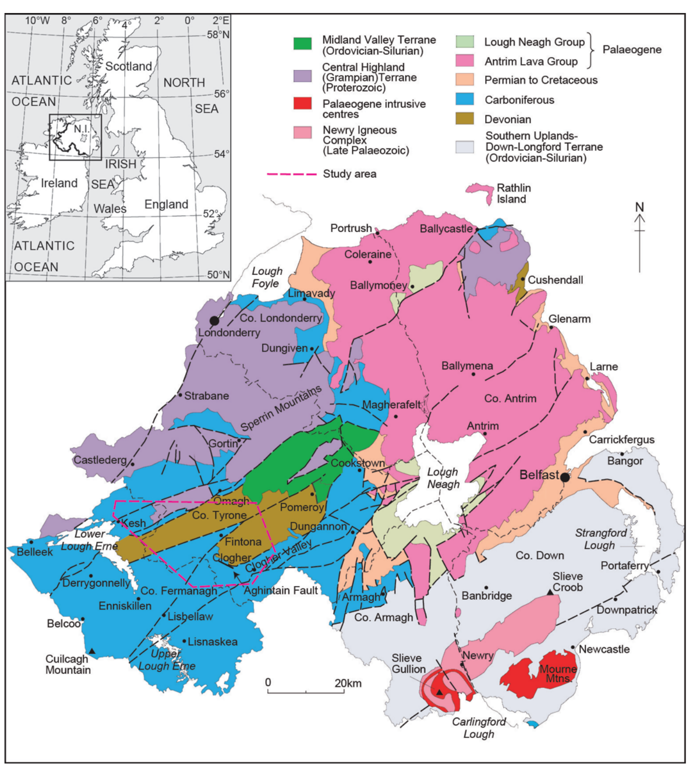


**Figure 1—Geological map of Northern Ireland (simplified after GSNI, 1997). The location of the study area is shown**

Previous geochemical surveys undertaken in the 1970s and 1980s revealed anomalies of lead, barium, and silver together with galena and barite in heavy mineral samples (Smith *et al.*, 1996). The faulted sedimentary rocks of the Clogher Valley share similar features and a geological environment of Lower Carboniferous shallow-water sediments near a basin margin fault, with many of the major lead-zinc deposits of the Irish Midlands (e.g. Navan, County Meath - Republic of Ireland. Although the identification of anomalies in the 1970s stimulated prospecting in the area, the exploration methods available at that time lacked the sensitivity necessary to identify subtle geochemical anomalies in the areas covered by thick superficial deposits. This study aims to investigate whether the integration of high-resolution airborne geophysical and detailed geochemical data using a Bayesian updating approach can improve precision in mapping the geochemical variables of economic interest.

Due to the diverse nature of the geology on a regional scale in Northern Ireland, a local-scale study area was selected to investigate the Bayesian updating approach. The Clogher Valley area (shown on Figure 1) in the county of Fermanagh and the southern part of County Tyrone was chosen because:

- The area comprises lithostratigraphically distinctive Carboniferous and Devonian rocks
- The stratigraphy is controlled by basement tectonics and early Devonian terrane-bounding faults as evidenced by the fault-bounded Fintona Block
- This area is of particular economic interest due to an inferred relationship between base metals and basin faulting (Arthurs and Earls, 2004).

## Survey data acquisition

The Tellus Project, managed by the Geological Survey of Northern Ireland (GSNI) and funded by the Department of Enterprise Trade and Investment and the EU's Building Sustainable Prosperity Fund, was the most detailed geological mapping project ever undertaken in Northern Ireland. The data comprised multi-source airborne geophysics collected by a low-level geophysical survey aircraft and a ground geochemical survey of soil, stream sediments, and stream waters (GSNI, 2013). The airborne geophysical data included magnetics, electrical conductivity, and radiometrics. The results provide new insights into the geological underlay and delineation of faults and dykes, particularly where bedrock is obscured by glacial cover and peat. The geochemical data data provide a baseline standard for 60 elements across rural Northern Ireland. The improved mapping of geology, soils, and natural resources has prompted a renewed interest in mineral prospecting and new licences have been awarded as a result. On release of the Tellus data, mineral licensing in Northern Ireland surged, with total coverage increasing from 17% to a peak of 73%. Prospecting junior Metallum Exploration was awarded a number of licences across Northern Ireland, including the MR4 licence which is focused on the Clogher Valley and the MR2 and MR3 licences which have their northern limits in the area (GSNI 2012). The company sees the area as prospective for potential 'Irish style' base metal mineralization.

### Geochemical survey

The GSNI Tellus Survey, completed between 2004 and 2006, provides a data-set combining comprehensive spatial soil sampling coverage with an extensive suite of soil geochemical analysis (Smyth 2007). Regional 'rural' soil samples were collected on a grid of one sample site every 2 $km^2$ across the rural areas of Northern Ireland. A parallel 'urban' soil sampling programme at a sample density of 4 sites per $km^2$ was completed across a number of selected urban areas, but these data are not used in this study. The soil samples collected at each rural sampling site included a surface sample collected from 5 cm to 20 cm below ground level (discarding surface organic litter and root zone where present) and a deep soil sample collected from 25 cm to 50 cm below ground level. The samples collected at each site represent a composite sample of five auger abstractions (completed with hand-held auger) at corner points and centre of a 20 m × 20 m sampling square. Rural soil samples were disaggregated prior to sieving to a 2 mm fraction and a representative sub-sample was obtained and milled for subsequent chemical analysis. Further details on the Tellus soil sampling programme, sampling methodology, and sample preparation including quality control procedures are summarized in Smyth (2007).

