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ORIGINAL ARTICLE

Developing number sense with Fingu: a preschooler's embodied mathematics during interactions with a multi-touch digital game

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Abstract

Early number sense, including subitizing and composition, is a foundation for mathematics, and bodies, especially fingers, are integral to number sense. Multi-touch technology offers innovative opportunities for developing and studying number sense, especially using conceptually congruent gestures that match the mathematics. However, there have been few investigations of the development of early number sense, particularly in embodied forms. Therefore, this mixed-methods study explores a preschooler's development of early number sense during a month of interactions with the multi-touch digital mathematics game Fingu. Key findings related to the development of early number sense include relevance of configuration and quantity, relationships among gestures and quantities, and development of estimation and precision. This research adds new perspectives to our understandings of early number sense research and practice, calling for consideration of embodiment and conceptually congruent gestures.

Keywords Embodied cognition · Number sense · Subitizing · Composition · Digital games · Multi-touch technology

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Introduction

Extensive research in mathematics education, cognitive psychology, neuroscience, and other fields supports the importance of early number sense, including as a foundation for arithmetic and more advanced mathematics (e.g., Anobile et al. 2016; Butterworth, 2005; Sarama and Clements 2009). Use of multi-touch technology (e.g., iPads) with digital games is prevalent in educational settings, and emerging research indicates that this can sometimes lead to positive performance outcomes, including those related to early number sense (e.g., Broda et al. 2018; Holgersson et al. 2016). Gestures matching the content can support learning mathematics (Segal et al. 2014), and such gestures are involved in interactions with some multi-touch technology (Tucker 2018; Sinclair and de Freitas 2014). To date, relatively little early number sense research has featured children in classroom settings, involved interactions with multi-touch technology, or examined development over time. Therefore, this study explores a preschooler's development of early number sense during a month of interactions with a multi-touch digital mathematics game, adding new perspectives to our understandings of early number sense with implications for research and practice.

Conceptual framework

We situate our study within embodied cognition and in relation to research on recognition and representation of quantity, and learning mathematics while using multi-touch technology, including emerging research involving the early number sense multi-touch digital game Fingu.

Embodied cognition

This study primarily focuses on learning in action(s) while also accounting for results of the learning. Embodied cognition is an appropriate theoretical foundation for this research because it focuses on physical actions that occur as one engages in activity. In embodied cognition, cognitive functions (e.g., thinking) are related to sensorimotor functions (e.g., movement) (Radford 2014). In the context of learning mathematics, this means that physical engagement in mathematical practices is equivalent to mathematical thinking, so changes in these mathematical practices are mathematical learning (Nemirovsky et al. 2013). In other words, one is doing mathematics (e.g., see $2 + 3$: count out five fingers), and changes in these actions (e.g., see $2 + 3$ again: immediately raise five fingers) are learning. Embodiment varies, but conceptually congruent gestures, which involve actions that match the mathematics, have been shown to support mathematics learning (Segal et al. 2014). Two examples of conceptually congruent gestures are, to indicate the quantity **** (four): (a) sequentially raising one finger at a time until four fingers are raised, and (b) simultaneously raising four fingers. The former is an example of "finger counting"; the latter is "all-at-once" (Sinclair and de Freitas 2014). A non-conceptually congruent gesture would be to indicate the quantity **** (four) by pointing to the numeral "4". Each gesture involves representing **** (four), but the conceptually congruent gestures involve actions with fingers matching the quantity (i.e., a group of four fingers), whereas the non-conceptually congruent gesture relies on a symbol, rather than the action, to represent the quantity. Conceptually congruent gestures may be part of embodied mathematics in various contexts, including as one interacts with technology.

Quantity: recognition and representation

The foundation of number sense involves recognizing and representing quantity. This involves an array of related constructs, including subitizing and composition, and multiple possible representations. Subitizing is the process of immediately recognizing small quantities (e.g., Kaufman et al. 1949; Sarama and Clements 2009), with speed measured in milliseconds (e.g., Ester et al. 2012). Although subitizing can include an oral indication of quantity (e.g., see ***, say "three") (Sarama and Clements 2009), consistent with embodied cognition, we also consider an embodied indication of quantity appropriate (e.g., see ***, immediately raise three fingers). Furthermore, research has found that preschoolers' gestures are more accurate than their speech for quantities beyond their counting cardinality range (Gunderson et al. 2015), emphasizing the importance of attending to gesture.

Researchers disagree on the extent of subitizing, with many limiting subitizing to 3–4 objects before transitioning to counting (e.g., Ester et al. 2012; Logan and Zbrodoff 2003), "groupitizing" subitizable quantities into larger sets (Starkey and McCandliss 2014), or estimation (e.g., Anobile et al. 2016; Revkin et al. 2008). Others consider subitizing and counting as levels along a continuum (e.g., Piazza et al. 2002). The field of mathematics education generally recognizes two overarching categories of subitizing: conceptual and perceptual (e.g., Clements et al. 2019; Sarama and Clements 2009). Perceptual subitizing is immediate recognition of quantity without grouping, usually extending to four or five objects. Conceptual subitizing involves unconscious use of partitioning or grouping extending beyond 5, though grouping strategies may become apparent after recognition. Thus, conceptual subitizing involves (re)grouping multiple groups of perceptually subitized objects. This relates to composition and decomposition, which involve making a whole quantity from parts and breaking a whole quantity into parts (e.g., $3 + 4 = 7$; $7 = 3 + 4$ or $6 + 1$, etc.). Although quantity recognition begins at birth, most subitizing and composition development occurs from 5 to 8 years of age, influenced by experiences as well as innate quantity recognition (Clements and Sarama 2009). Most mathematics education research considers subitizing to be distinct from counting, as the latter involves sequential enumeration rather than immediately quantifying by groups (e.g., Sarama and Clements 2009).

Extensive research has examined subitizing speed and accuracy, including effects of configuration (i.e., object arrangement). Recognition speed and accuracy decrease as quantity increases (e.g., Logan and Zbrodoff 2003), and people recognize familiar, often "canonical" (i.e., dice) configurations more quickly and accurately than unfamiliar or non-canonical configurations of the same quantity (e.g., Dehaene and Cohen 1994; Krajcsi et al. 2013). In each case, effects are stronger for quantities greater than three. However, subitizing speed and accuracy can increase through repeated exposures as configurations become familiar (e.g., Broda et al. 2018; Lassaline and Logan 1993). Research-based recommendations for teaching and learning early number sense endorse experiencing multiple configurations of the same quantity to support recognition of that quantity in various forms (Clements and Sarama 2009), linking perceptual and conceptual subitizing via composition and decomposition. Despite the potential for subitizing to build number sense, including arithmetic skills, instruction often overemphasizes counting (Clements et al. 2019).

Fingers are often used to represent quantity. Whereas touches in series are ordinal (numbers-in-sequence), all-at-once touches are cardinal (designating a quantity of a set) (Sinclair and de Freitas 2014). These are conceptually congruent to counting and subitizing, respectively. Gestures similar to all-at-once have been given various names (e.g., cardinal number gestures: Gunderson et al. 2015), but may occur after counting orally and/or gesturally. Here, "all-at-once" is used to emphasize the explicit lack of counting involved. The relevance of fingers and the act of gesturing reinforces the embodied nature of early number sense, with research indicating that fingers are key to developing understandings of number concepts (e.g., Butterworth 2005) and their use may influence early mathematics achievement (e.g., Gracia-Bafalluy and Noël 2008). However, bodily involvement in counting varies across cultures (Bender and Beller 2012), and there may be spatial (e.g., left-right) associations with counting and involvement of fingers and hands (Fischer and Brugger 2011). Therefore, it is important to consider embodiment when investigating early number sense.

