***Forthcoming in Richard Wagner (ed.)*** *James M. Buchanan: A Theorist of Political Economy and Social Philosophy*. Palgrave Macmillan.

# *On the definition and nature of fiscal coercion*

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## Abstract

We introduce ideas about how coercion in public finance can be formally defined, building on recent work in the literature. Our discussion illustrates the connection between selected aspects of this research and earlier seminal work on coercion by Wicksell, Lindahl, and Buchanan and Tullock. We also attempt to contribute modestly towards a fuller understanding of the nature of coercion in a public finance setting. We use a Lindahl solution as the counterfactual social state relative to which coercion inherent in any situation is to be judged in order to evaluate and compare the nature of coercion imposed by a social planner and in an electoral equilibrium.

**Key words**: coercion, fiscal coercion; calculus of consent; individual-as-dictator; individual-in-society; individual-as-planner; correlations of income, tastes and political influence; Lindahl solution; electoral equilibrium; optimal taxation

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## 1. Introduction

In this chapter we introduce some ideas about coercion in public finance using recent work in the literature as a foundation, while briefly illustrating the connection between selected aspects of this research and earlier seminal work on coercion by Wicksell, Lindahl, and Buchanan and Tullock. We also attempt to contribute modestly towards a fuller understanding of the nature of coercion in a public finance setting.

Coercion in public finance arises from two essential sources: (i) external control of individuals and that of the country exercised through threats of violence and sanctions; and (ii) as a by-product of the compromises that citizens must agree to in a democratic society. In this chapter we focus on the second source or type of coercion, assuming that the fiscal systems we consider are compatible with a stable democratic society in which the state has a legitimate and constitutionally circumscribed monopoly on violence.[1] This assumption is of course a big one. Nonetheless, as we hope will become clear in what follows, important issues of definition and analysis still remain before a full understanding of the nature of coercion in modern fiscal systems can be achieved.

To fix ideas, it is useful to begin with an example that we have used in earlier work (Winer, Tridimas and Hettich 2014, hereafter WTH 2014). Consider a sizeable group of citizens who have come together in a room for a common purpose and who must collectively set the temperature on a thermostat and pay for the resulting use of energy. Inevitably in such a group, some people will be too hot and some too cold, and even those for whom the temperature is just right may be unhappy with the balance they face between what they pay and what they get.

[1] This and the next section make use of some ideas from Martinez-Vazquez and Winer, eds. (2014) and Winer Tridimas and Hettich (2014). For exploration of the connections between the two fundamental sources of coercion and the implications of this relationship for public finance from differing points of view, see Skaperdas (2014) and Wallis (2014).

Individuals can escape this situation if they move rooms or leave the building that represents the collectivity in the example. But if they stay, they must cope with the coercion implied by their assent to the collectively made decision. Coercion for any individual in this example - roughly speaking, the difference between what they get and what he or she *thinks they deserve* at the tax-price that they have to pay - cannot be avoided whatever practical collective choice process is used.

Fiscal coercion of this kind, which arises naturally in all liberal democratic societies, is one of the foundations of what is perhaps the most famous diagram in Buchanan and Tullock's *Calculus of Consent* (1962, fig 3, p.71). This diagram endogenizes the constitutional choice of a decision rule for the making of fiscal and other decisions as the outcome of minimization of the sum of two types of costs: expected external costs that fall with the proportion of citizens required for a decision to be taken; and expected decision-making costs that rise with this proportion. As they also argue, there is no obvious reason why the optimal, cost-minimizing solution should require a simple majority.

'External costs' in the *Calculus of Consent* are the equivalent of coercion in our stylized example, though they are not referred to as such in the book. Despite the centrality of coercion to the *Calculus*, an exact definition of coercion is not provided nor has it been in their subsequent work. More generally, while philosophers and legal experts have explored its nature at length, work on coercion in economics has lagged behind that in other disciplines even though a concern with it often lies beneath the surface, especially when taxation is involved.

The exception in economics is the literature on mechanism design, recently reviewed by Ledyard (2014), which is built on the early work of Wicksell (1896) and his student Lindahl (1919). The early work was aimed at establishing a fiscal system with public goods that is economically efficient while at the same time minimizing (Wicksell), or even eliminating (Lindahl) fiscal coercion. In the mechanism design literature, the objective is similar to that of Lindahl's, with participation constraints formally requiring that all equilibria or solutions involve the absence of coercion. Whether such a solution with public goods is possible, and how one may be achieved under alternative assumptions about what citizens know about each other's 'type' remains an active area of research.

In the next section we briefly summarize our understanding of how fiscal coercion may be formally defined and used in fiscal analysis when citizens are constrained to remain in the room, so to speak, based on our earlier work. That exit from the community is prevented (or prohibitively costly) is a second important underlying assumption of the present analysis. We then develop an alternative definition of coercion in section three that aims at insuring the aggregate compatibility of individual views about coercion when individual tastes for public goods and individual incomes are both heterogeneous as well as correlated, and we explore some of its implications for fiscal analysis.

