

# Estimating seismic dispersion from prestack data using frequency-dependent AVO analysis

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## Abstract

Recent laboratory measurement studies have suggested a growing consensus that fluid saturated rocks can have frequency-dependent properties within the seismic bandwidth. It is appealing to try to use these properties for the discrimination of fluid saturation from seismic data. In this paper, we develop a frequency-dependent AVO (FAVO) attribute to measure magnitude of dispersion from pre-stack data. The scheme essentially extends the Smith and Gidlow (1987)'s two-term AVO approximation to be frequency-dependent, and then linearize the frequency-dependent approximation with Taylor series expansion. The magnitude of dispersion can be estimated with least-square inversion. A high-resolution spectral decomposition method is of vital importance during the implementation of the FAVO attribute calculation. We discuss the resolution of three typical spectral decomposition techniques: the short term Fourier transform (STFT), continuous wavelet transform (CWT) and Wigner-Vill Distribution (WVD) based methods. The smoothed pseudo Wigner-Ville Distribution (SPWVD) method, which uses smooth windows in time and frequency domain to suppress cross-terms, provides higher resolution than that of STFT and CWT. We use SPWVD in the FAVO attribute to calculate the frequency-dependent spectral amplitudes from pre-stack data. We test our attribute on forward models with different time scales and crack densities to understand wave-scatter induced dispersion at the interface between

25 an elastic shale and a dispersive sandstone. The FAVO attribute can determine the maximum magnitude of  
26 P-wave dispersion for dispersive partial gas saturation case; higher crack density gives rise to stronger magnitude  
27 of P-wave dispersion. Finally, the FAVO attribute was applied to real seismic data from the North Sea. The  
28 result suggests the potential of this method for detection of seismic dispersion due to fluid saturation.

29 **Keywords:** frequency dependent AVO; spectral decomposition; prestack; seismic dispersion

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31

## 32 **Introduction**

33 Frequency-dependent attenuation and dispersion are attracting more and more interests because they are believed  
34 to be directly associated to rock properties such as scale length of heterogeneities, rock permeability and  
35 saturating fluid. Theoretical studies of rock physics models (White, 1975; Chapman et al., 2003; Müller and  
36 Rothert, 2006; Gurevich et al., 2009) and Laboratory measurements of fluid saturated rocks (Murphy, 1982;  
37 Gist, 1994; Quintal and Tisato, 2013) suggested that wave-induced fluid flow between mesoscopic-scale  
38 heterogeneities is a major cause of P-wave attenuation and velocity dispersion in partially saturated porous  
39 media.

40

41 Since seismic attenuation is more sensitive to rock properties than velocity dispersion is, direct Q value  
42 estimation and tomography have been widely studied as a seismic attribute for reservoir characterization. The  
43 classical spectral ratio (Bath, 1974; Hauge, 1981; Dasgupta and Clark, 1998; Taner and Treitel, 2003) utilizes the  
44 ratio of seismic amplitude spectra at two different depths varies as a function of frequency to estimate Q value.  
45 Central frequency shift method (Quan and Harris, 1997) calculates Q from the decrease in centroid frequency of  
46 a spectrum of seismic wave traveling through a lossy medium.

47

48 The amplitude-versus-offset (AVO) as a lithology and fluid analysis tool has been utilized for over twenty years.  
49 However, the AVO theory is based on Zoeppritz equation and Gassmann's theory, in which attenuation and  
50 dispersion is generally not accounted for. Application of spectral decomposition techniques allows the  
51 frequency-dependent AVO behavior due to fluid saturation to be detected on seismic data, because reflections  
52 from hydrocarbon-saturated zone are thought to have a tendency of being low-frequency (Castagna et al., 2003).

53

54 Chapman et al. (2006) performed a theoretical study of reflections from the interface between a layer which  
55 exhibits fluids-related dispersion and an elastic overburden, and showed that in such cases the AVO response  
56 was frequency-dependent. Class I reflections tend to be shifted to higher frequency while class III reflections  
57 have their lower frequencies amplified. Recently, the frequency-dependent AVO (FAVO, Wilson et al., 2009;  
58 Wilson, 2010) inversion is introduced in an attempt to allow a quantitative measure of dispersion to be derived  
59 from pre-stack data. In this paper, we test the FAVO inversion scheme on synthetic and real seismic data to  
60 obtain a FAVO attribute. We begin by mathematically formulating the FAVO inversion theory based on Smith  
61 and Gidlow's (1987) two-term AVO approximation. Then the resolution of three typical spectral decomposition  
62 techniques: the short term Fourier transform (STFT), continuous wavelet transform (CWT) and the smoothed  
63 pseudo Wigner-Ville Distribution (SPWVD) based methods, have been discussed. The SPWVD with higher  
64 resolution is used to calculate the frequency-dependent spectral amplitudes from pre-stack data. We also discuss  
65 the effect of time scale parameter that control the frequency dispersion regime and crack density on the  
66 magnitude of dispersion estimation. Finally, the FAVO attribute is applied to seismic data from the North Sea.

67

## 68 **FAVO attribute for seismic dispersion**

69 Linear approximations to the exact Zoeppritz reflection coefficients can provide useful insights into subsurface  
70 properties. Smith and Gidlow's (1987) removed the density variation ( $\Delta\rho/\rho$ ) from Aki and Richards (1980) by  
71 using Gardner et al., (1974) relationship between density and P-wave velocity for water-saturated rocks. Then  
72 the approximation becomes two-terms and the P- and S-wave reflectivities ( $\Delta V_p/V_p$  and  $\Delta V_s/V_s$ ) can be inverted  
73 using parameters that are either known or can be estimated with Least-Square inversion. The reflection  
74 coefficient R of Smith and Gidlow's (1987) approximation can be written as:

$$75 \quad R(\theta) \approx A(\theta) \frac{\Delta V_p}{V_p} + B(\theta) \frac{\Delta V_s}{V_s}, \quad (1)$$

76 where  $\theta$  is the angle of incidence, the two offset-dependent constants  $A$  and  $B$  can be derived in terms of  $V_p, V_s$   
77 and the angle of incidence ( $\theta_i$ ) which can be calculated by way of ray tracing. Following the theory of Wilson et  
78 al.(2010), the coefficients  $A$  and  $B$  are frequency-independent and do not vary with velocity dispersion, the  
79 reflection coefficient  $R$  and the P- and S-wave reflectivities  $\Delta V_p/V_p$  and  $\Delta V_s/V_s$ , are considered to vary with  
80 frequency due to attenuation and dispersion at the interface or through the hydrocarbon saturated reservoir, then  
81 (1) can be written as:

$$82 \quad R(\theta, f) \approx A(\theta) \frac{\Delta V_p}{V_p}(f) + B(\theta) \frac{\Delta V_s}{V_s}(f). \quad (2)$$

83 Expanding (2) as first-order Taylor series around a reference frequency  $f_0$ :

$$84 \quad R(\theta, f) \approx A(\theta) \frac{\Delta V_p}{V_p}(f_0) + (f - f_0)A(\theta)I_a + B(\theta) \frac{\Delta V_s}{V_s}(f_0) + (f - f_0)B(\theta)I_b, \quad (3)$$

85 where  $I_a$  and  $I_b$  are the derivatives of P- and S-wave reflectivities with respect to frequency evaluated at  $f_0$ :

$$86 \quad I_a = \frac{d}{df} \left( \frac{\Delta V_p}{V_p} \right); I_b = \frac{d}{df} \left( \frac{\Delta V_s}{V_s} \right). \quad (4)$$

