Essays on Improvements to the Regulatory Capital Framework for Credit Risk

Inauguraldissertation
zur
Erlangung des Doktorgrades
der
Wirtschafts- und Sozialwissenschaftlichen Fakultät
der
Universität zu Köln

2020

vorgelegt
von

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aus

Malang

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Tag der Promotion: 20.08.2020

Acknowledgements

I am extremely grateful for my supervisor and my co-author, Prof. Dr. Thomas Hartmann-Wendels, without whose guidance this long journey would not be possible. I would also like to thank Prof. Dr. Dieter Hess for his willingness to co-referee this thesis.

This research would have been impossible without the support from Leaseurope and Global Credit Data. I would like to express my gratitude to both institutions for granting access to their database. Moreover, I also thank for the fruitful discussions on the related topics with the colleagues of both institutions, especially Hayley McEwen from Leaseurope, Nina Brumma and Nunzia Ranione from Global Credit Data.

Many thanks for the colleagues in the University of Cologne for the countless valuable supports and feedbacks, namely Ann-Christine Brunen, Tjark Eschenröder, David Fritz, Arndt-Gerrit Kund, Oliver Laubach, Florian Neitzert, and Matthias Petras.

Lastly, I would like to thank members of my family and friends for the constant emotional support.

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Chapter 1

Main Introduction

The capability of banks and financial institutions to protect them from a downturn period is a necessity for financial stability. The high interconnectivity of the financial ecosystem contributes to a domino effect leading to mass defaults. Although other risk types are not to be underestimated, the credit risk has been the major cause for several crisis events, including the financial crisis and the Euro sovereign crisis. The credit risk framework of the Basel III Accord has been improved on and updated to address some of its issues and to make it more robust. With time, the framework is becoming increasingly complex and extensive.

Along with other qualitative measures, the Standardised Approach and the (Foundation and Advanced) Internal Ratings-Based Approach underlie the quantitative methodology to calculate the risk weight of each exposure. While the Standardised Approach values simplicity and practicability, the Internal Ratings-Based Approach focuses on risk sensitivity and flexibility for the institutions. Addressing unresolved issues from the underlying model and unintended effects along the various revisions and reforms of these approaches have dominated the discussion on this topic. This cumulative thesis consists of three essays with focus on related issues concerning both of these approaches in the credit risk framework.

The first essay discusses the performance of these approaches for non-traditional exposures. Both approaches are primarily designed for banks. So it is not surprising to observe that some rules are especially tailored for loans. For other innovative financial instruments, the adequateness is uncertain to some degree. Although leasing is not particularly a recent invention, the empirical study in the credit risk context has been difficult due to lack of data.

Based on a unique dataset of 2.4 million active leasing contracts during 2007-2011 originated from twelve major European leasing companies, we analyse the unexpected losses of simulated leasing portfolios and compare them to the capital requirements of the Standardised Approach and the Internal Ratings-Based Approach. To the best of our knowledge, there is currently no empirical study on this topic with a leasing dataset of comparable size. The results are relevant for regulatory authorities which plan on a treatment change for leases but cannot judge whether the changed treatment is adequate and neutral. The analysis confirms that the current framework is excessively conservative and a 30% reduction in the risk weight for lease contracts still fulfils these criteria, i. e. the required capital covers the unexpected loss adequately and does not create an incentive for institutions to offer leases instead of secured loans due to the regulatory capital requirement alone.

The second essay concentrates on the downturn LGD, which is required for the use of the Internal Ratings-Based Approach. A published guideline by the EBA serves as the baseline how the downturn LGD has to be estimated. Unfortunately, the downturn definition adopted by the conditional PD under the Internal Ratings-Based Approach and the downturn definition by the downturn LGD guideline are inconsistent (latent variable based versus macroeconomic based). This mismatch will potentially result in a risk underestimation and inadequate capital coverage.

This underestimation is confirmed by Monte Carlo simulations based on an 18 years default database of over 50 international large banks. The simulation shows that the current regulatory downturn LGD does not pass the minimum survival probability of 99.9% as traditionally required in the Internal Ratings-Based Approach. An alternative method is offered, which incorporates latent variables to address the aforementioned inconsistency. It performs with a survival rate of 99.9%. Further, it also outperforms the Foundation Internal Ratings-Based Approach in terms of accuracy. In contrast to other conditional LGD models in the literature, our method applies not only to market-based LGD but to workout LGD as well.

