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Problem solving in the teaching of single variable differential and integral calculus: Perspective of mathematics teachers

Resolución de problemas en la enseñanza del cálculo diferencial e integral en una variable: Perspectiva de los docentes de matemática

Resolução de problemas no ensino do cálculo diferencial e integral em uma variável: Perspectiva dos docentes de matemática

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Abstract

There is a wide diversity of approaches to solving problems in the teaching of mathematics. In particular, the meaning of "problem solving" differs between theory and practice. In the teaching of higher mathematics, problem solving is frequently used in single variable differential and integral calculus, as indicated by course contents and the number of university programs that include it in their curricula. We therefore investigated the ways in which mathematics teachers use problem solving in the teaching of single variable differential and integral calculus. A questionnaire was applied to teachers with experience in teaching single variable differential and integral calculus from the Universidad de Costa Rica, the Universidad Nacional de Costa Rica, the Instituto Tecnológico Costa Rica, and the Universidad Estatal a Distancia. The results reveal contradictions between teachers' conceptions of what a mathematical problem is and their implementation of problem solving in the classroom.

Keywords: Teaching single variable differential and integral calculus; problem solving; higher mathematics; mathematics education.

Resumen

Las líneas de investigaciones en resolución de problemas en la enseñanza de las matemáticas son muchas y con similitudes y discrepancias entre sí. En particular, lo que se entiende por resolución de problemas presenta diferencias entre la teoría y la práctica. En la enseñanza de la matemática superior, el cálculo diferencial e integral en una variable es un área rica para el uso de la resolución de problemas por sus con-

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tenidos y la cantidad de carreras universitarias que la incluyen en sus planes de estudios. Por esto se quiso indagar sobre el uso que le dan docentes de matemática a la resolución de problemas en la enseñanza del cálculo diferencial e integral en una variable. Para esto se aplicó un cuestionario a docentes con experiencia en la enseñanza del cálculo diferencial e integral en una variable de la Universidad de Costa Rica, la Universidad Nacional, el Instituto Tecnológico de Costa Rica y la Universidad Estatal a Distancia. Los resultados revelan contradicciones entre las concepciones de docentes sobre lo que es un problema matemático y su implementación en el aula.

Palabras claves: Enseñanza del cálculo diferencial e integral en una variable; resolución de problemas; matemática superior; educación matemática.

Resumo

As linhas de pesquisa na resolução de problemas no ensino da matemática são muitas e com semelhanças e discrepâncias entre si. Particularmente, o que se entende por resolução de problemas apresenta diferenças entre a teoria e a prática. No ensino da matemática superior, o cálculo diferencial e integral em uma variável é uma área rica para o uso da resolução de problemas por seus conteúdos e pela quantidade de carreiras universitárias que a incluem em seus planos de estudos. Por isso, quis-se indagar sobre o uso que os docentes de matemática dão à resolução de problemas no ensino do cálculo diferencial e integral em uma variável. Para isso, foi realizado um questionário com os docentes experientes no ensino do cálculo diferencial e integral em uma variável da Universidade da Costa Rica, da Universidade Nacional, do Instituto Tecnológico da Costa Rica e da Universidade Estatal a Distância. Os resultados revelam contradições entre os conceitos dos docentes sobre o que é um problema matemático e sua implementação na sala de aula.

Palavras-chaves: Ensino do cálculo diferencial e integral em uma variável; resolução de problemas; matemática superior; educação matemática.

Many theoretical and field studies have been carried out related to problem solving in teaching in different areas of mathematics. Many of these investigations have followed extensive initial work by Polya, Brousseau and Schoenfeld on problem solving (Santos-Trigo, 2007).

In a study by Törner, Schoenfeld and Reiss (2007), whose aim was to document the "state of the art" of problem solving in different countries, the authors found that the topics and definitions related to this theory varied strongly from one culture to another, and even within the same culture. Arcavi and Friedlander (2007) also found that there is no consensus among curriculum developers, teachers and mathematical researchers on what constitutes a problem or what teaching problem solving implies.

