Published for SISSA by Springer

Received: *November 20, 2017*
Revised: *April 5, 2018*
Accepted: *April 11, 2018*
Published: *April 20, 2018*

DIS off glueballs from string theory: the role of the chiral anomaly and the Chern-Simons term

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Abstract: We calculate the structure function $F_3(x, q^2)$ of the hadronic tensor of deep inelastic scattering (DIS) of charged leptons from glueballs of $\mathcal{N} = 4$ SYM theory at strong coupling and at small values of the Bjorken parameter in the gauge/string theory duality framework. This is done in terms of type IIB superstring theory scattering amplitudes. From the AdS_5 perspective, the relevant part of the scattering amplitude comes from the five-dimensional non-Abelian Chern-Simons terms in the SU(4) gauged supergravity obtained from dimensional reduction on S^5. From type IIB superstring theory we derive an effective Lagrangian describing the four-point interaction in the local approximation. The exponentially small regime of the Bjorken parameter is investigated using Pomeron techniques.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence

ArXiv ePrint: 1711.06171

https://doi.org/10.1007/JHEP04(2018)118

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Contents

1 Introduction

The holographic dual description of deep inelastic scattering (DIS) of charged leptons from glueballs in the $\mathcal{N} = 4$ SU(N) SYM theory with an IR cutoff has been proposed by Polchinski and Strassler in [1]. In the planar limit, at strong 't Hooft coupling ($1 \ll \lambda \ll N$), $\mathcal{N} = 4$ SU(N) SYM theory is dual to type IIB supergravity on AdS$_5 \times S^5$. The compactification of type IIB supergravity on S^5 leads to the maximally supersymmetric five-dimensional supergravity with gauged SU(4) symmetry [2–6], and also there are Kaluza-Klein modes. This dimensional reduction induces a five-dimensional Chern-Simons term [3–5], related to the chiral anomaly in the dual $\mathcal{N} = 4$ SYM theory [6–8]. In the calculation of the hadronic tensor the chiral anomaly is reflected in the fact that it appears a structure function F_3.

In the study of DIS in terms of the AdS/CFT correspondence there are a few distinct parametric regions which depend on the relation between the Bjorken parameter x and the 't Hooft coupling of the gauge theory, $\lambda = g_{YM}^2 N$, where g_{YM} is the coupling of the gauge theory. For the parametric region where $1/\sqrt{\lambda} \ll x < 1$ the process is well described in terms of type IIB supergravity. For $\exp(-\sqrt{\lambda}) \ll x \ll 1/\sqrt{\lambda}$ excited strings are produced, therefore it is necessary to consider type IIB superstring theory scattering amplitudes in the

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holographic dual description. For exponentially small values of x, diffusion effects become important and Pomeron techniques can be used.

It is also possible to go beyond the tree-level approximation using type IIB supergravity. Particularly in reference [9] $1/N^2$ corrections to DIS of charged leptons off glueballs at strong coupling have been obtained, which correspond to a DIS process where there are two-hadron final states. Cutkosky rules allow us to calculate the imaginary part of an amplitude by considering scattering amplitudes of the incoming and outgoing states into all possible on-shell states. The result of that calculation is very interesting, namely: the large N limit and the limit in which the momentum transfer of the virtual photon is much larger than the IR cutoff do not commute. This indicates that in the high energy limit two-particle intermediate states (in terms of the Cutkosky rules) give the leading contribution.[1]

Moreover, the holographic dual description of DIS from flavor Dp-brane models has been carried out very successfully. Among the interesting results, it is worth emphasizing that holographic dual dynamical mesons show universal properties for the structure functions [11, 12]. This is particularly important because it should hold for scalar and polarized vector meson structure functions for QCD itself, at least in the large N limit.[2] The relevance of this comes from the fact that the discovery of properties such as relations among the structure functions (for example those similar to the Callan-Gross relation) provides essential information about the internal structure of hadrons, which can be helpful in order to study other scattering processes. In addition, universal behavior suggests deep underlying connections among different confining relativistic quantum field theories. In this work we find new Callan-Gross type relations for the antisymmetric structure function $F_3(x, q^2)$.

For scalar and polarized vector mesons new and very interesting developments have been done in [11–13]. Then, by using these results for mesons a comparison with lattice QCD data[3] has been carried out, finding good agreement (within accuracy of 10% or better) for an overall fitting of the first three moments of the F_2 structure function of the pion, and (within 21% or better) for the first three moments of the F_1 structure function of the ρ-meson [15]. These calculations have been extended to one-loop level type IIB supergravity for the D3D7-brane system, finding an impressive improvement with respect to the tree-level results, now fitting lattice QCD data within 1.27% (or better) for the first three moments of F_2 of the pion [16].

While most of the investigations outlined above concern the calculation of the symmetric structure functions $F_1(x, q^2)$ and $F_2(x, q^2)$, in the present work the interest is focused on the antisymmetric structure function $F_3(x, q^2)$. We consider DIS of charged leptons from glueballs in the $\mathcal{N} = 4$ SU(N) SYM theory with an IR cutoff energy scale Λ, and describe it in terms of its string theory dual model. It is interesting to recall the origin of

[1] In the paper [10] also $1/N^2$ corrections have been considered. However, they have used an effective model given by a scalar-vector Lagrangian, which has a very small number of modes and interactions among them in comparison with the actual possible field fluctuations of type IIB supergravity which we have included in our paper [9].

[2] It could also hold for the first sub-leading term in the $1/N$ expansion.

[3] Lattice QCD results of the first moments of the pion and rho meson structure functions are presented in reference [14].

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the antisymmetric structure functions which appear in the hadronic tensor in this gauge theory. $\mathcal{N}=4$ SU(N) SYM theory has an SU$(4)_R$ R-symmetry group. The field content of the gauge theory includes six real scalars transforming in the representation **6**, and also there are four complex Weyl spinors transforming in the fundamental representation of the R-symmetry group with the chirality part $(0,1/2)$ in the **4** and $(1/2,0)$ in the $\mathbf{4}^*$ [17]. This SU$(4)_R$ symmetry is anomalous, i.e. it is broken at quantum level. The anomaly can be calculated exactly at one-loop level, being the corresponding Feynman diagram the one with three external points, connected by three chiral fermion propagators. This is the so-called triangle Feynman diagram, which is related to the three-point function. The precise value of the chiral anomaly obtained perturbatively from $\mathcal{N}=4$ SU(N) SYM theory is [6, 18, 19]

$$\frac{\partial}{\partial z_\rho} < J^A_\mu(x) J^B_\nu(y) J^C_\rho(z) >_- = -\frac{N^2-1}{48\pi^2}\, i\, d^{ABC}\, \varepsilon_{\mu\nu\alpha\beta}\, \frac{\partial}{\partial x_\alpha}\, \frac{\partial}{\partial x_\beta}\, \delta^4(x-z)\, \delta^4(y-z)\,, \quad (1.1)$$

where the subindex minus indicates the abnormal piece of the three-point function, i.e. the one which leads to the chiral anomaly [6]. ε is the Levi-Civita tensor, d^{ABC} and f^{ABC} are the SU$(4)_R$ symmetry group symbols defined by $\mathrm{Tr}(T^A T^B T^C) \equiv \frac{1}{4}(i f^{ABC} + d^{ABC})$, where T^A are hermitian generators of SU$(4)_R$, which are normalized as $\mathrm{Tr}(T^A T^B) = \frac{1}{2}\delta^{AB}$. Considering the minimal coupling $\int d^4x J^A_\mu(x) A^{A,\mu}(x)$, where $A^{A,\mu}(x)$ are background fields, equation (1.1) can be rewritten as an operator equation

$$(D^\mu J_\mu(x))^A = \frac{N^2-1}{96\pi^2} i\, d^{ABC}\, \varepsilon^{\mu\nu\rho\sigma}\, \frac{\partial}{\partial x^\mu} \left(A^B_\nu \partial_\rho A^C_\sigma + \frac{1}{4} f^{CDE} A^B_\nu A^D_\rho A^E_\sigma \right) . \quad (1.2)$$

This anomaly is reflected in the bulk theory in a very nice way, namely: since the global boundary SU$(4)_R$ symmetry corresponds to a gauge SU(4) symmetry in the AdS$_5$, the corresponding action in the bulk is not gauge invariant [7]. It can be easily seen by looking at the gauge sector of the action in AdS$_5$, which after dimensional reduction of type IIB supergravity on the five-sphere leads to the maximal SU(4) gauged supergravity on AdS$_5$. The action of this supergravity contains a Chern-Simons term, thus it is not gauge invariant. Moreover, the AdS/CFT correspondence calculation shows the matching with the chiral anomaly of the boundary theory [6–8]. Let us recall how this works. The starting point is type IIB supergravity in ten dimensions. In fact if one considers type IIB superstring theory it turns out that the $1/N^2$ corrections only come from the Kaluza-Klein modes arising from the dimensional reduction on S^5, i.e. the N^2-1 overall factor in the chiral anomaly.[4] After dimensional reduction on S^5 it leads to the action for the SU(4) gauge fields $A^A_m(x,z)$

$$S_{5d}[A] = \int d^5x \left[\sqrt{-g_{\mathrm{AdS}_5}}\, \frac{1}{4g^2_{SG}}\, F^A_{mn} F^{A,mn} + \frac{i\,\kappa}{96\pi^2} (d^{ABC}\, \varepsilon^{mnopq}\, A^A_m \partial_n A^B_o \partial_p A^C_q + \dots) \right] , \quad (1.3)$$

where κ is an integer and we set $R=1$. Both the SU(4) gauged supergravity coupling g_{SG} as well as κ are fixed in terms of the boundary theory R-current correlators which are

[4]This was suggested by Witten [7] and a more complete AdS/CFT calculation was done by Bilal and Chu [8].

exactly known [6]. Parentheses in the action (1.3) indicate the Chern-Simons term. Notice that Latin indices stand for AdS_5 coordinates, while Greek indices denote the boundary gauge theory coordinates. The Chern-Simons term is proportional to the SU(4) symmetric symbol d^{ABC}. Thus, this is the origin of the quantum chiral anomaly in the dual $\mathcal{N}=4$ SU(N) SYM theory. From the Chern-Simons term above a three-point interaction in AdS_5 is derived, which leads to the three-point R-symmetry current correlator by using the AdS/CFT correspondence, obtaining the following equation

$$(D^\mu J_\mu(x))^A = \frac{i\,\kappa}{96\pi^2}\, d^{ABC}\, \varepsilon^{\mu\nu\rho\sigma}\, \frac{\partial}{\partial x^\mu} \left(A^B_\nu \partial_\rho A^C_\sigma + \frac{1}{4} f^{CDE} A^B_\nu A^D_\rho A^E_\sigma \right) , \tag{1.4}$$

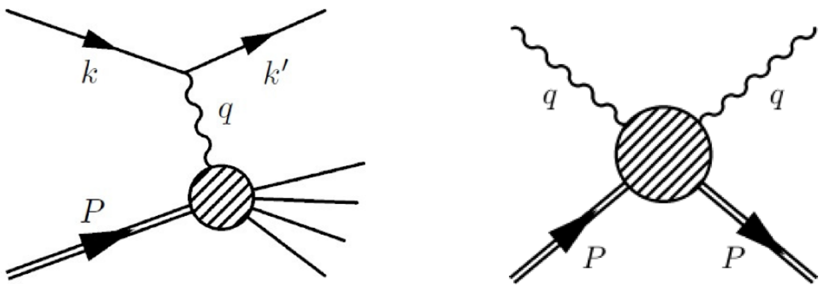
where we have considered the boundary values of the bulk gauge fields of the five-dimensional SU(4) gauged supergravity, $A^A_\mu(x) \equiv \lim_{z\to 0} A^A_\mu(x,z)$, which are sources for the boundary theory global $SU(4)_R$ symmetry currents $J^A_\mu(x)$. By matching equation (1.4) to equation (1.2) it leads to $\kappa = N^2-1$. In addition, the two-point R-symmetry current correlator fixes $g_{SG} = 4\pi/N$. This indicates that in terms of the Witten's diagrams the leading contributions from both terms in the action (1.3) come with the same factor N^2.

Now, let us explain the consequences of the Chern-Simons term for the calculation of the hadronic tensor of a scalar glueball in terms of the gauge/string theory duality.[5] The cubic part of the Chern-Simons term implies that in the holographic calculation of the hadronic tensor, at small values of the Bjorken parameter, the propagation of an $U(1) \subset SU(4)_R$ gauge field in the t-channel is allowed. In the general Lorentz covariant tensor decomposition of the current-current correlator (which enters the definition of the hadronic tensor) this term generates a tensor structure of the form $\varepsilon^{\mu\nu\alpha\beta}\, q_\alpha\, P_\beta/(2P\cdot q)$, proportional to the $F_3(x,q^2)$ structure function. This tensor is not invariant under parity transformations, thus a non-conserving parity structure function appears in $\mathcal{N}=4$ SU(N) SYM theory, and at small values of the Bjorken parameter we find that this is of the same order as the $F_1(x,q^2)$ and $F_2(x,q^2)$ structure functions. On the other hand, at larger values of the Bjorken parameter we find that $F_3(x,q^2)$ is subleading in comparison with $F_1(x,q^2)$ and $F_2(x,q^2)$.