The last essay of this thesis addresses the strong assumption in the Internal Ratings-Based Approach that the asset correlation stays constant throughout the years. This parameter holds the information on how the asset values of firms influence each other. With a high correlation, so is the co-default probability as well. Argumentatively, co-defaults will lead to a financial crisis. As the demand in the loan market soared shortly before the financial crisis due to concerns of the solvency in the banking sector, it may effectively have an impact on the asset correlation in a financial ecosystem. Corporates with active credit lines may max out their debt to secure their liquidity. This phenomenon (referred to as a *credit run*) ensures that there are more loans from the same borrowers, causing a systematic shift in banks' portfolio compositions.

Due to its constant asset correlation assumption, the Internal Ratings-Based Approach is not capable of grasping the impact of a credit run fully. The correlation coefficients in the Internal Ratings-Based Approach have not been recalibrated since their introduction in the Basel II Accord. In the literature, studies on asset correlations seem to show inconsistent estimates as well. Many of the available models require indirect proxies or questionable assumptions. The straightforward explanation for it may be because the underlying assumption may be oversimplified. The analysis offers evidence that a credit run increases the asset correlation value. Consequently, the concept *downturn* asset correlation may be necessary.

The results of this thesis are aimed for the improvement to the current regulatory framework regarding credit risk. In its current state, the credit risk framework is in urgent need of improvement. The recent reform on the Basel Accord includes many rudimentary fixes to addressed some of the related issues, but it cannot be the long-term solutions. Further research for an alternative approach might be appropriate considering the number of issues.

Chapter 2

Does the Finalised Basel III Accord Treat Leasing Exposures Adequately? Evidence from a European Leasing Dataset

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2.1 Introduction

Leasing has been understood as a viable alternative to loans and serves as the backbone of SME finance (Kraemer-Eis and Lang (2012)). The incentive for leasing is often price- or tax-related. Firms may also prefer a lease to a financing loan because of the flexibility that a lease can offer. Sharpe and Nguyen (1995) show that leasing reduces the cost of financing, especially for firms with low credit quality. This premise also implies that lessees are more likely to be firms with a low credit score. They show that the total lease share of low-rated firms that pay no dividends is 25% higher than the total lease share of high-rated firms that pay dividends. Other literature, such as Lease et al. (1990), points out that many characteristics of leasing contracts remind of the characteristics of junk bonds. This observation can be explained by the popularity of leases in SMEs, which typically have lower creditworthiness compared to large corporates.

On the other hand, Eisfeldt and Rampini (2009) argue that the repossession process of leased assets by lessors is easier than the foreclosure on the collateral of a secured loan. Realdon (2006) even argues that the credit spread of a financial lease may even decrease in default probability, as opposed to a secured loan. Comparing both instruments, the relevant differences in the risk context may include easier asset seize process in case of defaults due to legal asset ownership by the lessor, internal expertise about the assets, assets' revenue generation, and higher seniority. From the overall perspective, leasing may have a higher default risk, but it may also be coupled with a higher recovery rate as well. From the regulatory perspective, it is difficult to judge whether the higher recovery rates can compensate for the possible higher default risk.

This question is especially important for the Standardised Approach (SA) since the risk weight (RW) of the SA is supposed to reflect the aggregated risk profile combining both the default risk of lessees and the recovery capability of leased assets. Following the finalisation of the Basel III framework by the Basel Committee on Banking Supervision (2017a), this essay reviews the adequateness not only of the SA but also the Internal Ratings-Based Approach (IRBA) to cover unexpected losses (UL) from a portfolio of leasing exposures. Our analysis is based on a dataset of 2.4 million lease contracts active during 2007-2011, from which over 112,000 are defaulted, originated from twelve major European leasing companies operating across 25 European countries. Compared to its peers from other studies (see table 2.1), this dataset contains more contracts and most importantly also includes the 2008/2009 financial crisis. Due to the extensive amount of data, a Monte Carlo based approach (a Bootstrapping method with a simulation, to be exact) is chosen for the portfolio analysis. Similar methodologies with related context can also be found in Carey (1998); Schmit (2004, 2005).