## 2. The individual-in-society, the individual-as-dictator and imposition of coercion constraints in a social planning problem

A formal definition of fiscal coercion for an individual requires that a counterfactual be defined, so that an actual situation in which a taxpayer finds himself can be compared to one that the individual regards as non-coercive.[2] This counterfactual may be one in which the individual receives in public services what he or she thinks they deserve at the tax-price that must be paid, a formulation of the counterfactual implicitly used in the example stated earlier, or, analogously, one in which he or she pays what they think is appropriate for the public services actually

[2] 'Non-coercive' does not necessarily mean that same thing as 'voluntary'. For a deeper discussion of related issues in the definition of coercion, see Congleton (2014).

provided. The former approach is the one used by Breton (1996) and is implicit, we think, in work by Buchanan, for example in his *Demand and Supply of Public Goods* (1968, 145-146) in which he stresses the importance of the individual's recognition that he or she is part of a social situation. It is the approach used in WTH (2014). Adoption of a counterfactual in which the individual pays what he thinks is appropriate for the services actually received is suggested by the work of Lindahl, and is the counterfactual experiment embodied in the computable equilibrium study of fiscal coercion in the U.S. state of Georgia by Sehili and Martinez-Vazquez (2014).

Both of these approaches are part of what we have referred to as individual-in-society definitions. To formalize the approach in which the individual takes as given the socially determined tax rate $t_i$ , let $V_i^*$ be the maximized utility that a citizen enjoys under specified counterfactual conditions, and $V_i$ be the utility he or she actually enjoys from the operation of the public sector. In this individual-in-society approach to defining coercion, the individual determines the level of public good $G*$ that maximizes her utility subject to income $Y_i$ that may be a function of the tax rate. Coercion is then defined as the difference between the resulting counterfactual utility and the utility conferred by the actual fiscal system:

$$[V_i^*(G_i^*, Y_i, t_i) - V_i], \quad \textit{where } G_i^* = \underset{\{G\}}{argmax}\, V_i(G, Y_i, t_i) \tag{1}$$

A second approach begins with the assumption that appropriate treatment of an individual by the fiscal system is what that person would want if he or she was a dictator. This is the individual-as-dictator approach, first suggested by Usher.[3] Coercion is then calculated as the difference between utility with the 'dictator's' preferred outcome and the actual utility

[3] Personal communication from Dan Usher.

experienced in the world as it is. In a simple version of this approach, the individual–as–dictator with income

$Y_i$ determines a proportional tax rate *t* and the level of the public good *G* by maximizing utility subject to the government budget constraint $\sum_i^N tY_i = G$, where $N$ denotes the number of taxpayers. Coercion is calculated as:

$$[V_i^*(G^*, Y_i, t^*) - V_i], \ \ where\ G^* = \underset{\{G\}}{argmax}\ V_j(G, Y_i, \frac{\sum_i^N Y_i}{N}) \tag{2}$$

Hintermann and Rutherford (2017) use this individual-as-dictator definition in a computable general equilibrium model to analyse coercion in their study of environmental policy.[4]

An additional issue to be decided using either of the two approaches outlined is whether only citizens who lose relative to the counterfactual are to be considered coerced, or whether, as in WTH 2014, all citizens for whom the differentials above are non-zero are to be included in the measure of coercion.

*2.1 Coercion constrained optimal policy*

A society interested in liberty will set limits on the coercion that can be imposed on its individual members by the state. Studying the implication of such limits is therefore of interest, and doing so is easier if there are analytically tractable definitions of coercion like those illustrated above. This brings us to the question of whether to apply coercion constraints at the level of the individual, or at some aggregate level.

In accordance with Wicksell, who advocated approximate unanimity among groups as a way of minimizing coercion, a constraint involving individuals or groups may be specified as

$$V_i^* - V_i \ \leq K_i \ \ , \tag{3}$$

where the subscript refers to individuals or to specific social groups.

[4] It may be noted that a median voter is essentially a dictator imposing coercion on everyone else.

A more relaxed approach that allows for stronger policy judgments, and a greater degree of coercion in whatever allocation emerges, bears some similarity to the Kaldor–Hicks criterion for potential compensation (in contrast to the strict Pareto criterion). This involves the use of a constraint on the *sum* of individual utility differences, such as

$$\sum_i (V_i^* - V_i) \leq K \ . \qquad (4)$$

A social planning problem with a simple fiscal system and coercion constraints can be written as follows, where $F$ is the social objective:

$$\text{Max } F\big(V_1(Y_1, t, G), \dots, V_n(Y_n, t, G)\big) \qquad (5)$$

$$\textit{such that} \quad \sum_i^N tY_i = G \quad \textit{and} \quad \{V_i^* - V_i \leq K_i, \forall\, i \quad \textit{or} \quad \sum_i (V_i^* - V_i) \leq K\}.$$

It may be noted that when an individual-in-society definition of the coercion is used to define the counterfactual, the degree of coercion is endogenous in this problem since the planner must observe coercion constraints, which affects the choice of fiscal instruments and so the nature of coercion in the solution. On the other hand, if an individual-as-dictator approach is used, the counterfactual depends only on preferences, technology and endowments, and is therefore independent of the planner's objective.

It is also interesting to note, as Munger (2014) points out, that Coasian (1960) bargaining solves the problem with individual constraints, while economizing on the government's or the planner's need to know anything about individual preferences or about what levels of coercion are acceptable to the parties involved.[5]

Imposing coercion constraints on a social planning problem is one way of investigating the implications of limitations on coercion for the nature of optimal fiscal systems. This procedure is similar to imposing equity constraints in an inquiry about the kind of tax system that

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[5] The issues involved in determining the practicality of Coasian bargaining are well known and will not be enumerated here.

is best suited to achieving an equitable tax burden. Indeed, investigations of these kinds may be regarded as complements in the present context. For Wicksell knowingly avoided the equity problem in his seminal pursuit of a fiscal system that is simultaneously efficient and coercion-minimizing, by assuming at the outset that the problem of distribution had somehow been solved before the legislature acted. Whether the related problems of coercion and of equity in tax design should be tackled simultaneously or in some specific sequence is an open and longstanding question.

