Real Valued Card Counting Strategies for the Game of Blackjack *

Mózes Vidámi[1], László Szilágyi[1,2,3], and David Iclanzan[1]✉

[1] Computational Intelligence Research Group
Sapientia Hungarian University of Transylvania, Târgu-Mureş, Romania
iclanzan@ms.sapientia.ro
[2] Physiological Controls Research Center, Obuda University, Budapest, Hungary
[3] Department of Control Engineering and Information Technology Budapest University of Technology and Economics, Budapest, Hungary

Abstract. Card counting is a family of casino card game advantage gambling strategies, in which a player keeps a mental tally of the cards played in order to calculate whether the next hand is likely to be in the favor of the player or the dealer. A card counting system assigns point values (weights) to the cards. Summing the point values of the already played cards gives a concise numerical estimate of how advantageous the remaining cards are for the player. In theory, any assignment of weights is permissible. Historically, card counting systems used integers and rarely the 1/2 and 3/2 fractions, as computation with these are easier and more tractable for the human memory.
In this paper we investigate how much advantage would a system using real valued weights provide. Using a blackjack simulator and a simple genetic algorithm, we evolved weights vectors for ace-neutral and ace-reckoned balanced strategies with a fitness function that indicates how much a given strategy empirically under or outperforms a simple card counting system. After convergence, we evaluated the systems in the three efficiency categories used to characterize card counting strategies: playing efficiency, betting and insurance correlation. The obtained systems outperform classical integer count techniques, offering a better balance of the efficiency metrics. Finally, by applying rounding and scaling, we transformed some real valued strategies to integer point counts and found that most of the systems' extra edge is preserved. However, because of the large weight values, it is unlikely that these systems can be played quickly and accurately even by professional card counters.

Keywords: card counting strategies · evolutionary computation.

1 Introduction

Blackjack is unique among casino games as it affords to an observant player an opportunity to have an advantage over the house. There is ample statistical

* This project was supported by the Sapientia Foundation Institute for Scientific Research.

evidence that high cards benefit the player, while the low cards are advantageous to the dealer. In his 1962 book, Beat the Dealer [17], Edward O. Thorp described a system and proved that it gave a blackjack player an edge over the house. While Thorp is considered the father of card counting, even before the publication of its seminal work, professional card counters were already exploiting casino blackjack games for a profit. Since the early days of card counting, a plethora of other systems were proposed with the aim of offering a better ease-of-use vs. profitability balance, or as responses and adaptations to the counter-measures taken by casinos to curb the profitability of card counting. The documentary film "The Hot Shoe"[4] provides a nice overview of the card counting history.

Blackjack in general [19] and optimal strategies (when to hit, double, stand or split) and count systems in special [12, 2, 8, 5, 3], have received considerable attention from the AI community. Most approaches use evolutionary algorithm (EA) to optimize the strategies over simulated hands, while others use neural network to develop complete blackjack players.

Historically, the manually developed count systems or the ones obtained via artificial evolution [5] were targeted for use by humans. Therefore, these systems restrict the point counts to only integers (and rarely simple fractions) so people can perform the calculations mentally, relatively simply. In this paper we investigate i) if a count strategy that use real valued weights offers any meaningful edge over the integer restricted ones; ii) and if it does, can the system be transformed into an integer point count system that preserves (part of) the additional advantage.

2 The Game of Blackjack

Blackjack, also known as 21, is a card game in which a player or players compete against the dealer or "house", by obtaining a sum of cards that is as close to 21 as possible, without exceeding that value (busting). The game is played with one to eight decks of 52 French cards. The rules of Blackjack can vary by country and even by casino.

First, the dealer shuffles the card, while the players make their bets that are in-between a minimum and maximum bet size and can not be changed once or taken back the first card is dealt. The house deals cards from left to right, one by one. Players start with two cards, both face up, while only one of the dealers card is visible. The values of the cards between two and ten are their pip value (2 to 10), Jacks, Queens and Kings are all worth ten while the Ace has two values: one or eleven. The value of a hand equals sum of the card values. While pursuing the goal of getting as close to twenty-one as possible, every player can draw, request as many cards as they wish, an action called Hit. The player can also choose to Stand - take no more cards, Double - double the bet and draw one last card or Split, to obtain two separate hands from an initially dealt set of pairs. The dealer cannot double down. Some casinos let the player Surrender after seeing the first two cards, for a portion of the bet.