Potential losses from a leasing portfolio usually consist of losses from the credit risk as well as the market risk. The credit risk of a leasing portfolio is linked to the default risk of the lessees and their ability to pay the leasing obligations. The market risk of a leasing portfolio is mostly associated with the residual value risk of the leased object and the selling ability of the lessors, either after the contract's termination or after seizing the object due to a default event. This essay focuses on the credit risk since it is the risk type, which usually takes the biggest portion of the required capital.

We primarily investigate the question whether the current capital regulation for leasing exposures cover the ULs adequately or any substantial under-/overestimation can be observed. The simulation confirms that leases, in general, are not as risky as the capital regulation suggests and we are not alone with this conclusion (Schmit (2004, 2005); Pirotte and Vaessen (2008); Eisfeldt and Rampini (2009)). However, it is well within the design that the Basel Accord is built to be conservative to act as a fail-safe. How much conservative this fail-safe should be is to some extent a political discussion. In our simulation, the risk weights (RW) can exceed five- to eight-fold the ULs and are only weakly risk-sensitive for the SA. Unfortunately, we do not have a comparable non-leasing dataset to investigate whether this level of conservatism can also be observed for loan exposures. An excessively conservative and weakly risk-sensitive capital requirement can be detrimental for the economy. Any risk reduction strategies should always be reflected accordingly in the capital requirement. Otherwise, there are no incentives to reduce the overall portfolio risk. Whether the new Basel III framework has already addressed these issues cannot be answered by similar methodology since some of the changes in the new framework are fundamental, e. g. corporate exposures have to be evaluated with the foundation IRBA, so a direct comparison is no longer relevant. Nevertheless, we can expect this gap between the RWs and the ULs to get wider. On average, a higher minimum regulatory requirement is to be expected for banks, as reported by the European Banking Authority (2019).

A change in the regulatory treatment is entirely the decision of the relevant regulatory authorities. Apart from the over-conservatism issue, other aspects may influence this decision as well. In this essay, two aspects are essential: *adequateness* and *neutrality*. Not only should a regulatory treatment be adequate to cover the ULs, but also neutral as the regulatory framework does not have the intention to favour one financial instrument over the others. We share the concern that by favouring a particular asset class from the regulatory perspective, there is more incentive for institutions to prefer this particular financing instrument to others, ultimately causing a systematic effect. Some institutions may shift their portfolio to a particular asset class, as some banks may not necessarily see big differences between a loan with a physical collateral and a finance/capital lease (Bayless and Diltz (1988)). This essay does not propose a concrete change in the regulatory treatment for leasing exposures. Instead, we look for the most such a treatment change can be, before it starts to give incentives for banks to offer leases rather than other similar financial instruments. Assuming everything else (risk profile, profit margin, demand, etc.) is similar, the decision whether to offer leases instead of other instruments boils down to the regulatory capital requirement for each exposure. We show that a 30% reduction in leases' RW is the maximum which fulfils the aforementioned conditions and confirms this result further by using a reverse stress test.

This essay is structured as follows: section 2.2 reviews the relevant literature on the credit risk profile of leasing, section 2.3 shows an overview of the dataset, section 2.4 explains our methodologies and assumptions within the simulation, section 2.5 shows the comparison of the current capital requirement regulations and the UL, section 2.6 presents the lowest bound of the RW reduction can be, which still fulfils the adequateness and neutrality conditions, and section 2.7 concludes the essay.

The PhD candidate is responsible for the code architecture as well as its implementation, a part of the methodology design, and the writing of this essay with consultations from and discussions with the co-author.

2.2 Literature Review

The research on the credit risk of leasing portfolios has been focused too much on the exposure itself but ignores who most likely leases. If firms resort to leases only as debt substitutes, because they cannot get a cheap loan, then a leasing portfolio may have a higher level of credit risk than a loan portfolio on average. From the lessee's perspective, it often does not play an important role whether the asset is financed through a finance lease or a debt. For financial institutions, a lease and a collateralised loan share many common characteristics. Bayless and Diltz (1988) find out through queries that financial institutions, which offer both, do not treat the outstanding of capital leases and debts differently in the case of a term loan decision. However, this perspective may change as the leasing industry grows with time and gains more acceptance and popularity.