Multi-touch Technology in Mathematics Learning

Multi-touch technology (e.g., iPads) can recognize multiple simultaneous touch inputs, affording application of innovative gestures that might influence conceptions of mathematics (e.g., Sinclair and de Freitas 2014), even by preschool-aged children (e.g., Nacher et al. 2015). Research involving use of digital mathematics games on multi-touch devices has found positive outcomes related to children's mathematics achievement (e.g. Moyer-Packenham et al. 2015; Riconscente 2013), and some studies identified patterns in the interactions that may contribute to these outcomes (e.g., Watts et al. 2016; Tucker et al. 2017; Bullock et al. 2017). However, little research has focused on the embodied mathematics that may contribute to the outcomes. Notable exceptions include work involving the multi-touch app TouchCounts (e.g., Baccaglini-Frank et al. 2020; Sedaghatjou and Campbell 2017; Sinclair and de Freitas 2014; Sinclair and Pimm 2015), which indicates that interactions with multi-touch technology may influence development of early number sense, including featuring gestures that may help children differentiate between sequential counting and "all-at-once" (i.e., subitizing).

Attending to previous research

Relevant mathematics education research is often largely or entirely ignored in reviews of research originating in fields such as cognitive science and neuroscience, such as those on early number sense (Anobile et al. 2016) and "embodied numerosity" (Moeller et al. 2012). Research on subitizing frequently involves adult participants in single sessions (e.g., Ester et al. 2012; Krajcsi et al. 2013). Related classroom-based research often focuses on outcomes rather than development and employs non-routine procedures, such as extensive one-on-one assessments (e.g., Starkey and McCandliss 2014). Importantly, the overwhelming majority of subitizing research does not effectively account for the embodied nature of number sense and the relevance of conceptually congruent gestures, instead involving seeing a quantity and pointing to a numeral or naming the quantity (e.g., Ester et al. 2012; Kaufman et al. 1949; Lassaline and Logan 1993; Piazza et al. 2002). Most early number sense research involving children producing gestures has focused on counting (e.g., Alibali and DiRusso 1999). Some research implicitly involves children using subitizing gestures, focusing on relation to speech and examining accuracy in children's current states, rather than development or embodiment (e.g., Gunderson et al. 2015). Such research is insightful, but it leaves space to investigate development of embodied number sense in authentic forms.

Technological innovations afford promising angles for investigation. School-based number sense research has begun to feature multi-touch digital games integrated into students' routines. For example, preschoolers have been observed estimating quantity while using the Ladybug Count, which allowed serial finger placement to reach the correct response (Baccaglini-Frank and Maracci 2015). Children often placed too few or too many fingers on the screen before adding or removing fingers until matching the quantity, usually beginning at 5 or 10 for quantities 7 to 10. While interacting with TouchCounts, preschoolers exhibited various counting and subitizing behaviors when asked to indicate a quantity, including sequential screen touches, counting on fingers to the target quantity then touching the screen, and using a "lots of fingers" (i.e., all-at-once) gesture without finger counting (Sinclair and Pimm 2015). Importantly, preschoolers using TouchCounts have been observed: "preparing their fingers" in ways that relate to both their understandings of number, their knowledge of finger configurations used to indicate number, and their finger control (Baccaglini-Frank et al. 2020).

The iPad app Fingu is a multi-touch early number sense digital game that presents timed prompts featuring 1–10 objects and requires conceptually congruent gestures in response (see "Methods" for detailed description). Emerging research indicates that regular interactions with Fingu can positively affect students' performance on standardized early number sense assessments (Holgersson et al. 2016), and can improve students' subitizing task speed and accuracy with different effects by gender and age but not handedness (Broda et al. 2018). Initial descriptions of students' interactions with Fingu identified counting and subitizing strategies (Tucker et al. 2017; Baccaglini-Frank and Maracci 2015). Reporting on design and implementation of Fingu, Holgersson, et al. (2016) included brief but intriguing insights into children's preferred finger patterns (i.e., fingers displayed to indicate a quantity) for correct responses. For quantities of 1–5, children usually responded with adjacent fingers from the same hand. For quantities of 6–10, responses were either: a) semi-decimal (i.e., 5&2 for 7), b) symmetrical (e.g., 3&3 for 6), or c) mapped (e.g., 4&2 for 4 + 2). One five-year-old preschooler developed preferred patterns for quantities of 6, shifting from semi-decimal responses to symmetrical responses, except when presented with 5 + 1. However, research has just begun to explore development of such embodied number sense.

Methods

The purpose of this study was to follow one preschooler's number sense development during regular interactions with a multi-touch mathematics digital game over a 4-week period. The following research question guided the study: How does a preschool student's embodied number sense develop throughout regular interactions with Fingu?

Study context and participants

The data for this case study was taken from a larger mixed-methods study examining preschoolers' interactions with multi-touch technology. The study occurred in a full-day early childhood learning center near an urban university in the Eastern USA. Prior to data collection, researchers trained the teachers to support the study (i.e., facilitate interaction sessions). Participants were recruited from two preschool classrooms. During information sessions, researchers explained the study to parents, obtaining permission for students to participate. Parents of the 18 4-to-5-year-old participants completed a brief child demographic survey (e.g., gender, age, handedness).

Akin to other number sense research (e.g., MacDonald 2015), during data analysis from the larger project we chose one participant for in-depth analysis because their data provided opportunities to investigate key themes. We chose Maya (4y9m, right-handed) for this case study because her interactions provided evidence of number sense development over time and because she interacted with Fingu during every video recording session, reaching Level 5 and encountering quantities to 10. Backend data indicated that Maya encountered 1669 tasks.

Materials and data collection

Data collection occurred in May and June, toward the end of a preschool year that began in August. Study materials included eight iPads featuring the digital mathematics game Fingu and eight tripod-mounted digital cameras to record the participants' interactions with Fingu. During the initial classroom visit (Day 0), researchers introduced Fingu, modeling the interactions while providing every child with hands-on access. This included how to interact with Fingu (e.g., "figure out how many pieces of fruit you see and use that many fingers to touch the screen all at once") without providing specific strategies or answers (e.g., "count… this is five… touch with five fingers"). Participants were allowed to interact with Fingu during center time for up to 20 min 3 times per week for the following 4 weeks. Each student was assigned an avatar on a specific iPad for data tracking. Teachers and researchers supervised the interaction sessions without assisting children in task completion.

Researchers digitally recorded participants' interactions with Fingu five times: every Friday from Day 0 through Week 4. The tripod-mounted cameras focused on the interaction space, which included the iPad screen and, usually, children's hands (see Fig. 1). Fingu recorded backend data, including accuracy and response time for every task each child encountered, which also generated playbacks of finger touches (see Fig. 2). All students were given the opportunity to interact with the app, but video data was not recorded for non-participants and their backend data was not saved.

Fingu

Fingu is a digital mathematics game designed to support the development of early number sense. (See Holgersson et al. 2016 for a detailed description of Fingu design,

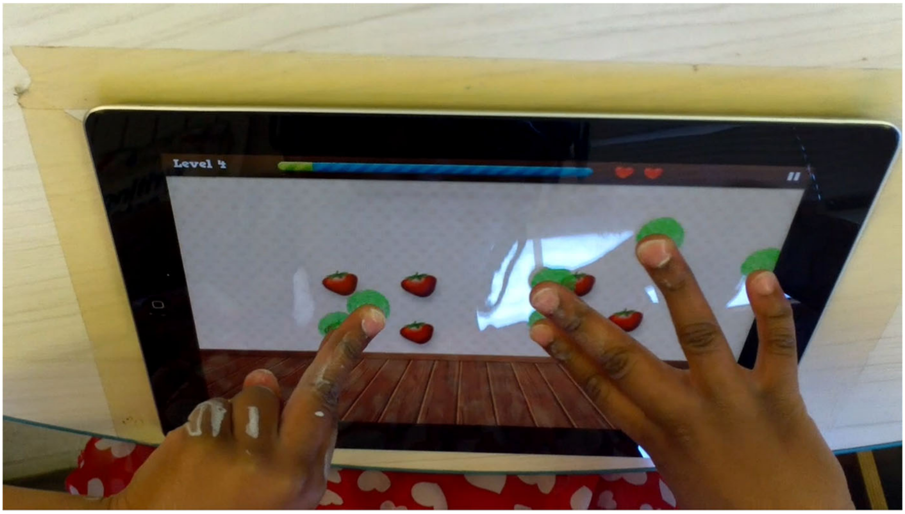


Fig. 1 Screenshot of interaction space

development, default settings, etc.) Fingu features seven increasingly difficult levels. Each level presents a consistent subset of developmentally appropriate tasks in random order from the 60 possible tasks (i.e., level 1 always included the same tasks but in random order). One must complete the 20–30 tasks with fewer than five mistakes in order to complete the level (e.g., Level 1, minimum 16/20), unlocking the next level.