87 For a typical CMP gather with  $n$  receivers denoted as a data matrix  $s(t, n)$ . Coefficients  $A$  and  $B$  at each sampling  
88 point, denoted as  $A_n(t)$  and  $B_n(t)$ , can be derived with the knowledge of velocity model through ray tracing.  
89 Spectral decomposition is performed on  $s(t, n)$  to derive the spectral amplitude  $S(t, n, f)$  at a series of frequencies.  
90 However,  $S$  contains the overprint of seismic wavelet, so we perform spectral balance, by which the spectral  
91 amplitudes at different frequencies are matched to the spectral amplitude at the reference frequency  $f_0$  through a  
92 strong continuous reflection caused by elastic interface, to remove this effect with a suitable weight function  $w(f,$   
93  $n)$ :

$$94 \quad D(t, n, f) = S(t, n, f)w(f, n). \quad (5)$$

95 where  $D(t, n, f)$  is the balanced spectral amplitude.  $w(f, n)$  is calculated from a defined window with  $k$  sampling  
 96 points using the ratio of RMS amplitudes at the chosen reference frequency  $f_0$  and other frequencies as shown in  
 97 (6),

$$98 \quad w(f, n) = \frac{\sqrt{\sum_k S^2(t, n, f_0)}}{\sqrt{\sum_k S^2(t, n, f)}} \quad (6)$$

99 Giving the fact that the seismic amplitudes can be associated with the reflection coefficients through convolution  
 100 with a seismic wavelet in the AVO analysis. The relationship between spectral amplitude and reflectivity  
 101 depends on the spectral decomposition we used. According to (2), we derive  $\Delta V_p/V_p$  and  $\Delta V_s/V_s$  at the reference  
 102 frequency  $f_0$  by replacing  $R$  with  $D$ . Considering  $m$  frequencies  $[f_1, f_2, \dots, f_m]$ , equation (3) can be expressed as  
 103 matrix form:

$$104 \quad \begin{bmatrix} D(t, 1, f_1) - A_1(t) \frac{\Delta V_p}{V_p}(f_0, t) - B_1(t) \frac{\Delta V_s}{V_s}(f_0, t) \\ \vdots \\ D(t, 1, f_m) - A_1(t) \frac{\Delta V_p}{V_p}(f_0, t) - B_1(t) \frac{\Delta V_s}{V_s}(f_0, t) \\ \vdots \\ D(t, n, f_1) - A_n(t) \frac{\Delta V_p}{V_p}(f_0, t) - B_n(t) \frac{\Delta V_s}{V_s}(f_0, t) \\ \vdots \\ D(t, n, f_m) - A_n(t) \frac{\Delta V_p}{V_p}(f_0, t) - B_n(t) \frac{\Delta V_s}{V_s}(f_0, t) \end{bmatrix} \approx \begin{bmatrix} (f_1 - f_0)A_1(t) & (f_1 - f_0)B_1(t) \\ \vdots & \vdots \\ (f_m - f_0)A_1(t) & (f_m - f_0)B_1(t) \\ \vdots & \vdots \\ (f_1 - f_0)A_n(t) & (f_1 - f_0)B_n(t) \\ \vdots & \vdots \\ (f_m - f_0)A_n(t) & (f_m - f_0)B_n(t) \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix}, \quad (7)$$

105 which can be denoted as:

$$106 \quad RR = RM \begin{bmatrix} I_a \\ I_b \end{bmatrix}. \quad (8)$$

107 Then the attributes of  $I_a$  and  $I_b$  can be calculated with least-squares-inversion:

$$108 \quad \begin{bmatrix} I_a \\ I_b \end{bmatrix} = (RM^T RM)^{-1} RM^T RR \quad (9)$$

109

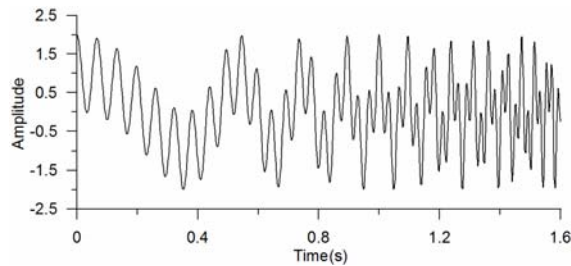
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111 **Choice of spectral decomposition techniques**

112 Application of spectral decomposition techniques allows the frequency-dependent AVO behaviour to be  
 113 detected from seismic data. A high resolution method is of vital importance for the accuracy and robustness of  
 114 estimating seismic dispersion. Here we study and compare three different spectral decomposition techniques:  
 115 short-time Fourier transform (STFT), continuous wavelet transform (CWT) and Wigner-Ville distribution  
 116 (WVD) based method. The STFT introduced by Gabor (1946) can be expressed as,

117 
$$STFT(t, f) = \langle x(\tau), \varphi(\tau - t)e^{j2\pi f\tau} \rangle = \int_{-\infty}^{\infty} x(\tau)\overline{\varphi}(\tau - t)e^{-j2\pi f\tau} d\tau, \quad (10)$$

118 where  $\varphi$  is the window function centred at time  $\tau = t$ , and  $\overline{\varphi}$  is the complex conjugate of  $\varphi$ . The STFT is a type  
 119 of linear Time-Frequency Representation (TFR). The choice of the width of window functions leads to a  
 120 trade-off between time localization and frequency resolution (Cohen, 1989).



121  
 122 **Fig.1 Quadratic FM signal comprised of two frequency components. The frequency of the signal increases**  
 123 **with time.**

124 As shown in Fig.1, the signal consists of two quadratic FM signals with different frequency components.  
 125 The frequency increases with time in quadratic trend, while the amplitude keep unchanged. We use the regularly  
 126 used window functions: Hamming window, Hanning window, Gauss window and Nuttall window for STFT  
 127 spectral analysis, for which the expression of window functions are as follows:

128 Hamming window:  $w(n) = 0.54 - 0.46 \times \cos(2\pi \frac{n}{N})$ ,  $0 \leq n \leq N$  ;

129 Hanning window:  $w(n) = 0.5(1 - \cos(2\pi \frac{n}{N}))$ ,  $0 \leq n \leq N$  ;

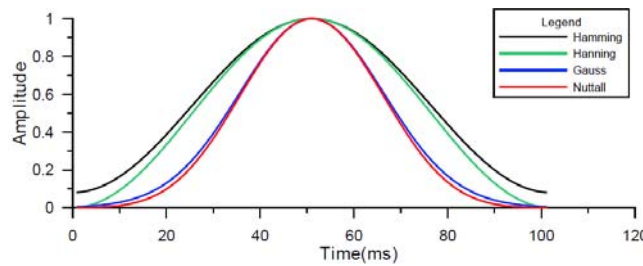
130 Gauss window:  $w(n) = e^{-\frac{1}{2}(\alpha \frac{n}{N/2})^2}$ ,  $-\frac{N}{2} \leq n \leq \frac{N}{2}$ , the length of the window is N+1;

131 Nuttall window:

132 
$$w(n) = a_0 - a_1 \times \cos(2\pi \frac{n}{N}) + a_2 \times \cos(4\pi \frac{n}{N}) - a_3 \times \cos(6\pi \frac{n}{N}), \quad 0 \leq n \leq N$$

133 where  $a_0=0.3635819$ ;  $a_1=0.4891775$ ;  $a_2=0.1365995$ ;  $a_3=0.0106411$ .