Any sort of constraint, whether directed at equity or coercion that is imposed on an optimizing planner will reduce social welfare (Kaplow, 2001). From a social planning point of view, this issue could be dealt with by folding coercion constraints and equity constraints into a social welfare function, leading then to an efficient or socially optimal degree of coercion and to an efficient degree of inequity. However, doing so may not be the best way to proceed if the concerns behind these constraints serve broad social objectives that are not clearly subsumed by the usual utilitarian approach to public finance.

## 3. An individual-as-planner definition of coercion and some of its implications.

A potential problem with both the individual–in–society and individual–as–dictator definitions is that in the counterfactual, each individual desires a different fiscal mix. With individuals having different incomes and tastes for the public good, the desired levels of the fiscal instruments are almost surely inconsistent with each other in the aggregate. This inconsistency suggests that an analysis based on such definitions of coercion contain within them an element of social instability.

For this reason, we introduce a third definition of coercion, the difference between individual utility in a Lindahl equilibrium and the actual utility conferred by the prevailing fiscal mix. This is an example of an individual-as-planner approach to the definition of coercion suggested by Boadway (2014). In a Lindahl solution, there is no coercion and all decisions are mutually consistent. In this section we specify a simple Lindahl-like equilibrium in which all citizens must contribute, and then use this as basis for defining coercion and comparing its nature in an optimal tax system and in an electoral equilibrium. In this investigation, individuals are heterogeneous; they differ in their (exogenously defined) incomes, tastes for a single pure public good, and in their degree of political influence.

We begin this comparative analysis with the specification of a simple fiscal system. Assume there is a society of $N$ citizen–taxpayers indexed by $i$. Each individual maximizes a Cobb–Douglas utility function defined over private consumption $C_i$ and a public good $G$, has an (exogenous) income $Y_i$ and pays a proportional income tax at rate $t$. Thus,

$$U_i = (1-\alpha_i)lnC_i + \alpha_i lnG \text{ , } where \;\; C_i = (1-t)Y_i \qquad (6)$$

Here the parameter $0 < \alpha_i < 1, \alpha_i \in [\alpha_m, \alpha_M],\; with \; 0 < \alpha_m, \alpha_M < 1,$ denotes the intensity of taste for the public good of each citizen-taxpayer, and has mean $\bar{\alpha} = \frac{\sum_i^N \alpha_i}{N}$.

Normalizing the unit price of the public good to unity, the budget constraint of the government is

$$\sum_i^N tY_i = G. \qquad (7)$$

We note that along with the stability of fiscal institutions and the costliness of exit from the community, the exogeneity of incomes is a third major assumption of our analysis.

### *3.1 A Lindahl-like solution*

If coercion is to be eliminated in the Lindahl solution, each person must face a personalized price for the public good, $\tau_i$, such that $\sum_i^N \tau_i = 1$ and such that each person optimizes their own welfare at that tax-price with exactly the same level of the public good provided to everyone.

Each individual maximizes their utility (6) subject to their Lindahl budget constraint $C_i + \tau_i G = Y_i$ , leading to the reduced form utility function

$$U_i = (1 - \alpha_i) ln(Y_i - \tau_i G) + \alpha_i lnG . \qquad (8)$$

Maximization of (7) with respect to $G$ gives $G = \frac{\alpha_i Y_i}{\tau_i}$, the size of the public good that $i$ prefers at the tax-price $\tau_i$ . Inverting the latter yields the condition that defines the maximum non-coercive tax-price (the demand price) at which every citizen is content with the same, utility maximizing level of the public good, that is, $\tau_i = \frac{\alpha_i Y_i}{G}$.

Now let the covariance between citizen income $Y_i$ and taste for the public good $\alpha_i$ be written as

$$\sigma_{Y\alpha}^2 = \frac{\sum_i^N \alpha_i Y_i}{N} - \frac{\sum_i^N \alpha_i}{N} \frac{\sum_i^N Y_i}{N},$$

where if the rich have less (more) intense tastes for the public good than the poor, $\sigma_{Y\alpha}^2 < (>)\ 0$. Then using the condition $\sum_i^N \tau_i = 1$ (tax-prices sum to 1) and the covariance formula, it can be seen that size of the public good in the Lindahl solution $G^L$ has the general form

$$G^L = N(\bar{\alpha}\bar{Y} + \sigma_{Y\alpha}^2). \qquad (9)$$

In view of (9), each individual pays a Lindahl tax of $\tau_i G^L = \alpha_i Y_i$.[6]

[6] We may use the latter to calculate an economy-wide average income tax rate as follows. Funding $G^L$ requires a tax revenue of $t \sum_i^N Y_i = G^L$ which implies that the notional tax rate is $t = \bar{\alpha} + \frac{\sigma_{Y\alpha}^2}{\bar{Y}}$. However, this is not the actual rate levied on taxpayers in a Lindahl solution. Each individual pays a personalized tax tailored to their preferences.