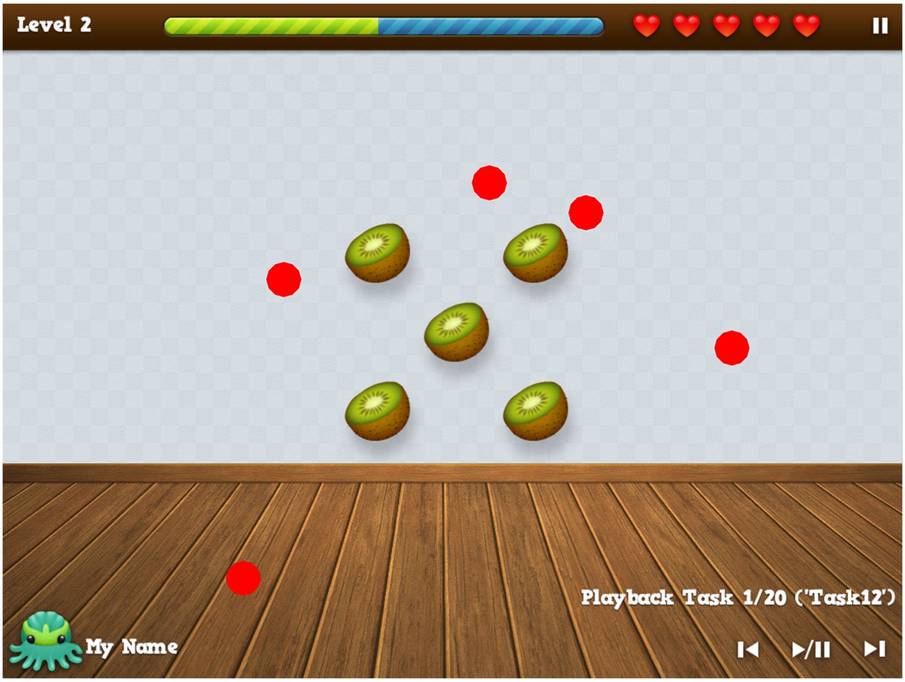


Fig. 2 Backend data playback screenshot

Fingu permits customization of many settings, including incorrect responses allowed, available time per task, and touch sensitivity. We activated the backend data recording but kept all other default settings.

Each task in Fingu presents a quantity of 1–10 total objects in the form of collections (i.e., sets of objects) of stylized fruit. There are two task types: undifferentiated wholes featuring one collection and differentiated wholes featuring two collections (see Fig. 3a and b). Each collection is arranged in a specific configuration, which may be canonical (dice) or non-canonical, with two possible configurations for each collection greater than 2 (e.g., 4a, 4b) (see Table 1). The configurations and collections are designed to encourage perceptual and conceptual subitizing by recognizing quantities and composing or decomposing the whole quantity (Holgersson et al. 2016). Each of the 60 tasks could be presented in two permutations (e.g., 3a + 1 or 1 + 3a), which we consider the same task (e.g., "3a + 1"), consistent with Fingu's designers (Holgersson et al. 2016). For initial task encounters, configurations are visible for 4 s before disappearing, allowing another 6 s for a response (10 s total). After repeated encounters, the same task may be presented with visibility reduced to 2 s followed by 4 further seconds to respond (6 s total). Users may respond at any time during the task. A response registered by Fingu ends the task with immediate feedback. To respond, users must indicate the quantity of fruit by using the corresponding number of fingers to simultaneously touch the screen in an all-at-once gesture that is conceptually congruent to subitizing. Finger patterns need not match the fruit configuration (e.g., 1&1 may be one finger from each hand or two fingers from either hand). Fingu shows a registered response by displaying green 'fingerprints' and immediately provides feedback graphics indicating whether the registered response was correct or incorrect, or if no response was registered before time expired (see Fig. 4a and b). Fingu's features are designed to encourage subitizing and discourage counting.

Data analysis

In this study, iterative data analyses involved complementary, simultaneously collected qualitative, and quantitative components. These included weaving together microgenetic learning analysis and descriptive statistics. Qualitative analysis involved microgenetic learning analysis (MLA), which is used to examine fine-grained, incremental learning (Parnafes and diSessa 2013). MLA involves flexible application and

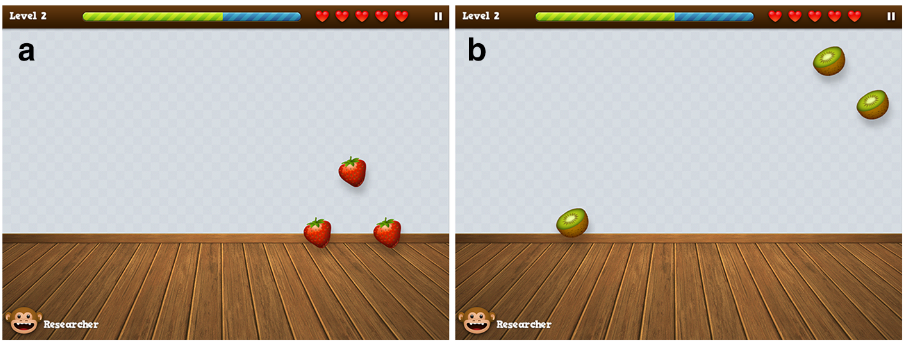
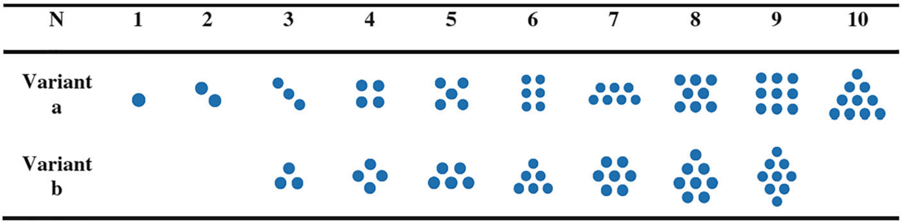


Fig. 3 **a** and **b** Undifferentiated whole (**a**: 3b+0) and differentiated whole (**b**: 1 + 2)

Table 1 Configurations for individual collections used in Fingu



Reprinted, with permission, from Holgersson, et al. (2016), p. 134

development of theory, occurs in any context, and focuses on fine-grained changes to find evidence of learning over extended periods of time. Consistent with embodied cognition, we examined actions and products of actions, here as they occurred in children's routine, classroom-based interactions with Fingu.

Analyzing video data for the wider project involved combining a form of field notes with analytic memoing to iteratively record and refine our interpretations, using phases of individual and dyadic video analysis to enhance idea development and increase trustworthiness of findings (Saldaña 2015). Generating descriptive field notes instead of using a formal transcription system allowed us to remain closer to the actions in the video (i.e., the embodiment) while providing text-based anchor points for searches. Analytic memoing interpreted both video and field notes, providing insights into emergent patterns and themes, links to relevant theoretical and empirical literature, and a basis for discussion. Granularity varied from seconds-long actions to full session videos, affording opportunities to examine number sense across multiple time scales. For this study, we used the same techniques and focused our analysis cycle on Maya's data. We watched her videos multiple times, created additional field notes and analytic memos, and interpreted the videos, notes, and memos. Where relevant, we also analyzed playbacks generated from backend data to examine evidence of finger touches from sessions between video recordings.