Our findings are interesting since, to our knowledge, this is the first result of the non-preserving parity structure function F_3 for a scalar hadron of $\mathcal{N}=4$ SU(N) SYM theory. We have obtained this in two different ways: firstly from a heuristic calculation in five-dimensional SU(4) gauged supergravity, and then from a first principles type IIB superstring theory calculation. Specifically, for small-x values we obtain $F_3 \propto 1/x$, while for the exponentially small-x region, dominated by the t-channel Reggeized particle exchange, using Pomeron techniques we find $F_3 \propto (1/x)^{1-\frac{1}{2\sqrt{\lambda}}}$. Notice that for QCD in the case of pure electromagnetic interaction $F_3 = 0$ since parity is preserved (see for instance [21, 22]).

This work is organized as follows. In the Introduction we describe DIS in Yang-Mills theories and its description in terms of the gauge/string duality. In section 2 we show a heuristic derivation of the effective Lagrangians from supergravity, which includes symmetric contributions as well as antisymmetric contributions. Then, in section 3 we

[5]There is a previous calculation of the $F_3(x,q^2)$ structure function [20], however this only contains a heuristic five-dimensional approach and it has been done for spin-1/2 hadrons.



(a) (b)

Figure 1. Schematic pictures of DIS (a) and forward Compton scattering (b) processes.

carry out a derivation of the effective action directly from type IIB superstring theory which specifically leads to the antisymmetric structure function F_3. This includes the derivation of the Chern-Simons interaction from the superstring theory scattering amplitude. In section 4 we calculate the antisymmetric structure function F_3 at small x and comment on the exponentially small-x regime. In section 5 we discuss our calculations and results.

1.1 Deep inelastic scattering in Yang-Mills theories

Let us consider a charged lepton with four-momentum k^μ scattered from a hadron with four-momentum P^μ as schematically shown in figure 1.a. The virtual photon carries four-momentum q^μ. The associated differential cross section is proportional to the $l_{\mu\nu}W^{\mu\nu}$ contraction, where $l_{\mu\nu}$ is the leptonic tensor calculated from perturbative QED. In contrast, the hadronic tensor $W^{\mu\nu}$ involves soft processes, therefore it cannot be calculated in perturbation theory. Its matrix elements are defined as two-point functions of a commutator of electromagnetic currents between the initial and final hadronic states with polarizations h and h'

$$W^{\mu\nu}_{hh'} \equiv i \int d^4x \; e^{iq\cdot x} \langle P,h|[J^\mu(x), J^\nu(0)]|P,h'\rangle \,. \tag{1.5}$$

Time-reversal and translational invariance, hermicity restrictions and Ward identities lead to several identities for the hadronic tensor. As a result, it can be written as a sum of Lorentz covariant tensor structures multiplied by the so-called structure functions, which can be seen as functions of the virtual photon momentum transfer q and the Bjorken parameter

$$x = -\frac{q^2}{2P\cdot q}\,, \tag{1.6}$$

whose physical values belong to the range $0 \leq x \leq 1$. The DIS regime corresponds to $q^2 \gg P^2$, keeping x fixed. The hadronic tensor can be decomposed in symmetric and antisymmetric terms under $\mu \leftrightarrow \nu$. In particular, for scalar hadrons this decomposition leads to

$$\begin{aligned}W^{\mu\nu}(x,q^2) &= W_{\rm S}^{\mu\nu}(x,q^2) + i\, W_{\rm A}^{\mu\nu}(x,q^2) \\ &= \left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) F_1(x,q^2) - \left(P^\mu - \frac{P\cdot q}{q^2} q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2} q^\nu\right) \frac{F_2(x,q^2)}{P\cdot q} \\ &\quad + i\,\varepsilon^{\mu\nu\rho\sigma} q_\rho P_\sigma \frac{F_3(x,q^2)}{2P\cdot q} \\ &= \eta^{\mu\nu} F_1(x,q^2) + P^\mu P^\nu \frac{2x}{q^2} F_2(x,q^2) - i\,\varepsilon^{\mu\nu\rho\sigma} q_\rho P_\sigma \frac{x}{q^2} F_3(x,q^2) + \dots \,.\end{aligned} \tag{1.7}$$

The last line of this equation has been rewritten in terms of x and q^2. Also, dots indicate terms proportional to q_μ which can be omitted since after contraction with the leptonic tensor they do not contribute to the DIS differential cross section. Notice, that the third term would not be included if we had imposed parity conservation. However, for $\mathcal{N}=4$ SYM theory a non-vanishing F_3 structure function is expected even for *electromagnetic* DIS.

Since there are contributions from soft processes to the DIS, the structure functions cannot be obtained from pertubative SYM theory. Fortunately, in certain parametric regimes DIS structure functions can be obtained by using the gauge/string theory duality. DIS is related to the forward Compton scattering (FCS) process through the optical theorem. The related amplitude can be written in terms of a tensor defined by the time-ordered expectation value of two electromagnetic currents inside the hadron as follows

$$T^{\mu\nu}_{hh'} \equiv i \int d^4x e^{iq\cdot x} \langle P,h|\hat{\rm T}\{J^\mu(x)J^\nu(0)\}|P,h'\rangle \,. \tag{1.8}$$

The precise relation between the two tensors is given by the following two equations

$$W_S^{\mu\nu} = 2\pi\,{\rm Im}\left[T_S^{\mu\nu}\right]\,, \qquad W_A^{\mu\nu} = 2\pi\,{\rm Im}\left[T_A^{\mu\nu}\right]\,. \tag{1.9}$$

The planar limit of $\mathcal{N}=4$ SU(N) SYM theory is dual to a particular solution of type IIB superstring theory, namely: AdS$_5\times S^5$ background, with a constant dilaton and N units of the flux of the five-form field strength through S^5. It is precisely in this context that the holographic dual picture of DIS was developed in [1]. Moreover, the procedure can be extended to other string theory dual models. In particular, we will focus on the planar limit of $\mathcal{N}=4$ SU(N) SYM theory. In order to break conformal invariance and induce color confinement the standard procedure requires to introduce an IR scale Λ. Then, the hadron is represented by a state of mass $M\sim\Lambda$. On the other hand, conformal symmetry is asymptotically recovered in the UV limit, and at least at tree-level the details of the IR structure are not important. An analogue to the virtual photon of the DIS process is generated by gauging a U(1) subgroup of the SU(4)$_R$ R-symmetry group under which the scalars and the fermions are charged. The conventional choice is to use the $T^3 = {\rm diag}(1/2,-1/2,0,0)$ generator. This leads to charges $\pm 1/2$ for two of the Weyl fermions and charge 1/2 for two complex scalars and the resulting gauge theory is anomaly free since $d_{333}=0$ [23]. In this work we use the three diagonal generators. It leads to a non-vanishing Chern-Simons term in the dual supergravity description.

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The explicit expression for the full non-Abelian conserved current J^A_μ (with $A = 1, \dots, 15$) in terms of the matter fields is given in [20, 23]. DIS of charged leptons from glueballs in the large N limit of $\mathcal{N} = 4$ SU(N) SYM theory has been described in detail in [1], in terms of the operator product expansion (OPE) of the two electromagnetic currents inside the hadron. At weak 't Hooft coupling the OPE is dominated by single-trace twist-two operators. However, at large coupling these operators develop large anomalous dimensions and the main contribution to the OPE is given by double-trace operators together with some specific protected operators such as the energy-momentum tensor and the conserved currents.

On the one hand, one can see that for moderate values of x the characteristics of the scattering are somewhat different in comparison with QCD, namely: the relevant double-trace operators can only create or annihilate an entire hadron, not being able to probe its internal structure. This is related to the fact that particle creation is suppressed in the bulk for $N \to \infty$. One-loop level $(1/N^2)$ corrections within this regime allow for the photon to strike a secondary hadron from the surrounding cloud of hadrons. On the other hand, for much smaller values of the Bjorken parameter, in the $q^2 \to \infty$ limit the OPE is dominated by the protected operators. This is in analogy with the Pomeron description of the Regge regime of QCD. As we will see in detail, this is dual to the t-channel graviton/gauge boson exchange dominance in the bulk.

1.2 Deep inelastic scattering and the gauge/string duality

The holographic dual model to the planar limit of $\mathcal{N} = 4$ SYM theory is given by a solution of type IIB supergravity on AdS$_5 \times S^5$, with radius R and the metric[6]

$$ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) + R^2 d\Omega_5^2 \,. \tag{1.10}$$

In terms of these coordinate the UV boundary is located at $z \to 0$. The relation between the number of color degrees of freedom, N, the 't Hooft coupling λ of the gauge theory, and the parameters of the string theory is given by

$$\frac{R^2}{\alpha'} = \sqrt{4\pi\lambda}\,, \qquad g_s \equiv g^2_{YM}\,, \tag{1.11}$$

where $\alpha' = l_s^2$ is the string length and g_s is the string coupling.

The introduction of an IR scale Λ in the gauge theory corresponds to a cutoff in the small z region. Since the details of the IR are not important, we use an over-simplified deformation known as the hard-wall model, in which the anti-de Sitter description is assumed to be exactly valid up to the point $z_0 = 1/\Lambda$. Since hadronic states at the boundary are dual to normalizable modes in the bulk, by imposing Dirichlet boundary conditions at this point leads to a restriction for the dual hadron mass. In this work, we will focus on glueballs created by operators which are dual to normalizable modes in the Kaluza-Klein (KK) tower associated to the ten-dimensional dilaton field ϕ. For example, for the

[6] We use the following conventions: $M, N, \dots = 0, \dots, 9$ are the ten-dimensional indices, $m, n, \dots = 0, \dots, 4$ are AdS$_5$ indices, $\mu, \nu, \dots = 0, \dots, 3$ are flat four-dimensional indices and $a, b, \dots = 1, \dots, 5$ are S^5 indices.

incoming mode the solution corresponding to a state created by an operator with scaling dimension Δ has a KK mass $R^{-2}\Delta(\Delta-4)$ from the point of view of the five-dimensional theory. Thus, the ten-dimensional field is given by

$$\phi_i(x^\mu, z, \Omega) = c_i \frac{\sqrt{P\Lambda}}{R^4} e^{iP\cdot x} z^2 J_{\Delta-2}(Pz) Y_\Delta(\Omega) \approx \frac{c_i}{\Lambda R^4} e^{iP\cdot x} \left(\frac{z}{z_0}\right)^\Delta Y_\Delta(\Omega), \tag{1.12}$$

where in the last expression we have expanded near the boundary. c_i is some numerical normalization constant and $Y_\Delta(\Omega)$ is a scalar spherical harmonic on the five-sphere.[7] On the other hand, the holographic dual of the virtual photon is given by a non-normalizable mode of a gauge field A_m in the bulk.[8] For the ingoing field, the solution to the associated Einstein-Maxwell equations on AdS$_5$ and the corresponding boundary conditions are

$$\begin{aligned} A_\mu(x^\nu, z) &= n_\mu e^{iq\cdot x}\, qz K_1(qz)\,, & A_z(x^\nu, z) &= -i(n\cdot q)\, e^{iq\cdot x} \frac{z^3}{R^2} K_0(qz)\,, \\ A_\mu(x^\nu, z\to 0) &= n_\mu\, e^{iq\cdot x}\,, & A_z(x^\nu, z\to 0) &= 0\,. \end{aligned} \tag{1.13}$$

We can set the transversal polarization condition $n\cdot q = 0$. The Bessel function of the second kind $K_1(qz)$ vanishes exponentially as qz increases in the bulk, which indicates that the interaction must occur at $z_{\text{int}} \sim 1/q$, leading to a suppression factor $(\Lambda^2/q^2)^{\Delta-1}$, at least when x is not exponentially small.