Since the most popular reason to lease is tax-related, it is reasonable to believe that lessees are typically firms, which otherwise cannot fully profit from the tax shield by buying the asset. Without getting into details which accounting, taxation, or bankruptcy law are discussed, the right for depreciation is generally only granted to the book owner of the asset. Although Finucane (1988); Mehran et al. (1999) report that tax-related factors are not significantly associated with the leasing level of firms, the contrary is reported by Barclay and Smith (1995); Sharpe and Nguyen (1995); Graham et al. (1998). They find out that companies with a high proportion of tax losses rely more on leasing as financing means. Lasfer and Levis (1998) argue that both can be true after controlling for firm size. They find that tax savings are not a major determinant for leasing decisions for small firms, but lessees tend to have higher tax losses. By analysing financial statements, they also find other typical lessees' characteristics, which are a higher debt-to-equity ratio, larger in size, and invest more than non-lessees. While SMEs see leases as substitutes for debt, large firms use leases complementary to debts, which is also reported by Ang and Peterson (1984). Further, Lasfer and Levis (1998) observe that small firms lease due to their growth opportunities, while large firms lease because of tax incentives. Moreover, they report that small less-profitable companies are more likely to lease, while large lessees are generally more profitable. Krishnan and Moyer (1994) report that less stable firms (cash-flow-wise) are more likely to use leasing. Sharpe and Nguyen (1995) also come to the same conclusion in regard to the credit quality. Newer evidence from Eisfeldt and Rampini (2009) also shows that financially constrained firms lease more.

While Krishnan and Moyer (1994) argue that leases reduce bankruptcy costs, Sharpe and Nguyen (1995) report the leasing's reduction effect of financing cost, especially for firms with a low credit score. Ang and Peterson (1984) report that firms with more leases tend to be more levered. Eisfeldt and Rampini (2009) argue that leasing preserves capital. Although these effects are positive for lessees, it can also be argued that e. g. firms with low debt capacity are more likely to lease in order to preserve their debt capacity. While the legal ownership of leased assets can be seen as an advantage by removing the ownership from the right to use, it also creates an agency problem. Flath (1980); Wolfson (1985) argue that lessees have reduced incentives to preserve the asset value. In practice, adequate insurance coverage or a purchase option may alleviate the problem (Krahnen (1990)) and are often mandatory requirements. Contradicting to this argument, Hendel and Lizzeri (2002) report that leased cars are of higher quality than non-leased ones. Lease et al. (1990) observe that the actual salvage values of assets from leasing contracts are significantly higher than the expected residual values, although unexpected inflation may also play a role. In summary, the literature highlights a possible higher default chance of lessees, but also a higher recovery rate in defaulted leasing contracts.

Although Schmit (2004); De Laurentis and Riani (2005) report a high recovery rate in general for leasing exposures, this fact should not be generalised since Han and Jang (2013); Frontczak and Rostek (2015) point out that the recovery rates depend on many factors such as the actions taken during the workout processes, the internal disposal costs, and the lender's disposal policies, which may differ from one lessor to another. Lessors have to deal not only with the credit risk (default and recovery rate) but also with the market risk (residual value) at the same time. By studying an automotive leasing dataset, Pirotte and Vaessen (2008) not only suggest that physical collaterals should be recognised in the capital adequacy regulation to reflect its low-risk profile better, but also that the residual value risk should not be separated from the credit risk model for a better risk valuation. In contrast, Miller and Töws (2018) use a multi-step approach which differentiates between asset-based recovery and miscellaneous recovery, to accommodate a better understanding of both types of recovery and to acquire a stable and more accurate recovery estimation. Pirotte and Vaessen (2008)'s analysis on automotive leasing shows an anti-cyclical effect for recovery rates. Other works, such as Schmit and Stuyck (2002);

Hartmann-Wendels and Honal (2010), confirm the independence of vehicle leasing's recovery rates from macroeconomic variables. While others see the residual values of leased assets as a risk, Tsay (2003) proposes residual values to be potential hedging components. By extending the lease valuation model by Grenadier (1996), Realdon (2006) shows that a financial lease's credit spread may decrease in the lessee's default probability, as opposed to secured loans, if the model considers initial prepayments or terminal options which are typical for leasing contracts. From a qualitative standpoint, the repossession of a leased asset is easier than the foreclosure on the collateral of a secured loan, as argued by Pirotte and Vaessen (2008). With a method similar to this essay's, Schmit (2004, 2005) confirms the excessiveness of the Basel capital requirements for leasing exposures. He argues that the retail portfolio loss based on a leasing dataset in nature is more idiosyncratic than systematic. While we agree that the current Basel capital requirement might be excessive for leasing, we lack an adequate comparison to confirm that the estimated ULs from leasing exposures are indeed lower as opposed to the ULs of e. g. secured loans or bonds.