[4] `https://www.imdb.com/title/tt9414698/`

The most valuable hand is an Ace paired with a ten value card. This is called a Blackjack. It depends on the casino whether or not this beats, draws or loses to the Blackjack of the dealer. A player automatically loses, if they draw more than twenty-one. Once all players completed their hands, the dealer turns over his hidden card and is obligated to draw until his hand values is at least seventeen, where they stop and they compare their hand to the ones obtained by the players. A player is considered a winner if is closer to twenty-one than the house. In case their hand value is equal, it's a draw, otherwise the player loses.

When the dealer's up card is an Ace, the players are allowed to take an "insurance" bet. If they two, the price is half the bet. If the dealer face down card is a ten, the insurance bet pays 2:1. The maximum size of the insurance bet is half of the current bet size. The odds of the dealer making a Blackjack is 9:4, therefore insurance can become profitable only if the player counts the cards and knows that there are proportionally above average ten-point cards still left in the shoe.

Edward O. Thorp used computer simulations to test each distinct situation in a blackjack game and derive the best action the player can take. This collection of rules is called Basic Strategy and when strictly followed, it decreases the edge of the house from 4% to 0.5% [9]. Since then the game has changed, now it is usually played with more than one deck of cards (to reduce the efficiency of card counting systems). Nevertheless, each blackjack game still has a Basic Strategy[5], which describes the optimal method of playing any hand against whatever the dealers up card is. Rarely, casino promotions such as limited 2:1 blackjack payouts enables players to have an edge over the house just by playing the basic strategy.

2.1 Card Counting Principles

Card counting strategies are built upon the observation that high cards benefit the player more than the dealer, while the opposite is true for the low cards. 5s help the dealer the most, thus many such cards remaining in the shoe is very disadvantageous for the player. Higher concentration of high cards benefit the player because it increases the player's chances of hitting a Blackjack, which pays out at a 3:2 rate, while the dealers Blackjack is valued at 1:1. When the deck is stacked in such way the player has the option of Doubling down on additional hands, to increase the expected profit, while the dealer is restricted from Doubling. It also leads to more splitting opportunities for the player, while the dealer, again, is restricted from Splitting. Also, a high enough concentration of 10's increases the probability of the dealer making a blackjack from 4/9 to over 0.5, making the insurance bet profitable.

A concentration of low cards benefit the dealer, since according to the rules the dealer must continue Hitting until he reaches 17. For the common hand values of 12-16, the dealer would bust for every 10-valued card, while low cards provide safety, and hand values close to or spot-on 21.

[5] `https://en.wikipedia.org/wiki/Blackjack##Basic_strategy`

Casinos have implemented many changes to the game rules and casino policies, in an effort to combat bleeding money to professional card counters. While the edge of the card counter player can be severely reduced, it can not be completely eliminated. Countermeasures include increasing the number of decks or shoe count, preferential shuffling - shuffling when the remaining cards are deemed to favour the player, decreasing deck penetration by reshuffling early, no mid-shoe entry into the game, continuous shuffling etc.

Card counting systems assign a positive, negative, or zero point value to each card value available. Once a card is dealt, the so called running count, which starts from 0, is adjusted by that card's point value. Low cards usually have positive point values and raise the value of the count, signaling the increased percentage of high cards in the remaining decks. Conversely, high cards have negative values and they decrease the count for the opposite reason. System that assign 0 point values to cards (usually 7-9s, sometimes aces) consider them neutral and they do not affect the running count.