The gravity counterparts for the different parametric regimes described above from the field theory viewpoint can be obtained by looking at the center-of-mass energy. There is the following parametric relation [1]

$$\tilde{s} \lesssim \frac{z_{\text{int}}^2}{R^2}\, s = \frac{1}{\sqrt{4\pi\lambda}\,\alpha'} \left(\frac{1}{x} - 1\right)\,, \tag{1.14}$$

where s is the four-dimensional Mandelstam variable and $\tilde{s}$ is its ten-dimensional counterpart. Thus, the IIB supergravity description of the bulk dynamics corresponds to the range $1 > x \gg \lambda^{-1/2}$ on the gauge field theory side. In this case the leading amplitude of the dual FCS process is given by an s-channel diagram in type IIB supergravity. In this parametric regime the photon strikes the entire hadron. Then, the DIS structure functions are obtained from the imaginary part of the two-point current correlator by considering the on-shell propagator. In contrast, when $x \ll \lambda^{-1/2}$ we see that $\alpha'\tilde{s}$ is order one, therefore the type IIB superstring theory dynamics becomes relevant, and consequently the exchange of a Reggeized graviton mode dominates. In the $\exp(-\lambda^{1/2}) \ll x \ll \lambda^{-1/2}$ regime the interaction can be thought of as local, thus it can be described in terms of an effective action deduced from flat-space string theory scattering amplitudes. For the smallest parametric region, i.e. when $x \leq \exp(-\lambda^{1/2})$ diffusion effects in the radial direction become important and the interaction cannot be considered local. This region can be described in terms of the Pomeron [1, 25].

[7]The normalization condition is given in the appendix of [24]. In their conventions the spherical harmonic Y_Δ is normalized over the unit five-sphere.

[8]In this work we use the convention $A^3_m \equiv A_m$.

In the holographic picture, the $\mathcal{N} = 4$ SYM R-symmetry group corresponds to the isometry group of the five-sphere, $\mathrm{SU}(4) \sim \mathrm{SO}(6)$. It can be gauged in order to construct a five-dimensional gauged supergravity on AdS$_5$ [2, 5]. From the ten-dimensional perspective, the corresponding gauge fields arise as perturbations of some particular fields which are expanded in modes on S^5. The details of the five-dimensional reduction are given in reference [2]. The excitations of the graviton h_{ma} and the Ramond-Ramond (RR) 4-form a_{mabc} with one index in the AdS$_5$ can be written as

$$h_{ma} = \sum_k B_m^{(k)}(x^n)\, Y_a^{(k)}(\Omega)\,, \qquad a_{mabc} = \sum_k \tilde{B}_m^{(k)}(x^n)\, \epsilon_{abc}{}^{de}\, \nabla_d Y_e^{(k)}(\Omega)\,, \tag{1.15}$$

where ϵ_{abcde} is the Levi-Civita tensor density on S^5, $Y_a^{(k)}(\Omega)$ are vector spherical harmonics, where $k \geq 1$ label their corresponding $\mathrm{SU}(4) \approx \mathrm{SO}(6)$ representations, while $B_m(x^n)$ and $\tilde{B}_m(x^m)$ are vector fields in AdS$_5$. At the lowest level, $k = 1$, the spherical harmonics correspond to the S^5 Killing vectors K_a^A. After diagonalization of the equations of motion associated with these modes, the fifteen AdS$_5$ massless gauge fields arise as the following linear combination

$$A_m^A \equiv B_m^A - \frac{16}{R}\tilde{B}_m^A\,. \tag{1.16}$$

The second contribution can be ignored in the supergravity calculations and also in the construction of the effective action that leads to the symmetric structure functions in the small-x regime [1]. However, for the holographic dual description of the antisymmetric structure functions the second contribution of equation (1.16) must be included.

2 Heuristic effective Lagrangian from supergravity

In the high center-of-mass energy regime, *i.e.* $x \ll 1$, the holographic dual description of DIS in the bulk is given by the exchange of excited strings states. In this situation it is necessary to go beyond the supergravity description. Thus, it requires considering string theory scattering amplitudes, which can be expressed as the product of a pre-factor $\mathcal{G}(\alpha'; s, t, u)$, which contains the α' dependence, and a kinematic factor $\mathcal{K}$. This amplitude is calculated in order to build an effective Lagrangian from which the hadronic tensor can be calculated after evaluating on the solutions of the fields in AdS$_5$.

The effective Lagrangian may be obtained in a heuristic way by analyzing the five-dimensional gauged supergravity diagram of the photon-dilaton to photon-dilaton scattering at tree level, where the leading diagram in the high energy limit is given by the t-channel. This heuristic method was discussed in [26], where non-forward Compton scattering amplitudes for dilatons have been calculated.

The supergravity action on AdS$_5$, with indices $m, n = 0, \ldots, 4$, can be written as

$$S_{5d} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g_{\mathrm{AdS}_5}} \left(-\mathcal{R} + \frac{1}{2}(\partial_m \phi)^2 - \frac{1}{4}\left(F_{mn}^A\right)^2 + \cdots \right) + S_{\mathrm{CS}}\,. \tag{2.1}$$

In this section and in the following one we set $R = 1$, thus $2\kappa_5^2 = 16\pi^2/N^2$. Also, F_{mn}^A is the non-Abelian field strength associated with the gauge fields, ϕ is the dilaton and $\mathcal{R}$

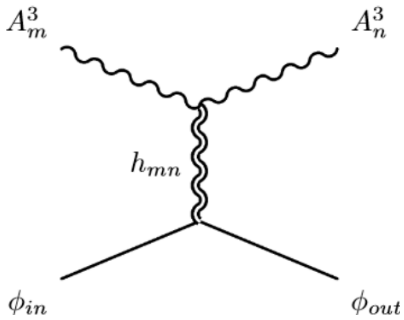

Figure 2. Feynman diagram corresponding to the graviton exchange contribution to the DIS (FCS) process for small values of the Bjorken parameter x.

the Ricci scalar which includes the graviton h_{mn}. Dots include the kinematic terms of the fields not relevant for our analysis, and also the interaction terms of the type $(\phi\phi h)$, (AAh) and $(A\phi\phi)$. The last factor S_{CS} is the Chern-Simons term defined in (1.3).

In this section we calculate the heuristic Lagrangian obtained from the $\phi A \to \phi A$ scattering mediated by a graviton, and show that it coincides with the one calculated in [1] from closed string amplitudes. Then, we will use the same techniques to calculate the effective Lagrangian from which the leading contribution to the antisymmetric structure function $F_3(x, q^2)$ can be obtained. The Lagrangian arises from the $\phi A \to \phi A$ scattering with the exchange of a gauge field in five-dimensional gauged supergravity on AdS$_5$.

2.1 Symmetric contributions

The idea is to calculate the four-point scattering amplitude where the ingoing and outgoing states are given by a dilaton ϕ dual to the hadron, and a gauge field A_m^3 dual to the U(1) current, interacting throughout the exchange of an AdS$_5$ graviton. The process is schematically shown in figure 2. Notice that in reference [26] only the AdS$_5$ components of the field decomposition have been considered, thus ignoring the Kaluza-Klein modes coming from the S^5 because they only contribute with a global constant. Given that the graviton couples to the energy-momentum tensor T_{mn}, the scattering amplitude in terms of the perturbations[9] is given by

$$\mathcal{A} = \kappa_5^2 \int d^5x \, d^5x' \, T^{\phi}_{mn}(x) \, G^{mnkl}(x, x') \, T^{A}_{kl}(x') \,, \tag{2.2}$$

where the AdS$_5$ graviton propagator in the high energy limit can be expressed as [28, 29]

$$G^{mnlk}(x, x') = \left(g^{mk} g^{nl} + g^{ml} g^{nk} - \frac{2}{3} \, g^{mn} g^{kl} \right) G_{\text{grav}}(x, x') \,, \tag{2.3}$$

with $G_{\text{grav}}(x, x')$ being some function that is not relevant in the present case, while the dilaton and gauge field energy-momentum tensors are given by

$$T^{\phi}_{mn} = (g_{mp} g_{nq} + g_{mq} g_{np} - g_{mn} g_{pq}) \partial^p \phi^* \partial^q \phi \,, \qquad T^{A}_{kl} = g^{pq} F_{kp} F_{lq} - \frac{1}{4} g_{kl} F_{pq} F^{pq} \,, \tag{2.4}$$

[9] We parameterize the perturbations as $\Phi \to \Phi_0 + \sqrt{2}\kappa_5 \Phi$, thus neither the energy-momentum tensor nor the propagator have κ_5 factors.

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respectively. The contraction of these three tensors leads to

$$
\begin{aligned}
T^{\phi}_{mn}(x)\, G^{mnlk}(x,x')\, T^{A}_{kl}(x') = {} & G_{\rm grav}(x,x') \\
& \times \Big[2\,(\partial^k\phi^*(x)\partial^l\phi(x) + \partial^l\phi^*(x)\partial^k\phi(x))F_{kp}(x')F_{lq}(x')g^{pq} + \cdots\Big]\,, \qquad (2.5)
\end{aligned}
$$

where we only write the leading terms in the $s\to\infty$ and $t\to 0$ limits. After integration, this expression matches the index structure of $\mathcal{K}|_{t\simeq 0}$ of equation (2.38) of [13].[10]

Since the derivation of this section is heuristic, in order to obtain the same action as in reference [1] we must multiply by the factor $\mathcal{G}(\alpha',\tilde{s},\tilde{t},\tilde{u})\,\tilde{s}^2$ included in the four-point string theory scattering amplitude, where

$$
\mathcal{G}(\alpha',\tilde{s},\tilde{t},\tilde{u}) = -\frac{\alpha'^3}{64}\frac{\Gamma\left(-\alpha'\tilde{s}/4\right)\Gamma\left(-\alpha'\tilde{t}/4\right)\Gamma\left(-\alpha'\tilde{u}/4\right)}{\Gamma\left(1+\alpha'\tilde{s}/4\right)\Gamma\left(1+\alpha'\tilde{t}/4\right)\Gamma\left(1+\alpha'\tilde{u}/4\right)}\,. \qquad (2.6)
$$

While at this level of derivation this is an *ad hoc* factor, it naturally appears when considering the four-point string theory scattering amplitude. It leads to the possibility of exchanging a whole tower of excited string states. This factor is particularly relevant because it leads to a finite contribution from equation (2.2) to the imaginary part of the scattering amplitude. Thus, the effective action turns out to be

$$
S_{\rm eff}^{\rm Sym} = {\rm Im}\left[\mathcal{G}\tilde{s}^2\right]\ \kappa_5^2\, C_{S^5}\int d^5x\,\sqrt{-g_{AdS_5}}\,\partial^k\phi^*\,\partial^l\phi\,F_{kp}\,F_{lq}\,g^{pq}\,, \qquad (2.7)
$$

where the ten-dimensional solutions for the scalars depend on the S^5 coordinates and C_{S^5} is a constant coming from the reduction on S^5.

Note that in (2.7) all fields are evaluated at the same spacetime point, namely: we have built an effective four-point interaction. This is referred as the ultra-local approximation. In the supergravity picture the scattering amplitude can be schematically written in terms of the quantum mechanical operator language as

$$
\mathcal{A}\sim\kappa_5^2\sum_x\sum_{x'}\langle T^{\phi}|x\rangle\langle x|G|x'\rangle\langle x'|T^A\rangle\sim\kappa_5^2\sum_x\langle T^{\phi}|x\rangle\langle x|G|T^A\rangle\,, \qquad (2.8)
$$

where expressions of the form $G_{\rm grav}(x,x')$ correspond to the matrix elements $\langle x|G|x'\rangle$. Now, for the solutions that we have described in the introduction the ten-dimensional curved space Mandelstam variables act as second order differential operators defined by

$$
\tilde{s}_5 = z^2 s + \nabla^2_{z(s)}\,, \qquad (2.9)
$$

$$
\tilde{t}_5 = z^2 t + \nabla^2_{z(t)}\,. \qquad (2.10)
$$

In the graviton propagator $G_{\rm grav}\sim\tilde{t}_5^{-1}$, however the full the string theory pre-factor we included in the previous paragraph depends on both $\tilde{t}_5$ and $\tilde{s}_5$. In the DIS regime at strong coupling, the latter can be thought of as a number instead of an operator since the second term in the r.h.s. of equation (2.9) can be neglected with respect to the first one. However, this is not the case for $\tilde{t}_5$. Nevertheless, at first order there is no $\tilde{t}_5$-dependence in the amplitude, due to the fact that we only have to consider the imaginary part of $\mathcal{G}$. Thus, in this context $\mathcal{G}$ can be thought of as function instead of a differential operator. Therefore

[10] In equation (2.38) of [13] we have corrected several mistakes in equation (82) of [1].

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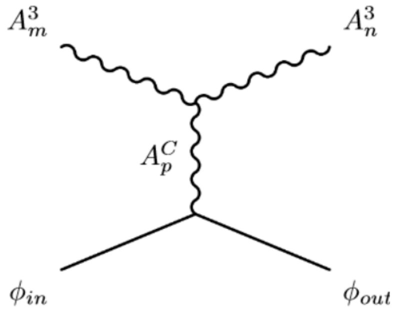

Figure 3. Feynman diagram corresponding to the gauge field exchange contribution to the FCS process for small values of the Bjorken parameter x.

the amplitude can be considered local. More details will be given in section 4. Note that as mentioned in the introduction, this approximation breaks down in the exponentially small-x regime where the last term in equation (2.9) cannot be neglected.