This paper uses data from the rural surface soil geochemical data set comprising a total of 6862 soil samples across Northern Ireland. As part of the Tellus Project, the rural soil samples were analysed for a range of up to 50 determinants. The soil samples used in this study were analysed using pressed pellet X-ray fluorescence spectrometry (XRF) for determination of major oxides and trace elements using wavelength dispersive XRF spectrometry (WD-XRF) and energy dispersive/polarized XRF spectrometry (ED-XRF) by the British Geological Survey (BGS), Keyworth, Nottingham. In a recent study, Dempster *et al.* (2012) demonstrated a very close geochemical match between soils, underlying glacial tills, and a range of bedrock types. Soil geochemistry will also reflect anthropogenic effects, anomalies associated with precious and base metal mineralization, and secondary environmental effect (i.e. absorption onto secondary Fe and Mn hydroxides, organic matter etc.). The Tellus data-set provides the basis for a comprehensive study with relevance for the whole of the UK.

### Geophysical survey

The airborne surveys measured three geophysical parameters, obtaining high-resolution magnetic, radiometric, and electromagnetic data. The survey was carried out with a Twin Otter aircraft of the Joint Airborne-Geoscience Capability (JAC), a partnership of BGS and the Geological Survey of Finland (GTK). Flight lines had a line spacing of 200 m and an average flight altitude of 56 m (240 m over urban areas (Jones and Scheib, 2007). The high-resolution imagery generated by the geophysical survey provides complementary information on the geology and soil types of Northern Ireland and is particularly valuable where the bedrock is obscured by glacial cover and peat. In particular, the airborne survey improves the delineation of faults and dykes (Cooper *et al.*, 2012). The high-resolution terrestrial gamma radiation was used in this study. Variations in gamma-ray emissions characterize near-surface rocks and soils. Terrestrial radiation was sampled every second using a gamma-ray spectrometer (Explorium GR-820/3), which resolves radiation emitted from the radioisotopes and daughters of potassium (K), uranium (eU), and thorium (eTh) (IAEA, 2003). These radioisotopes, due to their abundance and half-lives, are the main contributors of gamma radioactivity in rocks (Dypvik and Eriksen, 1983). Total count (TC) measurements, in units of counts per second (cps), were collected across the energy window from 0.41 to 2.81 MeV (Beamish *et al.*, 2007). TC data comprise a spectral summation, including contributions from both natural and artificial radioactive sources, and so provide a higher signal/noise ratio than the individual radioisotope data (Beamish 2013).

## Geostatistical methodology

The value of the Tellus data lies not only in the individual geophysical and geochemical data-sets produced, but equally in the potential for a comprehensive integrated mapping approach, as is undertaken in this paper. Airborne geophysics combined with soil geochemistry provides the opportunity to analyse and integrate data from multiple sources generated at varying spatial resolutions. The potential outcome of this is to maximize information generated by the Tellus project, generating even deeper and more accurate knowledge to support exploration and development of the natural resources of Northern Ireland.

Conventional geostatistical mapping comprises kriging data such as soil geochemistry to interpolate between sampled locations. Coefficients from variography are used to inform the kriged maps. Local uncertainty in the estimates is given by the kriging variance. In geostatistics, secondary data data are often used to provide a quantitative measure at unsampled locations. For example, secondary information such as seismic data and geological interpretation are often used to improve the 2D modelling of sparse well data in the interwell regions (Ren *et al.*, 2007). The geochemical survey forms only part of the Tellus geoscientific mapping project. Combining information from the Tellus soil survey with the results from the airborne radiometric survey affords the opportunity to significantly reduce the uncertainty in the estimation of key topsoil major and trace elements. Several geostatistical techniques can be used to integrate different types of data into a geostatistical model, including Gaussian-based Bayesian updating, indicator cokriging, and in particular collocated cokriging (Xu *et al.*, 1992). Coregionalization was used by Rawlins *et al.* (2007) to model the direct and cross-variograms for soil measurements of K and Th and the corresponding radiometric measurements. However, as the number of variables increases, fitting the direct and cross-variograms simultaneously becomes increasingly difficult. With a large number of variables the inference of the cross-covariance model required for cokriging can be demanding in terms of effort and computational time (Ren *et al.*, 2007). Use of a Bayesian updating approach is explored in this study due to the simplicity of data integration and the ability of the approach to take in account joint spatial and multivariate correlations between variables. This is particularly relevant for integrating the Tellus geophysical and geochemical data-sets, in that correlation between radiometric measurements and all of the geochemical elements can be explored and used if deemed appropriate. The strength of correlation between different geochemical elements along with the relationship between the radiometric and geochemical data can be taken into account and used to maximize the information generated from the geochemical survey data.

### *Theory of the Bayesian updating approach*

The Bayesian updating technique for data integration decomposes the collocated estimate into a production of two models: the prior and likelihood models. Doyen *et al.* (1996) were the first to apply this form of Bayesian updating approach to integrate spatially correlated primary and secondary data. The prior model is built from primary information and the likelihood model is built from secondary information. The prior model is then updated with information from the likelihood model to build the final '*updated*' model. The framework is Bayesian in the sense that different types of data are integrated together into a unified distribution of uncertainty. The terms 'likelihood' and 'prior' are somewhat arbitrary, but the mathematical framework of integration rests on a combination of these distributions in a (perhaps) non-convex manner; and therefore not a simple averaging. The approach by Doyen *et al.* enabled updating with only one secondary variable at a time. This approach for the Tellus Survey data would be time-consuming and ineffective in utilizing the correlation between the airborne radiometric survey elements and the multivariable geochemical data-set. Deutsch and Zanon (2004) proposed a similar approach that also allows the simultaneous integration of multiple secondary variables into the mapping of the primary variable. The advantage of this approach is easy implementation of mapping of multiple parameters using multiple secondary variables.