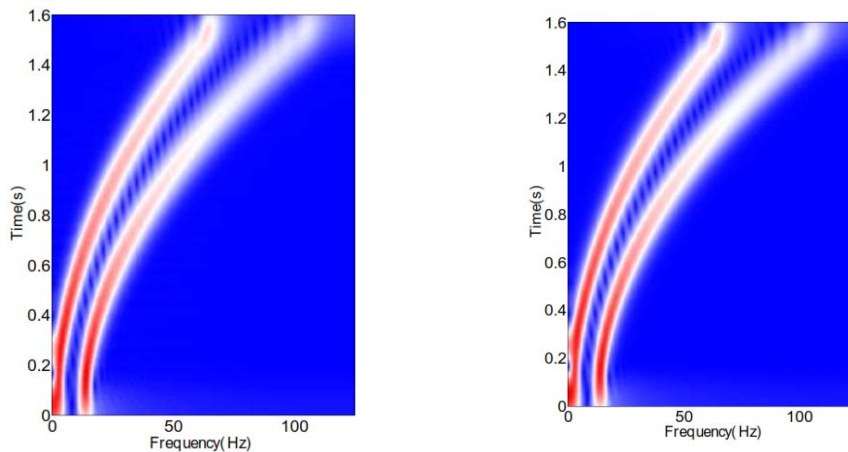
134 The shapes of the four window functions are shown in Fig.2. The Hamming window and Hanning window are  
135 wider than the Gauss and Nuttall windows.



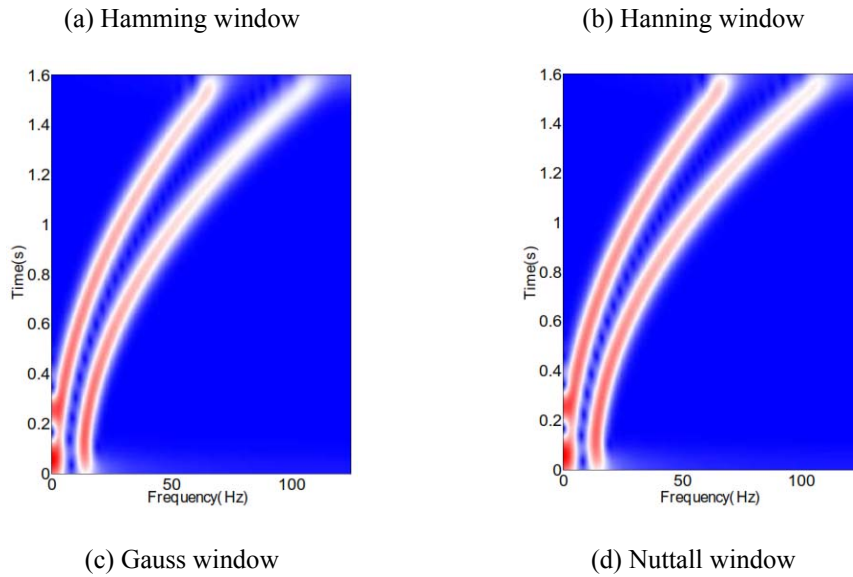
136

137 **Fig.2 The shapes of four different window functions. The Gaussian and Nuttall windows are wider than**  
138 **Hamming and Hanning windows.**

139 Fig.3 displays the STFT spectra with the four different window functions. The spectra with the Hamming  
140 window and Hanning window have higher frequency resolution especially at low frequencies but low temporal  
141 resolution as indicated by the stripes between the two signal components; while the Gauss window and Nuttall  
142 window show high temporal resolution especially at high frequency but relatively low frequency resolution at  
143 low frequency. However, we can see that Gauss and Nuttall windows provide a better TFR than Hamming and  
144 Hanning windows for this signal.







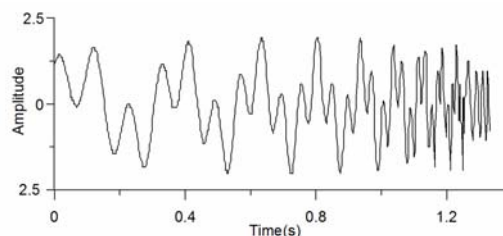
**Fig.3 STFT spectra with different window functions. The spectra using Gauss and Nuttall windows have higher temporal resolution than that of using Hamming and Hanning (Windows length: 400ms).**

145 The Continuous Wavelet Transform of a signal  $s(t)$  is defined as the inner product of a family of  
 146 wavelets  $\psi_{a,b}(t)$  and  $s(t)$  (Mallat, 1993, Sinha, et al., 2005):

$$147 \quad S(a,b) = \langle s(t), \psi_{a,b}(t) \rangle = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} s(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt, \quad (11)$$

148 where  $a$  is the dilation parameter (corresponding to frequency information),  $b$  is the translation  
 149 parameter (corresponding to temporal information),  $\overline{\psi(t)}$  is the complex conjugate of  $\psi(t)$  .

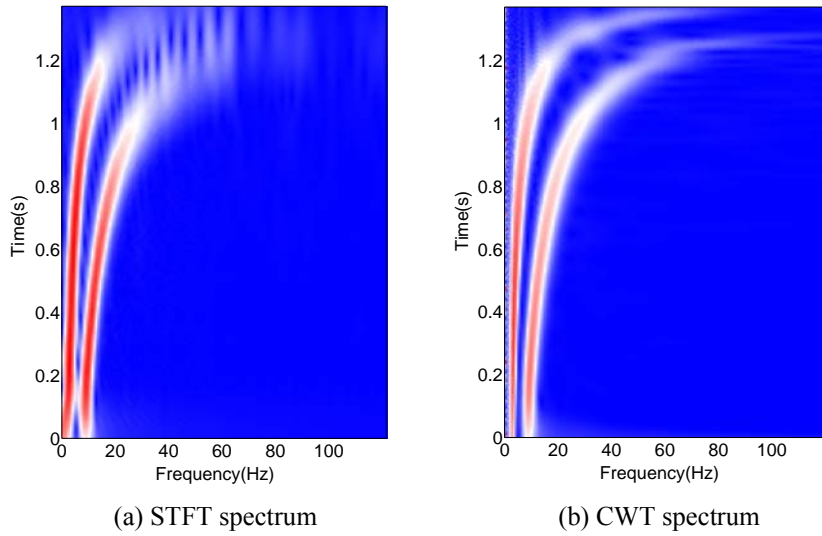
150  $S(a,b)$  is the variation of the original signal  $s(t)$  with the observation area under different scales at  
 151 time  $t=b$ .



152  
 153 **Fig.4 Hyperbolic FM signal with two frequency components. The frequency of the signal becomes higher  
 154 with increase of time.**

155 Consider another hyperbolic FM signal with two different frequencies as shown in Fig.4. A 400  
 156 ms Hamming window is used for STFT and the Morlet wavelet is used for CWT to obtain

157 time-frequency spectra as shown in Fig.5. From Fig.5 (a), we can see STFT spectrum displays high  
 158 resolution at low frequencies but low resolution at high frequencies due to a predefined window size.  
 159 Fig.5(b) is the result of CWT, we can see the event is thinner than that of STFT. Frequency resolution  
 160 at low frequencies is improved. Temporal resolution at high frequency is significantly improved as  
 161 well due to the dilation of wavelet function.