In order to obtain the structure functions from equation (2.7) the on-shell effective action must be calculated by inserting the AdS$_5$ solution for each field and carrying out the integrals.

2.2 Antisymmetric contributions

Up to now we have considered the exchange of a spin-2 field because the amplitude of the process scales as $\tilde{s}^j$ [20, 29, 30]. Now, in order to investigate the leading order contribution to the antisymmetric DIS structure functions at high energy it is necessary to consider the exchange of gauge fields. The action (2.7) derived in the last subsection gives the leading contribution to the symmetric structure functions for the glueball. However, it gives no information about the antisymmetric ones. In the case of QCD one would not expect these structure functions to be present in the electromagnetic DIS. For $\mathcal{N}=4$ SYM theory at $x \simeq 1$ these antisymmetric structure functions are sub-leading in comparison with the symmetric ones F_1 and F_2. However, we can see that the situation is different in the $x \ll 1$ regime, due to the Chern-Simons term present in the supergravity action (1.3). From this term the antisymmetric structure functions arise when a gauge field is exchanged in the t-channel, instead of the usual graviton exchange. Following the procedure of the subsection 2.1, we will derive an effective Lagrangian from which the glueball antisymmetric structure function $F_3(x, q^2)$ can be obtained, giving a contribution of the same order as the symmetric ones.

Since the incoming and outgoing states correspond to two A^3_m gauge fields, through the Chern-Simons term they couple to another A^C_m gauge field which propagates in the AdS$_5$ space. This state couples to two dilatons in the bulk with coupling $\mathcal{Q}^C$. In addition, there are the following eigenvalue equations for the spherical harmonics of the dilaton[11]

$$K^C_a\, \partial^a Y(\Omega) = -\mathcal{Q}^C\, Y(\Omega)\,, \tag{2.11}$$

for K^C associated to the diagonal SU(4) generators.

[11] Note that the equation below differs from the conventional eigenvalue equation $K^a \partial_a \phi = iQ\phi$, see appendix A. This is due to the convention of the generators of SU(4).

Then, the supergravity amplitude becomes

$$\mathcal{A} = \kappa_5^2 \int d^5x d^5x' \; \mathcal{J}_C^m(x)\, G_{mn}^{CD}(x,x')\, J_D^n(x')\,, \tag{2.12}$$

where $\mathcal{J}_C^m$ is the current associated with the Chern-Simons term, while J_D^n is the current associated with the dilaton. These currents are defined as

$$\mathcal{J}_C^m(x) = \frac{i}{6} d_{ABC}\, \varepsilon^{mnopq} \partial_n A_o^A \partial_p A_q^B\,, \qquad J_D^n(x') = -\mathcal{Q}_D\left(\phi\, \partial^n \phi^* - \phi^* \partial^n \phi\right)\,. \tag{2.13}$$

Also, the gauge field propagator can be expressed as[12]

$$G_{mn}^{CD}(x,x') = g_{mn}\, \delta^{CD}\, G_{\text{gauge}}(x,x')\,. \tag{2.14}$$

Then, the integrand of the amplitude (2.12) becomes

$$\mathcal{J}_C^m(x) G_{mn}^{CD}(x,x') J_D^n(x') = -\frac{i}{6} d_{ABC} \mathcal{Q}^C\; \varepsilon^{mnopq} \partial_n A_o^A \partial_p A_q^B\; \left(\phi\, \partial_m \phi^* - \phi^* \partial_m \phi\right)\,. \tag{2.15}$$

As mentioned, the incoming gauge fields correspond to photons A_m^3 related to $K^3 = K_a^3\, \partial^a$, which is the generator of one of the $\mathrm{U}(1)$ subgroups of the $\mathrm{SU}(4)_R$ group. Then, the relevant components of the symmetric symbol are of the form d_{33C}. Only d_{338} and $d_{33,15}$ contribute, which are related to the K^8 and K^{15} diagonal generators of $\mathrm{SU}(4)_R$.

Now, in order to obtain the antisymmetric structure function $F_3(x,q^2)$ in the $x \ll 1$ regime we have to build an effective Lagrangian with the tensor structure of equation (2.15). Then, similarly to the symmetric case described in the previous subsection, we must multiply by the string theory factor. The effective Lagrangian is

$$\begin{aligned} S_{\text{eff}}^{\text{Asym}} &= -\frac{i}{6} d_{ABC} \mathcal{Q}^C\, \mathrm{Im}\left[\mathcal{G}\tilde{s}^2\right]\, \kappa_5^2 \\ &\qquad \int d^5x\, \varepsilon^{mnopq}\, \partial_m A_n^{*A}\, \partial_o A_p^B\, \left(\phi\, \partial_q \phi^* - \phi^* \partial_q \phi\right)\,, \end{aligned} \tag{2.16}$$

where $A = B = 3$ and $C = 8, 15$ for the relevant case.

Next step must be the evaluation of the effective Lagrangian on the AdS_5 solutions. We present the calculation of $F_3(x,q^2)$ in section 4.

Although at this point $\mathcal{G}(\alpha', \tilde{s}, \tilde{t}, \tilde{u})$ has been included as an *ad hoc* pre-factor, it can be understood from the fact that it appears in the four-point scattering amplitudes calculated directly from string theory. Also, by multiplying by an extra $\tilde{s}^2$ we obtain an effective Lagrangian proportional to $1/\tilde{t}$, which is expected when a gauge field is exchanged in the t-channel. This is still a heuristic approach. In the next section we will show explicitly how these factors emerge from closed superstring theory scattering amplitudes.

3 Antisymmetric effective action from string theory

The Lagrangian we have obtained in the previous section from the Chern-Simons term of the five-dimensional SU(4) gauged supergravity on AdS_5 can be obtained from type

[12] There is also a pure gauge component which does not contribute to this process [29].

IIB superstring theory. Then, the *ad hoc* pre-factor can be straightforwardly obtained from a first principles derivation. For that purpose we have to calculate a four-point closed type IIB superstring theory scattering amplitude in the $\tilde{t} \to 0$ limit with insertions corresponding to two dilatons and two gauge fields A^3. In the case of a graviton exchange, the gauge fields are encoded in metric perturbations polarized in a particular way [1]. We start from the string theory scattering amplitude of the form $\mathcal{A}(h, h, \phi, \phi)$. String theory scattering amplitudes include all the possible interchanged modes. Then, a question one should ask is why the leading antisymmetric contributions we found heuristically in the previous section cannot be derived from $\mathcal{A}(h, h, \phi, \phi)$. The subtlety lies in the fact that, as emphasized in [2], the massless gauge fields that appear after the S^5 reduction are actually linear combinations of graviton and RR 4-form field modes. The precise relation is given in equation (1.16). This means that we also have to consider a process with ingoing RR states, i.e. a scattering amplitude of the form $\mathcal{A}(\mathcal{F}_5, \mathcal{F}_5, \phi, \phi)$.

As a consistency check, these RR modes should be associated with the derivation of the Chern-Simons term. In the next section we will show how it can be obtained from the amplitude $\mathcal{A}(\mathcal{F}_5, \mathcal{F}_5, h)$. Then, in subsection 3.2 we will derive the effective Lagrangian from this term. This Lagrangian will be used in section 4 for the calculation of the leading contribution to the structure function $F_3(x, q^2)$ for glueballs.

3.1 Chern-Simons interaction from the superstring amplitude

In this section we derive the structure of the Chern-Simons term of five-dimensional gauged supergravity from type IIB string theory on the $\mathrm{AdS}_5 \times S^5$ background. Firstly, we calculate a three closed string scattering amplitude on flat space-time, and then we evaluate the incoming closed string states on a certain specific *Ansatz*. The *Ansatz* corresponds to the S^5 compactification from the ten-dimensional type IIB supergravity solution generating the effective SU(4)-gauged supergravity on AdS_5 [2, 27, 31]. In this work we will mainly follow the first two references.

Let us focus on the *Ansatz* for the graviton and the RR 4-form field perturbations. The relevant ten-dimensional type IIB supergravity action is given by

$$S_{10d}^{\text{IIB sugra}} = \frac{-1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left(\mathcal{R}_{10\text{d}} - \frac{1}{240} \mathcal{F}_5^2 \right) + \cdots , \tag{3.1}$$

together with the self-duality condition $\mathcal{F}_5 = *\mathcal{F}_5$, where $*$ is the Hodge dual operator in ten dimensions and G_{MN} is the ten-dimensional metric. Recall that $2\kappa_{10}^2 = \mathrm{Vol}\left(S^5\right) 2\kappa_5^2$ with $\mathrm{Vol}\left(S^5\right) = \pi^3$. In this notation the five-form field strength and the self-duality condition are written as

$$\mathcal{F}_{M_1 \dots M_5} = 5\, \partial_{[M_1} a_{M_2 \dots M_5]} , \qquad (*\mathcal{F})_{M_1 \dots M_5} = \frac{1}{5!} \sqrt{-G}\, \varepsilon_{M_1 \dots M_5 N_1 \dots N_5}\, \mathcal{F}^{N_1 \dots N_5} . \tag{3.2}$$

Type IIB supergravity action can be consistently reduced on S^5, obtaining the five-dimensional SU(4) gauged supergravity action (2.1). In [2] it was pointed out that the linearized equations of motion of graviton and four-form field excitations are decoupled from other fields, which means that it is consistent to turn off all other fields and work only with these perturbations. By expanding the fields in scalar, vector and tensor spherical harmonics on S^5, it has been shown that only a particular linear combination of the fundamental modes of both the graviton and the four-form field gives rise to the massless vector modes A_m^A [2]. The form of the relevant perturbations is given by

$$h_{ma} = A_m^B K_a^B \;, \; a_{mabc} \sim A_m^B Z_{abc}^B \tag{3.3}$$

up to a numerical constant, and where the K_a^B are the 15 Killing vectors on S^5 (in other words, the lowest vector spherical harmonics, thus giving the usual Kaluza-Klein *Ansatz* of the metric components). Z_{abc}^A is a pseudo-tensor on S^5 defined from these Killing vectors, the volume form ϵ and the covariant derivatives ∇_a as

$$Z_{abc}^A \equiv \epsilon_{abcde} \nabla^d K^{eA} \,. \tag{3.4}$$

The Levi-Civita tensor is given by

$$\epsilon_{abcde} = \sqrt{g_{S^5}}\; \varepsilon_{abcde} \,, \qquad \epsilon^{abcde} = \frac{1}{\sqrt{g_{S^5}}}\; \varepsilon^{abcde} \,,$$

where ε is the totally antisymmetric symbol such that $\varepsilon_{12345} = \varepsilon^{12345} = 1$.

The starting point is the following flat-space three-point closed superstring theory scattering amplitude[13]

$$\mathcal{A} \sim \int \prod_{i=1}^{3} d^2 z_i \, \langle V_{\rm RR}^{(-\frac{1}{2},-\frac{1}{2})}(z_1,\bar z_1)\, V_{\rm RR}^{(-\frac{1}{2},-\frac{1}{2})}(z_2,\bar z_2)\, V_{\rm NSNS}^{(-1,-1)}(z_3,\bar z_3)\rangle \,, \tag{3.5}$$

where the vertex operators on the two-sphere and the corresponding conventions can be found for example in [32–34] and references therein. In the case we are interested in, the RR modes correspond to self-dual five-form field strength perturbations while the NSNS mode is the graviton. The expression has been explicitly obtained in [34]

$$\mathcal{A}(\mathcal{F}_5^{(1)}, \mathcal{F}_5^{(2)}, h) = -\frac{2i\kappa_{10}}{3}\, h^{MN} \mathcal{F}^{(1)}_{MM_1\ldots M_4} \mathcal{F}_N^{(2)\;M_1\ldots M_4} \,, \tag{3.6}$$

and it corresponds to an interaction term in the type IIB supergravity action which can be obtained by perturbing the $\mathcal{F}_5^2$ term using $G_{MN} \to G_{MN} + h_{MN}$.

Now, the extension of this term to the curved spacetime background can be written as

$$\frac{1}{3\,\kappa_{10}^2} \int_{{\rm AdS}_5\times S^5} d^{10}x \sqrt{-G}\, h^{MN} \mathcal{F}_{MM_1\ldots M_4} \mathcal{F}_N^{\;M_1\ldots M_4} \,. \tag{3.7}$$

By plugging the perturbations (3.3) in the above equation, it is easy to see that the result has the following structure

$$\sqrt{-G}\mathcal{F}_{MM_1\ldots M_4}\mathcal{F}_N^{\;M_1\ldots M_4} h^{MN} \sim \left[\varepsilon^{mnopq}\partial_m A_n^A \partial_o A_p^B A_q^C\right] \left[\sqrt{g_{S^5}}\epsilon_{abcde}\nabla^a K_A^b \nabla^c K_B^d K_C^e\right] . \tag{3.8}$$

Thus, from the ten-dimensional point of view the five-dimensional Chern-Simons term on AdS$_5$ comes with an integral over S^5. The explicit computation of this integral leads to the symmetric symbol d_{ABC}. For that we use equation (A.8) given in the appendix A of the present work.