The development of the statistical theory of merging prior and likelihood distributions for the Bayesian updating approach is provided in detail by Doyen *et al.* (1996), Zanon and Deutsch (2004), Neufeld and Deutsch (2004), and Ren *et al.* (2007). In summary, Bayesian updating requires four distinct steps:

(1) Calibration of the primary and secondary data to define the likelihood distribution
(2) Calculation of the prior distribution using kriging
(3) Updating the prior distribution with the likelihood distribution to obtain the updated distribution
(4) Post-processing the posterior distribution, including summarizing local uncertainty and simulation for larger scale uncertainty.

#### *Notation and Bayesian updating equations*

A random function $Z(u_i)$ is the primary data available at $n$ locations in the area of interest where $u$ is the location vector and $i$=1, ..., $n$. There are then $m$ random functions $S_j(u)$ $j$=1, ...,$m$ as secondary data available at all locations in the entire study area. Bayesian updating is a Gaussian-based technique, so transformation of data is an integral stage of the approach. Following transformation, the $Y(u_i)$value is the transformed primary data $Z(u_i)$ and the $X_j(u)$ notation represents the transformed secondary data $S(u)$. In the context of Bayesian statistical analysis, the results of simple kriging using only the primary data provide the mean and variance parameters for a prior distribution of uncertainty parameterized by:

$$y_{\rho}^{*}(u)=\sum_{i=1}^{n}\lambda_i\, y(u_i) \quad [1]$$

The variogram is modelled by an authorized function (McBratney and Webster, 1986) and the weight $\lambda$ is calculated from the simple kriging (or normal) equations:

$$\sum_{i=1}^{n}\lambda_i C(u_i-u_k)=C(u-u_k),\quad k=1,\ldots n \quad [2]$$

where $C(u_i - u_k)$ is the covariance between primary data $y(u_i)$ and $y(u_k)$ at distances $h$ away, and $C(u - u_k)$ is the covariance between estimated location $y(u)$ and primary data $y(u_k)$ at distances $h$ away. The kriging variance is given by

$$\hat{\sigma}_{\mathrm{sk}}^{2}(u)=\sigma^2-\sum_{i=1}^{n}\lambda_i C(u-u_i) \quad [3]$$

The mean (Equation [1]) and variance (Equation [3]) define a Gaussian distribution at each location. The secondary data can be mathematically combined to provide an alternate estimate of uncertainty in the primary data at each location using the same normal equations (or simple cokriging equations) with the weights calculated from the correlations between different variables and between the secondary variable and primary variable:

$$y_{L}^{*}(u)=\sum_{j=1}^{m}\lambda_j x_j(u) \quad [4]$$

Here the weights $\lambda_j (j = 1, \ldots , n)$ are given by:

$$\sum_{j=1}^{m}\lambda_j\rho_{j,k}=\rho_{j,0},\quad k=1,\ldots m \quad [5]$$

where $\rho_{j,k}$ is the correlation between different types of secondary data $\rho_{j,0}$ and $\rho_{j,0}$ is the correlation between the secondary and primary data. The likelihood estimation variance is then given by:

$$\hat{\sigma}_{\mathrm{L}}^{2}(u)=1-\sum_{j=1}^{m}\lambda_j\rho_{j,o} \quad [6]$$

The mean (Equation [4]) and variance (Equation [6]) define a second Gaussian distribution of uncertainty at each location. This is referred to as a likelihood distribution, a term which is somewhat arbitrary; it is a second Gaussian distribution

The prior information (first distribution) and the likelihood information (second distribution) are then combined to yield a combined distribution of uncertainty that simultaneously considers sparse primary data and multiple secondary data

$$y_{U}^{*}=\frac{y_{L}^{*}\sigma_{P}^{2}+y_{P}^{*}\sigma_{L}^{2}}{(1-\sigma_{L}^{2})(\sigma_{P}^{2}-1)+1} \quad [7]$$

and the corresponding variance is calculated as:

$$\sigma_{U}^{2}=\frac{\sigma_{P}^{2}\sigma_{L}^{2}}{(1-\sigma_{L}^{2})(\sigma_{P}^{2}-1)+1} \quad [8]$$

These results give the parameters of an updated distribution, also called the posterior distribution. These parameters define a Gaussian distribution that must be back-transformed to original units. The updating equation is interesting because it is not commonly applied in geostatistics for updating or merging two conditional distributions. These combination equations to obtain the updated mean and variance are the same as colocated cokriging (Chiles and Delfiner, 2012), that is, the underlying assumption behind their derivation is a Markov screening assumption. Primary data (referred to as 'prior' in this paper) would screen the influence of lower quality secondary data (referred to as 'likelihood' in this paper). Of course, if the secondary data data were of sufficient quality that the variance approached zero, then it would screen the influence of primary data at some distance away. Reference to 'prior' and 'likelihood' is arbitrary in this context; they are two different conditional distributions from two different sources that must be combined.

The potential usefulness of this approach for the multi-source Tellus data is evident and would enable the relationships between the topsoil major and trace geochemical elements and the airborne radiometric survey elements (K, eTh, and eU) to be fully explored and used. In the Bayesian updating technique, data generated by both the geochemistry soil survey and airborne geophysical surveys are used to produce more detailed maps and in particular maximize information for mapped estimates of minerals of economic interest.