Substitution of (9) into (6) gives the indirect utility of individual $i$ in the Lindahl solution that we shall use in our comparison of coercion in the optimal tax and electoral equilibrium situations described below:

$$V_i^L = (1-\alpha_i)ln(1-\alpha_i)Y_i + \alpha_i lnN(\bar{\alpha}\bar{Y} + \sigma_{Y\alpha}^2). \qquad (10)$$

Before proceeding, it is of interest to derive the analogue to the formula in Buchanan (1964) that shows when the Lindahl tax share will rise, remain constant, or fall with income - that is, be progressive, proportional or regressive with respect to income.[7] Given $G^L = \frac{\alpha_i Y_i}{\tau_i}$, it can be seen that $\frac{dlnG^L}{dY_i} = \frac{d\alpha_i}{dY_i}\frac{1}{\alpha_i} + \frac{dY_i}{dY_i}\frac{1}{Y_i} - \frac{d\tau_i}{dY_i}\frac{1}{\tau_i}$ . Since in a Lindahl equilibrium $\frac{dG^L}{dY_i} = 0$, because everyone demands the same level of $G$, we can multiply through by $Y_i$ to put this in elasticity form. Thus we can write: $\frac{d\tau_i}{dY_i}\frac{Y_i}{\tau_i} = 1 + \frac{d\alpha_i}{dY_i}\frac{Y_i}{\alpha_i}$. In words, in the Lindahl solution, we have that the elasticity of the tax share with respect to income = 1 + the elasticity of $\alpha_i$ with respect to income. Thus if the latter elasticity is greater than 0, the Lindahl tax price schedule (if we can think of it as such) will be progressive in our model economy.

### *3.2 The optimal tax solution*

In the traditional social planner or optimal tax approach (OT), the government sets the proportional tax rate in (7) at a level that is completely unconstrained by the coercive character of its actions, maximizing a social welfare function that we assume is the unweighted sum of individual utilities:

$$S = \sum_i^N U_i \qquad (11)$$

[7] Buchanan (1964) pp. 229-230: "A more general statement of the necessary condition (for a Lindahl solution - our addition) is as follows: The income elasticity of demand for the public good divided by the price elasticity of demand must be equal to, and opposed in sign to, the income elasticity of the tax-price schedule. Full neutrality is present when this condition is met throughout the range of possible incomes."

Maximizing $S$ with respect to $G$ and using (7), we obtain the social welfare maximizing size of the public good, and the corresponding proportional tax rate:

$$G^{OT} = N\bar{Y}\bar{\alpha} \quad \text{and} \quad t^{OT} = \bar{\alpha} \,. \tag{12}$$

The indirect utility of citizen-voter $i$ in this optimal tax scheme then can be stated as

$$V_i^O = (1-\alpha_i)ln(1-\bar{\alpha})Y_i + \alpha_i lnN\bar{Y}\bar{\alpha} \,. \tag{13}$$

*3.3 An electoral equilibrium*

Before calculating and comparing coercion levels under social planning and in a political equilibrium, we must also solve for indirect utility in the electoral equilibrium. When the fiscal mix is decided by the outcome of competitive elections, policy outcomes reflect a balancing of the heterogeneous economic interests of citizens. This sort of balance can be modeled using a probabilistic spatial voting model (see Coughlin 1992, or Mueller 2003). In such a setting, electoral equilibrium can be replicated using a Representation Theorem of the sort described by Coughlin (1992), by Hettich and Winer (1999) and by others. This involves maximization of a synthetic political support function defined over individual indirect utilities, where the weights on each citizen's utility reflect their relative political influence in the electoral equilibrium.

We proceed assuming that such a representation theorem applies. Let $w_i$ denote the normalized relative political influence of citizen $i$ in the electoral equilibrium, so that $\sum_i^N w_i = 1$. Equilibrium values of $G$ and $t$ maximize the support function

$$S = \sum_i^N w_i U_i \,. \tag{14}$$

This support function looks like a social welfare function, but it is not. The weights do not reflect a normative view about the distribution of welfare, but rather are determined in the Nash electoral equilibrium. In this case, in the version of the theorem used here, the outcome also lies on the Pareto frontier, though not the one consistent with the OT solution in which each individual's welfare is weighted equally.

Let $\sigma^2_{w\alpha}$ denote the covariance between citizen influence $w_i$, and taste for the public good $\alpha_i$,

$$\sigma^2_{w\alpha} = \frac{\sum_i^N w_i\alpha_i}{N} - \frac{\sum_i^N w_i}{N}\frac{\sum_i^N \alpha_i}{N},$$

where if those with low (high) $\alpha_i$ have more political influence, then $\sigma^2_{w\alpha} < (>) 0$. Maximizing (14) and using $\sigma^2_{w\alpha}$ we obtain the equilibrium fiscal system with one pure public good and a proportional tax on income:

$$G^P = N\bar{Y}(\bar{\alpha} + N\sigma^2_{w\alpha}) \quad \text{and} \quad t^P = \bar{\alpha} + N\sigma^2_{w\alpha}. \qquad (15)$$

Substituting into the utility function (6) leads to the the indirect utility of voter-taxpayer $i$ in this electoral equilibrium,

$$V_i^P = (1-\alpha_i)ln(1-\bar{\alpha} - N\sigma^2_{w\alpha})Y_i + \alpha_i lnN\bar{Y}(\bar{\alpha} + N\sigma^2_{w\alpha}). \qquad (16)$$

## 4. Who is coerced, and when?

We now proceed with an analysis of coercion in OT and in the electoral equilibrium using the Lindahl solution as the standard of reference to define coercion in each case. We begin with the optimal tax solution.