[13] For details see appendix B.

3.2 The $A + \phi \to A + \phi$ scattering amplitude

The results in the previous section indicate that in order to calculate the effective Lagrangian (2.16) there are two relevant contributions to the $A + \phi \to A + \phi$ scattering amplitude, and particularly we need the one coming from the massless RR-RR-NSNS-NSNS four-point closed string scattering amplitude $\mathcal{A}(\mathcal{F}_5, \mathcal{F}_5, \phi, \phi)$ obtained in type IIB superstring theory. For small values of the Bjorken parameter we have to focus on the $\tilde{t} \to 0$ limit.

The scattering amplitude is given by the worldsheet correlation function[14] of four vertex operators

$$\mathcal{A} \sim \int \prod_{i=1}^{4} d^2 z_i \, \langle V_{\rm RR}^{(-\frac{1}{2},-\frac{1}{2})}(z_1,\bar{z}_1) V_{\rm RR}^{(-\frac{1}{2},-\frac{1}{2})}(z_2,\bar{z}_2) V_{\rm NSNS}^{(-1,-1)}(z_3,\bar{z}_3) V_{\rm NSNS}^{(0,0)}(z_4,\bar{z}_4)\rangle \,. \tag{3.9}$$

Details of the computation can be found in [32]. The final result in the case where the RR-modes correspond to five-form field strength perturbations and the NSNS-modes correspond to the dilaton, is given by the product

$$\mathcal{A}(\mathcal{F}_5^{(1)}, \mathcal{F}_5^{(2)}, \phi^{(3)}, \phi^{(4)}) = \mathcal{G}\,\mathcal{K} \,, \tag{3.10}$$

with $\tilde{s} = -2k_1 \cdot k_4$, $\tilde{t} = -2k_1 \cdot k_2$ and $\tilde{u} = -2k_1 \cdot k_3$ the ten-dimensional Mandelstam variables ($k_1 + k_2 + k_3 + k_4 = 0$ and $\tilde{s} + \tilde{t} + \tilde{u} = 0$), and $\mathcal{K}$ is the kinematic factor

$$\mathcal{K} = -80\kappa_{10}^2 \tilde{s}\,\tilde{u}\,\phi_3\,\phi_4\,\mathcal{F}^{(1)}_{MM_1\ldots M_4}\mathcal{F}_N^{(2)\;M_1\ldots M_4} k_4^M k_4^N \,. \tag{3.11}$$

For the small-x regime and within the ultra-local approximation, we are interested in considering the small-$\tilde{t}$ limit (which is trivial for this particular $\mathcal{K}$ except for the fact that we can take $\tilde{u} = -\tilde{s}$) and constructing an effective four-point interaction Lagrangian that reproduces this scattering amplitude. The Lagrangian associated with $\mathcal{A} = \mathcal{G}\,\mathcal{K}$ in the Einstein frame turns out to be

$$\mathcal{L}_{\mathcal{F}_5\mathcal{F}_5\phi\phi} = -20\kappa_{10}^2 \left[\mathcal{G}(\alpha', \tilde{s}, \tilde{t} \to 0, \tilde{u})\,\tilde{s}^2\right] \mathcal{F}_{MM_1\ldots M_4}\,\mathcal{F}_N{}^{M_1\ldots M_4}\,\partial^{(M}\phi\,\partial^{N)}\phi \,. \tag{3.12}$$

Finally, the full effective action written in terms of the gauge fields and the Killing vectors associated with the expansion on S^5 is obtained by writing the curved-space version of the effective action corresponding to (3.12) and inserting the explicit form of the $\mathcal{F}_5$ perturbations (3.3). This yields an integrand of the form

$$\left[\mathcal{G}(\alpha', \tilde{s}, \tilde{t} \to 0, \tilde{u})\,\tilde{s}^2\right] \sqrt{g_{S^5}} \left(\epsilon^{abcde}\nabla_a K_b^A \nabla_c K_d^B\right) \left(\varepsilon^{mnopq}\partial_m A_n^A \partial_o A_p^B\right) \partial_{(e}\phi\,\partial_{q)}\phi \,. \tag{3.13}$$

By using the relation (2.11) and the Killing vector identity (A.9) presented in appendix A, in the ingoing/outgoing convention we see that both the symmetric symbol d_{ABC} and the current associated with dilaton come from

$$\left(\epsilon^{abcde}\nabla_a K_b^A \nabla_c K_d^B\right) \partial_{(e}\phi\,\partial_{q)}\phi^* = \frac{4i}{R}\,d_{ABC}\,K_C^e \partial_{(e}\phi\,\partial_{q)}\phi^* = \frac{2i}{R}\,d_{ABC}\,J_q^C \,, \tag{3.14}$$

[14] In this section we use the standard convention for string theory scattering amplitudes where all external states are ingoing. We will switch back to the $A + \phi \to A + \phi$ notation in the next section, where two of the states will be taken to be outgoing.

where J_m^C is the second of the currents (involving dilatons) given in equation (2.13). Also, the d_{ABC} factor combined with the second parentheses of equation (3.13) renders the Chern-Simons current $\mathcal{J}_n^D$. This means that we obtain the structure anticipated in equation (2.15). These results are in full agreement with the effective action (2.16) we predicted in section 2.3 using heuristic arguments. Finally, remember that for the particular process studied in this paper we will focus on the contribution proportional to d_{33C}.

4 Antisymmetric structure function F_3 at small x

In this section we obtain the antisymmetric structure function $F_3(x, q^2)$ for the glueball, following the conventions of reference [1].[15] We recover R factors wherever it corresponds. As explained in the introduction, in the $e^{-\sqrt{\lambda}} \ll x \ll \lambda^{-1/2}$ regime the holographic method consists in evaluating the on-shell amplitude associated with the effective supergravity process and taking its imaginary part. Then, if we separate the hadronic tensor into its symmetric and antisymmetric parts as $T^{\mu\nu} = T^{\mu\nu}_{sym} + iT^{\mu\nu}_{\rm asym}$ (and the same for $W^{\mu\nu}$) the AdS/CFT dictionary implies [20, 35]

$$-\,iS_{\rm eff}^{\rm Asym} \equiv n_\mu n_\nu^* \,{\rm Im}\left(T^{\mu\nu}_{\rm Asym}\right) = \frac{1}{2\pi} n_\mu n_\nu^* W^{\mu\nu}_{\rm Asym}\,, \tag{4.1}$$

where the last equality follows from the optical theorem. The calculation of F_3 is similar to the one corresponding to the symmetric structure functions F_1 and F_2 presented in [1]. The starting point is the effective action proposed in section 2 from heuristic arguments and derived from first principles in section 3. Considering two ingoing states and two outgoing states, this on-shell action is given by

$$\begin{aligned} S_{\rm eff}^{\rm Asym} &= i\frac{R}{6}\, d_{33C}\,\mathcal{Q}^C\, {\rm Im}\left[\mathcal{G}(\alpha', \tilde{s}, \tilde{t}\to 0, \tilde{u})\, \tilde{s}^2\right] \\ &\quad\times \int d^5\Omega\,\sqrt{g_{S^5}} \int d^5x\, \varepsilon^{mnopq}\partial_m A_n^{3*}\partial_o A_p^3 \left(\phi\,\partial_q\phi^* - \phi^*\partial_q\phi\right)\,. \end{aligned} \tag{4.2}$$

The AdS$_5$ solutions we have to insert are given by (1.12) and (1.13). Also, let us recall that the relation between the ten-dimensional invariant $\tilde{s}$ and the four-dimensional one is

$$\alpha'\tilde{s} \approx \alpha' s\frac{z^2}{R^2}\,, \tag{4.3}$$

in the regime under consideration and up to corrections from the radial and S^5 components of order $\alpha'/R^2 \sim \lambda^{-1/2}$ which can be neglected.

As in the symmetric case, by taking the $\tilde{t}\to 0$ limit, the imaginary part of the prefactor can be replaced by a sum over excited string states [1]. The exact result is

$${\rm Im}_{\rm exc}\left[\mathcal{G}(\alpha', \tilde{s}, \tilde{t}, \tilde{u})\ \tilde{s}^2\right]|_{\tilde{t}\to 0} = \frac{\pi\alpha'}{4}\sum_{m=1}^{\infty}\delta\left(m - \frac{\alpha'\tilde{s}}{4}\right)(m)^{\frac{\alpha'\tilde{t}}{2}}, \tag{4.4}$$

[15] Note that in [1] the normalization of the fields is such that the interaction term between the dilaton and the gauge field is

$$S_{\rm int} = iQ^C\int d^{10}x\sqrt{g}A^m_C\left(\phi^*\partial_m\phi - \phi\partial_m\phi^*\right).$$

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where the last factor can be ignored if x is not exponentially small, i.e. when $e^{-\sqrt{\lambda}} \ll x \ll \lambda^{-1/2}$. This sum can be expressed in terms of $\omega = qz$ as

$$\sum_m \delta\left(m - \frac{\alpha' \tilde{s}}{4}\right) = \sum_{\omega_m} \left(\frac{2q^2R^2}{\alpha' s\,\omega}\right) \delta\left(\omega - \omega_m\right) , \quad \omega_m^2 \equiv m\frac{2R^2q^2}{\alpha' s} , \tag{4.5}$$

which is well approximated by an integral for $x \ll \lambda^{-1/2}$.

Plugging the solutions for the gauge fields and the dilaton current together with equation (4.5) in the on-shell effective action (4.2), and working out the integration over the full ten-dimensional spacetime, we find

$$n_\mu n_\nu^* W^{\mu\nu}_{\text{asym}} = |c_i'|^2 \frac{\pi^2}{3} \frac{\mathcal{Q}\,\mathcal{I}_\Delta}{\sqrt{4\pi\lambda}} \left(\frac{\Lambda^2}{q^2}\right)^{\Delta-1} n_\mu n_\nu^*\, \varepsilon^{\mu\nu\rho\sigma} \frac{q_\rho P_\sigma}{2P\cdot q}\frac{1}{x} \tag{4.6}$$

where the charge is $\mathcal{Q} = d_{33C}\mathcal{Q}^C$, while $\mathcal{I}_\Delta$ stands for the ω integral over the Bessel functions

$$\mathcal{I}_\Delta = \int d\omega\, \omega^{2\Delta+2}\, K_0(\omega)\, K_1(\omega) = \frac{\sqrt{\pi}}{4} \frac{\Gamma\left(\Delta+1\right)^2 \Gamma\left(\Delta+2\right)}{\Gamma\left(\Delta+\frac{3}{2}\right)} . \tag{4.7}$$

Now, by comparison of equation (4.6) with the general decomposition (1.7) we obtain the antisymmetric structure function for the glueball

$$F_3(x,q^2) = \frac{1}{x}\left(\frac{\Lambda^2}{q^2}\right)^{\Delta-1} \frac{\mathcal{Q}\,|c_i'|^2\pi^2}{3\sqrt{4\pi\lambda}}\,\mathcal{I}_\Delta . \tag{4.8}$$

Let us recall that for the dilaton in the $\exp(-\sqrt{\lambda}) \ll x \ll \lambda^{-1/2}$ regime, one obtains the following symmetric structure functions

$$F_1(x,q^2) = \frac{1}{x^2}\left(\frac{\Lambda^2}{q^2}\right)^{\Delta-1} \frac{\pi^2\rho_\Omega|c_i|^2}{4\sqrt{4\pi\lambda}} I_{1,2\Delta+3} , \qquad F_2(x,q^2) = 2x\frac{2\Delta+3}{\Delta+2}F_1(x,q^2) , \tag{4.9}$$

were ρ_Ω is a dimensionless constant coming from the angular integral of the symmetric effective action, defined in equation (88) of [1], and

$$I_{1,2\Delta+3} = \int d\omega\,\omega^{2\Delta+3}\,K_1^2(\omega) \; = \frac{(2\Delta+2)(\Delta+2)}{2\Delta+3}\mathcal{I}_\Delta . \tag{4.10}$$

We find new Callan-Gross like relations that can be expressed as:

$$F_3(x,q^2) = \frac{\mathcal{Q}}{\rho_\Omega}\frac{4}{3}(\Delta+1)F_2(x,q^2) = \frac{\mathcal{Q}}{\rho_\Omega}\frac{8}{3}\frac{(2\Delta+3)(\Delta+1)}{(\Delta+2)}xF_1(x,q^2) . \tag{4.11}$$

One subtle difference arises from the $\mathcal{Q}$ factor: for F_3 to be non-vanishing the hadron must be charged under the U(1) groups associated to the T_8 and T_{15} generators, while this is not necessary for the symmetric functions.