As part of the wider research project, we compiled and cleaned all backend data (see Broda et al. 2018). For this study, we used Excel to organize and quantitatively analyze Maya's backend data, focusing on task exposure frequency and accuracy rates for the quantities and configurations encountered. Examining the quantitative backend data made three valuable contributions: (a) basic, descriptive quantitative analysis (i.e., accuracy rates, task exposure frequency), (b) identifying potential patterns for qualitative exploration (e.g., "increase in accuracy, but why?"), and (c) interrogating qualitative findings (e.g., "did she often respond with five fingers, or just in the video?"). We intentionally used each data set as

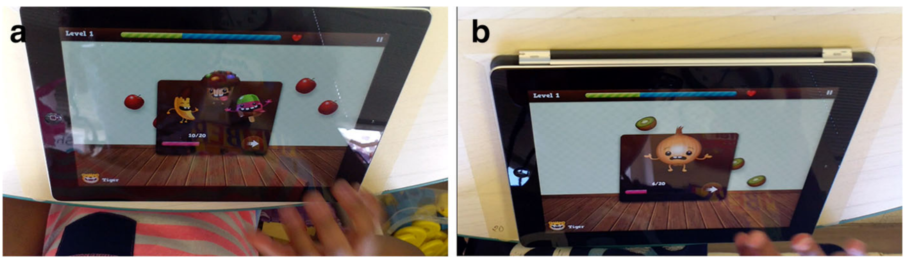


Fig. 4 **a** and **b** Feedback after task registered as correct (**a**) and incorrect (**b**)

appropriate based on our ongoing, interwoven, iterative analyses to provide rich insights into the embodied number sense Maya developed throughout.

Delimitations

This design has inherent delimitations. Rather than experimenting or intervening, our single-case study allows for deep, descriptive insights of development over time. We tell *a* story, not *every* story, though it is a story we believe to be relevant beyond itself. Although video provides opportunities to see actions develop across time scales (e.g., seconds, minutes, weeks), interactions also occurred between video recordings, leaving us to draw inferences from corresponding backend data (i.e., watching playback, reviewing speed and accuracy). We did not interview Maya and we avoided providing mathematical guidance, which may have been insightful and influential. The classroom setting was chaotic and messy / authentic and enjoyable. The design is intentional, providing a foil to most extant early number sense research.

Findings

The results are presented by week to illustrate Maya's developmental progression over the course of the study. The qualitative descriptions were generated from watching the weekly video recording sessions. Quantitative findings from backend data are integrated into the results to provide support for the video observations and later inferences. In this context, we do not differentiate between hand choice (e.g., a response of 3&1 may be one finger on the right hand and three fingers on the left, or switched), as Maya almost exclusively represented her greater quantity on her right hand, regardless of how the quantity was presented (e.g., 1 + 3a, 3a + 1, 4a + 0). Maya's interactions on Day 0, the initial classroom visit, served as a baseline performance. Each subsequent section describes findings from that week's interactions leading to and including the weekly recording.

Day 0

Throughout her first three attempts to complete Level 1, Maya experimented with ways to interact with Fingu (e.g., tapping or dragging fruit). During Maya's third attempt of Level 1, the researcher cued her to match her fingers to the quantity presented. For the next few tasks, Maya tried to count the fruit before they disappeared. Soon, Maya's accuracy improved, such as when she placed two fingers from her right hand on the screen in response to 1 + 1. By her fifth attempt at Level 1, Maya no longer appeared to count. For the rest of Day 0, Maya exclusively used her right hand to respond, though sometimes she struggled to simultaneously touch all intended fingers to the screen. At times Maya allowed a task to time out instead of responding after the fruit disappeared. Several times, Maya rapidly placed all five fingers of her right hand on the screen immediately or as time was about to expire. Soon, however, Maya began completing quantities of 1–3 with increasing accuracy. For example, on the fifth attempt of Level 1, she correctly responded to 2 + 1 then 3a + 0, each time placing her middle three fingers on the screen (see Fig. 5). Maya often exuberantly responded to feedback indicating a correct answer.

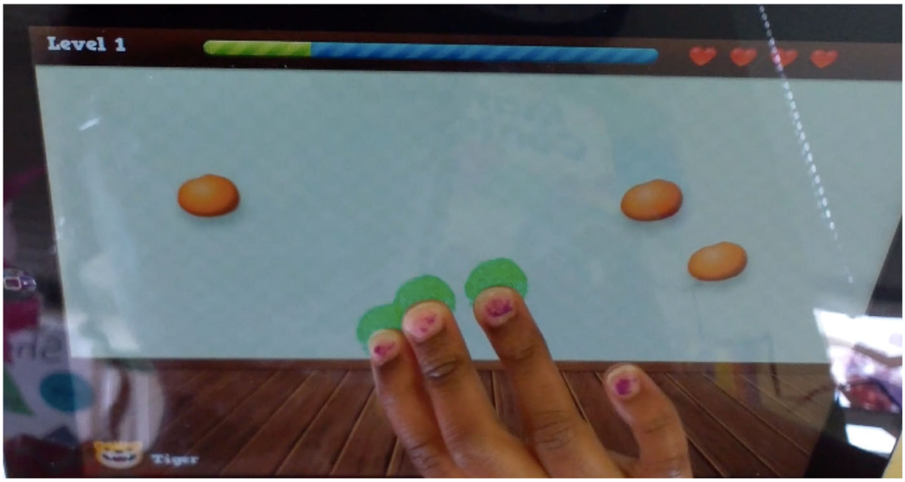


Fig. 5 Response of three middle fingers registered as correct for 2 + 1

Yet Maya's accuracy was inconsistent when presented with quantities of 4 and 5 (see Table 2). Although Maya was consistently inaccurate across quantities of 4, her responses within each task varied. Of the ten registered responses to 4a + 0, there were six 3's, no 4's, and no 5's. Of the eight registered responses to 3a + 1, there were no 3's, no 4's, and six 5's. Of the nine registered responses to 2 + 2, there were four 3's, two 4's, and no 5's. Some of Maya's errors appeared to be indicative of incorrectly quantifying the sum. For example, several times she quickly placed her middle three fingers with her thumb and pinky tucked under her palm when presented with 4a + 0, 2 + 2, 5a + 0, and 3a + 2 (i.e., a response of 3&0 to quantities of 4 or 5). However, Maya's responses were not always registered as they appeared to be intended. For example, at times when presented with quantities of 5, Maya appeared to try to place five fingers on the screen, but

Table 2 Weekly task accuracy, by quantity

	Percent accuracy (frequency of exposure)				
Quantity	Day 0	Week 1	Week 2	Week 3	Week 4
1	64 (11)	97 (33)	100 (4)	100 (1)	
2	88 (16)	85 (72)	88 (34)	100 (3)	
3	60 (20)	38 (72)	86 (76)	91 (43)	100 (3)
4	7 (27)	20 (115)	49 (99)	70 (93)	85 (33)
5	45 (20)	68 (75)	72 (121)	66 (85)	93 (55)
6			3 (106)	31 (120)	59 (64)
7			14 (36)	32 (79)	29 (90)
8					30 (56)
9					50 (4)
10					100 (2)

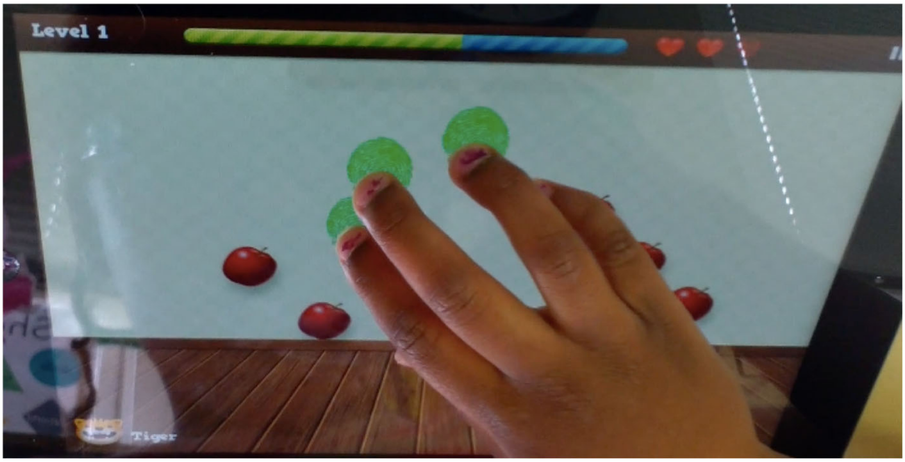


Fig. 6 Maya's pinky and thumb did not touch screen simultaneously with her other fingers

her pinky, thumb, or both were both delayed, resulting in her response being incorrect (see Fig. 6). Yet for 2 + 2, she sometimes tried to place all the fingers of her right hand on the screen but her pinky was delayed, resulting in her response being registered as correct. At times after receiving feedback indicating an incorrect response, she counted the fruit, verifying the quantity before moving to the next task. In one sequence, Maya fidgeted after an incorrect response of three middle fingers for 4a + 0, celebrated by clapping and cheering after her pinky was too late to turn her registered 4 into her intended 5 for 2 + 2, and was again incorrect on 4a + 0 with three middle fingers. This ended her attempt to complete the level, to which she exclaimed: "I can do the [tasks] up to three. I can't do more than three!" Yet Maya continued for another 8 min, celebrating an array of correct responses, including on quantities more than three. After 13 min, Maya stopped interacting with Fingu.