In the area of calculus and analysis, in particular, research on problem solving (Abarca, 2007; Cuevas and Pluvinage, 2009; Lois and Milevicich, 2008) have concentrated on the search for methodological strategies that allow each individual to go beyond learning algebraic algorithms to achieve a clear conceptualization of the processes underlying mathematical content. Investigations in this area address different types of content in differential and integral calculus, in different situations and contexts, using a variety of didactic

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and technological resources. For example, Lois and Milevicich (2008), in an electrical engineering course at the Universidad Tecnológica Nacional de Argentina, incorporated technological and historical elements in teaching the integral. The authors designed a software package that allowed an approach to integral calculus from a geometric perspective, addressing historical processes in the development of this topic and establishing connections between the conceptualization of integration and its application in engineering. The activities designed by the authors promoted student experimentation, analysis, extrapolation, argumentation, the exchange of ideas, commitment, and discussion.

On the other hand, in Mexico, Cuevas and Pluvinage (2009) suggested a first course in calculus based on the historical development of the subject, starting with the study of real functions such as polynomials, fractional, radicals, discontinuous and transcendental functions. They state that this structure allowed them to introduce the concepts of calculus in a simple and gradual manner, and facilitated the modeling of real situations without unnecessary algebraic and numerical complexity. The authors designed different activities that simulated real situations in which it was possible to model a certain function. The Calc Visual software (Cuevas and Mejía, 2003) was a great help in simulating situations presented and for a detailed study of these situations.

In Costa Rica, the Higher Education Council approved in 2012 the new mathematics for secondary education curriculum, in which problem solving plays a fundamental role in the construction of mathematical knowledge. In particular, reference is made to problem formulation and resolution in real contexts that allow identification, use and construction of mathematical models (Morales-López, 2017). To accomplish this, the Costa Rican Ministry of Public Education has provided training for secondary mathematics teachers to promote the use of problem solving as a teaching and learning strategy. Likewise, problem solving is promoted at a higher level as a means for constructing concepts in most of the programs of single real variable differential and integral calculus courses in Costa Rican public universities. This approach is present in the theoretical foundations of curricula for training mathematics teachers.

With increasingly clear national and international trends toward teaching focused on problem solving and existing resources, it would be obvious to think of mathematics classes with methodologies other than the traditional one, whose classroom activities are directed at experimentation, exploration, argumentation, thinking and discussion of mathematical concepts and procedures.

Differential and integral calculus are among the first courses taken by the majority of students in Costa Rican higher education. At the Universidad de Costa Rica, for example, the curricula of at least 25 programs of study include courses in differential and integral calculus, as well as 10 programs of the Universidad Nacional de Costa Rica. This, together with the nature of the contents in this area, make these courses excellent candidates for the use of problem solving as a methodological strategy for the development of these subjects.

There is evidence that, in order to understand problem solving in theory and practice, it is first necessary to understand and define the elements that characterize it (Santos, 2008). That is, it must be clear what a mathematical problem is, what it means to do mathematics, what is understood as problem solving, and what conditions are necessary

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to achieve the objectives of problem solving in mathematics teaching.

Based on these considerations, the need arose to investigate the use of problem solving in single variable differential and integral calculus by mathematics teaching staff; in particular, to understand teachers' perspectives on what a mathematical problem is, the objectives and characteristics of a mathematical problem, and the use of problem solving in teaching single variable differential and integral calculus. A questionnaire was therefore applied to mathematics teachers with experience in teaching single variable differential and integral calculus in four of the five Costa Rican state universities: the Universidad Nacional de Costa Rica (UNA), the Universidad de Costa Rica (UCR), the Instituto Tecnológico de Costa Rica (ITCR) and the Universidad Estatal a Distancia (UNED).

It is hoped that the results presented here will be useful to guide actions that respond to the needs detected in this population.

Theoretical framework

Problem solving has served as an umbrella under which investigations with various orientations have been carried out (Schoenfeld, 1992), ranging from the development of activities that allow cognitive and mathematical skills development, to the use of problem solving for explaining the development of students' mathematical thinking (Santos, 2008). The problem is that most theories emerging from this research do not broadly explain the processes that involve the development of basic concepts or skills, or how they relate to problem solving. The different theories are rather for the most part "recycled" without a proper evaluation of their limitations or scope (English, Lesh and Fennewald, 2008).