### *4.1 Coercion under the OT social planner*

Comparing (10) and (13), we have

$$V_i^L - V_i^O = (1-\alpha_i)ln(1-\alpha_i) - (1-\alpha_i)ln(1-\bar{\alpha}) + \alpha_i ln\left(1 + \frac{\sigma^2_{Y\alpha}}{\bar{\alpha}\bar{Y}}\right) \qquad (17)$$

We may say that when $V_i^L > V_i^O$ , citizen $i$ is coerced by the social planner. On the other hand, when $V_i^L < V_i^O$ , citizen $i$ benefits from the coercion forced on the rest of the polity.

From (17) we see that the sign of the utility differential $V_i^L - V_i^O$ depends crucially on $\bar{\alpha}$ relative to $\alpha_i$, and the sign of $\sigma_{Y\alpha}^2$, the correlation between individual incomes and tastes for the public good. There are three cases to consider, namely, (i) richer citizens have a relatively lower taste for the public good; (ii) the opposite case, where richer citizens have a relatively higher taste for the public good; and (iii) the case in which income and preferences for the public good are independent.

**(i)** When richer citizens have a relatively lower taste for the public good, $\sigma_{Y\alpha}^2 < 0$, in which case $ln\left(1 + \frac{\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}}\right) < 0$. Since for small values of $\alpha_i$ we may use the approximation $ln(1 - \alpha_i) \approx -\alpha_i$, the difference in (18) yields the following quadratic equation

$$V_i^L - V_i^O = {\alpha_i}^2 - \left(1 - ln\left(1 + \frac{\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}}\right) - ln(1 - \bar{\alpha})\right)\alpha_i - ln(1 - \bar{\alpha}). \qquad (18)$$

Denoting $ln(1 - \bar{\alpha}) \equiv -k; k > 0$ and $ln\left(1 + \frac{\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}}\right) \equiv -\theta;\ \theta > 0$ , (18) can be rewritten as

$$V_i^L - V_i^O = {\alpha_i}^2 - (1 + \theta + k)\alpha_i + k. \qquad (18')$$

Solving the latter yields $\alpha_i = \frac{1}{2}\left[1 + \theta + k \pm \theta\sqrt{1 + \frac{2(1+k)}{\theta} + \frac{(1-k)^2}{\theta^2}}\right]$. Using the approximation $\sqrt{1 + px + qx^2} \approx 1 + \frac{p}{2}x + \frac{1}{2}\left(q - \frac{p^2}{4}\right)x^2$ we obtain the roots

$$\alpha_1 = \frac{ln(1-\bar{\alpha})}{ln\left(1+\frac{\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}}\right)} > 0 \quad \text{and} \quad \alpha_2 = 1 - ln\left(1 + \frac{\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}}\right) - ln(1 - \bar{\alpha}) - \frac{ln(1-\bar{\alpha})}{ln\left(1+\frac{\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}}\right)}$$

Of the above, only the sign of $\alpha_1$ is unambiguously positive, but at this level of generality we cannot tell whether it is larger or smaller than one. As for $\alpha_2$ , we note that neither its sign nor its size is unambiguous. We therefore list all possible combinations and the corresponding signs of $V_i^L - V_i^O$. Figure 1 below illustrates graphically what is involved in each case:

(a) $\alpha_1 < 1$ and $\alpha_1 < \alpha_2 < 1$ . Then

For $0 < \alpha_i < \alpha_1$ $\Rightarrow$ $V_i^L > V_i^O$

For $\alpha_1 < \alpha_i < \alpha_2$ $\Rightarrow$ $V_i^L < V_i^O$

For $\alpha_2 < \alpha_i < 1$ $\Rightarrow$ $V_i^L > V_i^O$

(b) $\alpha_1 < 1$ and $\alpha_1 < 1 < \alpha_2$ . Then

For $0 < \alpha_i < \alpha_1$ $\Rightarrow$ $V_i^L > V_i^O$

For $\alpha_1 < \alpha_i < 1$ $\Rightarrow$ $V_i^L < V_i^O$

(c) $\alpha_1 > 1$ and $\alpha_2 < 0$. Then

For $0 < \alpha_i < 1$ $\Rightarrow$ $V_i^L < V_i^O$

(d) $\alpha_1 > 1$ and $0 < \alpha_2 < 1$ . Then

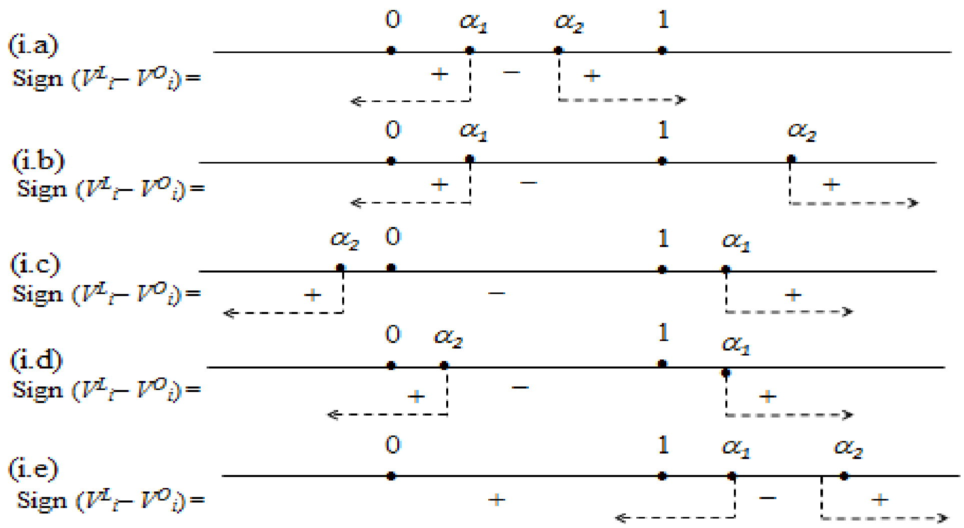
or $0 < \alpha_i < \alpha_2$ $\Rightarrow$ $V_i^L > V_i^O$

For $\alpha_2 < \alpha_i < 1$ $\Rightarrow$ $V_i^L < V_i^O$

(e) $\alpha_1 > 1$ and $\alpha_2 > 1$ . Then

For $0 < \alpha_i < 1$ $\Rightarrow$ $V_i^L > V_i^O$.