**Fig.5 Comparison of STFT and CWT spectra for the FM signal in Fig.4.**

162 A third time-frequency representation in seismic signal analysis is the Wigner-Ville distribution  
 163 (WVD, Classen and Mecklenbräuker, 1980; Cohen, 1989). The WVD of the signal  $x(t)$  can be defined  
 164 as,

$$165 \quad WVD(t, f) = \int_{-\infty}^{\infty} X(t + \tau / 2) \bar{X}(t - \tau / 2) e^{-j2\pi f \tau} d\tau \quad (12)$$

166 Where  $\tau$  is the time delay variable,  $X(t)$  is the analytical signal associated with the real signal  $x(t)$ ,

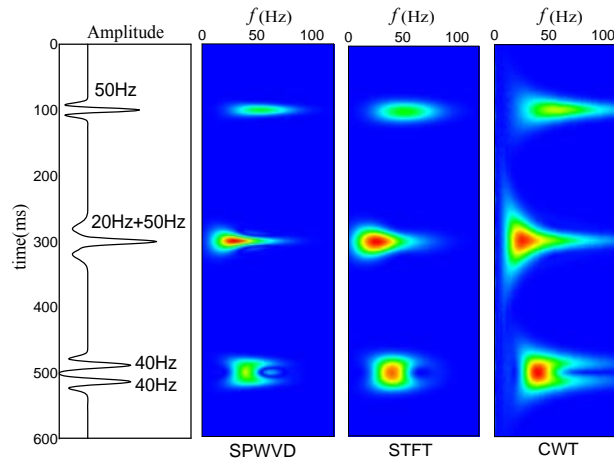
$$167 \quad X(t) = x(t) + jH[x(t)] \quad (13)$$

168  $H[x(t)]$  is the Hilbert transform of  $x(t)$  as the imaginary part of  $X(t)$ . The WVD avoids the STFT  
 169 trade-off between time and frequency resolution. However, this improvement comes at the cost of the  
 170 well-known cross-term interference (CTI) caused by WVD bilinear characteristic. One of the

171 improvements is the smoothed pseudo Wigner–Ville distribution (SPWVD), using both a time  
 172 smoothing window and a frequency smoothing window independently, expressed as

$$173 \quad SW_{g,h,X}(t, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(t + \frac{\tau}{2}) \bar{X}(t - \frac{\tau}{2}) g(\nu) h(\tau) e^{-j2\pi f\tau} d\nu d\tau \quad (14)$$

174 where  $\nu$  is the time delay and  $\tau$  is the frequency offset.  $g(\nu)$  is the time smoothing window,  $h(\tau)$  the  
 175 frequency smoothing window on condition that  $g(\nu)$  and  $h(\tau)$  are both real symmetric functions and  
 176  $g(0)=h(0)$ . Then it is possible to attenuate the CTI presented in the WVD, by independently choosing  
 177 the type of these two windows.



178  
 179 **Fig.6. A synthetic trace constructed by Ricker wavelets of different centre frequencies and its spectra**  
 180 **generated with different methods. The SPWVD shows superior resolution than the other methods for this**  
 181 **signal.**

182 As shown in Fig. 6, a synthetic seismic trace comprised of three components is constructed by  
 183 adding Ricker wavelets of different centre frequencies. We perform SPWVD with a 30 ms and a 60  
 184 ms length Gauss low-pass filter as a time and frequency smoothing window, STFT with a 80 ms  
 185 length Hamming window and CWT with Morlet wavelet. We can see the SPWVD method provides  
 186 the highest resolution, while both temporal and frequency resolutions are not high enough to  
 187 demonstrate the spectral characteristics with STFT method. The CWT result shows that the temporal  
 188 resolution is high but the frequency resolution is low at low frequencies, and the frequency resolution

189 is high but the temporal resolution is low at high frequencies. We use the SPWVD for spectral  
 190 decomposition of prestack data in the FAVO attribute estimation.

191

## 192 **Numerical modelling of dispersion estimation**

193 We study a case that considers the effect of variation of timescale  $\tau$  on the magnitude of dispersion caused  
 194 by scattering at the interfaces. Timescale  $\tau$  controls the frequency range over which dispersion occurs. The  
 195 choice of material parameters is based on the models created by Chapman et al. (2006). The case is a  
 196 two-layer model, in which the top elastic shale has P- and S-wave velocities of  $2743 \text{ ms}^{-1}$  and  $1394 \text{ ms}^{-1}$ ,  
 197 and a density of  $2.06 \text{ gcm}^{-3}$ . The lower sandstone is considered to have a P- and S- wave velocities of  $2835$   
 198  $\text{ms}^{-1}$  and  $1472 \text{ ms}^{-1}$ , and a density of  $2.08 \text{ gcm}^{-3}$  under water saturation and then change to partial gas  
 199 saturation with wood equation to calculate the mixed fluid moduli. The parameters of the model are listed  
 200 in Table 1.

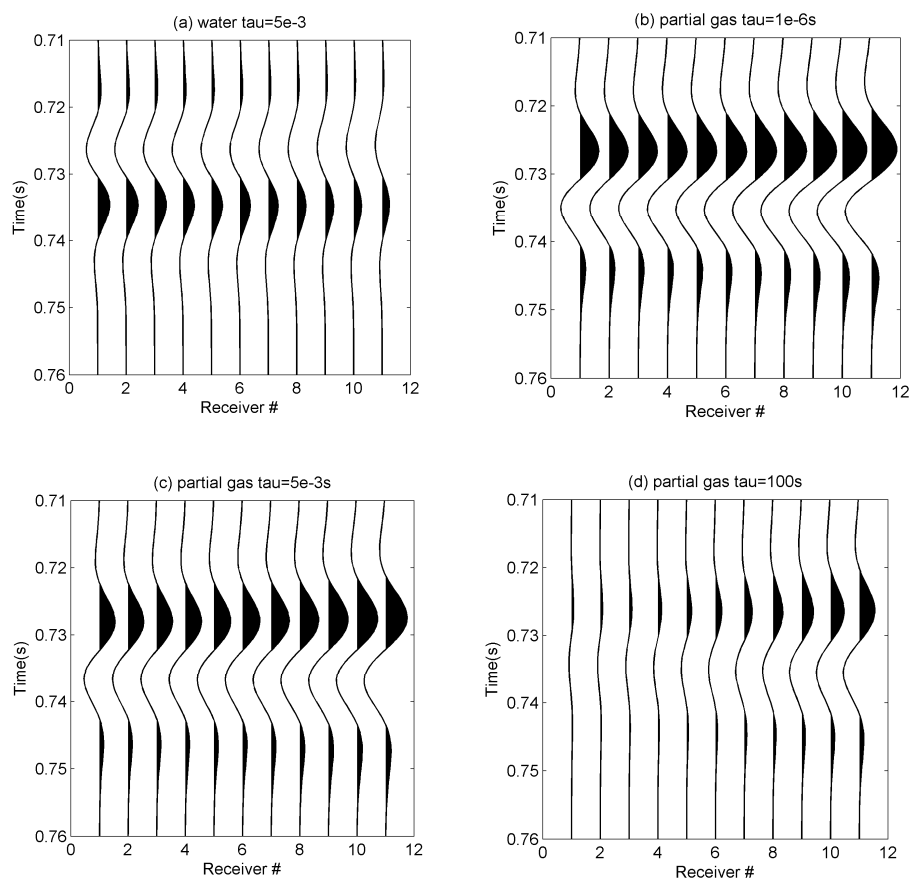
201 Table 1 Material parameters for the two-layer model as described in Chapman et al. (2006).