Consensus does not exist either in the case of implementation and curriculum design for problem solving. It is not yet clear what the fundamental contents are, or how they should be structured in terms of problem solving activities, and still less clear how to make this theory visible in educational proposals (Santos, 2008). The author states: "the recognition that there may be several ways to organize a proposal for a curriculum that promotes problem solving implies the need to clearly explain how the principles of this perspective are distinguished in the organization and structure of content" (Santos, 2008, p. 18). The challenge thus consists in generating conditions for students, so that they see in these opportunities for investigation several ways to solve problems and to extend or formulate other problems.

It is therefore necessary to understand and define the fundamental components that distinguish this theory – that is, it must be clear what a mathematical problem is; what it means to do mathematics, what is meant by problem solving, and what conditions are necessary to achieve the objectives of problem solving in teaching mathematics.

What is a mathematical problem and what does it mean to do mathematics?

There are varied and sometimes contradictory definitions of what a mathematical problem is.

For a long time a mathematical problem was thought of as a confusing and difficult situation that required mathematical procedures and calculations in order to solve, characterized by having only one solution (Schoenfeld, 1992).

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In many research efforts, this definition has evolved into a more dynamic one that involves different cognitive processes, contexts, and skills. The approach of Mason (2016) is particularly interesting in considering that a situation may or may not represent a mathematical problem for the student, depending on when and how it is used in the process of teaching and learning.

A mathematical problem is understood as a situation that allows students to learn a new concept (Brousseau, 1986) through tasks which are "significantly" different from the routine, in which the challenge is to decide how to tackle the problem and what kinds of knowledge or mathematical resource to use (Burkhardt and Bell, 2007; Santos-Trigo, 2007). The situation should allow him or her to think mathematically through the processes of discussion, negotiation, and speculation about possible examples and counterexamples that help to confirm or reject an idea (Santos-Trigo, 2007; Schoenfeld, 1985).

When using this conception of a mathematical problem, what is meant by "doing mathematics" takes another course. The National Research Council (1999) defines the process of "doing mathematics" as an activity that "involves observing patterns, proving conjectures, and estimating results" (p. 31). These patterns can be numerical, and can involve reasoning and communication, patterns of movement and change between figures or geometric shapes, patterns of symmetry and regularity, and patterns of positions (Devlin, 1996). They can also be real or imaginary, visual or mental, dynamic or static, qualitative or quantitative, and of utilitarian or recreational interest. These patterns can also refer to the world that surrounds us or be a pure reflection of the individual's mind (Santos, 2008).

What is meant by problem solving?

There are also different and incoherent definitions of what a mathematical problem is, what it means, and what problem solving is used for in teaching and learning in mathematics (Santos-Trigo, 2007).

Although the topic is somewhat ignored in the conversations of mathematics teachers, the solution of lists of exercises already discussed in class is still a reality for students, requiring the use of algorithms and explanations provided by their teacher.

However, given the new understanding of what a mathematical problem is, current trends indicate that solving a problem must involve other, more demanding mental activities, oriented towards student participation in the search for solutions (Benítez and Benítez, 2013).

Stanic and Kilpatrick (1989) recognize three purposes of solving problems in mathematics teaching: (a) as a context to stimulate and motivate students and achieve concept learning; (b) as a skill in itself that must be taught in the curriculum; or (c) as a means to "do mathematics."

Puig (1998) characterizes the solution process as "the mental and manifest activity that the solver carries out from the moment in which, confronted with a problem, decides that he or she is facing a problem and wants to solve it, working until the task is finished" (p. 31).

Lesh and Zawojewski (2007) define problem solving as "the process of interpreting a situation mathematically, which involves several interactive cycles of expressing, testing, and revising interpretations, and ordering, integrating, modifying, revising, or redefining groups of mathematical concepts from various areas within and beyond mathematics" (p. 782, quoted by Santos,

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2008). This suggests cycles of reflection, definition and review of ideas and concepts by students. For these authors, the final goal of solving a problem is not the answer, but "to identify and contrast different ways of representing, exploring and solving the problem" (Santos, 2008, p. 4).