2.3 Data

Our analysis is based on a historical leasing dataset containing active[1] contracts during 2007-2011 originated from twelve major European leasing companies, which is collected by Leaseurope[2]. The dataset contains more than 2.4 million lease contracts with mobile assets and an outstanding sum of over €45 billion by the end of 2011. The portfolio covers 25 European countries[3]. To the best of our knowledge, there is no literature based on a leasing dataset with more than 100,000 defaulted contracts (see table 2.1). Note also that the source of the dataset is twelve major European bank-owned leasing companies, which have survived the pe-

[1] All contracts in the portfolio of the participating companies during these years, which are not yet closed.

[2] Leaseurope is a European federation of leasing company associations with currently 45 member associations across 32 countries. For details: `http://www.leaseurope.org`

[3] These include: Austria, Belgium, Cyprus, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Luxembourg, the Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, and the UK.

Table 2.1: Dataset size comparison on some selected literature with similar context.

	Dataset Size	Years
Schmit and Stuyck (2002)	37,259 defaulted lease contracts	issued between 1976-2002
Schmit (2004)	46,732 completed lease contracts	issued between 1990-2000
De Laurentis and Riani (2005)	1,118 financial lease operations	written off in 2000
Schmit (2005)	35,861 auto. lease contracts	issued between 1990-2000
Pirotte and Vaessen (2008)	4,828 defaulted auto. lease contracts	issued between 1990-2001
Miller and Töws (2018)	1,493 defaulted lease contracts	executed between 1996-2009

riod 2007-2011. This case is a typical example of survivorship bias, i. e. the dataset consists only of survivors, although the analysis aims to study the non-survivors. However, we argue that the magnitude remains small in this case. Nevertheless, it remains difficult to investigate this issue further since obtaining a portfolio dataset from a defaulted leasing company or any defaulted institution is not possible in most situations. Other bias types, e. g. bank-ownership, cannot be ruled out either. Independent leasing companies or captives may exhibit different behaviours or strategies. However, the relevance to the capital adequacy regulation for them is to some extent also limited.

From originally thirteen leasing companies, we exclude one due to its extremely low default rate and abnormally high recovery rate. Benford's test is performed on the outstanding amounts of the remaining dataset; no significant deviation can be observed from the Benford's distribution. Exclusion of single contracts may occur, e. g. if there are missing variables or multiple data points with the same contract ID. Granular statistics are avoided for the economic interest of the participating companies as well as to enable further data collection for future research. The dataset is the same as used by Hartmann-Wendels and Imanto (2019) and largely overlaps with the one used by Deloitte (2013b,a).

The dataset is gathered explicitly with the purpose to assess the implicit RWs of the leasing activity. The definition of a lease may differ across different accounting standards. In this essay, the definition of a lease is oriented towards the IFRS definition (at the time of data collection, the relevant standard was IAS 17), which is an agreement whereby the lessor conveys to the lessee in return for a payment or series of payments for the right to use an asset for an agreed

Table 2.2: Number of active leasing contracts in the dataset split by active years and all-time default status. The default definition relies on the Basel's default definition, which is either unlikeliness to pay or 90 days past due. The total number of contracts is lower than the sum of each year because the statistic is calculated by active years.

	2007	2008	2009	2010	2011	Total
Defaulted	6,085	25,657	57,967	71,368	75,719	112,726
Non-defaulted	659,865	1,211,075	1,392,199	1,420,417	1,400,477	2,289,076
Totals	665,950	1,236,732	1,450,166	1,491,785	1,476,196	2,401,802

period of time. Furthermore, the default definition relies on the Basel definition at the time of the data collection. Table 2.2 depicts the composition of the dataset. Note that the total number is lower than the sum over all the years. Since the analysis is portfolio-based, it is crucial to view the dataset by active years (instead of e. g. contract begin). One of the consequences is also that the ratio of defaulted contracts to the total number of contracts of a particular year will not result in the default rate but rather the default proportion in the portfolio. In general, default rates are lower than default proportions in the portfolio, if the workout processes take more than one year on average. From the regulatory requirement perspective, the default proportion is more important since these defaults have to be backed with capital, regardless of how long the workout processes will take.