**Figure 1. Coercion when rich people have a lower taste for the public good, $\sigma^2_{\alpha Y}$<0**
Sign of the difference $V^L_i - V^O_i$ for different values of the taste for public goods $\alpha_i$.
*A positive sign indicates coercion in comparison to Lindahl equilibrium*




**(ii)** When rich people have a higher taste for the public good, we have $\sigma^2_{Y\alpha} > 0$ . In this case $ln\left(1+\frac{\sigma^2_{Y\alpha}}{\bar{\alpha}\bar{Y}}\right) > 0$. Since for small values of $\alpha_i$ we may use the approximation $ln(1-\alpha_i) \approx -\alpha_i$ , the difference in (18) yields the following second order polynomial

$$V_i^L - V_i^O = {\alpha_i}^2 - \left(1 - ln\left(1+\frac{\sigma^2_{Y\alpha}}{\bar{\alpha}\bar{Y}}\right) - ln(1-\bar{\alpha})\right)\alpha_i - ln(1-\bar{\alpha}). \qquad (19)$$

Working as above, the roots of the quadratic equation are

$$\alpha_1 = \frac{ln(1-\bar{\alpha})}{ln\left(1+\frac{\sigma^2_{Y\alpha}}{\bar{\alpha}\bar{Y}}\right)} < 0 \text{ and } \qquad \alpha_2 = 1 - ln\left(1+\frac{\sigma^2_{Y\alpha}}{\bar{\alpha}\bar{Y}}\right) - ln(1-\bar{\alpha}) - \frac{ln(1-\bar{\alpha})}{ln\left(1+\frac{\sigma^2_{Y\alpha}}{\bar{\alpha}\bar{Y}}\right)} .$$

The negative root $\alpha_1$ does not make economic sense. We then have

(a) $\alpha_1 < 0$ and $\alpha_2 < 0$. Then

For $0 < \alpha_i < 1$ $\Rightarrow$ $V_i^L > V_i^O$

(b) $\alpha_1 < 0$ and $0 < \alpha_2 < 1$ . Then

For $0 < \alpha_i < \alpha_2$ $\Rightarrow$ $V_i^L < V_i^O$

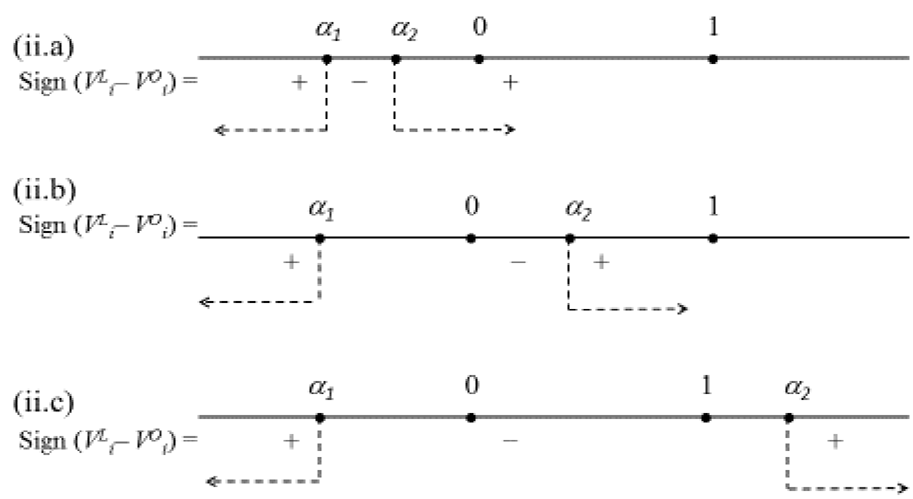
For $\alpha_2 < \alpha_i < 1$ $\Rightarrow$ $V_i^L > V_i^O$

(c) $\alpha_1 < 0$ and $0 < 1 < \alpha_2$ . Then

For $0 < \alpha_i < 1$ $\Rightarrow$ $V_i^L < V_i^O$.

Figure 2 illustrates these cases.

**Figure 2. Coercion when rich people have a higher taste for the public good, $\sigma^2_{\alpha Y}>0$**
Sign of the difference $V^L_i - V^O_i$ for different values of the taste for public goods $\alpha_i$.
*A positive sign indicates coercion in comparison to Lindahl equilibrium*



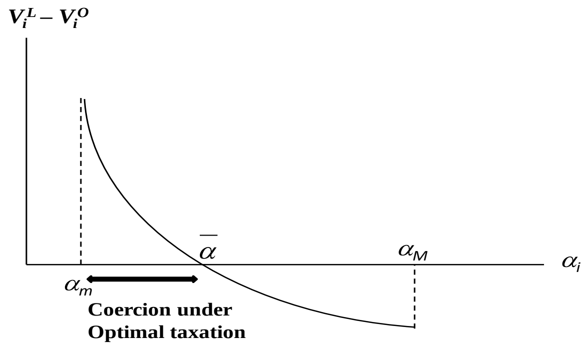

**(iii)** In the case where income and preferences for the public good are independent of each other, $\sigma^2_{Y\alpha} = 0$ , so that (17) yields

$$V_i^L - V_i^O = (1-\alpha_i)\big(ln(1-\alpha_i) - ln(1-\bar{\alpha})\big) > (<)\ 0\ for\ \alpha_i < (>)\ \bar{\alpha}\ . \qquad (20)$$

That is, taxpayers with a public good taste smaller than the mean $\bar{\alpha}$ lose in the counterfactual relative to OT.