| lithofacies | $V_p(\text{ms}^{-1})$ | $V_s(\text{ms}^{-1})$ | $\rho_w(\text{g.cm}^{-3})$ | $\rho_G(\text{g.cm}^{-3})$ | $\phi(\%)$ | cd(%) | Thickness(m) |
|-------------|-----------------------|-----------------------|----------------------------|----------------------------|------------|-------|--------------|
| Shales      | 2743                  | 1394                  | 2.06                       | -                          | -          | -     | 1000         |
| Sandstone   | 2835                  | 1472                  | 2.08                       | 2.04                       | 15         | 5     | Half space   |

202

203 11 receivers were synthesized with a trace spacing of 100 m and 40 Hz Ricker wavelet as the  
 204 explosive source. Fig.7 displays the four synthetic gathers generated by ANISEIS software package with a  
 205 Fortran program to create the frequency dependent layer. Two dispersive cases have the same  $\tau$  value of  
 206  $5 \times 10^{-3}$  s under water and partial gas saturation respectively. Such a  $\tau$  value corresponds to the transition  
 207 frequency being located at the seismic band. The low frequency case ( $\tau = 1 \times 10^{-6}$  s) and the high frequency

208 case ( $\tau = 100$  s) have no attenuation and correspond to elastic case. From Fig.7(a), we can see the typical  
 209 Class I AVO feature that the amplitudes decrease with offset under water saturation. However, Class I  
 210 AVO changes to Class III AVO when the fluid is substituted with partial gas saturation. Another feature  
 211 when partial gas saturation is that the amplitudes decrease with increasing  $\tau$  value. This indicates the  
 212 reduction of velocity difference and acoustic impedance difference between the two layers from low  
 213 frequency to high frequency.

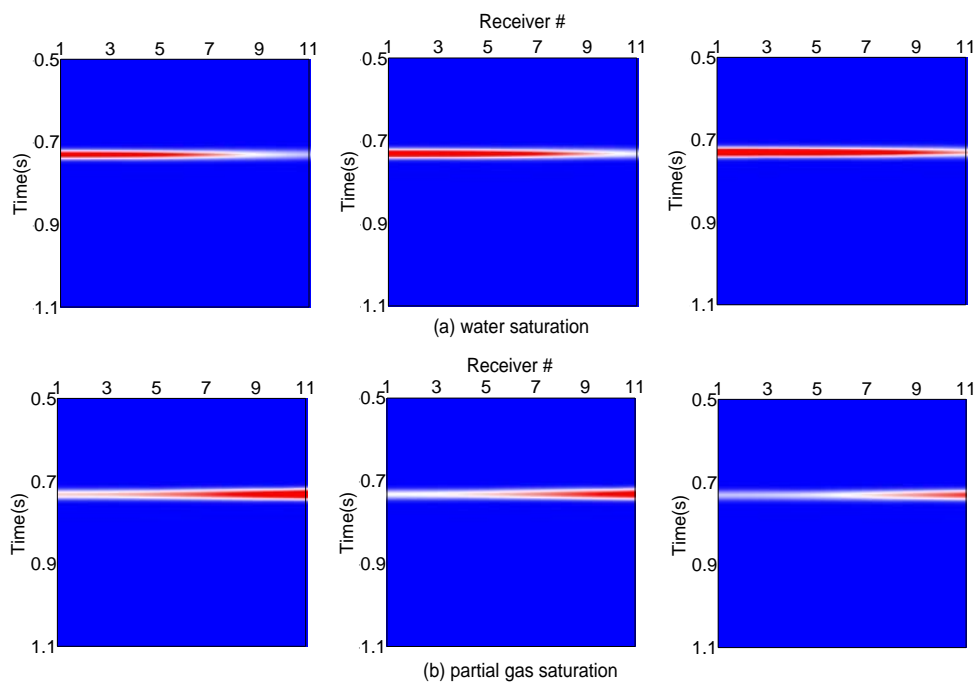


**Fig.7 Four synthetic gathers under full water saturation when (a): $\tau = 5 \times 10^{-3}$  s, and partial gas saturation when (b): $\tau = 1 \times 10^{-6}$  s, (c):  $\tau = 5 \times 10^{-3}$  s and (d):  $\tau = 100$  s.**

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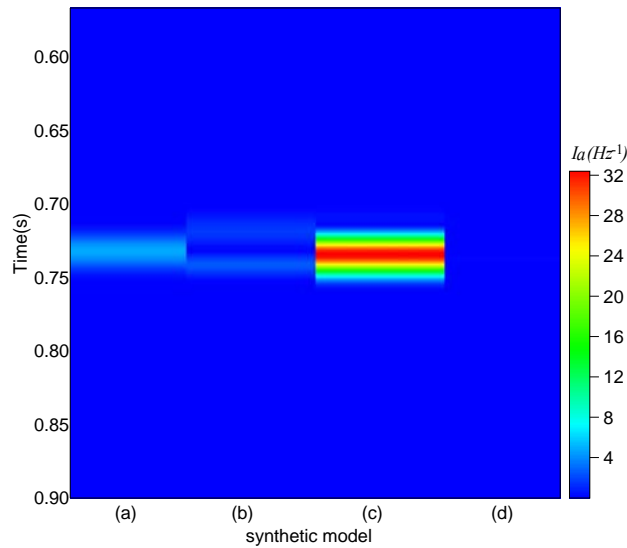
215 Using 40Hz (dominant frequency of the source wavelet) as the reference frequency, we calculate the  
 216 FAVO attribute with the spectral amplitudes at a series of frequencies 25Hz, 30Hz, 40Hz, 50Hz, 60Hz,  
 217 70Hz and 80Hz. A set of weights were derived in the elastic model by matching the maximum spectral

218 amplitude at non-reference frequencies to the maximum spectral amplitude at 40Hz. These same weights  
 219 are applied to the dispersive model to remove the wavelet overprint. Since the shear modulus is decoupled  
 220 from the saturating fluids, we only calculate the derivative of P-wave reflectivity  $I_a$ . Fig.8 displays a  
 221 comparison between the balanced isofrequency sections at 25Hz, 50Hz and 80Hz for the two dispersive  
 222 models under water and partial gas saturation respectively. We can see that for partial gas, energy decreases  
 223 with increasing frequency, whereas energy increases with increasing frequency when saturated with water.



224  
 225 **Fig.8 Isofrequency sections at 25Hz, 50Hz and 80Hz for the dispersive models under water and partial gas**  
 226 **saturation respectively ( $\tau = 5 \times 10^{-3}$ s).**

227 Fig.9 displays the  $I_a$  attribute for the four models. We can see that the magnitude of P-wave dispersion  
 228 for the partial gas saturation when  $\tau = 5 \times 10^{-3}$ s (c) is the highest. This is much more significant than low and  
 229 high frequency cases (b and d), of which there is almost no dispersion occurring. The result for dispersive  
 230 model under water saturation (a) is a little weak compared with (c), indicating the magnitude of dispersion  
 231 is weaker than partial gas saturation. This can be seen from the isofrequency sections.