Note that our result of equation (4.8) is in agreement with the behavior found in [36] for the spin-1/2 case given by a dilatino mode. In the mentioned work, the antisymmetric structure functions are computed in the exponentially small-x regime, but one can extrapolate the result by considering the ultra-local approximation.

4.1 Comments on the exponentially small-x regime

In the exponentially small-x regime the ultra-local approximation does not hold due to diffusion effects in the radial direction of AdS_5 become important. This happens because the last factor in equation (4.4) cannot be neglected. Thus, one must consider the full differential operator of equation (2.10) [1, 13, 25, 37–39].

In the symmetric case, this leads to the interchange of a Pomeron. Let us start by reviewing this in the conformal limit. The differential operator acts on $\partial_\mu\phi\,\partial_\nu\phi^*$. More concretely, it is given by the spin-2 Laplacian, and the exponent reads

$$\frac{\alpha'\tilde{t}}{2} = \frac{1}{2}\frac{\alpha'}{R^2}z^2 t + \frac{1}{2}\frac{\alpha'}{R^2}\Delta_2 = \frac{1}{2}\frac{\alpha'}{R^2}z^2 t + \frac{1}{2\sqrt{\lambda}}\left[z^2\partial_z^2 + z\partial_z - 4\right]. \tag{4.12}$$

We will set $t = 0$. Δ_2 is a particular case of the Hodge-de Rham operator, defined more generally by

$$R^2\Delta_j = z^2\partial_z^2 + (2j-3)z\partial_z + j(j-4)\,. \tag{4.13}$$

It can be evaluated in terms of an auxiliary quantum mechanical problem where $u = -\log(z/z_{\mathrm{ref}})$ plays the role of time and $H = -z^2\partial_z^2 - z\partial_z + 4 = -\partial_u^2 + 4$ is the Hamiltonian. In the conformal case z_{ref} is an arbitrary scale and there is no cut-off in the AdS spacetime. One can then diagonalize this operator in terms of its eigenfunctions, which are plane waves in u with energies $E_\nu = \nu^2 + 4$. Then, the scattering amplitude can be written in terms of a kernel which in the $\tilde{t} \to 0$ limit takes the form

$$\mathcal{K}(u, u', t = 0, j = 2) = \left(\alpha'\tilde{s}\right)^{2-\frac{2}{\sqrt{\lambda}}}\sqrt{\frac{\lambda^{1/2}}{2\pi\tau}}e^{-\frac{\sqrt{\lambda}}{2\tau}(u-u')^2}\,, \tag{4.14}$$

where $\tau = \log(\alpha'\tilde{s})$. Note that the s^2 factor was already present in the ultra-local approximation of the scattering amplitude. It reflects the appropriate scaling with the center-of-mass energy for a graviton exchange. The $(u-u')^2$ dependence in the exponential is known as the diffusion factor and the inverse of its coefficient gives the associated characteristic diffusion time. The final DIS amplitude is obtained by evaluating the rest of the gauge-field part of the effective Lagrangian at u and the ϕ-field part at u', and integrating. For example, the part of the on-shell effective action that contributes to the $F_1(x,q^2)$ structure function reads[16]

$$\begin{aligned}
n_\mu n_\nu^* T^{\mu\nu}_{\mathrm{Sym}}|_{F_1} &= \frac{\pi\alpha'\rho_\Omega R^2}{2}\int dz\sqrt{-G}F^{mn}F^{p*}_{n}|_{F_1}\left(\frac{\alpha'\tilde{s}}{4}\right)^{\alpha'\tilde{t}/2|_{t=0}}\partial_m\phi^*\partial_p\phi \\
&= \frac{\pi\rho_\Omega}{8}\lambda^{1/2}\int\frac{dz}{z}\left[qzK_1(qz)\right]^2\left(\alpha'\tilde{s}\right)^{2+\alpha'\tilde{t}/2|_{t=0}}\left[\frac{R^4}{z}|\phi(z)|\right]^2 \\
&= \frac{\pi\rho_\Omega}{8}\lambda^{1/2}\int\frac{dz}{z}\frac{dz'}{z'}\left[qzK_1(qz)\right]^2\left(\alpha'\tilde{s}\right)^{2+\frac{1}{2\sqrt{\lambda}}\Delta_2}\delta(u(z)-u'(z'))\left[\frac{R^4}{z'}|\phi(z')|\right]^2 \\
&\equiv \frac{\pi\rho_\Omega}{8}\lambda^{1/2}\int du\,du'P_A^{(1)}(u)\mathcal{K}(u,u',t=0,j=2)P_\phi(u'),
\end{aligned} \tag{4.15}$$

[16] This expression is valid after the angular integration on S^5. Also the scalar solution ϕ does not include the scalar spherical harmonic. Finally, notice that in these expressions we have absorbed a factor of 4π in the definition of the 't Hooft coupling λ.

where $P_A^{(1)}(u(z)) = q^2 z^2 K_1^2(qz)$ and $P_\phi(u(z)) = R^8 z^{-2}|\phi(z)|^2 \approx (z\Lambda)^{2\Delta-2}$ are scalar factors that only depend on the corresponding incoming solutions, and all contractions are made with the curved metric. In the last step we have written everything in terms of u and inserted an identity of the form $\int du' \delta(u-u') = \int du' \int \frac{d\nu}{2\pi} e^{i\nu(u-u')}$, which naturally leads to the appearance of the spin-2 kernel. Now, due to the optical theorem $F_1(x,q^2)$ is obtained simply by multiplying by a 2π factor. A similar expression can be found for $F_2(x,q^2)$ by replacing $P_A^{(1)} \to P_A^{(2)}(u(z)) = q^2 z^2 (K_0^2(qz) + K_1^2(qz))$ and inserting an extra factor of $2x$. Of course, in these final formulas the x-dependence is hidden in the s and τ factors since in this regime the four-dimensional Mandelstam invariant is $s \approx q^2/x$. Also, the result for the parametric region $\exp(-\lambda^{1/2}) \ll x \ll \lambda^{-1/2}$ is formally recovered in the large λ limit.

When the cut-off at z_0 is introduced in the AdS$_5$ spacetime to induce confinement general steps of the above calculation remain valid, but one has to impose boundary conditions on z_0, consistent with energy-momentum conservation. Taking the reference value as $z_{\mathrm{ref}} = z_0$, the boundary condition on the Pomeron modes h_{++},[17] and the resulting eigenfunctions are

$$\partial_z(z^2 h_{++})|_{z_0} = 0 \;\Rightarrow\; h_\nu(u) = \frac{1}{\sqrt{2}}\left[e^{i\nu u} + \left(\frac{\nu-2i}{\nu+2i}\right)e^{-i\nu u}\right]. \tag{4.16}$$

Therefore, in the $t \to 0$ limit the conformal kernel must be replaced by

$$\mathcal{K}_\Lambda(u,u',t=0,j=2) = \left(\alpha'\tilde{s}\right)^{2-\frac{2}{\sqrt{\lambda}}}\sqrt{\frac{\lambda^{1/2}}{2\pi\tau}}\left[e^{-\frac{\sqrt{\lambda}}{2\tau}(u-u')^2} + F(u,u',\tilde{\tau})e^{-\frac{\sqrt{\lambda}}{2\tau}(u+u')^2}\right] \tag{4.17}$$

where $\tilde{\tau} = (4\lambda)^{-1/2}\tau$ and we have defined the function

$$F(u,u',\tilde{\tau}) = 1 - 4\sqrt{\pi\tilde{\tau}}e^{\eta^2}\mathrm{erfc}(\eta)\,, \tag{4.18}$$

with

$$\eta = \frac{u+u'+4\tilde{\tau}}{\sqrt{4\tilde{\tau}}}\,,\quad \mathrm{erfc}(\eta) = 1-\mathrm{erf}(\eta) = \frac{2}{\sqrt{\pi}}\int_\eta^\infty dx\, e^{-x^2}. \tag{4.19}$$

Note that $-1 < F(u,u',\tilde{\tau}) < 1$. These results are important to understand the holographic DIS process at high energies. In fact, the structure of the amplitude at strong coupling written in terms of the Pomeron kernel has a striking formal resemblance with the one obtained at weak coupling. Also, the comparison with the available data for DIS at small x leads to some very interesting results [39].

The process we have been analyzing is such that the leading contribution to the F_3 antisymmetric structure function comes from the exchange of a Reggeized gauge field. This was also pointed out in the DIS from dilatinos in reference [20].[18] As we have seen in section 2, this vector mode interacts with the currents instead of the energy-momentum tensors, implying that we have to consider the spin-one differential operator. Thus, in this case we have

$$\frac{\alpha'\tilde{t}}{2} = \frac{1}{2}\frac{\alpha'}{R^2}z^2 t + \frac{1}{2}\frac{\alpha'}{R^2}(\Delta_1+3) = \frac{1}{2}\frac{\alpha'}{R^2}z^2 t + \frac{1}{2\sqrt{\lambda}}\left[z^2\partial_z^2 - z\partial_z\right]. \tag{4.20}$$

[17] We are using light-cone coordinates. Also, we consider only modes which are relevant in the high energy limit.

[18] Since the analysis of [20] is similar to what we need for the dilaton case, we only outline the important steps.

By introducing $\rho = 2u = -2\ln(z/z_{\mathrm{ref}})$ we can rewrite $\Delta_1 + 3 = 4(\partial_\rho^2 + \partial_\rho)$. After diagonalization of the relevant operator, we obtain a conformal kernel of the form

$$\mathcal{K}(\rho, \rho', t = 0, j = 1) = \left(\alpha' \tilde{s}\right)^{1-\frac{1}{2\sqrt{\lambda}}} e^{-\frac{1}{2}(\rho+\rho')} \sqrt{\frac{\lambda^{1/2}}{2\pi\tau}} e^{-\frac{\sqrt{\lambda}}{8\tau}(\rho-\rho')^2}. \tag{4.21}$$

The Regge slope is now $1 - 1/(2\sqrt{\lambda})$ since both the scaling with the center-of-mass and its curvature correction change. Note that this implies that $F_3(x, q^2)$ grows more rapidly as $x \to 0$. Also, diffusion in $\rho(z)$ is still present. Now, let us consider the effect of introducing a cut-off in the AdS$_5$ spacetime. The boundary condition on the Reggeized gauge field modes A_+ is

$$\partial_z \left(z A_+\right)|_{z_0} = 0\,. \tag{4.22}$$

However, the eigenfunctions are modified in such a way that this condition is actually analogous to the one above, leading to an identical modification of the kernel. Therefore, we obtain

$$\begin{aligned}\mathcal{K}_\Lambda(\rho, \rho', t = 0, j = 1) &= \left(\alpha' \tilde{s}\right)^{1-\frac{1}{2\sqrt{\lambda}}} e^{-\frac{1}{2}(\rho+\rho')} \\ &\times \sqrt{\frac{\lambda^{1/2}}{2\pi\tau}} \left[e^{-\frac{\sqrt{\lambda}}{8\tau}(\rho-\rho')^2} + F(\rho/2, \rho'/2, \tilde{\tau}) e^{-\frac{\sqrt{\lambda}}{8\tau}(\rho+\rho')^2}\right]\end{aligned} \tag{4.23}$$

The final form of the structure function in this regime is given by complicated integrals in ρ and ρ'. The formal expression obtained for F_3 can be split in the conformal $F_3^{\mathrm{conformal}}$, i.e. from the complete AdS$_5$ spacetime with no IR cut-off, plus the contribution from the deformation induced by the IR cut-off:

$$F_3 = F_3^{\mathrm{conformal}} + F_3^{\mathrm{deformation}}\,. \tag{4.24}$$

The explicit result is obtained by following the same steps that led us to equation (4.15), together with the insertion of the hard-wall spin-1 kernel instead of the conformal one. Thus, $F_3(x, q^2)$ is given by

$$\begin{aligned}F_3(x, q^2) &= \frac{\mathcal{Q}\pi^2}{12} \int d\rho\, d\rho'\, \mathcal{P}_A(\rho, q) \\ &\times \left\{ \left(\alpha' \tilde{s}\right)^{1-\frac{1}{2\sqrt{\lambda}}} \sqrt{\frac{\lambda^{1/2}}{2\pi\tau}} \left[e^{-\frac{\sqrt{\lambda}}{8\tau}(\rho-\rho')^2} + F(\rho/2, \rho'/2, \tilde{\tau}) e^{-\frac{\sqrt{\lambda}}{8\tau}(\rho+\rho')^2}\right] \right\} P_\phi(\rho', \Lambda)\,,\end{aligned} \tag{4.25}$$

where

$$\mathcal{P}_A(z, q) = q^3 z^3 K_0(qz) K_1(qz)\,, \quad P_\phi(z', \Lambda) = R^8 z'^{-2} |\phi(z')|^2. \tag{4.26}$$

The information about the x-dependence is contained in the $\tilde{s}$ and τ factors. Note that the contribution from the IR cut-off is model dependent, in the sense that it is sensible to how the AdS$_5$ space is deformed near z_0. However, the conformal contribution is model independent.