### *Development of Bayesian updating approach for this study*

The Bayesian updating method was used in this study as a technique to improve the resolution of the more widely spaced soil geochemistry data (sampled on a grid of one sample per 2 km$^2$) by integrating more closely-measured airborne geophysical data (resolution 65 x 200 m). The local case study for the Clogher Valley area, in the southwest of Northern Ireland, was used to investigate the effectiveness of the approach for integrating multiple variables of different types and different sources. The directly measured ground geochemistry survey data were considered as the primary data in the Bayesian approach and the airborne radiometric data was used as secondary data. For the Clogher Valley study area, the geochemistry survey comprised 589 data points (19 geochemical elements were selected) and the geophysical survey comprised 202 891 data measurements for four radiometric variables (eTh, eK, eU, and TC measurements). Within the study area, matching of paired geochemistry and geophysics was achieved for 589 points using a 1000 m radius. These points were used for the correlation matrix, which is discussed in due course.

### Results - Bayesian updating approach applied to the case study

The Bayesian updating approach utilized soil geochemistry for 19 elements and airborne radiometric data (as illustrated later in the production of the correlation matrix for the likelihood model). The mapped outputs display the results for geochemical elements of particular economic interest: Cu (mg/kg), Zn (mg/kg), and Pb (mg/kg). In addition, the outputs for $K_2O$ (%) are shown to exemplify the Bayesian updating approach. Summary statistics and histogram distributions for the selected elements for the Clogher Valley case study are displayed in Figure 2. The histogram for $K_2O$ (Figure 2a) clearly indicates a bimodal pattern: K levels across the study area display a normal distribution of K in topsoils but with the presence of a secondary peak of low values. The summary statistics for Pb, Zn, and Cu indicate that the study area contains locations that are significantly anomalous with respect to these elements. Maximum levels for Pb, Zn and Cu are recorded as 487.5 mg/kg, 307.5 mg/kg, and 390 mg/kg respectively, set against a background pattern of lower values with minimum values of 7.5 mg/kg, 7 mg/kg, and 1 mg/kg for Pb, Zn, and Cu respectively). This is reflected in the strongly skewed histogram distributions (Figure 2b, c, and d). All data were transformed using normal score transformation prior to application of the Gaussian-based Bayesian technique (Deutsch and Journel, 1998).

### *Calculation of the prior distribution*

Simple kriging (SK) was used to estimate the prior distribution of the geochemistry data. As the data had already been standardized (in the first pre-analysis step), the assumption of a random variable with known zero mean, as in SK, is justified. SK is equivalent to the normal equations and provides the exact solution to the parameters of a conditional distribution in a multivariate Gaussian context. If ordinary kriging was applied in this framework, this could lead to a bias in the estimates if the input distributions are highly skewed. Coefficients from variography were used for kriging the geochemistry data and enabled interpolation between the sampling locations. Omnidirectional variograms of the normal score transformed data for the selected geochemical elements ($K_2O$, Pb, Zn, and Cu) are displayed in Figure 2 (e–h). The variograms show very stable experimentally calculated points due to the large amount of available data. There is a relatively high nugget effect due to