On the other hand, Arcavi (2000) states that problem solving is the activity that "leads to the development or construction of inquisitive thinking, where mathematical knowledge is conceptualized in terms of dilemmas or questions that demand the use and ways of thinking that are consistent with the endeavors of the subject area" (p. 19). Thus, problem solving should allow students to ask questions, identify conjectures or relationships, and support and communicate results. These activities should be developed within a learning community where individual and collaborative work is valued.

What conditions are necessary to achieve the objectives of problem solving?

From the previous discussion, it follows that for a problem solving process seen as a skill in itself, it is necessary, among other things, to have knowledge of mathematical content and problem solving strategies, a clear plan of self-monitoring, and a proactive approach to formulating and solving problems.

In this regard, the National Council of Teachers of Mathematics (2000) states:

> Teaching problem solving requires even more from teachers, since they must be able to promote such knowledge and attitudes in their students. ... Teaching itself is a problem-solving activity (p. 341).

According to Schoenfeld (quoted by Abarca, 2007), for students to learn to do mathematics, it is necessary to have an environment or situation in which those ways of thinking are promoted. According to the author, each situation must simulate a real task in which knowledge and mathematical skills are essential, but not the only things needed, in the search for solutions to the problem. The author proposes four dimensions that influence the problem solving process: (a) students must have mastery of previous and current mathematical knowledge; (b) cognitive strategies, including decomposition of a problem into simpler cases, reversing a problem, and diagramming; (c) meta-cognitive strategies, related to the monitoring of the processes used to solve a problem; and (d) a belief system, including student ideas about mathematics and how to solve problems, their fears and concerns.

The problems raised must provide students with a sense of the discipline – of its field of action, of its power, and of its history – and should generate a sense of what mathematics is and how mathematics is done, at a level appropriate to their understanding and experience.

Lesh, Doerr, Cramer, Post and Zawojewski (2003), on the other hand, define five fundamental principles for the formulation of mathematical problems: (a) the principle of personal meaning – the situation presented must represent a real, familiar situation for students, to encourage them to work on it; (b) the principle of model construction – the task must present students with the need to construct, refine, extend or modify a model; (c) the principle of self-assessment – the situation should allow students to judge their answers and procedures for themselves; (d) the principle of externalization of the model – the situation should generate

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in students the need to document their ideas and progress; and (e) the principle of a simple prototype – the situation must be simple but must generate the need for a significant mathematical model whose solution serves as a prototype for other situations.

A fundamental element to consider in problem solving is the incorporation and use of technological resources. Much has been said about technological literacy and the need for teachers of scientific disciplines to provide technological content, with such content not presented in technology-specific courses but rather in a cross-cutting and integrated fashion (Lois and Milevicich, 2008). However, few concrete results have been seen as a result of this methodological change in the teaching of mathematics (Lois and Milevicich, 2008). The Inter-American Development Bank (2006) attributes this situation to insufficient institutional plans to support such reform, high prices for access to Information and Communications Technologies (ICTs), and lack of digital education to interact with ICTs. Santos-Trigo (2016) also mentions the importance of considering the forms of reasoning that will be stimulated in students through the use of technology in the processes of learning mathematical concepts through problem solving.

The rapid development of technology, and of the internet and online resources, changes the nature of the tools available and their use in doing and learning mathematics. It is consequently necessary to rethink the mathematical situations (Trouche, 2009). “Mathematical laboratories” as a resource for the investigation and creation of class networks allow articulation, research and construction in the practice of mathematics. The use of visualization, simulation, calculation and representational tools opens the possibility of experimentation, dialogue, and debate, and changes the nature of mathematics teaching (Trouche, 2009).

In problem solving, the use of computational tools can not only facilitate implementation of strategies, but also enhance or extend the repertoire of heuristics (Santos-Trigo, 2007). In this context, use of technology directly influences the conceptualization and way of interacting with problems and as a consequence, contributes to the development of a theory that explains student competences. Moreno-Armella and Santos-Trigo (2008) state that the use of digital tools has allowed the introduction and consideration of new mathematical cognitive aspects in the development of students’ competences and, as a result, offer the potential for rethinking and structuring new research agendas.