It is difficult to obtain an analytical expression for the above integral (4.25). However, some approximations obtained from simplification of the scalar and gauge field external solutions lead to interesting results [39]. In that reference the wave-functions products were approximated by Dirac delta functions determined by the relevant scales

$$\mathcal{P}_A(z,q) \approx \frac{1}{q}\delta\left(z-\frac{1}{q}\right) , \quad P_\phi(z') \approx \frac{1}{M}\delta\left(z'-\frac{1}{M}\right) , \tag{4.27}$$

where M stands for some relevant mass scale, for example the proton mass. It should be directly related to the IR cut-off scale identified as $\Lambda \sim \Lambda_{\mathrm{QCD}}$. With this approximation, the final expression for the conformal contribution takes the form

$$\begin{aligned} F_3(x,q^2) &\approx \frac{\mathcal{Q}\pi^2}{3}(\alpha'\tilde{s})^{1-\frac{1}{2\sqrt{\lambda}}}\sqrt{\frac{\lambda^{1/2}}{2\pi\tau}}e^{-\frac{\sqrt{\lambda}}{2\tau}\log^2(q/M)} \\ &\approx \frac{1}{x}\frac{\mathcal{Q}\pi^2}{3\sqrt{\lambda}}\left(\frac{q}{M}\right)(\alpha'\tilde{s})^{-\frac{1}{2\sqrt{\lambda}}}\sqrt{\frac{\lambda^{1/2}}{2\pi\tau}}e^{-\frac{\sqrt{\lambda}}{2\tau}\log^2(q/M)}, \end{aligned} \tag{4.28}$$

while the hard-wall contribution can be obtained using a similar simplification. Note that in the context of the kernel notation $\tilde{s}$ (and τ) should be thought of as symmetrized in z and z', and in (4.28) this implies $\alpha'\tilde{s} = \frac{s}{\sqrt{\lambda}}\frac{1}{qM} = \frac{1}{\sqrt{\lambda}}\frac{1}{x}\frac{q}{M}$ [25, 39].

5 Discussion

In this work we describe how the antisymmetric structure function $F_3(x,q^2)$ is obtained in the dual holographic description of DIS of charged leptons from glueballs at small-x in $\mathcal{N}=4$ SYM theory deformed by the introduction of the IR scale Λ. The reason for the non-vanishing $F_3(x,q^2)$ comes from the chiral anomaly of $\mathcal{N}=4$ SYM theory, which does not depend on the IR deformation. This anomaly can be seen from the three-point correlation function of current operators, and is proportional to the symmetric symbol d_{ABC}. From the string theory point of view this comes from the S^5 dimensional reduction of type IIB supergravity, which leads to the SU(4) gauged supergravity on AdS$_5$. Its action contains the Chern-Simons term proportional to the symbol d_{ABC}. Thus, there is a deep connection between the chiral anomaly of the $\mathcal{N}=4$ SYM theory and the emergence of $F_3(x,q^2)$. On the other hand, the fact that the chiral anomaly is related to the Chern-Simons term in the bulk is reflected in the fact that $F_3(x,q^2)$ has the power dependence in the Bjorken variable which comes from the propagation of a gauge field in the t-channel Feynman-Witten diagram of SU(4) gauged supergravity in the bulk.

In QCD F_3 is zero for the electromagnetic DIS, i.e. a charged lepton scattered from a hadron with exchange of a virtual photon, due to the fact that this particular structure function does not preserve the parity symmetry. Of course, this would not be the case when considering an interaction mediated by a $W^\pm$ or Z^0 gauge boson such as in neutrino DIS. However, though QCD and IR-deformed $\mathcal{N}=4$ SYM theories may share some generic features in the large-N, these gauge theories are essentially different. In particular, $\mathcal{N}=4$ SYM theory is chiral. The R-symmetry current associated with the global $\mathrm{U}(1)_R \subset \mathrm{SU}(4)_R$ can be gauged in order to describe the *electric* current, therefore allowing for the construction of a bulk dual photon which mediates the DIS process. The F_3 structure function was not analyzed in the original calculation developed in [1], but it was taken into account to some extent in related papers such as [20, 35] from a heuristic viewpoint for the case of spin-1/2 hadrons. From our results it is not possible to derive implications for neutrino DIS within $\mathcal{N} = 4$ SYM theory. A possibility to deal with a holographic dual description of neutrino DIS is to consider a non-chiral holographic dual such as the model based on the D4D8anti-D8-branes constructed by Sakai and Sugimoto from type IIA superstring theory [40], in that case for instance scalar and polarized vector mesons have been described, as well as baryons as skyrmions [41].

In the supergravity regime, i.e. when $\lambda^{-1/2} \ll x < 1$, the amplitude is dominated by the s-channel diagram and the corresponding contributions to F_3 are sub-leading in comparison with the symmetric structure functions F_1 and F_2 for glueballs. However, for polarized spin-1/2 hadrons $F_3 = F_2 = 2F_1$ due to the form of the associated AdS$_5$ solutions [35]. The scattering process in this context is the same as for the parity-preserving structure functions F_1 and F_2.

It is interesting to describe how our results for the supergravity regime $(1/\sqrt{\lambda} \ll x < 1)$ are related to a superstring theory analysis $(\exp(-\sqrt{\lambda}) \ll x \ll 1/\sqrt{\lambda})$. First of all, we should notice that in the supergravity regime the ten-dimensional Mandelstam s-channel variable satisfies the condition $\alpha' \tilde{s}_{10D} \ll 1$, which implies that for moderate values of the Bjorken parameter only supergravity modes take part in the dynamics. On the other hand, for smaller values of x, massive string modes become relevant, therefore a string theory analysis is required.

A very interesting connection between the string theory and the partonic regimes was proposed in the original paper by Polchinski and Strassler [1]. Let us briefly recall it. Consider a closed string, in an inertial frame its tension is constant T_{10}. However, as seen by a four-dimensional observer the string tension T_4 is proportional to the metric warp factor, i.e. $T_4 = r^2/R^2\, T_{10}$, being $z = R^2/r$. Thus, from a four-dimensional perspective the string tension increases as the string approaches the AdS$_5$ boundary, and it therefore shrinks. This implies that the more efficient way for the string to scattered is to tunnel to large enough r values, being its size of order $1/q$. This leads to a power law suppression in the scattering amplitude, which indeed goes like $(\Lambda^2/q^2)^{\tau-1}$, being τ the twist of the hadronic scattered state. Since the whole string tunnels from IR towards UV, this means that for large λ the four-dimensional hadronic state does not contain point-like partons. In the large N limit hadron production is suppressed, while for finite values of N the probability that the virtual photon strikes a hadron surrounding its *parent* hadron is high enough so that, for sufficiently large virtual photon momentum transfer q, the terms proportional to $1/N^2$ become dominant in the OPE of two electromagnetic currents (JJ OPE). On the other hand, for small λ the probability that the photon strikes a parton is high, therefore the appropriate description is perturbative quantum field theory (QFT). Also, these ideas were very nicely discussed in [1] in the context of the JJ OPE in terms of the interplay between the anomalous dimensions of twist-two operators, which dominate for the perturbative QFT regime, and the protected operators, which are the leading ones in the non-perturbative regime.

For the scalar glueball we have the following schematic behavior. On the left we consider small values of x (string theory description) and on the right of the arrow we write down the behavior for larger values of x (supergravity description),

$$F_1 \propto \frac{1}{x^2}\left(\frac{\Lambda^2}{q^2}\right)^{\tau-1}\frac{1}{\sqrt{\lambda}} \to 0\,, \tag{5.1}$$

$$F_2 \propto \frac{1}{x}\left(\frac{\Lambda^2}{q^2}\right)^{\tau-1}\frac{1}{\sqrt{\lambda}} \to \left(\frac{\Lambda^2}{q^2}\right)^{\tau-1} x^{\tau+1}(1-x)^{\tau-2}\,, \tag{5.2}$$

$$F_3 \propto \frac{1}{x}\left(\frac{\Lambda^2}{q^2}\right)^{\tau-1}\frac{1}{\sqrt{\lambda}} \to \mathcal{O}(N^{-2})\,. \tag{5.3}$$

Thus, while for the $1/\sqrt{\lambda} \ll x < 1$ regime $F_1 = 0$ since it is proportional to the Casimir of the scalar glueball under the Lorentz group, F_2 leads to a non-vanishing function of x, keeping the same q^2-power fall-off. In addition, the behavior of F_3 for moderate values of x becomes sub-leading, i.e. it is suppressed by powers of N. This can be straightforwardly seen from the fact that in the small-x regime the dominant contribution comes from a t-channel tree-level Witten diagram. On the other hand, for the supergravity regime the dominant s-channel Witten diagram which leads to contributions to the antisymmetric part of the hadronic tensor implies the exchange of at least two on-shell particles, thus it is suppressed by $1/N^2$, and in the large N limit it does not contribute to F_3.

In the exponentially small-x regime the situation changes drastically because excited strings must be included as intermediate states. The dominant diagrams are given by t-channel Reggeized particle exchange. In the original description these modes belong to the tower of states associated with the graviton. This leads to the x-dependence for F_1 and F_2 of the form[19] $x^{-2+2/\sqrt{\lambda}}$ and $x^{-1+2/\sqrt{\lambda}}$, respectively. However, this process only gives contributions to the structure functions which characterize the symmetric part of the hadronic tensor $W^{\mu\nu}$. This is not the right place to look for F_3. After the graviton exchange, the next-to-leading order contribution in terms of center-of-mass energy scaling is given by gauge field exchange. As we have shown, it is in this context that the leading antisymmetric contributions appear.

As originally suggested in [20], the presence of the cubic Chern-Simons interaction in the five-dimensional gauged supergravity theory is crucial, as it leads to the possibility of a gauge field exchange with the necessary four-dimensional index structure. We have described the corresponding scattering amplitude from two different perspectives. On the one hand, after describing the technique in the well-known symmetric case, we have constructed an effective local four-point interaction Lagrangian by considering symmetry properties, starting from the five-dimensional SU(4) gauged supergravity Lagrangian [5]. In addition, confirming our heuristic results, we have arrived to the same effective action directly from the analysis of a four-point type IIB superstring theory scattering amplitude. The difference with the symmetric case comes from the fact that one needs to consider RR vertex operators in order to include the t-channel Chern-Simons contribution. This is due to the role that the ten-dimensional self-dual five-form field strength $\mathcal{F}_5$ plays in the construction of the gauge fields (described at the linear level in [2]) when reducing the theory on S^5. More specifically, in the symmetric case the relevant modes are given by

[19] We omit further corrections from the Pomeron kernel.

two dilaton and two graviton perturbations (with specific polarizations), whereas in the antisymmetric case we find that the relevant scattering amplitude is of the form

$$\mathcal{A}(\mathcal{F}_5, \mathcal{F}_5, \phi, \phi)\,, \tag{5.4}$$

as suggested by the analysis of section 3.1.

Focusing on the dependence in the Bjorken parameter, the precise calculation of the amplitude leads to

$$F_3(x) \sim \left(\frac{1}{x}\right)^{1-\frac{1}{2\sqrt{\lambda}}}\,. \tag{5.5}$$

This means that DIS of a charged lepton from a scalar has a non-zero F_3 even when it was subleading for larger values of x. The result we show in equation (5.5) leads to two interesting conclusions. Firstly, in the small-x region F_3 does not vanish even for scalar hadrons. Furthermore, the first term in the exponent implies that F_3 is not sub-leading since it grows as F_2 does. Secondly, the last term of the exponent shows that an $\mathcal{O}(\lambda^{-1/2})$ shift appears in the exponentially small-x region as in the symmetric case. However, the differential operator in the t-channel Laplacian is now associated to spin-one fields as opposed to the spin-2 operator considered in [1, 25]. Thus, it leads to a different shift. The particular value is of the same sign, but it is smaller, which means that F_3 actually grows faster than F_2 for extremely small values of the Bjorken parameter.

In the symmetric structure functions, at some point, the fast rising of the single-Pomeron exchange results when $x \to 0$ will fulfil the Froissart bound. In order to restore unitarity, it is necessary to consider the contribution from loop diagrams, *i.e.* sub-leading contributions in the $1/N^2$ expansion. In the high energy limit these contributions are dominated by multi-Pomeron exchange. As it is known, the formalism used above can be readily generalized to include these diagrams by using the eikonal notation. The eikonal formula resumes the full class of ladder diagrams, where the exchanged particles lead to the inclusion of Pomeron propagators, build from the Pomeron kernel.[20] From these one can construct the eikonal phase χ. The saturation regime is reached when $\chi \sim 1$ [36, 39, 42]. We think that similar features would take place for the antisymmetric contributions studied in this paper. However, one should be cautious in performing the eikonal approximation for the $j \approx 1$ exchange since there are some subtleties that should be taken into account [30].