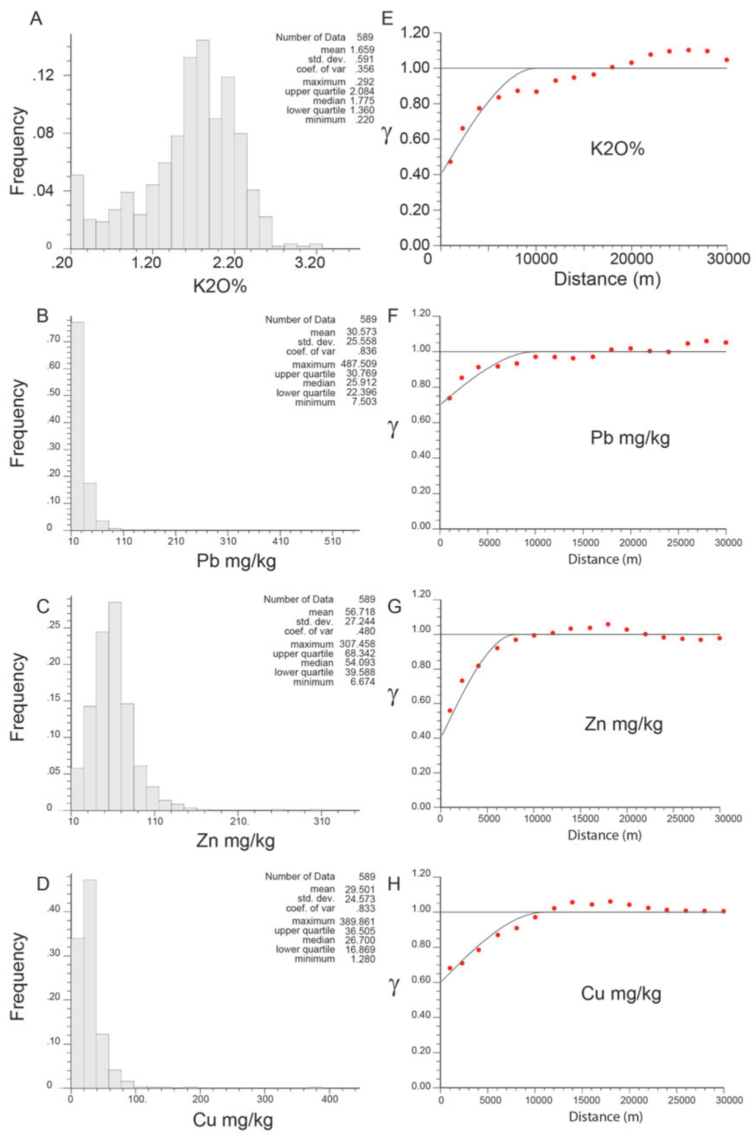


**Figure 2—Summary statistics and histograms for study area: (A) $K_2O$%, (B) Pb mg/kg, (C) Zn mg/kg, and (D) Cu mg/kg; omnidirectional variograms of the normal score transformed data estimated for (E) $K_2O$%, (F) Pb mg/kg, (G) Zn mg/kg, and (H) Cu mg/kg. γ is semivariance**

short-scale variability and the large distance scale. Variogram analysis indicates a nugget:sill ratio (Isaaks and Srivastava, 1989) of 40% or more for all the selected geochemical elements. The model fitted to the variogram estimated from the Pb data shows the highest nugget:sill ratio (70%). This indicates higher spatial variability over short distances for the study area and short-range variation in the distribution of the elements in the topsoils, which was not detected by the 2 km$^2$ sampling spacing. Similar range values or correlation distances (7–10 km) are indicated for the Pb, Zn, and Cu data-sets. This suggests that the spatial distribution of these geochemical elements in the topsoils is comparable.

The kriged outputs for the $K_2O$, Pb, Zn, and Cu geochemical data-sets for the Clogher Valley area are shown in Figure 3. The kriged output maps indicate an elongated zone, trending NE/SW, of elevated K, Zn, and Cu with a secondary area or anomaly of elevated levels to the southeast of the area. The distribution of Pb is more sporadic but a similar pattern, although less well defined, can be identified.

### *Calculation of the likelihood distribution*

The likelihood distribution defines the bivariate relationship between the primary and secondary data (Neufeld and Deutsch, 2004). Many of these relationships are too complex to be defined by a single parameter such as the correlation coefficient, so the production of the likelihood distribution comprised two parts: (1) the correlation matrix and (2) the prior distribution of the secondary data (in this case, the airborne radiometric data). The mean and variance for the likelihood data are derived from the combination of all the secondary data based on the correlation matrix.

#### Correlation matrix

The correlation matrix (Figure 4) was produced to provide the correlation coefficient for each pair of variables. This allowed joint spatial and multivariate correlations between the geochemical elements and the radiometric data to be taken into account in the Bayesian updating approach. For the

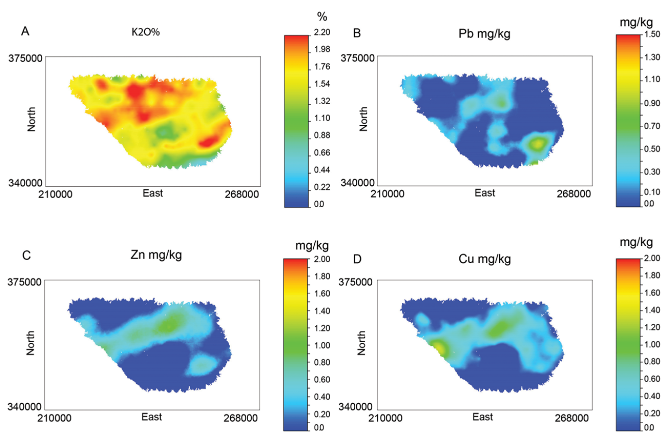


Figure 3—Kriged output maps for the primary geochemistry variables (A) $K_2O$%, (B) Pb mg/kg, (C) Zn mg/kg, and (D) Cu mg/kg

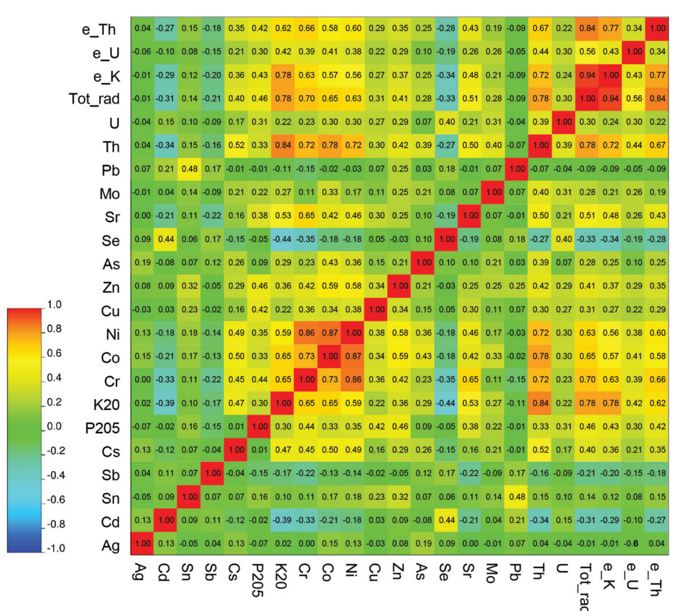


Figure 4—Global correlation matrix showing the correlation coefficient for each pair of variables. In the colour scale of the correlation matrix, cold colours are negative correlations and hot colours are positive. The correlation coefficient values are given in the squares