2.2 Efficiency Metrics

Good card counting strategies must perform well several objectives and metrics, that gauge different aspects of the game. Following the work of Peter A. Griffin [6], strategies aim to achieve a balance of efficiency in three categories:

1. Playing Efficiency (PE) or Strategic Efficiency. This metric indicates how well a counting system can be used to vary playing strategy, according to the actual composition of the remaining cards in the shoe. PE is particularly important in hand-held games that only use one or two decks of cards. Approximately 0.70 is the cap on the highest possible PE [6] for a single parameter counting system, that does not use side counts. PE is not relevant to unbalanced counting systems (the running count does not equal zero after all cards are dealt), therefore in this paper we only develop balanced strategies.
2. Betting Correlation (BC) gauges how well the system detects the player advantage based on the remaining undealt cards. Effective card counting system assign point values to each card that correlates well with the card's "effect of removal" as computed in [17], enabling a good estimation of the edge provided by the composition of cards still to be dealt. The player advantages in percentages, when removing card types from Aces, 2, 3 ... to ten-valued cards are: -2.42, 1.75, +2.14, +2.64, +3.58, +2.40, +2.05, +0.43, -0.41, +1.62. Larger ratios between the point values permit a higher correlation but they also result in an increased complexity, mentally more taxing computations of the count. By taking the ratio between the highest and lowest assigned point values of a system, counting strategies may be referred to as "level 1", "level 2" etc. The correlation value computed by the BC can approach 1.00.
3. Insurance Correlation (IC) expresses how well a counting strategy indicates whether an Insurance bet should be taken. A high IC offers additional value to a card counting system, as the expected gain from counting cards also comes from taking the insurance bet, when the count is high. A point value of -9 for tens and +4 for all other cards provides a maximal IC value.

To obtain a single valued overall metric that permits an easy comparison of strategies, in this paper we will use the Unified Performance Metric (UPM), that for a point value count vector w, simply computes the normalized averages of the above mentioned metrics:

$$UPM(w) = \frac{PC(w)/0.7 + BC(w) + IC(w)}{3} \tag{1}$$

2.3 Card Counting Strategies

Table 1 illustrates a few famous balanced card counting systems and their respective performance metrics.

Strategy	A	2	3	4	5	6	7	8	9	10JQK	PE	BC	IC	UPM
Hi-Lo	-1	1	1	1	1	1	0	0	0	-1	0.51	0.97	0.76	**0.8195**
Hi-Opt I	0	0	1	1	1	1	0	0	0	-1	0.61	0.88	0.85	**0.8671**
Hi-Opt II	0	1	1	2	2	1	1	0	0	-2	0.67	0.91	0.91	**0.9257**
Mentor	-1	1	2	2	2	2	1	0	-1	-2	0.62	0.97	0.8	**0.8852**
Omega II	0	1	1	2	2	2	1	0	-1	-2	0.67	0.92	0.85	**0.9090**
Revere Point Count	-2	1	2	2	2	2	1	0	0	-2	0.55	0.99	0.78	**0.8519**
Revere RAPC	-4	2	3	3	4	3	2	0	-1	-3	0.53	1	0.71	**0.8223**
Revere 14 Count	0	2	2	3	4	2	1	0	-2	-3	0.65	0.92	0.82	**0.8895**
Wong Halves	-1	0.5	1	1	1.5	1	0.5	0	-0.5	-1	0.56	0.99	0.72	**0.8366**
Zen Count	-1	1	1	2	2	2	1	0	0	-2	0.63	0.96	0.85	**0.9033**
Averages											**0.599**	**0.9454**	**0.8009**	**0.8674**

Table 1: Comparison of different balanced card counting strategies. The first 10 columns after the strategy name describe the card values used in counting, while the last 4 contain different performance metrics, namely the Playing Efficiency, Betting Correlation, Insurance Correlation and Unified Performance Metric defined in eq. 1

The Hi-Lo or the "Complete Point-Count System" balanced card counting strategy was first introduced by Harvey Dubner in 1963 at the Fall Joint Computer Conference in Las Vegas and was later refined by Julian Braun and discussed by Edward Thorp's famous book, Beat the Dealer [17] (pp. 93-101). The Hi-Lo is the most commonly used card counting strategy and the majority of simulations and studies are based on this count. Hi-Lo has a high BC of 0.97 but its PE is the smallest and the IC is also below average.