The dataset only includes mobile leasing contracts. Arguably, other asset categories such as real estates have a different risk profile and a different regulatory treatment. The assets in our dataset are categorised into:

- **Commercial Vehicles**: all registered commercial vehicles of all sizes. Unregistered vehicles fall under "Machinery & Industrial Equipment". Caravans and motorbikes are categorised under "Passenger Cars".
- **Passenger Cars**: all new and used, private or business-used cars, as well as caravans and motorbikes.
- **Machinery & Industrial Equipment**: all machinery for commercial, industrial, or agricultural use, including harvesters, tractors, and earthmovers. If the asset has a license plate, it should be considered as "Commercial Vehicles".

Table 2.3: Number of active leasing contracts split by asset type and all-time default status. Default status is based on the Basel framework.

	Defaulted	Non-Defaulted	Total
Commercial Vehicles	29,189	379,207	408,396
Passenger Cars	17,746	432,149	449,895
Machinery & Industrial Equipment	29,273	502,524	531,797
Computers & Business Machines	28,047	770,672	798,719
Ship, Aircraft, Railway, Rolling Stock	207	4,323	4,530
Other	7,693	180,714	188,407
Unknown	571	19,487	20,058
Totals	112,726	2,289,076	2,401,802

- **Computers & Business Machines**: all IT equipment and other business machines, such as photocopiers.
- **Ships, Aircraft, Railway, Rolling Stock**
- **Other**
- **Unknown**

The asset type "Ships, Aircraft, Railway, Rolling Stock" will be excluded from the analysis because the number of contracts is too small and therefore inappropriate for a Monte Carlo-based analysis. In contrast, "Other" and "Unknown" are combined for the analysis and are included. Although we believe that it is not the case with the participating companies, there is a concerning practice to categorise bad cases as either one of these asset types. We include these asset types to show that this concern is not justified within our dataset.

From the exposure type, the dataset consists of corporate, institution, retail, and sovereign exposures, as well as from unknown exposure type. The categorisation of exposure is based on the Basel II's asset classes. Typical for a leasing dataset, retail exposures make the majority of the population.

The average duration until contract termination is about 52 months. Typically, there is a difference in the duration between the asset types, which is proportionate to its amortisation period. Passenger cars are leased often for a shorter duration, while ships or aircraft for a longer duration.

Table 2.4: Number of active leasing contracts split by exposure type and all-time default status. Default status and exposure type are based on the Basel framework.

	Defaulted	Non-Defaulted	Total
Corporate	36,187	875,349	911,536
Institution	556	95,105	95,661
Retail	71,580	1,237,014	1,308,594
Sovereign	25	8,181	8,206
Unknown	4,378	734,427	77,805
Totals	112,726	2,289,076	2,401,802

Table 2.5: Average of initial outstanding by asset types and exposure types. Exposure type is based on the Basel framework. Initial outstanding is the nominal outstanding amount at the beginning of the contract.

	Average initial outstanding
Commercial Vehicles	€58,493
Passenger Cars	€30,596
Machinery & Industrial Equipment	€84,822
Computers & Business Machines	€21,555
Ship, Aircraft, Railway, Rolling Stock	€624,053
Other & Unknown	€68,324
Corporate	€72,003
Institution	€22,913
Retail	€34,114
Sovereign	€24,092

Table 2.5 shows that larger assets, such as machinery or commercial vehicle, exhibit a higher average of initial outstanding. Smaller assets with high depreciation rate, such as office equipment, have a substantially lower amount of initial outstanding due to their lower asset values. As previously argued, ship and aircraft may have a different risk structure since their asset values are significantly higher. The initial outstanding of corporates is comparatively higher than of other exposure types. There is a higher proportion of finance leases to operating leases in the dataset. Table 2.6 shows that the asset type influences the contract type, e. g. vehicles leases are often finance leases and office equipments leases are often operating leases.

