Book review

E. N. Mahmudov "Approximation and Optimization of Discrete and Differential Inclusions" Elsevier, 2011, pp. 394

The book consists of six chapters divided into sections and subsections and contains many results that have not been published in the monographic literature. The primary goals of this book are to present the basic concepts and principles of mathematical programming in terms of set-valued analysis (Chapters 2 and 3) and on the basis of the method of approximation, to develop a comprehensive optimality theory of problems described by ordinary and partial differential inclusions (DFI) (Chapters 4,5, and 6).

Convex sets and convex functions are studied in the setting of n-dimensional Euclidean space. However, the reader familiar with functional analysis can generalize the main results to the case of infinite-dimensional functional spaces. In spite of the fact that the stated notions and results are known, they play a decisive role for obtaining the main results in the next chapters of the book. The key issues of convex analysis in finite-dimensional spaces have been addressed in the books *Convex Analysis* by Rockafellar and *Convex Analysis and Extremum Problems* by Pshenichnyi. The identifications of convex functions and their epigraphs make it easy to pass back and forth between the geometric and analytical approaches. It is shown that convex sets and functions form classes of objects preserved under numerous operations of combination; pointwise addition, pointwise supremum and infimal convolution of convex functions are convex.

The key tools for analysis are based on the extremal principle and its modifications together with the Locally Adjoint Mapping (LAM) calculus; see, e.g., Mordukhovich (*Variational Analysis and Generalised Differentiation* Vol.I and II, Springer, 2006) for more developments and discussions.

Moreover, for problems described by ordinary non-convex DFI under the especially formulated monotonicity and t_1- transversality conditions, sufficient conditions for optimality are proven.

Employing LAM and the discrete approximations method in Hamiltonian and Euler-Lagrange forms are driven necessary and sufficient optimality conditions for various boundary value (Dirichlet, Neumann, Cauchy) problems for first order, elliptic, parabolic and hyperbolic type of discrete and partial DFIs. One of the most characteristic features of such approaches with partial DFI is peculiar to the presence of equivalents to the LAM. Such problems have essential specific features in comparison with the ordinary differential model considered in the second part of the book. For every concrete problem with partial DFI are established rather interesting equivalence results that shed new light upon both qualitative and quantitative relationships between continuous and discrete approximation problems.

In this book duality results are formulated and the conditions under which primary and dual problems are connected by such duality relations are searched for. For duality constructions of convex problems we employ the duality theorems concerning infimal convolution and the addition of convex functions.

Since many problems in engineering are reducing to such problems, the book will be of interest to mathematicians and non-mathematician specialists whose study involves the use of ordinary and partial differential equations (inclusions) and approximation methods and its applications, as well as to undergraduate, graduate and post-graduate students at universities and technical colleges. In other words the book is intended for a broad audience – students of universities and colleges with comprehensive mathematical programs, for engineers, economists, and mathematicians interested in solutions of extremal problems.

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