Hi-Opt I and Hi-Opt II are strategies developed by Lance Humble and its collaborators [7]. Because of their high PE they are very suited for single deck games. Hi-Opt II has the highest UPM and its still used by many professional blackjack players as it works very well in shoe games, outperforming many other systems [15].

Mentor [13] is a strategy developed with the aim of being suitable for both hand-held and shoe games. It achieves this balance with an above average PE and BC, and slightly below average IC.

Omega II [1] is a more complex counting system created by Bryce Carlson, with one of the highest PE and above average IC. It has the second highest UPM of the strategies from table 1.

Revere Advanced Plus-Minus, Revere Point Count, Revere RAPC and Revere 14 Count are balanced strategies developed by Lawrence Revere and described in its book Playing Blackjack as a Business [14]. Revere was originally a blackjack dealer, and trained many players to count cards with his advanced systems and also shared techniques meant to avoid detection by casinos. His counting systems have very high BC, thus are very suited for shoe games.

Wong Halves [20] is special counting system that also uses fractions, not just integers. It has a near perfect BC of 0.99, however the PE and IC is way below the average. In practice, many players double the tag values to remove the fractions.

Zen Count [16] is an advanced balanced counting strategy, with all 3 metrics well above average. Similarly to Mentor, it was designed to provide a balance between single-deck and multi-deck strategies.

3 Methods

3.1 Genetic Algorithm

For evolving the strategies we use a Genetic Algorithm [18] implemented with the help of the Distributed Evolutionary Algorithms in Python (DEAP[6]) [4] framework.

The solution are encoded as vectors of float numbers of length 9 in case of ace-reckoned strategies, one float weight for the cards ranging from Ace, 2, 3 to 9. In the case of ace-neutral strategies, the first weight is always zero, therefore the method optimizes the remaining 8 point counts.

As observed, the genetic algorithm does not search for the point value for the 10-J-Q-K cards. Instead, this value is computed from the other weights, in order to ensure that the strategies are all balanced, the count after playing all cards is zero:

$$w_{10} = -\frac{\sum_{i=1}^{9} w_i}{4} \tag{2}$$

The method uses a population size of 100 individuals, tournament selection of size 2 [10], crossover probability of 0.8. After crossover, individuals are mutated with a probability of 0.5; if mutations occurs, each allele is perturbed with a probability of 0.2 using a Gaussian mutation with $\mu = 0$ and $\sigma = 0.1$.

Objective Function Blackjack is a nonlinear potentially chaotic game[5], therefore attempts to actually calculate the expected gain from a particular system often rely on simulation techniques [11, 17].

[6] `https://deap.readthedocs.io/en/master/`

In this paper we also asses the quality of the evolved strategies according to bench–marks obtained from a simple but efficient blackjack simulator. The simulator takes a given strategy expressed as an array of 10 elements as input and simulates that strategy over a user selected shoe count (number of decks used for playing), allowed shoe penetration before reshuffling and number of random hands played. The bet sizes are adjusted according to the current count and the supplied bet spread. If the input is comprised only of zeros (no card counting), the simulator executes the basic strategy. The simulator returns a net overall result summing up all the game results.

The fitness function for a strategy characterized by a weight vector w is defined as the difference between its benchmarks results and the net Hi-Lo strategy, divided by the number of played hands:

$$F(x) = \frac{sim(w, hands) - sim([-1, 1, 1, 1, 1, 1, 0, 0, 0, -1], hands)}{length(hands)} \quad (3)$$

In our experiments, the number of hands used in the bench-marks is 10e6. Due to the high variability of blackjack game, these hands are re-sampled every generation. Therefore, the fitness function is noisy, an individual fitness can vary slightly from generation to generation. $F(w)$ is positive if the strategy encoded by w outperforms the Hi-Lo strategy on the actual *hands*. We have chosen the Hi-Lo system as the baseline as this is the most commonly used card counting strategy and the majority of previous studies and simulations were also based on this count. However, other strategies can also be used to provide a baseline.

3.2 Expert Advisor Mobile Application

For testing and training purposes we also developed an Expert Advisor that scans and recognizes the played cards and indicates what to play next, according to the actually loaded strategy. While card counting with the mind is legal, the use of an automatic card counting device in a casino game would be illegal in most jurisdictions.