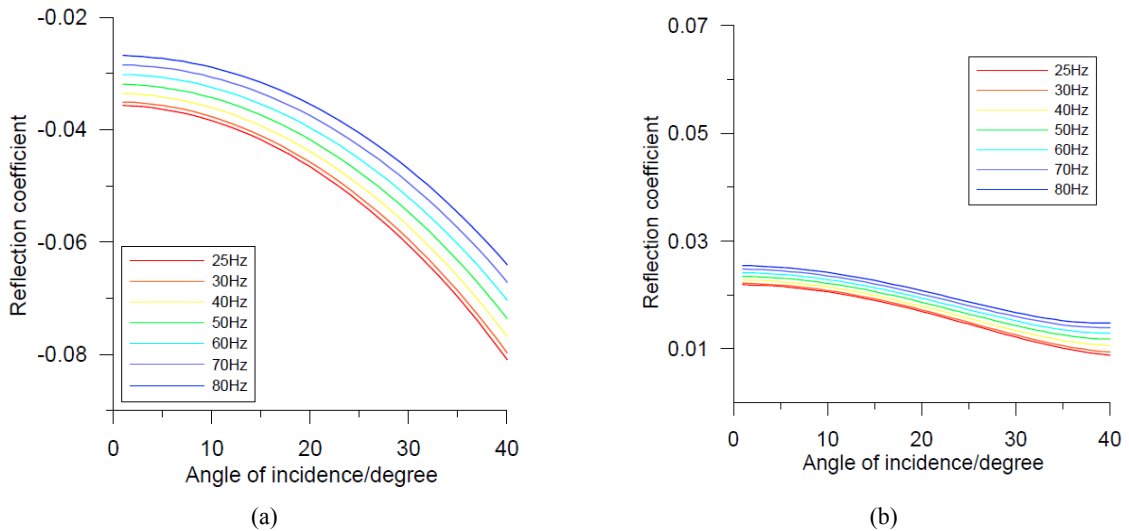


232  
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**Fig.9 The attribute of derivative of P-wave reflectivity  $I_a$  for the four synthetic models as displayed in Fig.7.**

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236  
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Fig.10 displays the reflection coefficients varying with angle of incidence at frequencies 25, 30, 40, 50, 60, 70 and 80Hz under the two dispersive models (Fig.7 a and c). Comparing the amount of separation between the curves of different frequencies, we can see that there is stronger dispersion within the seismic bandwidth in the synthetic seismogram under partial gas saturation. This corresponds to the result displayed in Fig.9.



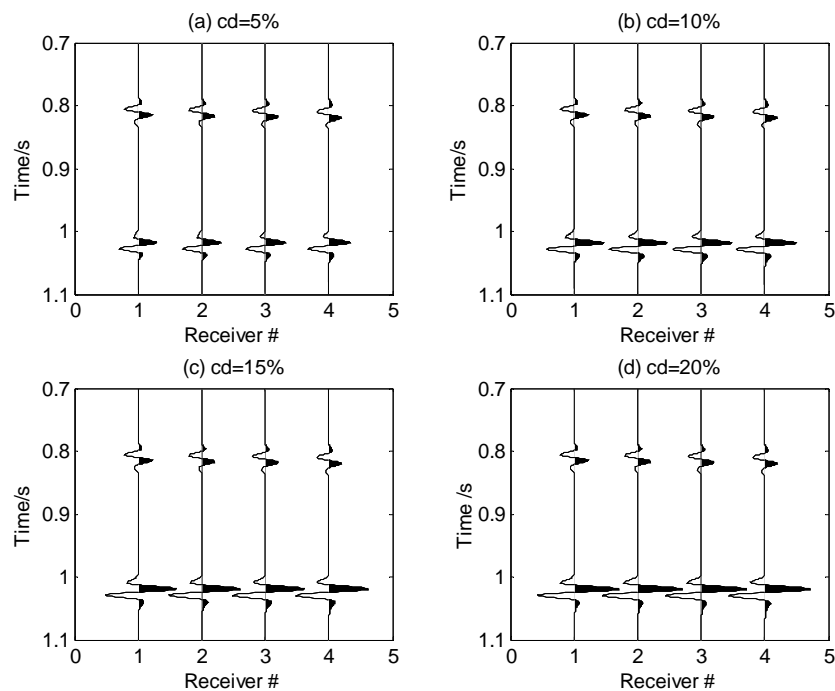
**Fig.10 The P-wave reflection coefficient versus angle of incidence for the two dispersive model under water and partial gas saturation respectively.(a)partial gas saturation;(b)water saturation.**

240

241 In the second case we create a three-layer model with varying crack densities of 5%, 10%, 15% and 20%  
 242 for the dispersive sandstone under gas saturation, in order to study the influence of crack density on the  
 243 magnitude of dispersion. The P-wave and S-wave velocity and density are the same as that of the previous  
 244 example, but an elastic overburden shale layer is added to form an elastic interface for spectral balance. All  
 245 the parameters are listed in Table 2. Using 40Hz Ricker wavelet as the source, we create the four models  
 246 with 4 receivers (1.1km – 1.4km) at a trace spacing of 100m. Fig.11 displays the synthetic seismograms  
 247 when varying crack densities. It is clear that reflection amplitude from the second interface increase with  
 248 increasing crack density.

249 Table 2 Material parameters for the three-layer model as described in Chapman et al. (2006).

| lithofacies | Vp(ms <sup>-1</sup> ) | Vs(ms <sup>-1</sup> ) | ρ <sub>w</sub> (g.cm <sup>-3</sup> ) | ρ <sub>G</sub> (g.cm <sup>-3</sup> ) | φ(%) | cd(%)      | Thickness(m) |
|-------------|-----------------------|-----------------------|--------------------------------------|--------------------------------------|------|------------|--------------|
| Shales      | 2500                  | 1250                  | 2.02                                 | -                                    | -    | -          | 1000         |
| Shales      | 2743                  | 1394                  | 2.06                                 | -                                    | -    | -          | 300          |
| Sandstone   | 2835                  | 1472                  | 2.08                                 | 2.04                                 | 15   | 5,10,15,20 | Half space   |



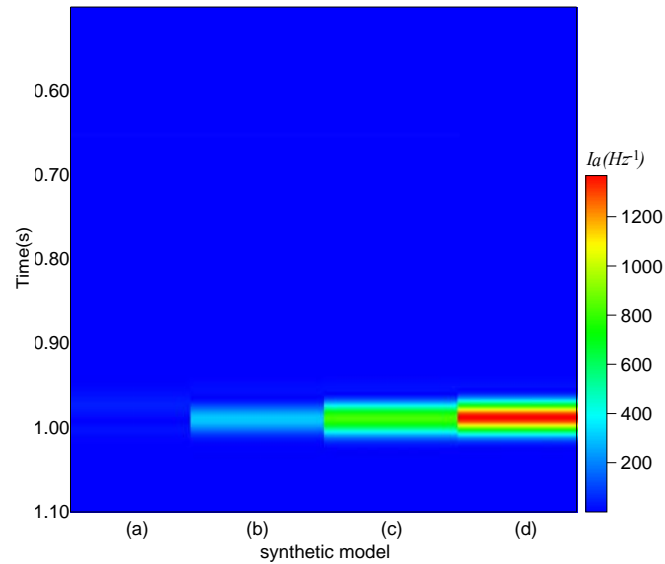
250

251

**Fig.11 Three-layer model with different crack densities.**



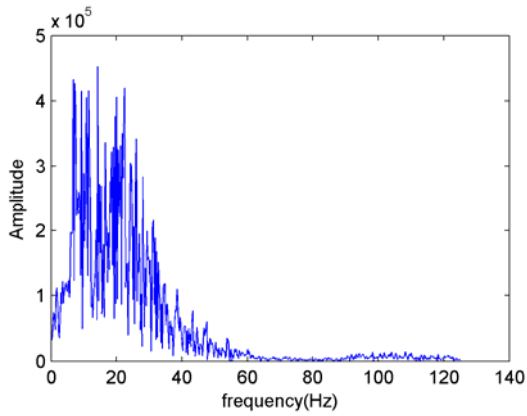
252 Using the same set of frequencies at 25Hz, 30Hz, 40Hz, 50Hz, 60Hz, 70Hz, 80Hz and 40Hz as reference  
 253 frequency for spectral balance, we implement FAVO attribute and obtain the  $I_a$  of the four models as  
 254 shown in Fig.12. We can find that there is no dispersion for the first interface at 0.8s, whereas there is  
 255 conspicuous dispersion for the second interfaces at 1.02s. The  $I_a$  value increases as crack density increases.