Figure 3 shows graphically what is involved when $\sigma^2_{Y\alpha} = 0$ and we count as coerced only those who lose relative to the Lindahl counterfactual. The difference between utility under the counterfactual, Lindahl-like solution and the optimal tax one is drawn against the intensity of taste for the public good from lower to higher. Coercion is highest when the taste for $G$ takes its lowest value $\alpha_m$, and declines thereafter up to a threshold value $\bar{\alpha}$. For individuals with $\alpha_i > \bar{\alpha}$, utility under the OT planner rises as their taste for the public good increases.

**Figure 3. Coercion in OT when income and taste for the public good are independent of each other ($\sigma^2_{\alpha Y}$ = 0).** Coercion = $V^L_i - V^O_i > 0$.




*4.2 Coercion in an electoral equilibrium*

Working as before, we obtain

$$V_i^L - V_i^P = (1-\alpha_i)ln(1-\alpha_i) - (1-\alpha_i)ln(1-\bar{\alpha} - N\sigma^2_{w\alpha}) + \alpha_i ln\left(\frac{\bar{\alpha}\bar{Y}+\sigma^2_{Y\alpha}}{\bar{\alpha}\bar{Y}+N\bar{Y}\sigma^2_{w\alpha}}\right). \quad (21)$$

As with equation (17), the sign of (21) is ambiguous and depends on $\bar{\alpha}$ relative to $\alpha_i$, the sign of $\sigma^2_{Y\alpha}$, the correlation between individual incomes and tastes for the public good, as well as the correlation between political influence and taste for the public good, $\sigma^2_{w\alpha}$. With each one of $\sigma^2_{Y\alpha}$ and $\sigma^2_{w\alpha}$ taking positive, zero and negative values, we have a total combination of nine possible constellations, each one leading to a number of sub-cases. So to go forward, we simplify further.

If it is plausible that the rich have lower intensity of preferences for the public good, we have $\sigma^2_{Y\alpha} < 0$ . If it is further assumed that those with a high taste for *G* are also politically more influential - that is, that the poor have greater influence than the rich, then $\sigma_{w\alpha}^2 > 0$ . We then have $ln(1-\bar{\alpha}-N\sigma_{w\alpha}^2) < 0$ and $ln\left(\frac{\bar{\alpha}\bar{Y}+\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}+N\bar{Y}\sigma_{w\alpha}^2}\right) < 0$. Using the latter, the expression in (21) yields a quadratic equation similar to (18′) and a similar range of solutions.

On the other hand, if the rich have lower intensity of preferences for the public good and they are also politically more influential or, equivalently, those with low taste for the public good are more influential, so that $\sigma_{w\alpha}^2 < 0$ , expression (21) yields the quadratic equation

$$V_i^L - V_i^P = {\alpha_i}^2 - \left(1 - ln(1-\bar{\alpha}-N\sigma_{w\alpha}^2) - ln\left(\frac{\bar{\alpha}\bar{Y}+\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}+N\bar{Y}\sigma_{w\alpha}^2}\right)\right)\alpha_i - ln(1-\bar{\alpha}-N\sigma_{w\alpha}^2) \quad (22)$$

The roots of (22) are

$$\alpha_1 = \frac{ln\left(\frac{\bar{\alpha}\bar{Y}+\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}+N\bar{Y}\sigma_{w\alpha}^2}\right)}{ln(1-\bar{\alpha}-N\sigma_{w\alpha}^2)} - ln(1-\bar{\alpha}-N\sigma_{w\alpha}^2) - ln\left(\frac{\bar{\alpha}\bar{Y}+\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}+N\bar{Y}\sigma_{w\alpha}^2}\right) \text{ and}$$

$$\alpha_2 = 1 - \frac{ln\left(\frac{\bar{\alpha}\bar{Y}+\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}+N\bar{Y}\sigma_{w\alpha}^2}\right)}{ln(1-\bar{\alpha}-N\sigma_{w\alpha}^2)} .$$

For concreteness we assume $\frac{ln\left(\frac{\bar{\alpha}\bar{Y}+\sigma_{Y\alpha}^2}{\bar{\alpha}\bar{Y}+N\bar{Y}\sigma_{w\alpha}^2}\right)}{ln(1-\bar{\alpha}-N\sigma_{w\alpha}^2)} > 0$ and $(1-\bar{\alpha}-N\sigma_{w\alpha}^2) > 0$ . A total of six cases are then possible as described below:

(i.a) $0 < \alpha_2 < 1$ and $\alpha_1 < 0$ . Then

- For $0 < \alpha_i < \alpha_2 \quad \Longrightarrow \quad V_i^L - V_i^P < 0$
- For $\alpha_2 < \alpha_i < 1 \quad \Longrightarrow \quad V_i^L - V_i^P > 0$