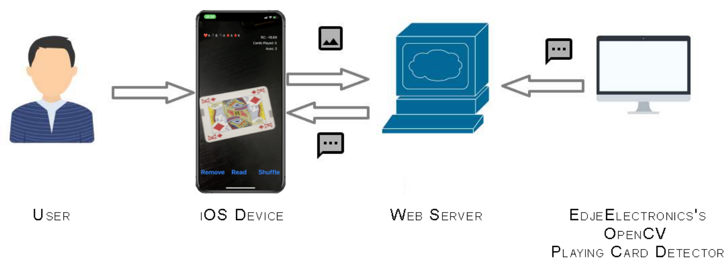


Fig. 1: Expert Advisor for testing and training purposes.

The flowchart of the card processing is depicted in fig. 1. To identify the cards we used EdjeElectronics's OpenCV Playing Card Detector [7]. The method first detects the card object in the image frame. Then it processes the card image determining its corner points and corrects for perspective and obtains a flattened 200x300 pixels sized image of the card. In the last step, the method isolates the card's suit and rank from the flattened image. The detector works best if it is provided with sample rank and suit images generated from the actual playing cards.

4 Experiments and Results

Some strategies count the ace (ace-reckoned strategies) while others do not (ace-neutral strategies). Counting the aces usually improves betting correlation since the ace is the highest value card in the deck for betting purposes. However, since the ace can either be counted as one or eleven, is both a small and a high card. Including it in the count decreases playing efficiency, therefore many experts prefer to assign a value of zero to the ace. To obtain strategies with emphasis on BC (more important in shoe games) but also ones that emphasize PE (more important in single- and double-deck games) more, we searched for both flavours of strategies with 150 runs of the genetic algorithm, each run spanning 50 generations. The simulator was configured with a shoe count of 6 and 10e6 played hands, the minimum bet size was 1 while the maximum was set to 100.

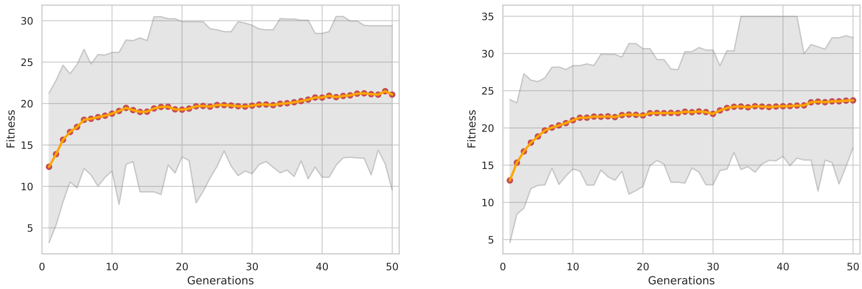
The average, minimum and maximum fitness values obtained from the runs is depicted in fig. 2. The runs show a great variability in the range of fitness values, which can be attributed to the noisy nature of the fitness function (the hands are re-sampled every generation) and also to the inherent variability of the game. The average fitness curve shows a steep increase in the first generations then a steady but small growth in the later ones.

Next we computed the PE, BC and IC performance metrics for each one of the 300 evolved strategies using `blackjackinfo.com`'s the free Card Counting Efficiency Calculator [8] then we computed the normalized averages per eq. 1 to obtain the UPM. The distribution of these values is depicted in fig. 3.

For both ace-neutral and ace-reckoned strategies the average values of 0.9261 and 0.87342 significantly exceeds the 0.8674 average value of the strategies summarized in table 1. The average is much higher in the case of ace-neutral strategies, even exceeding the 0.9257 maximum value from table 1, belonging to the Hi-Opt II strategy.