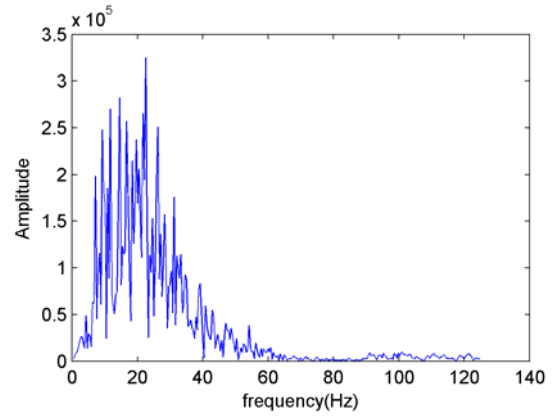
256  
 257 **Fig.12 The derivative of P-wave reflectivity  $I_a$  under different crack densities for the four synthetics**  
 258 **models as displayed in Fig.11.**

259  
 260 **Cases study**

261 Numerical test has demonstrated that the FAVO attribute scheme is able to quantitatively estimate seismic  
 262 dispersion and separate dispersive models from elastic models. In this section, we present the application of  
 263 FAVO attribute to real seismic data from the North Sea. Selection of optimal frequencies and reference  
 264 frequency for spectral decomposition is a key issue for the calculation of FAVO attribute. To this end, we  
 265 analyze the energy distribution an arbitrary pre-stack trace with Fast Fourier Transform (FFT). As shown in  
 266 Fig.13, the dominant frequency is around 15Hz with bandwidth from 0Hz to 40Hz for prestack trace. We  
 267 define the reference frequency  $f_{ref} = 15\text{Hz}$  and select a set of frequencies at 10Hz, 15Hz, 20Hz, 30Hz, 40Hz  
 268 for the FAVO attribute.



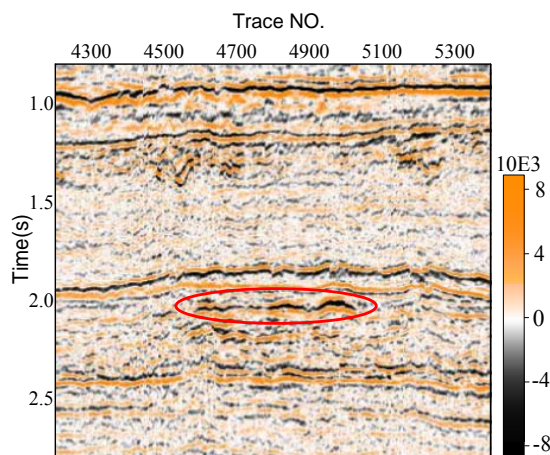
pre-stack trace



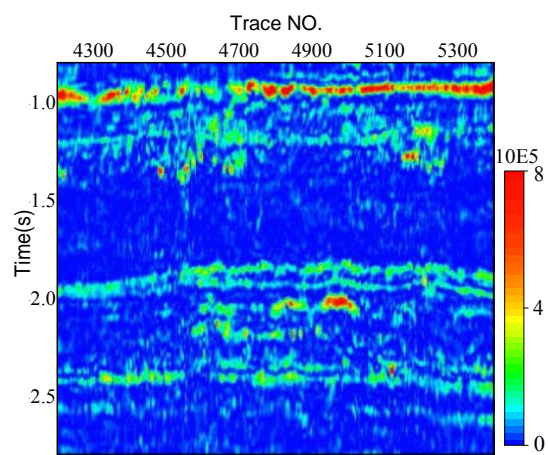
stacked trace

**Fig.13 Spectral analysis of a arbitrary prestack trace.(a)the whole trace (b)trace under 1.6s.**

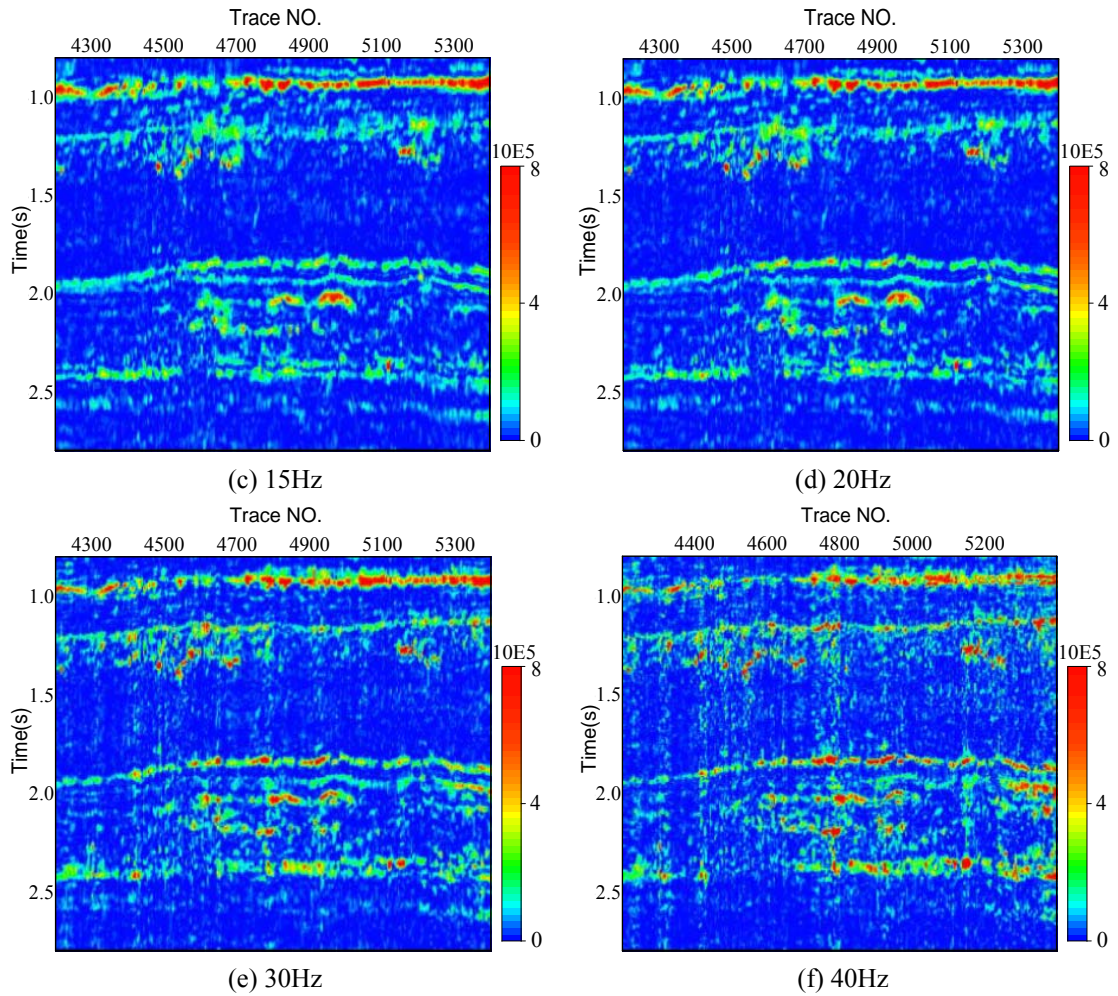
269 Fig.14 (a) displays the stacked section of Xline4921 from the North Sea. The target is around 2.0s,  
 270 corresponding to a relatively deep depth of about 2km. The positions of potential reservoir exhibit as  
 271 “bright spots”, where the pre-stack CMP gathers have the feature of Class III AVO. We perform SPWVD  
 272 on the stacked section to obtain isofrequency sections at 10Hz, 15Hz, 20Hz, 30Hz, and 40Hz. Continuous  
 273 reflections above 1.0s can be deemed to be caused by elastic interface, therefore can be used for spectral  
 274 balance with 15Hz as reference frequency. Fig.14 (b-f) display the balanced isofrequency section at 10Hz,  
 275 15Hz, 20Hz, 30Hz, 40Hz of the stacked section. We can see that spectral amplitudes at left and middle  
 276 anomalies have a trend of increasing with frequency, whilst spectral amplitudes at the right anomaly is  
 277 stable at 10Hz, 15Hz, 20Hz and then decrease at 30Hz and 40Hz.