(i.b) $0 < \alpha_2 < 1$ and $0 < \alpha_1 < 1$ . Then

- For $\alpha_i < min[\alpha_1, \alpha_2] \quad \Longrightarrow \quad V_i^L - V_i^P > 0$
- For $\alpha_i \in [\alpha_1, \alpha_2] \quad \Longrightarrow \quad V_i^L - V_i^P < 0$
- For $\alpha_i > max[\alpha_1, \alpha_2] \quad \Longrightarrow \quad V_i^L - V_i^P > 0$

(i.c) $0 < \alpha_2 < 1$ and $\alpha_1 > 1$ . Then

- For $\alpha_i < \alpha_2 \quad \Longrightarrow \quad V_i^L - V_i^P > 0$
- For $\alpha_2 < \alpha_i < 1 \quad \Longrightarrow \quad V_i^L - V_i^P < 0$

(ii.a) $\alpha_2 < 0$ and $\alpha_1 < 0$ . Then

For $0 < \alpha_i < 1$ $\Rightarrow$ $V_i^L - V_i^P > 0$

(ii.b) $\alpha_2 < 0$ and $0 < \alpha_1 < 1$ . Then

For $0 < \alpha_i < \alpha_1$ $\Rightarrow$ $V_i^L - V_i^P < 0$

For $\alpha_1 < \alpha_i < 1$ $\Rightarrow$ $V_i^L - V_i^P > 0$

(ii.c) $\alpha_2 < 0$ and $\alpha_1 > 1$ . Then

For $0 < \alpha_i < 1$ $\Rightarrow$ $V_i^L - V_i^P < 0$ .

*4.3 A comparison of OT and electoral equilibrium*

Finally we compare coercion with the OT solution and in an electoral equilibrium by considering the welfare differentials (17) and (21). We might expect coercion under a social planner to always exceed that in the electoral equilibrium, because the social planner is allowed to coerce anyone to any extent, as a matter of social solidarity, as long as social welfare increases.

However, this generalization does not hold in our simple model. After substituting from (17) and (21) and manipulating, we see that if $\sigma_{w\alpha}^2 < (>)\ 0$ , when

$$\alpha_i > \frac{ln(1-\bar{\alpha}) - ln\left(1-\bar{\alpha}-N\sigma_{w\alpha}^2\right)}{ln(1-\bar{\alpha}) - ln\left(1-\bar{\alpha}-N\sigma_{w\alpha}^2\right) + ln\left(\bar{\alpha}+N\sigma_{w\alpha}^2\right) - ln\bar{\alpha}}$$

it is the case that $V_i^L - V_i^O < (>)\ V_i^L - V_i^P$ . So coercion for an individual under the social planner may be lower or higher than in our democracy. Of course $V_i^L - V_i^O = V_i^L - V_i^P$ when $\sigma_{w\alpha}^2 = \sigma_{Y\alpha}^2 = 0$.

## 4. Concluding remarks

Social interaction necessarily requires limits on individual choices. As soon as we are part of a group, various opinions must be heard and compromises must be made. Difficult questions will inevitably arise about how limits to individual actions are to be determined, how such limits or rights are to be defined, and how they will be enforced once agreement on their nature is achieved. Coercion of the individual by the group is an inevitable outcome of our struggle to deal with these issues.

Although coercion is therefore essential to, and plays a key role in the *Calculus of Consent*, it has not been well-defined or studied extensively in economics. A careful definition requires the use of a counterfactual, non-coercive social state against which the coercion inherent in any particular situation is to be judged. We have outlined three different approaches to the choice of a counterfactual in the fiscal context: the individual-as-dictator, in which the counterfactual is what the individual would want if they alone decided everything; the individual-in-society, in which the counterfactual is what the individual would like to pay (or, to have in public services) taking as given the socially determined level of public goods (the socially determined tax rate they must pay); and the individual-as-planner counterfactual, which we have tentatively explored in this chapter. In the individual-as-planner approach, in contrast to the other approaches, all counterfactual positions are explicitly required to be mutually consistent. The Lindahl solution serves as one obvious choice for such a counterfactual, and it is the one that we have employed in our preliminary investigation.

Our analysis of the individual-as-planner approach to coercion has led to somewhat complex results about the nature of fiscal coercion. In the OT solution, if we treat only those who lose relative to the counterfactual as being coerced, the extent of coercion depends entirely on the nature of an individual's taste for the public good relative to a critical threshold that depends on average tastes, average income and the correlation of tastes and income. The sign of the correlation of income and tastes determines how low or high taste citizens fare relative to the counterfactual. In the electoral equilibrium, there is also a critical level that can be compared to an individual's taste for public goods to determine the nature of coercion, but now (and not surprisingly) the threshold taste level depends on the correlation of income and political influence as well as the correlation of income and tastes.

Some statements can be made about the comparative nature of coercion in OT and in the electoral equilibrium; in particular, it is not the case that the (coercion-unconstrained) social planner will always impose more coercion than occurs in the electoral equilibrium. But simple general rules about what does happen do not seem possible even in the stripped down model we have explored. Perhaps others can find sensible assumptions that lead to more definite results.

The analysis we have conducted is subject to two fundamental assumptions: that the power of the state is suitably restrained; and that exit from the community is prohibitively expensive. In addition, we have assumed that income is determined independently of the fiscal system. A full analysis of coercion in public finance and, in this respect, of the calculus of consent, awaits a more complete analysis that relaxes these assumptions while deriving general propositions about coercion that are relevant to modern fiscal systems.

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