The average metric values for the ace-neutral strategies were PE=0.6652 $\pm$ 0.0136, BC=0.8910 $\pm$ 0.0155, IC=0.9370 $\pm$ 0.0197 and for the ace-reckoned ones PE = 0.5784 $\pm$ 0.02369, BC=0.9610 $\pm$ 0.0168, IC=0.83291 $\pm$ 0.0330. As expected, considering the ace to be neutral leads to strategies with a high PE and lower BC, while the opposite happens when aces are also counted. Surprisingly, the evolved ace-neutral strategies have a very high IC, the biggest observed one

[7] `https://github.com/EdjeElectronics/OpenCV-Playing-Card-Detector`
[8] `https://www.blackjackinfo.com/card-counting-efficiency-calculator/`



(a) Ace-neutral strategies. (b) Ace-reckoned strategies.

Fig. 2: Average, minimum and maximum fitness values observed over the 2 x 150 runs of the Genetic Algorithm.

being 0.9735, while the average IC value of the strategies from table 1 is just 0.8. The average again exceeds even the maximum IC value of 0.91 belonging to Hi-Opt II.

	A	2	3	4	5	6	7	8	9	10JQK	PE	BC	IC	UPM
AN	0	2.8	2.68	3.55	4.67	3.52	2.99	1.32	-0.13	-5.35	0.6850	0.9012	0.9404	**0.9400**
AR	-2.36	2.7	3.9	4.62	4.93	4.51	3.1	0	-1.16	-5.06	0.6401	0.9702	0.8485	**0.9110**

Table 2: Interesting counting strategies obtained by the Genetic Algorithm.

Table 2 contains the best ace-neutral (AN) and ace-reckoned (AR) strategies, with the point counts truncated to two decimal points, an the value of 10-J-Q-K adjusted to maintain a balanced strategy. The AN outperforms all strategies from table 1 on PE, IC and UPM. The AN strategy provides a great balance between PE and BC, outperforming strategies like Mentor and Zen Count that were especially developed to be suitable for both hand-held and shoe games. The only metric where the evolved strategies did not beat classical ones is BC, the highest obtained value being 0.98902, while several published strategies have a BC of 0.99.

4.1 Integer weights

We also tested if the strategies can be converted to integer counts while also retaining their advantageous properties. For this we rounded each weight to the nearest quarter an then scaled each value by 4. Finally, we slightly increased-decreased some point counts, until the re-balancing for the 10-J-Q-K also yielded an integer value.

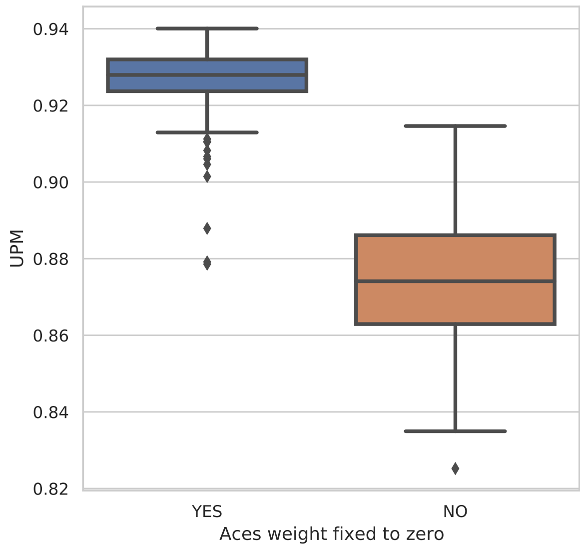


Fig. 3: Distribution of the evolves strategies UPM values

Strategy	A	2	3	4	5	6	7	8	9	10JQK	PE	BC	IC	UPM
AN_I	0	11	11	14	19	14	12	7	0	-22	0.6833	0.8953	0.9461	**0.9391**
AR_I	-9	11	16	18	20	18	12	0	-6	-20	0.6414	0.9704	0.8386	**0.9084**
Combined	-4	12	12	16	20	16	12	6	-6	-21	0.6725	0.9451	0.8907	**0.9321**

Table 3: Integer counting strategies derived from the real weights obtained by the Genetic Algorithm.

The resulting weights and the corresponding performance metrics can be found in the first two rows of table 3. As can be observed from the last column, the remapping to integers leads to only a slight decrease in UPM and other metrics, the edge of the original real valued systems are retained.