A very interesting question is what is the relation of the F_3 structure function calculated in the present paper, i.e. using the string theory analysis, with the corresponding ones for baryons. Following [1] one can consider a composite object built out of $\mathcal{N} = 4$ SYM gluinos, which might be interpreted as a holographic representation of a dual spin-1/2 baryon state. We have carried out the corresponding calculations in [43] obtaining a similar dependence on the Bjorken parameter, the virtual photon momentum transfer q and the 't Hooft coupling, but replacing the scaling dimension of the hadron state by its twist, as expected since in the case of glueballs they both are the same. Certain very remarkable phenomenological implications are developed in [43].

[20] In this context, one needs both the imaginary and the real part of the kernel.

As a general remark, whenever one considers the holographic dual calculation of DIS processes in $\mathrm{AdS}_5 \times W^5$, being W^5 a five-dimensional Einstein manifold, as pointed out in [1] the calculation will only be modified by the effects of W^5 isometries. This affects the parametrization of the graviphoton, $h^{\mu a} = A^\mu v^a$, in terms of the Killing vectors v^a of W^5. For instance, one in principle could carry out very similar calculation in the $\mathrm{AdS}_5 \times T^{1,1}$ conifold background for $\mathcal{N} = 1$ SYM theory [44], where the spectrum of type IIB supergravity is known [45]. Also, mild IR deformations of the AdS_5 spacetime will only change the overall normalization constant coming from the AdS-field wave-functions. Notice that in all these cases, as well as for probes flavor Dp-branes emulating matter fields in the fundamental representation of the gauge group [11–13, 15], by considering the planar limit of the SYM (or YM) theory, one always obtains structure functions scaling as N^0. This is expected since we only consider the confining phase of these theories.

A very interesting situation can be analyzed by studying an S^1 compactification of AdS_5 as considered by Witten [7, 46]. It corresponds to $\mathcal{N} = 4$ SYM at finite temperature T. Notice that supersymmetry is completely broken since anti-periodic boundary conditions along S^1 are imposed for fermions, while scalars get mass at one-loop level, rendering a (non-supersymmetric) Yang-Mills theory. In the planar limit Witten has considered the YM theory at finite T on $S^3 \times S^1$ and on $R^3 \times S^1$. In [7] it has been shown that the planar limit of the YM theory on $S^3 \times S^1$ has a confining phase at low T, while at high T the theory becomes unconfined, therefore its free energy is proportional to N^2. On the other hand, the YM theory on $R^3 \times S^1$ at any T is in the high-temperature phase, i.e. unconfined, therefore its free energy always scales with N^2, reflecting the contribution of N^2 species of gluons. The same scaling occurs for all the structure functions.

In the cases of F_1 and F_2 it has been found that, in a deconfined phase of a strongly coupled $\mathcal{N} = 4$ SYM plasma, they scale with N^2 [47] using the black-hole embedding. In addition, in [48] it has been obtained the corresponding $\mathcal{O}(\alpha'^3)$ corrections from type IIB superstring theory. One expects that for the YM theory on $R^3 \times S^1$, at any T, F_3 also should scale with N^2, following the same scaling arguments which hold for the calculations carried out in [47, 48].

Acknowledgments

We thank Carlos Núñez for a critical reading on this manuscript and comments. This work has been supported by the National Scientific Research Council of Argentina (CONICET) and the National Agency for the Promotion of Science and Technology of Argentina (ANPCyT-FONCyT) Grant PICT-2015-1525, and the PIP-UE CONICET Grant Búsqueda de nueva física.

A Conventions for the Killing vectors on S^5

In this appendix we describe the explicit relation between the SU(4) Gell-Mann matrices and the S^5 Killing vectors.

The Lie algebra of SU(4) describes the full set of 4×4 traceless hermitian matrices. The canonical basis given is by $\{T_A; A = 1, \dots, 15\}$, where for example

$$T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; T_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; T_{15} = \frac{1}{2\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}, \tag{A.1}$$

are the only diagonal elements. The T matrices satisfy the orthonormality condition and the commutation relations

$$\mathrm{Tr}(T_A T_B) = \frac{1}{2}\delta_{AB}\,, \qquad [T_A, T_B] = i f_{ABC}\, T_C, \tag{A.2}$$

where f_{ABC} are the completely antisymmetric structure constants. For SU($N \geq 3$) it is also useful to consider a completely symmetric symbol d_{ABC} which appears in the anti-commutations. In terms of traces of the generators, these objects are given by

$$f_{ABC} = -2i\,\mathrm{Tr}\left(T_A[T_B, T_C]\right)\,, \; d_{ABC} = 2\,\mathrm{Tr}(T_A\{T_B, T_C\})\,. \tag{A.3}$$

$\mathrm{SU}(4)_R$ is the R-symmetry group of $\mathcal{N} = 4$ SYM theory, and the d_{ABC} symbol appears for example in the anomaly of the three-point function of the R-currents. The d_{ABC} symbol appears in front of the Chern-Simons interaction in the action of the dual gravitational theory [6–8]. In gauge/gravity duality applications, the electromagnetic current in general is modeled by gauging a $\mathrm{U}(1)_R \subset \mathrm{SU}(4)_R$, whose generator is generally associated with T_3 [20]. Thus, in the *electromagnetic* DIS of $\mathcal{N} = 4$ SYM theory, for the antisymmetric structure functions we are only interested in the d_{33C} components. The only non-vanishing components are $d_{338} = 1/\sqrt{3}$ and $d_{33,15} = 1/\sqrt{6}$.

On the other hand, in terms of the gauge/gravity duality the R-symmetry is realized as the isometry group of the five-sphere, SO(6), which is isomorphic to SU(4). In this context, one has a different basis given by the 15 Killing vectors $K_{[ij]}$. Now, let us consider the canonical embedding of S^5 into the Euclidean space R^6. The Killing vectors are the rotation generators

$$K_{[ij]} = x^i \partial_j - x^j \partial_i\,, \quad i, j = 1, \dots, 6\,, \tag{A.4}$$

where x^i are Cartesian coordinates on R^6. For example, the precise mapping for the diagonal T generators is

$$\begin{aligned} &T_3 \leftrightarrow K_3 = 2i\left(K_{[14]} + K_{[26]}\right)\,, \quad T_8 \leftrightarrow K_8 = \frac{i}{2\sqrt{3}}\left(K_{[14]} - K_{[26]} + 2K_{[35]}\right)\,, \\ &T_{15} \leftrightarrow K_{15} = \frac{i}{\sqrt{6}}\left(-K_{[14]} + K_{[26]} + K_{[35]}\right)\,. \end{aligned} \tag{A.5}$$

The resulting Killing vectors are normalized as[21]

$$\int_{S^5} d^5\Omega\, \sqrt{g_{S^5}}\, K^a_A\, K^b_B\, g_{ab}(S^5) = -\frac{\pi^3 R^7}{6}\,\delta_{AB}\,. \tag{A.6}$$

[21]The normalization has a minus sign due to the imaginary unit included in equation (A.5).

Defining a new symmetric symbol as $\tilde{d}_{[ii'][jj'][kk']} \equiv \varepsilon_{ii'jj'kk'}$ (which of course only takes non-zero values ± 1), one has the identity

$$\varepsilon_{ii'jj'kk'} = \frac{3}{4\pi^3 R^6} \int_{S^5} d^5\Omega \, \varepsilon^{abcde} \, K_a^{[ii']} \, \partial_b K_c^{[jj']} \, \partial_d K_e^{[kk']} \,, \tag{A.7}$$

which leads to the expression for the Chern-Simons interaction given in [27]. In the $\{K_A\}$ basis, equation (A.7) becomes an integral expression for d_{ABC} in terms of the five-dimensional Levi-Civita symbol and the Killing vectors (together with their derivatives) given by

$$d_{ABC} = \frac{3i}{2\pi^3 R^6} \int_{S^5} d^5\Omega \, \varepsilon^{abcde} \, K_a^A \, \partial_b K_c^B \, \partial_d K_e^C \,. \tag{A.8}$$

This allows one to rewrite the Chern-Simons term in the action in the more familiar notation.

We can also write the additional identity

$$\varepsilon^{abcde} \, \partial_a K_b^A \, \partial_c K_d^B = \frac{4i}{R} \, d_{ABC} \, K_C^e \,, \tag{A.9}$$

which is usefull in the computation of the effective action in subsection 3.2.

Note that in this language the relevant combination is $d_{33C} \, K_C = (i/2) \, K_{[35]}$, which means that our final result for the structure function F_3 is proportional to the eigenvalue $\mathcal{Q} \equiv d_{33C} \, Q_C = (1/2) \, Q_{[35]}$ of the spherical harmonic that defines the dilaton solution with respect rotations on the internal $(3,5)$-plane. This means that F_3 is non-vanishing for hadrons charged with respect to $K_{[35]}$.

B Gamma matrix algebra in the three-point closed string scattering amplitude

The starting point is the flat-space three-point type IIB superstring theory scattering amplitude $\mathcal{A}^{(3)}$(RR,RR,NSNS), involving two RR-vertex operators and one NS-vertex operator. This is given in [34] as follows

$$\mathcal{A}^{(3)}_{\text{closed}} = -\frac{i}{2} \kappa_{10} \, h^1_{MN} \text{Tr} \left(\zeta^2 \mathcal{C} \Gamma^M \zeta^{3T} \mathcal{C} \Gamma^N \right) \,, \tag{B.1}$$

where ζ is given by

$$\zeta^{IIB}_{AB} = \mathcal{F}^{(5)}_{M_1 \cdots M_5} \left(\mathcal{C} \Gamma^{M_1 \cdots M_5} \right)_{AB} \,. \tag{B.2}$$

Now, we have to calculate the trace of twelve gamma matrices. We follow the notation of reference [33], appendix B. The conjugation matrix $\mathcal{C}$ raises and lowers the indices of the gamma matrices. The corresponding indices in the definition of the gamma matrices are $\left(\Gamma^M\right)_A{}^B$, being A and B Dirac indices. We have the following properties

$$\begin{aligned} \mathcal{C}^{-1}_{AB} \mathcal{C}^{BC} &= \delta^C_A, & \mathcal{C}^{AB} &= -\mathcal{C}^{BA}, \\ \left(\Gamma^M\right)_{AB} &= \mathcal{C}^{-1}_{BC} \left(\Gamma^M\right)_A{}^C, & \left(\Gamma^M\right)^{AB} &= \mathcal{C}^{AC} \left(\Gamma^M\right)_C{}^B, \\ \mathcal{C}^{AC} \left(\Gamma_M\right)_C{}^D \mathcal{C}^{-1}_{DB} &= -\left(\Gamma^T_M\right)^A{}_B, & \left(\Gamma_M\right)_A{}^B &= -\mathcal{C}^{-1}_{AC} \left(\Gamma^T_M\right)^C{}_D \mathcal{C}^{DB}, \end{aligned}$$

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also we use the fact that for any two matrices which are themselves products of gamma matrices

$$R_{AB}S^{BC} = -R_A{}^B S_B{}^C\,.$$

Other useful properties are listed below

$$\mathcal{C}^{-1}_{AB} = -\mathcal{C}^{-1}_{BA}\,, \qquad \left(\left(\Gamma^{M_1\cdots M_5}\right)^T\right)^A{}_B = -\mathcal{C}^{AC}\left(\Gamma^{M_5\cdots M_1}\right)_C{}^D\,\mathcal{C}^{-1}_{DB}\,. \tag{B.3}$$

Then, the scattering amplitude becomes

$$\mathcal{A}^{(3)}_{\text{closed}} = -\frac{i}{2}\kappa_{10}h^1_{MN}\mathcal{F}^2_{M_1\cdots M_5}\mathcal{F}^3_{N_1\cdots N_5}\,\mathrm{Tr}\left(\Gamma^{M_1}\cdots\Gamma^{M_5}\Gamma^M\Gamma^{N_1}\cdots\Gamma^{N_5}\Gamma^N\right)\,. \tag{B.4}$$

Next we can calculate the last trace using similar arguments as in appendix A of reference [32]. Therefore, one obtains the following contractions:

$$\mathcal{A}^{(3)}_{\text{closed}} = -\frac{i}{15}\kappa_{10}\,h^1_{MN}\left[5\left(\mathcal{F}^M_2\cdot\mathcal{F}^N_3 + \mathcal{F}^M_3\cdot\mathcal{F}^N_2\right) - g^{MN}\mathcal{F}_1\cdot\mathcal{F}_2\right]\,, \tag{B.5}$$

where omitted indices are contracted. The last term vanishes since h is traceless.

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