Week 1

Throughout the first week, Maya repeated Level 1. Backend data indicated that she had a high level of accuracy for quantities of 1 and 2 (see Table 2). Relative to Day 0, her accuracy increased for quantities of 4 and 5 but decreased for quantities of 3. During the video recording at the end of Week 1, she exclusively used her right hand to indicate quantity. She was successful during all encounters with quantities of 5 but was unsuccessful with all but one encounter with quantities of 3 or 4. The video data revealed some immediate responses of 5 (all 5&0), though Maya's difficulty with finger coordination also affected the accuracy of some responses. At times, she appeared to unintentionally place four fingers on the screen for quantities of 3 and five fingers for quantities of 4 and 5. On several encounters with 3b + 0 her thumb or pinky finger touched the screen as she tried to place the three middle fingers of her open right hand. During an encounter with 4a + 0 she attempted to place her right index, middle, ring, and pinky fingers but her thumb unintentionally touched the screen. On another encounter with 4a + 0 she tried a different gesture, attempting to place her right thumb, index, middle, and ring fingers, but her ring finger was delayed. Maya expressed frustration and ended the video-recorded session after 4 min.

Week 2

Maya progressed through multiple levels during Week 2. During the first session of Week 2, Maya successfully completed Level 1. Backend data logs showed that Maya correctly responded to all configurations of 1, 2, and 5. Her four incorrect responses were on 3b + 0, 2 + 1, and 3a + 1, with each response registered as one more than the quantity presented. During the second session of Week 2, Maya successfully completed Level 2. Accurate for quantities to 5, backend data logs show that Maya's four incorrect responses were for quantities of 6 (4a + 2 and 3a + 3a), which registered responses of 5. Overall, Maya's accuracy for quantities of 2–5 improved, yet her accuracy for quantities greater than 5 was very low. Within a quantity, accuracy varied by quantity configuration. For quantities of 5, her accuracy for 4a + 1 and 3a + 2 was considerably lower (60% and 57% respectively) than 5a + 0 (94%) (see Table 3). The only quantity of 6 Maya answered correctly was 5a + 1 (17%). Most (85%) of Maya's incorrect responses for quantities 6–9 registered as 5 or 10 (see Table 4).

At the outset of the Week 2 video recording session, Maya excitedly shared her progress with the researcher. During the session, Maya played only Level 3, which involved quantities of 3–7. Maya's accuracy had improved and some of her gestures had changed. She correctly responded to all encounters with 3a + 0 this session by placing her right thumb, index, and middle fingers. She also consistently used her right thumb, index, middle, and ring fingers to represent 4 (see Fig. 7), rather than changing among different gestures as previously. Additionally, she no longer responded to 4a + 0 with 5. At first, Maya's response when encountering quantities of 6 (5a + 1, 4b + 2, and 3a + 3b) or 7 (5a + 2, 4a + 3b) was immediately placing all five fingers of her right hand. Within the first 2 min of the video session, the researcher reminded Maya that she could use both hands when responding. Maya began sometimes rapidly placing 10 fingers on the screen for tasks presenting differentiated wholes greater than 5, yet she soon returned to favoring 5 instead of 10. She occasionally attempted to directly map the quantity presented, experiencing some success with tasks involving the configuration 5a (5a + 1, 5a + 2). Maya interacted with Fingu for 20 min during the Week 2 video recording.

Week 3

Throughout Week 3 Maya almost exclusively attempted Level 3, featuring quantities of 3–8. She maintained or improved her overall accuracy on quantities of 1, 2, and 3 while encountering most of these tasks less frequently (see Table 3). Her overall accuracy with quantities of 4 improved from week 2, but her accuracy was far greater for the undifferentiated whole (4a + 0: 91%) than the differentiated wholes (2 + 2, 3a + 1, 3b + 1: 52% or less). Her accuracy for quantities of 5 slightly decreased from week 2 (see Table 2), as the tasks on which she had been most accurate did not appear in Level 3. Although her accuracy on 3a + 2 increased to 90%, her accuracy for 4a + 1 fell to 39%. There was also considerable variation in her accuracy on quantities of 6, which was again much higher when the configuration 5a was present (5a + 1: 86%; 4b + 2 and 3a + 3b: 7%).

Table 3 Weekly task accuracy, by task

		Percent accuracy (frequency of exposure)				
Quantity	Task	Day 0	Week 1	Week 2	Week 3	Week 4
1	1 + 0	64 (11)	97 (33)	100 (4)	100 (1)	
2	2 + 0	88 (8)	88 (34)	87 (31)	100 (1)	
	1 + 1	88 (8)	82 (38)	100 (3)	100 (2)	
3	3b + 0	70 (10)	38 (37)	88 (32)	100 (2)	
	2 + 1	50 (10)	37 (35)	83 (29)	0 (2)	
	3a + 0			87 (14)	95 (39)	100 (4)
4	2 + 2	22 (9)	18 (38)	61 (33)	0 (2)	
	4a + 0	0 (10)	28 (39)	53 (15)	91 (45)	100 (4)
	3a + 1	0 (8)	13 (38)	37 (30)	50 (2)	
	3b + 1			48 (21)	52 (44)	40 (5)
	4b + 0					92 (24)
5	5a + 0	45 (11)	72 (36)	94 (35)	100 (2)	96 (23)
	3a + 2	44 (9)	64 (39)	57 (21)	90 (42)	100 (5)
	4a + 1			60 (40)	39 (41)	88 (24)
	3b + 2			72 (25)		
	5b + 0					100 (3)
6	5a + 1			17 (18)	86 (36)	86 (7)
	4a + 2			0 (28)		87 (23)
	4b + 2			0 (17)	7 (43)	75 (4)
	3a + 3a			0 (24)		
	3a + 3b			0 (19)	7 (41)	25 (4)
	6a + 0					100 (3)
	5b + 1					22 (23)
7	5a + 2			24 (17)	64 (39)	78 (27)
	4a + 3b			5 (19)	0 (40)	33 (6)
	7b + 0					0 (28)
	6b + 1					4 (27)
	4b + 3a					100 (2)
8	6a + 2					0 (4)
	5a + 3b					50 (2)
	5b + 3a					8 (24)
	4a + 4a					50 (24)
	4a + 4b					100 (2)
9	6b + 3a					0 (2)
	5b + 4b					100 (2)
10	5a + 5b					100 (2)

During the video recording session, Maya appeared focused but relatively subdued. Video 1 presents a sequence of consecutive responses encompassing examples of these

Table 4 Percentage of incorrect responses registered as 5, 10, or no response (frequency of occurrence)