If the lessee does not meet his obligation, the leasing object can legally be seized by the lessor. The seized asset can then be sold or re-leased. In some cases, it is not necessarily the best option to seize the leased assets, since it deprives the cure possibility of the defaulted lessee. Table 2.7 shows less than 50% selling rate for each asset categories.

Table 2.6: Distribution of initial outstandings split by contract and asset type. Contract type is based on the IFRS lease categorisation.

	Finance Lease	Operating Lease	Total
Commercial Vehicles	19.77%	2.46%	22.23%
Passenger Cars	9.24%	1.01%	10.25%
Machinery & Industrial Equipment	36.84%	3.98%	40.82%
Computers & Business Machines	8.66%	6.34%	15.00%
Ship, Aircraft, Railway, Rolling Stock	1.59%	0.19%	1.78%
Other	7.08%	2.11%	9.19%
Unknown	0.66%	0.07%	0.73%
Total	83.83%	16.17%	100.00%

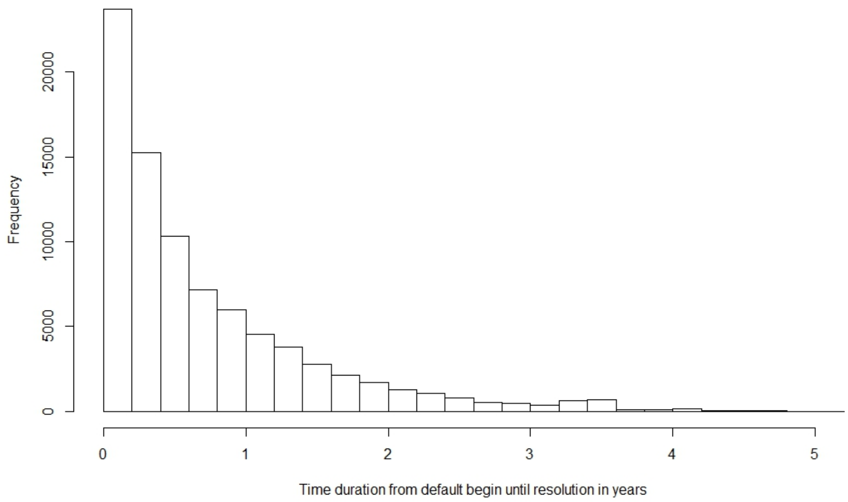

Figure 2.1: Histogram of resolution time of closed defaulted contracts

2.4 Methodologies

The main concept of the analysis is to replicate a representative leasing portfolio in a one-year period by the mean of a Monte Carlo simulation. A similar analysis can be found in Carey (1998); Schmit (2004). Based on historical data, a portfolio with a typical number of contracts will be drawn randomly with replacement. Since the information about the realised loss on each resolved contract is available, the loss distribution can be estimated empirically. The 99.9% percentile of the loss distribution is defined as the Value-at-Risk (VaR). The advantage of such a

Table 2.7: Number of defaulted leasing contracts broken down by asset type.

	Total	asset sold
Commercial Vehicles	29,189	12,754
Passenger Cars	17,746	6,027
Machinery & Industrial Equipment	29,273	8,476
Computers & Business Machines	28,047	10,923
Ships, Aircraft, Railway, Rolling Stock	207	37
Other	7,693	2,637
Unknown	571	130
Total	112,726	40,984

non-parametric method is that it only uses minimal assumptions. The estimation of the potential loss is realistic and is based on historical data. Since the Monte Carlo-based methods are based on historical data, the derived information should originate from the actual historical observation. While our dataset includes the financial crisis, which arguably contains information of downturn effect, others may argue that the 2008/2009 crisis is more about a real-estate bubble. Thus, it may only have a comparably weak downturn effect on leases. Although the dataset shows that these downturn years have some impact, we cannot ensure that a leasing-related downturn period will not have even worse consequences for lessors.

The IRBA is not designed to estimate the VaR directly, but rather the expected loss (EL) conditional on the latent factor at the 99.9% level. In this model, one single latent factor is used to represent all systematic factors. Roughly explained, the VaR represents the *worst* loss (out of 1,000 cases) while the conditional loss represents the associated loss under the most *distressed* period (out of 1,000 possible periods). Both are only comparable if the worst loss is also caused by a distressed period. In general, this association can never be guaranteed without assumptions. However, as proven by Gordy (2003), the VaR is asymptotically equivalent to the downturn loss at a given confidence level under the asymptotic-single-risk-factor model with some relevant assumptions.