Three aspects can be defined in the use of ICTs in higher education: (1) as a means of teaching, (2) as a means of learning, and (3) as a means of research and resolution of real problems (Hernández, Delgado, Fernández, Valverde and Rodríguez, 2001). Means of learning supported by use of specific software enrich the process of exploration, algorithm development and logical skill formation, and are more flexible than traditional media (whiteboards, video equipment, etc.). As a means of researching and solving problems, they facilitate the combination of academic and research components, the concepts to be learned and ways to carry this out (Lois and Milevicich, 2008).

The use of technologies as mediation strategies allows teachers to play the roles of mentor, motivator and stimulator of learning, assisting students in reasoning and searching (Lois and Milevicich, 2008). In turn, the relationship between teacher and student is strengthened, bringing it closer to “co-researching” and “co-learning” (Valverde and Garrido, 1999).

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Calculus and calculus teaching

The different investigations dealing with teaching and learning calculus show the existence of a "traditional model for calculus teaching ", in which contents are structured as a sequence of logically organized definitions, theorems and demonstrations, in such a way that previous concepts and procedures are necessary to understand those that follow (Salinas and Alanís, 2009). Even the most popular calculus textbooks show this kind of structure.

In this model of teaching it becomes necessary to teach the subject of limits before teaching derivatives, functions before limits and so on, while the chapters about applications are presented at the end of each topic to apply or replicate what has been learned (Salinas and Alanís, 2009).

Consequently, there is a lack of understanding and high levels of failure of courses, as well as negative attitudes of students towards learning mathematics (Salinas and Alanís, 2009). Students can, at best, calculate derivatives and integrate, but cannot understand how to use these concepts for solving problems (Cantoral and Mirón, 2000).

Efforts for "reform" in the teaching of calculus have succeeded in stimulating discussion of aspects related to what to teach (content) versus how to teach (context) (Steen, 2003).

With regard to what to teach, the discussion compares approaches that favor algorithms and procedures (techniques) with those that favor formal and theoretical approaches (theory). The first approach assumes that teaching mathematics is teaching algorithmic techniques to solve problems. On the other hand, the theoretical approach proposes that teaching mathematics is accomplished through presenting crystallized theories, and assumes that the way in which the theory is presented is how mathematics is learned (Gascón, 2001).

However, each approach separately does not seem to be providing the results expected (Artigue, 2001). Gascon (2001) states that there are great difficulties for the student in using a theorem properly, applying a technique or understanding if an object meets a definition. Likewise, in the technique-oriented approach students "tend to forget the 'real' problems, those whose main difficulty is to choose the right techniques to construct a 'solution strategy'" (Gascón, 2001, p. 136).

According to Cantoral, Cordero, Farfán and Imaz (1990), theoretical mathematical discourse is the less effective way for communicating the ideas of calculus, but they also explain that they are not in favor of teaching techniques as a way to ease the acquisition of knowledge or the use of routines.

Different educational approaches have directed research to the area of how to teach. The historical-epistemological approach, for example, emphasizes the role of mathematics as a human activity that is related to the need to solve real problems. In this approach, the teaching of calculus using history as a didactic tool takes the lead in giving meaning to the learning of students.

The technological approach to education, including the use of software, has also made contributions to the teaching of calculus. However, its use has been limited compared to its potential and, in some cases, it is reduced to showing the same, but in another way (Salinas and Alanís, 2009).

Likewise, problem solving, collaborative learning, and the instrumental utility of mathematical knowledge have been used in efforts to change the teaching of calculus.

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However, Salinas and Alanís (2009) state that although most of these efforts facilitate the introduction of subjects and try to involve students in the learning process, they end up predisposing them about the mathematical content in its formal version.

Methodology

The research carried out for this study is qualitative, and seeks to determine the perspectives of university mathematics teachers on the use of problem solving as a methodological strategy in teaching single variable differential and integral calculus.