Quantities	Response	Day 0	Week 1	Week 2	Week 3	Week 4
1–4	5	20.5 (8)	45.6 (68)	46.2 (30)	62.5 (20)	0.0 (0)
	10	0.0 (0)	1.3 (2)	6.2 (4)	0.0 (0)	0.0 (0)
	None	15.4 (6)	1.3 (2)	1.5 (1)	6.3 (2)	40.0 (2)
5	10	0.0 (0)	12.5 (3)	26.5 (9)	0.0 (0)	0.0 (0)
	None	9.1 (1)	0.0 (0)	0.0 (0)	10.3 (3)	50.0 (2)
6–9	5			74.6 (100)	70.1 (96)	23.7 (31)
	10			10.4 (14)	0.7 (1)	0.0 (0)
	None			0.7 (1)	2.2 (3)	44.3 (58)

Weeks 2 and 3, quantities 6–9 includes only 6 and 7, as the levels reached did not include 8 or 9. Maya encountered quantities of 10 twice, answering both correctly during Week 4

actions from early in the session, with Table 5 providing a description and corresponding screenshots. Overall, Maya accurately responded to most quantities of 3–5, even when presented as undifferentiated wholes. Two exceptions were 3b + 1 and 4a + 1, to which Maya responded in three ways: immediately, with four fingers on her right hand; immediately, with all five fingers on her right hand; or occasionally allowing the task to time out. When presented with differentiated wholes of 6–8, Maya's responses varied. Maya frequently directly mapped when tasks included the configuration 5a (5a + 1, 5a + 2), with little evidence of a delay between hands touching the screen. However, for other differentiated wholes of 6–8 (4b + 2, 3a + 3b, 4a + 3b), Maya often immediately placed all five fingers of her right hand rather than attempting to answer precisely or using 10. Occasionally, Maya allowed tasks to time out with no response attempted. Backend data confirmed that 5 was her most frequent incorrect response during Week 3 (see Table 3).

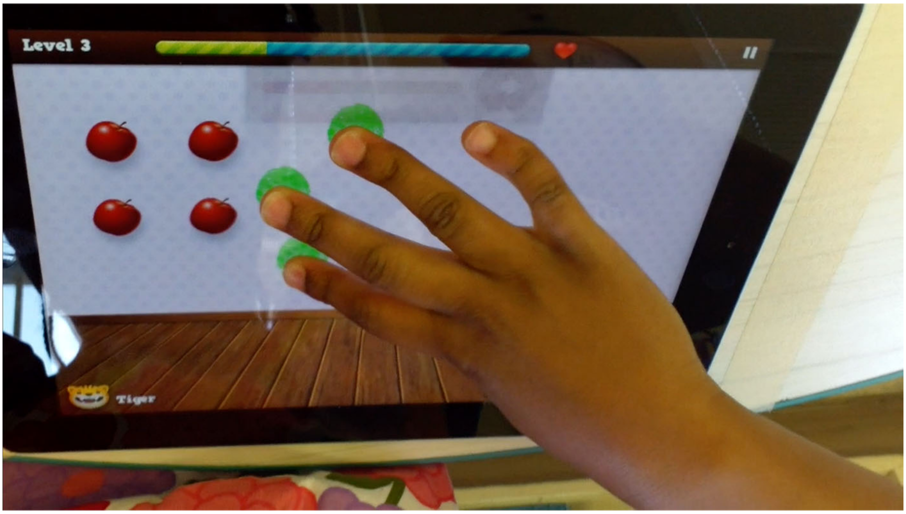
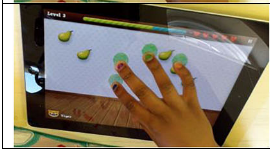
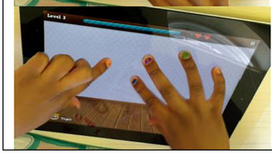


Fig. 7 Representing 4 using the right thumb (hidden by palm), index, middle, and ring fingers

Table 5 Description of and Images from Week 3 response sequence in Video 1

Maya's Response	Description
	3a+0: Maya places her right thumb, index, and middle fingers to indicate 3, all of which Fingu records (3: correct).
	3a+2: Maya places all five fingers of her right hand, all of which Fingu records (5: correct).
	3b+1: Maya allows the task to time out and does not touch the screen, even after the fruit disappears (no response: incorrect).
	4a+3b: Maya places all five fingers of her right hand, all of which Fingu records (5: incorrect).
	3a+3b: Maya allows the task to time out and does not touch the screen, even after the fruit disappears (no response: incorrect).
 	5a+2: Maya prepares 5 on her right hand, but changes her left hand. She starts with index/thumb, changes to index/middle, and switches back to index/thumb. As the fruit disappear, Maya places these fingers on the screen, all of which Fingu records (7: correct).

Audio was deleted from the video before dissemination due to frequent utterances of children's names

During her final three attempts to complete Level 3, Maya responded to every task before time expired. She continued to accurately respond to tasks involving 5a and often immediately responded with five fingers to other differentiated wholes of 6–8, and once correctly direct mapping for 4b + 2. After 7 min of interacting with Fingu, Maya switched centers.

Week 4

By Week 4, Maya no longer encountered quantities of 1 and 2, and only encountered 3, as 3a + 0, during the first session of Week 4. During that session, she successfully completed Level 3 and unlocked Level 4. Backend data logs indicated that Maya was incorrect on only four tasks, allowing 3b + 1 and 4a + 3b to time out and responding to 3a + 3b and 4b + 2 with 5. Level 4 introduced 4b + 0, a collection with a non-canonical configuration which she completed with a high degree of accuracy (see Table 3). Throughout Week 4, Maya responded to quantities of 4 and 5 with a high degree of accuracy (see Table 2), and achieved her highest accuracy on all tasks involving a collection of four. Her accuracy with quantities of 6 varied by configuration, though her accuracy on 4a + 2 increased since she had last seen it in Week 2 (0% to 87%). Maya's accuracy varied on quantities of 7–10, with higher accuracy on 5a + 2, low accuracy on 4a + 3b, and mixed accuracy on new tasks with relatively few exposures.

Maya began the final video recording session on Level 4. During this session, she quickly and accurately responded to all quantities of 4 or 5. When first presented with 5b + 1, Maya tried to count but the fruit disappeared, and she quickly placed all the fingers of her right hand before the task timed out. However, she usually responded to differentiated wholes of 6 by immediately attempting to directly map the quantities in each group, with particular success on 4a + 2 and 5a + 1. When Maya first encountered 4a + 4a, she attempted to place the index, middle, ring, and pinky fingers of both hands but a thumb touched the screen, resulting in an incorrect response. Soon, she honed the index, middle, ring, and pinky gesture and continued to correctly apply it for all configurations of 4 (e.g., the "4" in 4a + 2 and 4b + 0). This included directly mapping for 4a + 1, whereas previously her correct responses had been five fingers on her right hand. Maya also changed her gesture for 3, using her middle three fingers whenever correctly responding to quantities greater than 5 featuring a configuration of 3 (3a + 3b, 4b + 3a, 4a + 3b, 5b + 3a). Maya did not return to using her previously favored gestures for configurations of 3 or 4.

During her fourth attempt of Level 4, Maya correctly responded to all but four tasks. Maya twice responded to 7b + 0 by placing all 5 fingers of her right hand on the screen. After timing out during her first encounter with 6b + 1, Maya accidentally touched the screen while counting the fruit. Maya directly mapped for all correct responses. She showed little reaction upon completing Level 4, immediately returning to the menu and selecting the newly-unlocked Level 5. Maya attempted Level 5 three times, directly mapping to correctly respond to differentiated wholes involving two configurations of 4 or 5 (4a + 4b, 5a + 5b, and 5b + 4b). When faced with undifferentiated wholes involving a configuration greater than 5, her responses varied. For 6a + 0, she consistently, correctly placed her three middle fingers from each hand. For 7b + 0, Maya immediately responded by attempting to place six bunched fingers. However, she often allowed tasks featuring differentiated wholes with a configuration greater than 5 (6b + 1, 6a + 2, and 6b + 3a) to time out. After 10 min, Maya's teacher encouraged her to change centers.

Discussion: development of embodied number sense

Maya's embodied early number sense developed throughout the interactions, providing evidence of subitizing, (de)composition, counting, relationships among gestures, estimation vs. precision, and other relevant themes. Our subdivisions are not intended as a position statement on boundaries within number sense; rather, they emerge from convenience based on relationships identified throughout the findings.