Clogher Valley case study the correlation matrix comprised the 19 geochemical elements along with four radiometric variables (eTh, eK, eU, and TC). Within the study area, matching of paired geochemistry and geophysics data was achieved for 589 points. These paired observations were used in the following analysis. Correlations between geochemistry data and the secondary radiometric variables were used for the likelihood calculation. The strength of correlation between different geochemical elements was taken into account together with strength of correlation between geochemical elements and radiometric variables. For example, the strength of correlation between eK and $K_2O$% ($r$ = 0.78) could be used in updating the output not only for topsoil $K_2O$ but also for the geochemical elements of particular economic interest: Cu ppm, Zn ppm, and Pb mgkg$^{-1}$. Likewise, the strength of correlation between associated elements (e.g. $r > 0.7$ for Ni, Cr, and Co) can be used to reduce uncertainty in the final updated model. A moderate correlation is demonstrated between geochemical Th and eTh ($r$ = 0.67) and a weaker correlation for U ($r$ = 0.30). Strongly correlated variables were given more weight and weaker correlations less weight in the application of the Bayesian updating technique.

### *Secondary radiometric data*

The secondary data required for the likelihood distribution were obtained from the radiometric data. Coefficients from variography were used for kriging the radiometric data. The variograms and kriged outputs estimated from the radiometric data are provided in Figures 5 and 6. The variograms estimated for the topsoil geochemical Th and U are provided for comparison. The overall shape of the variograms estimated from the geochemical and radiometric data, eTh, eU, and eK, are comparable (see Figure 2e for comparison with $K_2O$). The higher resolution of the radiometric data-sets than the geochemical data is immediately evident in the kriged estimates. The elongated NE/SW trending zone is evident in all outputs, but is most apparent in the kriged outputs for eTh and total radiometrics. The distinct borders of the anomalous gamma emission zone suggest a fault-bounded control. All secondary data can be mathematically combined based on its correlations (following the methodology provided previously) to provide a likelihood model.

### *Bayesian updating of the likelihood and prior distributions*

Bayesian updating uses the mean and variance of the primary data given a specific value of the secondary data as the likelihood distribution (Neufeld and Deutsch 2004). The complex heteroscedastic and nonlinear relationships previously observed in the Tellus geochemistry data (McKinley and Leuanthong 2010) can be captured using this approach. The mean and variance for the prior distribution were derived from kriging the primary geochemistry data.

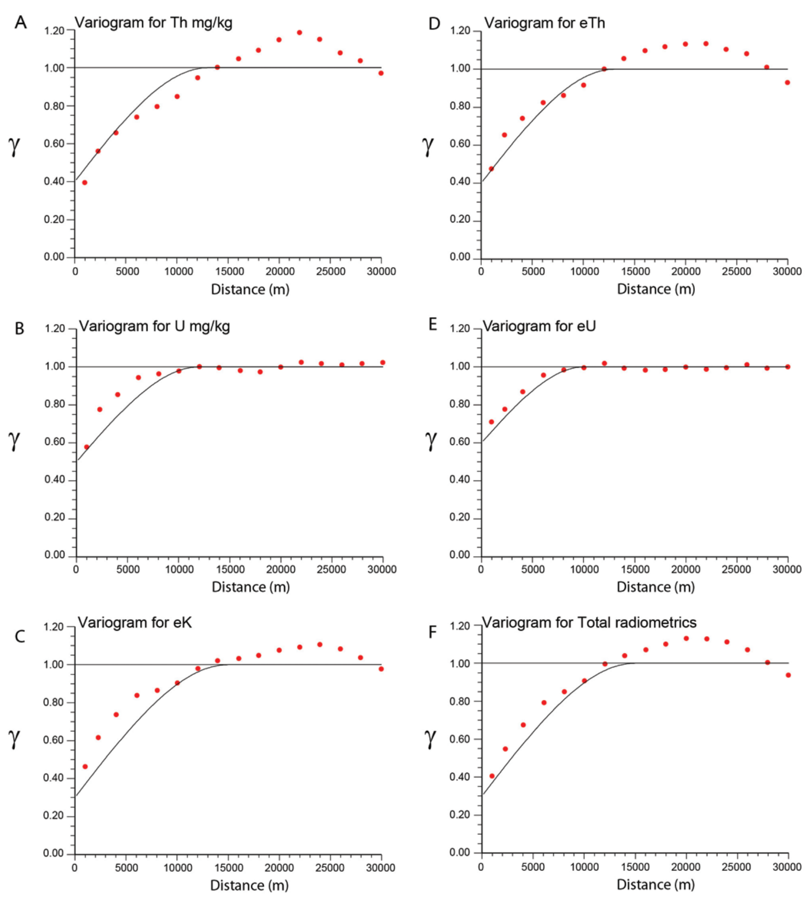


**Figure 5—Omnidirectional variograms of the normal score transformed data estimated for (A) Th mg/kg, (B) U mg/kg for geochemical data and radiometric variables (C) eK, (D) eTh, (E) eU, and (F) total radiometrics. $\gamma$ is semivariance**

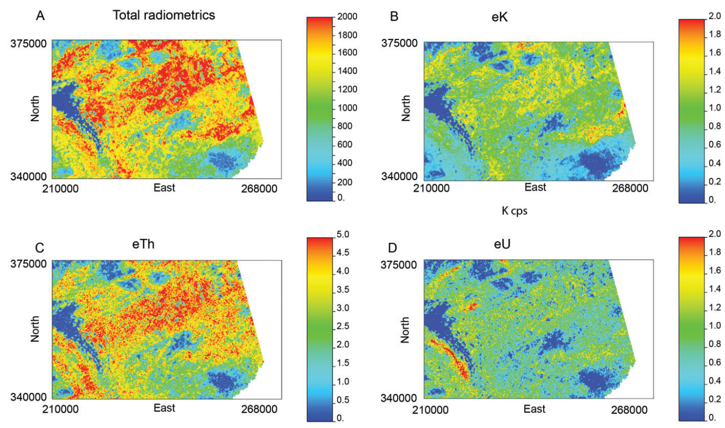


Figure 6—Kriged output maps for the secondary radiometric variables (A) total radiometrics, (B) eTh, (C) eU, and (D) eK. TM65 Irish grid coordinate system, is used. The location of the maps refers to the highlighted area in Figure 1