(a) Stacked section of Xline4921



(b) 10Hz



**Fig.14 The Stacked section of Xline4921 and its isofrequency sections.**

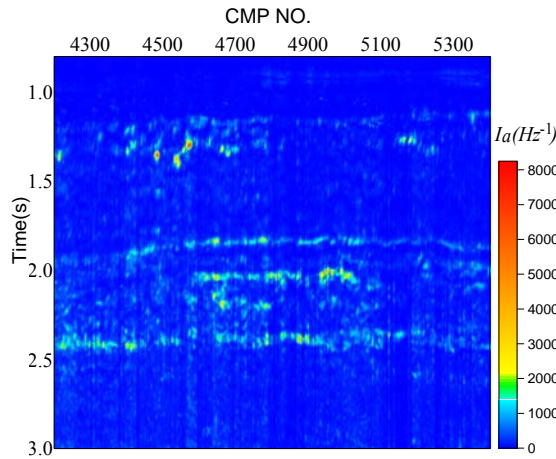
278

279 The pre-stack CMP gathers from No.4201 to No.5399 are extracted from the 3D seismic data for the  
 280 calculation of  $I_a$  attribute. In order to reduce the influence of NMO stretching, we use the first 45 traces for  
 281 the FAVO attribute.

282

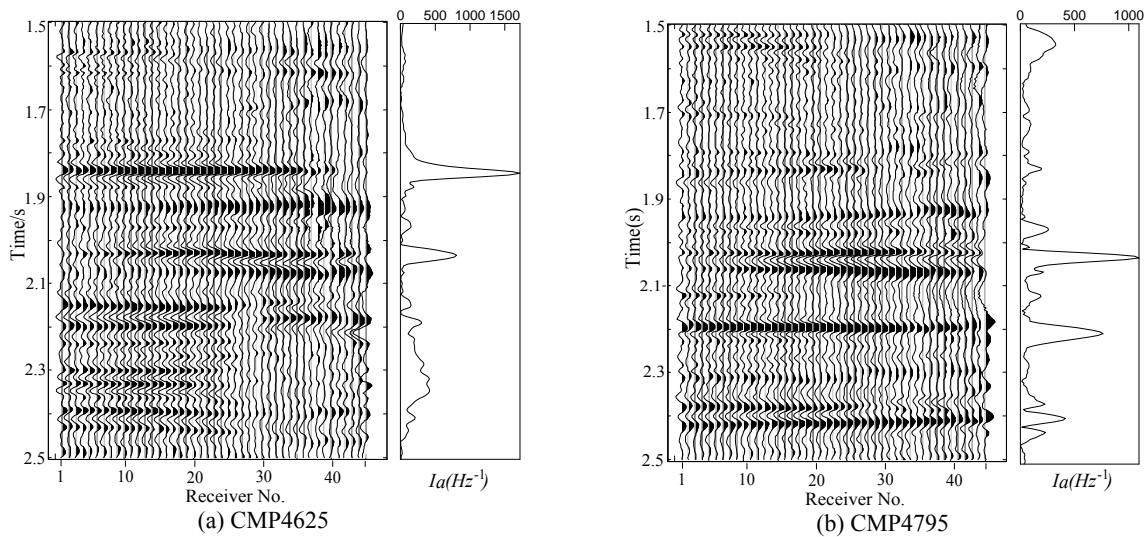
283 We perform the SPWVD on each pre-stack gather to obtain isofrequency sections at 10, 15, 20, 30 and  
 284 40Hz. A trace of  $I_a$  attribute can be calculated for each pre-stack gather. Figure 15 displays the  $I_a$  attribute  
 285 for the CMP numbers from 4201 to 5399. We can see strong reflection energy due to non-reservoir  
 286 interfaces in Figure 14 has been partially eliminated in this attribute. The zone of interest around 2.0s  
 287 shows significant magnitude of dispersion. Figure 16 displays the No.4625 and No.4795 CMP gather. The

288 amplitudes exhibit the typical feature of Class III AVO at the reservoir position. The inverted  $I_a$  attribute on  
 289 the right shows maximum amplitude around this area, which may be caused by fluids saturation.



290  
 291  
 292

**Fig.15 The  $I_a$  attribute for the CMP number from 4201 to 5399.**



**Fig.16 The calculated  $I_a$  attribute for the two CMP gathers.**

293  
 294

## Conclusions

295 In this paper, we have developed a FAVO attribute to demonstrate the possibility of inferring dispersion  
 296 properties directly from pre-stack data and linking this to fluid saturation. The attribute combines a  
 297 high-resolution spectral decomposition technique with Smith and Gidlow (1987)'s AVO approximation. We

298 illustrate the method through analysis of synthetic data. The real seismic example from the North Sea indicates  
299 the potential of this method for the detection of seismic dispersion resulting from fluid saturation.

300 We also compare three typical spectral decomposition techniques and use SPWVD in the FAVO attribute.  
301 forward modelling indicates that the FAVO attribute can determine the maximum magnitude of P-wave  
302 dispersion for dispersive partial gas saturation case. Higher crack density gives rise to stronger magnitude of  
303 P-wave dispersion. Real seismic data example from the North Sea suggests the potential of this method for  
304 detection of seismic dispersion due to fluid saturation.

305 It is worth to mention that the inversion scheme still may be affected by NMO stretching. For deep reservoirs,  
306 we use near-offset traces only in order to reduce the affect of NMO stretching, but for shallow reservoirs the  
307 NMO stretching should be corrected before inversion.

308

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313

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