Quantities 1–3

From the outset, Maya appeared to rapidly and accurately perceptually subitize quantities of 1, 2, and 3 presented as undifferentiated wholes (1 + 0, 2 + 0, 3b + 0). She also consistently composed quantities 1–3 configured as differentiated wholes, representing the entire quantity using her right hand (e.g., 2 + 1 using two fingers as 3&0; see Fig. 5). On these tasks, Maya demonstrated cardinality by combining the two collections into one whole. The early errors she experienced on these quantities often related to learning to use Fingu. The evidence indicates that for quantities of 1–3, Maya could both perceptually subitize an individual collection and conceptually subitize by combining the two perceptually subitized collections into one regrouped quantity (i.e., composing parts of a differentiated whole: 1 + 1 = 2, 2 + 1 = 3). These findings align with research indicating that humans can accurately subitize small quantities in unfamiliar configurations (Dehaene and Cohen 1994; Krajcsi et al. 2013). The gestures are similar to those reported by Holgersson et al. (2016), where children often used one hand to respond to quantities 1–5 (including 1–3). Maya's decreased accuracy on quantities of 3 during Week 1 coincided with closer attention to finger placement and attempts to distinguish between quantities of 3 and 4, suggesting that accuracy measures provide important but limited insights into development of number sense.

Quantities of 4–5

Maya also developed her subitizing for quantities of 4–5, influenced by both motor skills and quantity recognition. After she determined how to interact with Fingu early in Day 0, there was no video evidence of attempts to count quantities of 1–5 in any configuration. For differentiated wholes of 4–5, Maya consistently attempted to conceptually subitize by combining two perceptually subitized collections into a regrouped quantity on one hand, as she had done with quantities of 1–3. Despite attempts to apply cardinality through combined collections, her accuracy and gestures varied.

Maya commented on her initial difficulties with quantities of "more than three," implying that she recognized they were distinct from quantities 1–3, but that she sometimes struggled to precisely quantify or accurately represent what she saw. This extends research indicating that subitizing becomes more challenging once quantities exceed 3 (Logan and Zbrodoff 2003). As in other studies (Dehaene and Cohen 1994; Krajcsi et al. 2013), configuration influenced quantification. Maya was more accurate for undifferentiated wholes of 4 and 5 than for the corresponding differentiated wholes, with only one exception (week 1: 2 + 2). She eventually became regularly accurate on undifferentiated wholes of 4–5 (perceptual subitizing) and differentiated wholes of 5 (conceptual subitizing). Although Maya consistently struggled with differentiated wholes of 4, her accuracy on each was improving when they were phased out due to her overall progress. Taken together, this aligns with research indicating that subitizing accuracy can improve with repeated exposure (Broda et al. 2018; Lassaline and Logan 1993), and supports recommendations of exposure to various configurations of the same quantity (Clements and Sarama 2009). Yet the pattern in Maya's responses for quantities of 4 on Day 0 (e.g., 4a + 0 usually 3, 3a + 1 usually 5, and 2 + 2 usually either 3 or 4) suggest the relationship between configuration and quantification bears further investigation.

Quantities of 6–10

For quantities of 6–10, Maya's accuracy on many tasks was low (see Tables 2 and 3) but increased with exposure for some tasks, and her actions reveal important links related to configuration and gesture. Maya's accuracy increased for all six tasks with quantities 6–10 which she encountered both before and during Week 4, with all but one rising from below 25% the first week encountered to above 75% in Week 4 (4a + 3b: 5% to 33%). This reinforces that subitizing accuracy can increase with exposure (Broda et al. 2018; Lassaline and Logan 1993), even for greater quantities. Five tasks with quantities 6–10 were first encountered in Week 4 with at least 20 exposures. Of those five, she was most accurate on 4a + 4a, at 50%. Once Maya began to use two hands to respond on the final day of Week 2, her correct responses were almost always a direct mapping of the quantities in each group. The prevalence of direct mapping and infrequent decomposing or regrouping of these quantities (e.g., 4a + 4a as 4&4, not 5&3) makes it difficult to interpret if Maya ever conceptually subitized (i.e., composed) by combining the two perceptually subitized collections or if she only perceptually subitized two separate collections (e.g., 4 + 4 = 8 vs. 4 and 4 each as a different 4). Although Maya demonstrated awareness of cardinality for each individual collection, she may not have applied cardinality to the whole. Maya struggled when it was impossible to directly map because a collection was greater than 5 (e.g., 6a + 2), except for 6a, which Maya decomposed into the symmetrical 3&3. Her correct responses to other quantities of 6 involved direct mapping, which is different from the child reported in Holgersson, et al. (2016) who changed from using 5&1 to 3&3, except for 5a + 1 which remained 5&1. In addition to likely differences in the children's development and age (4y9m vs. "five years old"), contextual factors may have played a role. Over less time (4 weeks vs. 8 weeks), Maya attempted fewer tasks (1669 vs. 4572) and fewer levels (5 vs. 7). Exposure to additional tasks and configurations might have contributed to these differences. Such variations emphasize the potential for researchers to examine these and other findings across many children and contexts.

Maya's responses to tasks involving the configuration 5a also demonstrate the relevance of configuration (i.e., object arrangement, including number of collections and configuration of each collection) to number sense. After the initial session where she learned to interact with the app, Maya's accuracy on each task including 5a (e.g., 5a + 0, 5a + 1, 5a + 2) was consistently higher than every other task of equal quantity with a similar number of exposures. The semi-decimal pattern (i.e., involving all fingers on one hand) may have helped her directly map the quantity presented (e.g., 5a + 1 easier than 4a + 2), and she was likely more familiar with 5a than 5b. Many of Maya's incorrect responses on differentiated wholes involving 5a (5a + 1, 5a + 2, 5a + 3b) involved one hand touching before the other even though both had the correct fingers ready. As Maya progressed, her intended fingers touched the screen together more often and her accuracy increased, suggesting that at first, she may have attended to one collection of the differentiated whole more quickly than the other. This supports the argument that she initially perceptually subitized the two collections separately, though it does not confirm if she later attended to the whole (i.e., composed via conceptual subitizing). These findings build on the semi-decimal, symmetrical, and direct mapping responses to quantities of 6–10 presented as general categories by Holgersson et al. (2016), indicating that more detailed analyses are possible. Together, they also imply that not only does configuration influence how quickly and accurately one recognizes a quantity (Dehaene and Cohen 1994; Krajcsi et al. 2013), but that configuration might also influence how one represents a quantity.

Counting

Maya favored subitizing over counting throughout the video-recorded interactions, though she did occasionally attempt embodied counting. While still determining how to interact with Fingu, Maya used her right index finger to attempt to point to objects in sequence, often touching the screen during the process. This was an attempt at keeping track of counting while tagging the items (Alibali and DiRusso 1999), but touching the screen ended the task before Maya could complete the counting sequence. Quickly, Maya recognized that sequential individual touches were not an accepted response and abandoned this form of counting. Throughout, whenever Maya responded, she did so quickly, suggesting she was unlikely to be counting. She was not seen or heard counting (e.g., sequentially pointing individually or unfurling fingers without touching the screen, or reciting the counting sequence) again until a few Week 4 encounters with tasks featuring unfamiliar configurations (e.g., 5b, 6b). She soon discontinued this approach because she could not count all the objects before they disappeared. Maya's experiences align with the designers' intent that Fingu's features (e.g., individual task time constraints, all-at-once touch) should make subitizing more efficient than counting for these tasks (Holgersson et al. 2016). Although subitizing and counting may be related (Piazza et al. 2002), quantity, configuration, familiarity, and even time constraints may all be relevant influences.

Relationships among gestures and quantities

Maya's evolving gestures offer insights into development of embodied number sense, including relationships among gestures and quantities. Nascent research involving Fingu noted that gestures could change over time (e.g., Holgersson et al. 2016). Even as Maya's gestures changed, they remained consistent in some ways. Regardless of how the quantity was presented, Maya almost exclusively used her dominant right hand to represent more of the quantity (e.g., 2 + 0 and 0 + 2 as 0&2; 4a + 1 and 1 + 4a as 0&5; 4a + 2 and 2 + 4a as 2&4). Relationships among space and embodied quantification have been noted (Bender and Beller 2012; Fischer and Brugger 2011), though this application to conceptually congruent subitizing gestures is relatively novel. A related study found that handedness did not affect subitizing task accuracy or speed (Broda et al. 2018), but these findings show that handedness might nevertheless affect embodied response.