The aim is to give an overview of the teaching perspective with experience in teaching calculus on the use of problem solving in the development of their lessons.

Data collection techniques and instruments were designed based on the qualitative approach, so that data collected allowed the description and explanation of the objectives of this work.

Participants

A total of 58 teachers from the Schools or Departments of Mathematics of the UCR, UNA, ITCR and UNED participated in the investigation. These public universities are the oldest institutions of higher education in the country, with a great deal of experience in training professionals in various careers.

For the selection of teachers, the authorities of each school were consulted, who provided lists of names of teachers that could, in their opinion, make significant contributions to this investigation. Among the selection criteria used were: at least one year of experience in teaching single variable differential and integral calculus, satisfactory teacher evaluations by students, and experience in coordinating teaching of differential and integral calculus.

At the time of the interviews, the participating teaching staff had taught courses in single variable differential and integral calculus in the mathematics or other programs of one of the previously mentioned universities. On average, they had 11 years of experience teaching mathematics at university level, and had bachelors, masters or doctoral degrees in areas such as mathematics teaching, educational mathematics, pure mathematics, educational computing, statistics, actuarial sciences, and didactics of mathematics.

Information collection techniques and tools

A questionnaire was administered to the participants in order to determine their perspective on problem solving in teaching single variable differential and integral calculus in higher education; it was applied in the second half of 2015.

The questionnaire included open, closed, ordination and Likert scale questions. The instrument was validated with experts and with a small sample of mathematics teachers with experience in teaching differential and integral calculus that were not included in the main sample, who provided suggestions and observations that helped to refine the initial version of the instrument.

The questionnaire was applied digitally using the Universidad Nacional platform, and LimeSurvey software licensed to the university.

Criteria for information analysis

For information analysis, two categories were created: (a) Perspectives of mathematics teachers on what a mathematical problem is, their objectives in teaching single variable differential and integral calculus and its characteristics; (b) Perspectives of mathematics teachers on the use of problem solving in teaching single variable differential and integral calculus.

In the closed questions of the questionnaire, the LimeSurvey software provided relative and absolute frequencies of results. For open-ended questions, a list was compiled of all answers provided by the participating subjects, and each response was considered.

In the ordination questions, a statement was given to the teacher followed by five propositions which each participant was asked to order from highest to lowest as they moved away or closer to what he or she thought, specifying a value of 1 for the proposition that was farthest away from what they thought, and 5 for the propositions that were closest to what they thought. For the analysis of these questions, the total of responses of 4 and 5 was calculated (Total A, responses which were closest to what they thought), and the total of responses of 1 and 2 (Total B, responses which were farthest away from what they thought). Total A was then subtracted from Total B (the "Difference"). Finally, propositions were ordered from highest to lowest according to the Difference values, with the highest difference values indicating the propositions, which were closest to the opinions of the teachers. Responses of "3" were not used for ordination purposes.

In the questions that used a Likert scale, a number of propositions were presented to the interviewees, who were asked to indicate their degree of agreement with them on a scale of 1 to 5. For the analysis of these questions, the total of responses of 3, 4 and 5 (Total a favor) was calculated, as was the total of responses of 1 and 2 (Total against). These totals (for and against) were used to accept or reject a proposition in the analysis.

Analysis of results

Regarding the definition of a mathematical problem, 42 participating mathematics teachers considered "*A situation that provides students with the possibility of discussions and discoveries related to some subject*" to be the closest to their conception of the term, while 14 teachers did not agree with this definition.

To this definition they add the following: "*A situation that motivates students to learn new concepts or procedures*" and "a *context in which students can apply a concept or a mathematical procedure to a real situation*."

Nineteen teachers considered that a mathematical problem is: "*A situation that allows students to demonstrate if they have learned a concept or a procedure*". However, 36 disagreed with this definition, indicating that this definition does not accurately represent what a mathematical problem is. These teachers also disagreed with the following definition: "*A situation that allows students to develop new skills.*"

When considering the objectives of a mathematical problem in a course of single variable differential and integral calculus, 46 teachers stated that problems should be used mainly for developing skills that allow students to make conjectures and arrive at mathematical results, as well as promoting student interest and motivation in the topics covered. Only four participants did not consider this to be true. They also mentioned the fact that problems can be solved in more than one way as a topic that can be used to promote discussion among students, and the

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development of skills in the demonstration of results using the theory studied.