The strategy from row three was obtained as a combination of the first two and provides a great balance between PE, BC, and IC, outperforming the best PE strategies Hi-Opt II and OMEGA II not only on PE but also on BC.

Unfortunately, the integer point counts are very high, making the mental counting of the running count difficult.

5 Conclusions

We have shown that by using real valued weights in card counting strategies offers a significant extra edge in the game of blackjack. Many of the developed systems have a very high Insurance Correlation value while also matching and outperforming the classical systems on Performance Efficiency and Betting Correlation.

We also found, that when re-scaling these strategies to integer point counts, most of the edge is preserved. However, this leads to high level counting strategies, that are harder to mentally operate with, and may detract players from their ability to count cards quickly and accurately. Experts suggest, that the return of a simpler and less advantageous system that can be played flawlessly for hours, typically outperforms the return of more complex systems that are prone to user error.

Future work will experiment with unbalanced and suite aware counting strategies and the application of other intelligent search algorithms.

References

1. Carlson, B.: Blackjack for Blood. Pi Yee Press (2001)
2. Coltin, K.: Optimal strategy for casino blackjack: A markov chain approach. Tech. rep. (2012)
3. Curran, D., O'Riordan, C.: Evolving blackjack strategies using cultural learning in multi–agent systems. Tech. rep., National University of Ireland, Galway
4. De Rainville, F.M., Fortin, F.A., Gardner, M.A., Parizeau, M., Gagné, C.: Deap: A python framework for evolutionary algorithms. In: Proceedings of the 14th annual conference companion on Genetic and evolutionary computation. pp. 85–92 (2012)
5. Fogel, D.B.: Evolving strategies in blackjack. In: Proceedings of the 2004 Congress on Evolutionary Computation (IEEE Cat. No. 04TH8753). vol. 2, pp. 1427–1434. IEEE (2004)
6. Griffin, P.A.: The Theory of Blackjack: The Compleat Card Counter's Guide to the Casino Game of 21. Huntington Press (1999)
7. Humble, L., Cooper, K., Cooper, C.: The world's greatest blackjack book. Main Street Books (1987)
8. Kendall, G., Smith, C.: The evolution of blackjack strategies. In: The 2003 Congress on Evolutionary Computation, 2003. CEC'03. vol. 4, pp. 2474–2481. IEEE (2003)
9. Mezrich, B.: 21: Bringing Down the House-Movie Tie-In: The Inside Story of Six MIT Students Who Took Vegas for Millions. Simon and Schuster (2008)
10. Miller, B.L., Goldberg, D.E., et al.: Genetic algorithms, tournament selection, and the effects of noise. Complex systems **9**(3), 193–212 (1995)
11. Millman, M.H.: A statistical analysis of casino blackjack. The American Mathematical Monthly **90**(7), 431–436 (1983)
12. Pérez-Uribe, A., Sanchez, E.: Blackjack as a test bed for learning strategies in neural networks. In: 1998 IEEE International Joint Conference on Neural Networks Proceedings. IEEE World Congress on Computational Intelligence (Cat. No. 98CH36227). vol. 3, pp. 2022–2027. IEEE (1998)
13. Renzey, F.: Blackjack Bluebook II: The Simplest Winning Strategies Ever Published. Blackjack Mentor (2017)
14. Revere, L.: Playing Blackjack as a Business: A Proffessional Player's Approach to the Game of" 21". Lyle Stuart (2000)
15. Schlesinger, D.: Blackjack Attack: Playing the Pros' Way. HUNTINGTON Press (2018)

16. Snyder, A.: Blackbelt in Blackjack. Cardoza Publishing (1998)
17. Thorp, E.O.: Beat the Dealer: a winning strategy for the game of twenty one, vol. 310. Vintage (1966)
18. Whitley, D.: A genetic algorithm tutorial. Statistics and computing **4**(2), 65–85 (1994)
19. Widrow, B., Gupta, N.K., Maitra, S.: Punish/reward: Learning with a critic in adaptive threshold systems. IEEE Transactions on Systems, Man, and Cybernetics (5), 455–465 (1973)
20. Wong, S.: Professional blackjack. Pi Yee Press (1994)