At times, Maya could coordinate her fingers in ways that did not appear to involve counting (e.g., rearranging fingers, Table 5, Video 1). This extended coordination was infrequent and might have occurred as Maya shifted her gestures. Gesture development related to quantities and collections of 3 and 4 were especially revealing. Early on, Maya's thumb or pinky occasionally accidentally touched the screen while trying to indicate 3 with her middle three fingers. She also switched between thumb and pinky as her fourth finger for 4, with inconsistent accuracy. Maya's decreased accuracy on quantities of 3 may be related to gestural experimentation as she replaced her reasonably effective gesture. By the end of week 2, Maya consistently used her right thumb, index, and middle fingers for 3, adding her ring finger for 4. This attention to gesture might have helped her differentiate between quantities of 3 and 4, both in recognition and indication. It might also demonstrate awareness of connections between quantities of 3 and 4 since she changed both gestures even though her initial gesture for 3 often worked.

Maya continued adapting her gestures during Week 4. When correctly responding to quantities greater than 5 that included a configuration of 3 (3a + 3b, 4b + 3a, 4a + 3b, 5b + 3a) or 6a (6a + 0), Maya exclusively used her middle three fingers to represent the three. As she began successfully directly mapping differentiated wholes for quantities greater than 5 with both collections of 5 or fewer (e.g., 5a + 1, not 6b + 1), she changed her gesture for 4 to right index, middle, ring, and pinky fingers. She consistently used this gesture as part of her response to all tasks involving a collection of 4, including task 4a + 1, which she had previously combined onto her right hand (i.e., 5&0 changed to 4&1). Whereas one hand had worked for the early differentiated wholes she encountered (i.e., quantities 2–5), once most required two hands (i.e., quantities 6–10), she often used two hands even when one hand would work. Although Holgersson, et al. (2016) noted the prevalence of using one hand to indicate quantities 1–5, research had not revealed reversion to using two hands. Akin to children interacting with the multi-touch digital game TouchCounts (Baccaglini-Frank, et al. 2020), Maya coordinated (or 'prepared') her fingers in ways related to her understandings of number, her knowledge of finger configurations that could be used to indicate number, and her motor (finger) control, with changes in the finger coordination reflecting development in these areas. Maya changed her chosen gestures by context, suggesting they may relate to quantity, configuration, motor skills, and even exposure to related quantities and configurations. This also indicates that the response method (e.g., gestures, words) is relevant when learning and evaluating number sense.

Estimation and precision

Maya often used rapid responses of five fingers of one hand or all ten fingers to indicate an estimate, and her deployment of estimation developed throughout the interactions. Video data suggests that difficulty interacting with Fingu rarely contributed to the frequency of 5 or 10 as a response, especially after Day 0. It is impossible to definitively determine in every instance whether Maya was attempting to precisely quantify, but emergent patterns suggest Maya often intentionally used 5 and 10 as estimates. Although attempts have been made to differentiate among subitizing and estimation (e.g., Anobile et al. 2016; Revkin et al. 2008), exploration of development of embodied estimation and precision in relation to subitizing is novel.

Within the first day, Maya began estimating when facing difficulty precisely quantifying before the task time expired. Initially, she rapidly responded with 5 for tasks she did not immediately recognize, which often appeared linked to the overall quantity or unfamiliar configurations (see Table 4). For example, on Day 0, Maya's most frequent response—correct or incorrect—for 3a + 1 was 5. During weeks 1 and 2, Maya continued to often immediately answer 5 when encountering quantities of 4, though by the video session at the end of the week 2, she had stopped responding to 4a + 0 (undifferentiated) with 5. Combined with her honed gesture for four, this suggests Maya might have begun to recognize a quantifiable difference between a canonical undifferentiated 4 and a canonical undifferentiated 5 (i.e., configurations 4a vs. 5a). During Week 3, Maya's percentage of incorrect responses of 5 for quantities 1–4 increased, but this almost exclusively occurred on 3b + 1, which was the only task for a quantity of 1–4 she frequently answered incorrectly.

During week 2, Maya began to encounter quantities of 6–9. Her responses of 10 increased for all ranges, and 85% of her incorrect responses for quantities of 6–9 registered as either 5 or 10. During Week 3, Maya phased out using 10 as an estimate, but still often immediately responded with 5, especially for quantities of 6 or 7 that did not involve the configuration 5a (see Table 5 and Video 1). By Week 4 with continued exposure to quantities of 6–10, Maya estimated less frequently. She could be seen trying to precisely quantify for wholes of 6–10 where each configuration was 5 or fewer (e.g., 4a + 3b) by directly mapping. She often declined to answer when presented with a configuration greater than 5 (e.g., 7b + 0, 6a + 2), sometimes after attempting to count. This suggests that she did not use rapid responses of 5 or 10 to simply advance to the next task. Maya's use of estimation changed throughout the interactions as she developed her subitizing and composition skills. She began by using 5 as an estimate and briefly incorporated 10, phasing out each as she increasingly focused on precision overestimation. Whereas Baccaglini-Frank and Maracci (2015) found that children using another multi-touch digital game estimated and then added or removed fingers for precision (i.e., "counting on" or "counting back"), Fingu does not allow changing fingers after the registered touch in a task. This encourages strategies for estimation and precision related to subitizing (e.g., all-at-once). The novelty of these findings suggests that development of estimation and precision, particularly with links to embodiment, may be fruitful areas for number sense research.

Synthesis and future directions

Our study extends previous research while linking to embodied cognition and emergent findings. With Maya's development of embodied early number sense, we build on research indicating that configuration and quantity influence subitizing (e.g., Dehaene and Cohen 1994; Krajcsi et al. 2013), and that subitizing can improve with repeated exposure (e.g., Broda et al. 2018; Lassaline and Logan 1993). The interactions provided insights into relationships among perceptual and conceptual subitizing, (de)composition, and estimation, supporting calls for increased intentional integration of subitizing in early learning experiences (Clements et al. 2019; Clements and Sarama 2009).

Maya evolved her conceptually congruent "all-at-once" subitizing gestures throughout the interactions. This appeared to be linked not only to the quantities, but also to how the quantities were presented (e.g., differentiated vs. undifferentiated whole; collection configuration), her awareness of other quantities and configurations, and her representations of these quantities and configurations. Features of Fingu might have influenced development of embodied number sense, especially subitizing and composition, including presentation of a quick succession of tasks that were time-limited, developmentally appropriate, deliberately configured, and requiring conceptually congruent responses, paired with immediate feedback. Although confusion from unexpected feedback to unintended responses might have engendered frustration, it also might have encouraged attention to quantity and gesture.

Together, these findings suggest that using conceptually congruent gestures, particularly while interacting with intentionally-designed multi-touch technology, might support development of early number sense. Not only are number-related gestures cultural constructs (Bender and Beller 2012), but interactions with technology may influence development of these mathematical gestures (Sinclair and de Freitas 2014) and perhaps even what the mathematics itself means. This study adds to the growing body of work (e.g., Tucker 2018; Baccaglini-Frank et al. 2020; Holgersson et al. 2016; Segal et al. 2014) attending to embodiment in research and practice related to development of number sense. Number sense research originates in many fields and provides insights into how humans perceive and communicate quantity (e.g., Anobile et al. 2016; Bender and Beller 2012; Sarama and Clements 2009; Sinclair and de Freitas 2014). Although fields may embrace different evidence and use divergent definitions of constructs, it is important that they communicate with one another. To this conversation, we add the relevance of embodiment in the form of conceptually congruent gestures. We hope that contributors from various fields grab this with both hands.

Availability of data and material Not applicable.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no competing interests.

Code availability Not applicable.

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