When considering the characteristics that a mathematical problem must have, the teaching staff emphasized three components: the context or situation raised in a mathematical problem, the processes involved in searching for a solution to the problem, and the solution of the problem itself. With respect to each of these components, they state:

- The context of the problem: it must be a situation in which students cannot find the answer immediately and must make a cognitive effort to construct new knowledge. It should make sense for the students and within the discipline of study; it must represent a challenge for students, that allows them to create relationships between concepts and procedures; and it should be different from the list of exercises used to apply concepts or algorithms.
- The processes involved in problem solving: they should favor decision making, peer discussion, development of mathematical skills, concept application, nonlinear reasoning and data interpretation. They should also allow the application of several areas of knowledge, not just knowledge of the area under study. They must require prior knowledge and research by the students, and motivate them to evaluate what they have studied.
- The solution to the problem: the problem must have a solution and have the possibility of being solved in different ways.

The teachers interviewed state that they make use of problem solving in their classes on single variable differential and integral calculus classes, and emphasize three ways in which they do so: (a) 58 teachers stated that they use it at the end of the development of a mathematical concept for the application of theory, (b) 57 teachers use it before developing a mathematical concept as a motivation technique, and (c) 40 teachers used it to construct or illustrate some mathematical concept.

In the first case, teachers mentioned using optimization problems at the end of the topic of derivatives to apply the theory learned, or problems concerning solids of revolution for the application of integrals. In the second case, the calculation of areas under curves was also cited as an example of a type of problem that allowed them to introduce the topic of integrals. However, no teacher provided a problem or specific example of the third type of use of problem solving.

Fourteen teachers say they do not make use of problem solving in their courses.

Six teachers mentioned other uses of problem solving in their lessons, but did not provide details or examples of how they did so. For example, they claimed to use problem solving as a way of introducing the history of mathematics into dealing with a particular concept, or providing an open problem that allows students to expand their knowledge of the procedures and algorithms developed in the classroom.

Conclusions

In summary, with regard to perspectives on what a mathematical problem is, the group of teachers participating in this study appear to regard problem solving more as a way of doing mathematics than as a means to evaluate acquired knowledge. For the participants, problem solving in a course on

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single variable differential and integral calculus has as its fundamental objective the development of abilities to establish conjectures and arrive at mathematical results that develop student interest and motivation in the subjects covered, and promote the finding of different ways to solve problems and discussion among students. This coincides with the research carried out by Stanic and Kilpatrick (1989).

Although their views are in some ways close to the concept of problem solving as a way of doing mathematics, many of the characteristics of a mathematical problem they mention are opposed to this concept, showing that there is no consensus on what a mathematical problem should be.

The actual implementation of problem solving in courses on single variable differential and integral calculus in the classroom by the teachers participating in this study, is opposed to the concepts that they expressed in interviews. Most participants claim to use it at the end of the development of a mathematical concept to demonstrate the application of the theory, in agreement with the traditional teaching model mentioned by Salinas and Alanís (2009), while another group uses problem solving as a means of motivating students before developing a mathematical concept, and only a few of them claim to use it as a methodological tool to help in the construction of mathematical concepts.

While problem solving can be understood as a context and a means of motivation for the study of a mathematical concept, as a way of doing mathematics or as a way of acquiring various skills, the results of this research show that there is no consistency between the theoretical conceptualization of problem solving and its implementation.

These results are worrying if viewed in the light of the country's needs and the teaching approaches promoted by state universities.

It is necessary to create new research projects to generate coherence between existing conceptions about problem solving and their implementation in mathematics classes. It is important that the results of such research help to promote the use of problem solving in the construction of mathematical concepts in specific areas, and serve as a basis for the formulation of other proposals.

Spaces for the dissemination and review of these proposals for the training of teachers would also be useful, so that teachers can become familiar with, analyze and select activities on specific topics that allow them to teach from the problem solving approach.

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