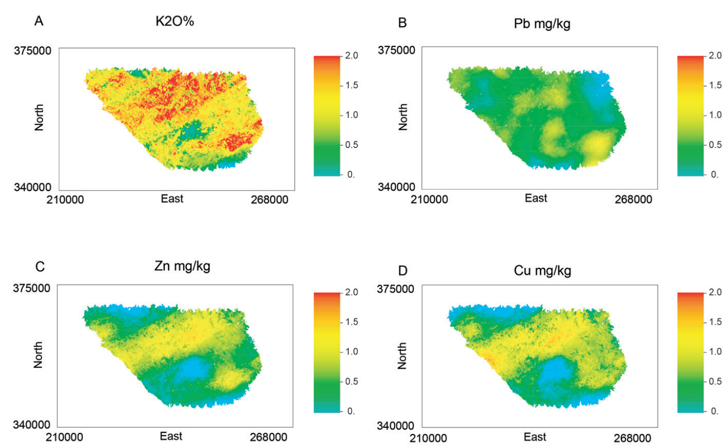


Figure 7—Updated maps for the primary geochemistry variables (A) $K_2O$%, (B) Pb mg/kg, (C) Zn mg/kg, and (D) Cu mg/kg. The TM65 Irish grid coordinate system is used. The location of the maps refers to the highlighted area in Figure 1

The mean and variance of the primary geochemistry data, together with the secondary radiometric data, were used to define the likelihood distribution. Merging the primary information and the secondary information in this way provided an updated distribution containing information from both data-sets. This model gives a best estimate (in the context of minimum uncertainty) based on the primary and secondary data. Back-transformation was achieved by reversing the normal score transform with many quantiles sampled from the updated (or merged) conditional distributions.

Updated maps for the K, Pb, Zn, and Cu geochemistry data-sets are shown in Figure 7. The updated maps illustrate the benefit of integrating the geochemistry and radiometric data-sets. From a comparison of the original kriged maps (Figure 3, produced using only geochemical data) with the updated outputs (Figure 7) it is evident that greater detail is available on the updated maps. This is most obvious for $K_2O$, but the mapped estimates of Zn, Cu, and to a lesser extent Pb also show greater definition of the elongated NE/SW trending zone. The strong correlation between K from the soil survey and radiometric K has contributed to maximizing the information in the updated map for geochemical $K_2O$.

However, the Bayesian updating approach has also provided better delineation of a fault-bounded control for Zn and Cu. To validate the output maps, the kriging variances (in Gaussian units) or the calculated variance of the back-transformed quantiles can be mapped (Neufeld and Deutsch 2004). Validation of variances using the theta statistic or standardized squared prediction errors has been demonstrated by Lark (2000).

## Discussion

The nature of the detailed mapping approach undertaken in the Tellus project and the systematic generation of ground survey and airborne data has provided a rich data-set of multi-source variables. In the Bayesian approach, the soil geochemistry survey data, measured on a 2 $km^2$ sampling spacing, provided primary information to build a prior model. Variography demonstrated evidence of spatial dependence in the selected geochemical data-sets of economic interest, Pb, Zn, and Cu. Consistency in correlation distances (range distances of 7–10 km) suggested that the spatial distribution of these geochemical elements in the topsoils is comparable. The kriged outputs provided smoothed estimate maps of the topsoil geochemistry. Similarity in the spatial distribution of the elemental concentrations could still be surmised, with the indication of an elongated NE/SW trending zone (most obvious in the kriged estimates of Zn and Cu, Figure 3). In the Bayesian context, secondary data, in this case the radiometrics, were combined with the primary data to provide an estimate of the geochemical data at each location using weights calculated from the correlations between different variables and between the secondary variable and primary variable. The usefulness of this approach is that strength of correlation between variables, for example, between eK and $K_2O$ ($r$ = 0.78) and between Ni, Cr, and Co ($r$ > 0.7) can be taken into account to reduce uncertainty in the final updated output maps for the geochemical elements of particular economic interest. Strongly correlated variables are given more weight and weaker correlations less weight. Using multiple secondary variables can reduce the uncertainty, but a refinement of the approach would include reducing the number of variables in the correlation matrix to include only those with moderate to high correlations. A moderate correlation was recorded between geochemical Th and eTh ($r$ = 0.67) and a weaker correlation for U ($r$ = 0.30). Rawlins *et al.* (2007), in a comparative study of radiometric and soil data across eastern England, attributed limited correlation between geochemical and radiometric Th and U to greater analytical errors for these elements for both XRFS and estimations based on gamma emissions. The correlation between K and Th estimated by radiometrics ($r$ = 0.77) is slightly lower than in the soil survey data ($r$ = 0.84) for the Clogher Valley case study area. This is contrary to the findings of Rawlins *et al.* (2007) for the analogous study across eastern England. Rawlins *et al.* (2007) surmised that gamma emission measurements overestimated Th concentrations over Triassic (Mercia) mudstone parent material. The strength of correlation found in the current study between estimations based on gamma emissions and the soil survey is attributed to the high resolution of the radiometric data collected by the Tellus project and the association of high K and Th in Upper Devonian sandstones, siltstones, and mudstones. Besides the correlation between variables, the reliability of the secondary data can affect the estimation and uncertainty assessment of the updated model. In this case, terrestrial gamma radiation data provided high-resolution secondary data to include in the likelihood model with consistent quality over the entire study area. The value of using the high-resolution radiometric data is evident in the kriged mapped estimates, in that elevated levels of eTh and total radiometrics concur with the presence of an elongated NE/SW trending zone that was indicated in the kriged estimates of topsoil geochemistry.

Merging the primary and secondary information via Bayesian updating has provided more detailed updated mapped estimates for the selected variables of economic interest. This is most evident for $K_2O$ due to the strong correlation between eK and topsoil K. However, the geostatistical method also has maximized information on mapped estimates of Zn, Cu, and to a lesser extent Pb. Greater delineation of an elongated NE/SW trending zone in the updated maps strongly suggests a stratigraphic control for the geochemical data. Elevated K (strongly correlated with Th), Zn, and Cu are principally associated with sandstones, siltstones, and mudstones of the Upper Devonian Shanmullagh Formation of the fault-bounded Fintona Block.

### *Economic significance*

Greater definition of the orientation of terrane-bounding faults such as the Fintona Block is of particular significance due to the inferred relationship between base metals and basin faulting in the Clogher Valley area (Arthurs and Earls, 2004; McKinley and Leuanthong, 2010). In particular, previous exploration was focused on a target area adjacent to an interpreted fault line, called the Aghintain Fault, defined by elevated base metal values identified in earlier BGS geochemical surveys. Coincident stream sediment samples and panned concentrates indicated zinc anomalies associated with elevated lead, barium (baryte), copper (malachite), and zinc (sphalerite). The majority of the anomalous stream sediment and panned concentrate samples were found in the area northwest of the Clogher Valley close to this interpreted Aghintain-Lislane Fault System (Figure 1). However, drilling in the Aghintain area failed to prove the existence of the interpreted northwest-trending fault. The interpretation was that if the fault is present it must have thrown the uppermost Ballyness Formation on the hangingwall against the stratigraphically lower Ballyness Formation on the footwall. As a result, detection of the fault proved very difficult. The mapped estimates of elemental Tellus geochemistry data using the Bayesian updating approach has maximized the amount of information available for the Clogher Valley area, and in particular the inferred location of the Aghintain fault (Figure 8). This has provided useful information for subsequent geological, economic, and environmental investigations. The integration of primary information (soil geochemistry) and secondary information (radiometric data) merged with information from joint spatial and multivariate correlations has made best use of data from the Tellus project and in particular increased the definition and sensitivity necessary to identify subtle anomalies indicating prospectivity for base metals in areas of thick drift cover such as in the Clogher Valley.

## Conclusions

The research has presented an innovative use of geostatistical analysis, and in particular a Bayesian updating approach, for the investigation, integration, and interpretation of geophysical and geochemical data from different sources. The advantage of the Bayesian updating technique is that multiple variables of different types and from different sources (in this case radiometric data and soil geochemistry) can be simultaneously integrated and applied to mapping of variables of interest. The primary information (in this case soil geochemistry) and the secondary information (radiometric data) can be shown separately, supported by a measure of uncertainty in the estimates, and the correlation between the data types and data sources can be tested. This research has utilized a global correlation to test the approach. A global correlation coefficient may be insufficient to describe local geological features and can be significantly affected by the presence of extreme values. Future work will include validating the uncertainty in the resulting model, using a jackknife or bootstrap approach and refinement of the approach to investigate the use of a locally varying coefficient to more accurately characterize complex local relationships between geochemical elements and radiometric data. A local

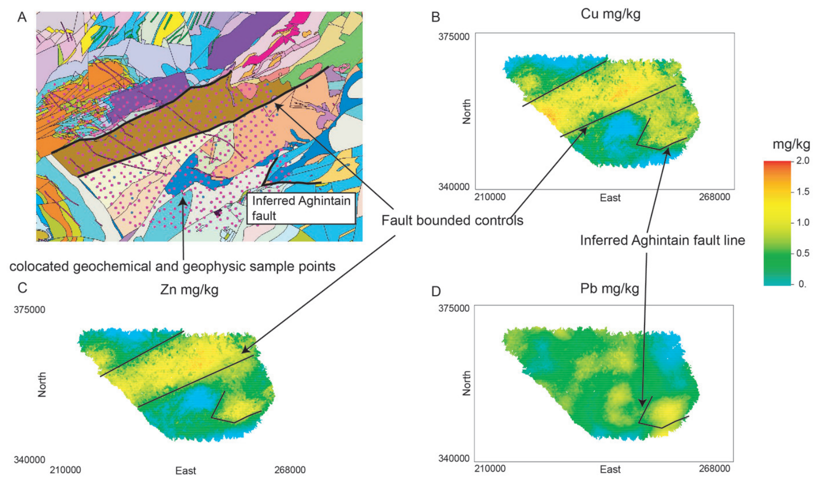


**Figure 8—Faulted-bounded controls inferred from geostatistical analysis. The TM65 Irish grid coordinate system is used. The location of the map refers to the highlighted area in Figure 1**

case study area has been selected using a limited data-set but the technique has the potential to be developed for the Tellus data for whole of Northern Ireland. This will enable a more meaningful interpretation of the nature of geochemical and radiometric variability and consequently any geological, environmental, and economic inferences.

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