Table 2.8: One-year default rates broken down by active year and exposure type. The default rate is calculated as a quotient with the denominator as the number of active contracts at t and the numerator as the number of defaulted contracts from the denominator, of which the default event occurred at t. Default definition and exposure type are based on the Basel framework.

	2007	2008	2009	2010	2011	All years
Global	1.12%	1.56%	2.61%	2.13%	1.72%	1.89%
Retail	1.56%	1.96%	2.93%	2.38%	1.74%	2.16%
Corporate	0.72%	1.24%	2.46%	2.11%	1.83%	1.76%
Sovereign	0.17%	0.04%	0.05%	0.13%	0.12%	0.09%
Institutions	0.21%	0.15%	0.33%	0.19%	0.42%	0.27%
Unknown	0.82%	1.22%	2.66%	1.27%	2.00%	1.56%

2.4.1 PD, LGD, and Recovery Rates of Leasing Collateral

The two primary components of the IRBA are the *conditional* probability of default (PD) and the *downturn* loss given default (LGD). Other components such as the exposure amount at default or the effective maturity usually do not need to be estimated. While the conditional PD is determined theoretically with a given formula, the downturn LGD is determined individually in compliance with the downturn LGD guideline (see EBA/GL/2019/03). In practice, the input parameters (mostly the observed PD and LGD of a particular asset segment) are based on historical data or external information, e. g. from a rating agency. In our case, where the dataset covers only five years, an out-of-sample analysis is quite difficult to do. Ideally, the regulatory capital requirement for the year t can only be calculated using information up to $t-1$. Using e. g. the observed default rate of 2008 for the PD input of 2008 is unrealistic and renders our analysis insufficiently conservative. However, to show that the current regulatory capital is too conservative for lessors, this flaw plays in our favour. Even under these obstacles and non-conservative assumptions, we are still able to show that the regulatory capital requirement far exceeds the simulated UL.

The dataset shows relatively low default rates with the clear impact of the financial crisis in 2008/2009, as shown in table 2.8. It seems that this contradicts with the conjecture that lessees are predominantly firms with low credit quality. However, the dataset is originated only from European major leasing companies. Lessor size and country effects may play a role, but cannot

Table 2.9: Long-run average default rates broken down by exposure type. The default definition and exposure type are based on the Basel framework.

	2007	2008	2009	2010	2011
Global	1.12%	1.34%	1.76%	1.85%	1.83%
Retail	1.56%	1.76%	2.15%	2.21%	2.11%
Corporate	0.72%	0.98%	1.47%	1.63%	1.67%
Sovereign	0.17%	0.11%	0.09%	0.10%	0.10%
Institutions	0.21%	0.18%	0.23%	0.22%	0.26%
Unknown	0.82%	1.02%	1.57%	1.49%	1.59%

be controlled in this essay. Compliant with the EBA/GL/2017/16, the long-run average default rates are used as input parameters, i. e. the arithmetic average of the past default rates. This procedure ensures to include a mix of good and bad years up to the most recent five years. Since our dataset only covers five years and includes the financial crisis, the long-run average default rate is calculated as the mean of the annual observed default rates. Further, it is worth mentioning that the long-run average default rates in table 2.9 are higher than the newly-set input floors in the Basel Committee on Banking Supervision (2017a). These long-run average default rates will be used as PD input parameters in the IRBA.

For calculating the LGD (or the downturn LGD), the realised loss of a defaulted leasing contract in the dataset has to be analysed. For closed defaults, information on the final realised loss for each default is available. The ratio of realised loss and its outstanding amount at the default time is the nominal LGD. The discounted LGD requires detailed information on payment time and interest rates, which are unfortunately unavailable. Especially for European assets, there should be no significant difference between the nominal and the discounted LGD due to the low long-term interest rates. In the case of leasing, the minimum lease payment[4] (MLP) is taken as the initial outstanding amount. For the unresolved cases, the final loss has to be estimated. This step is crucial to avoid resolution bias, i. e. negative bias originated from excluding defaults with a long default duration (with a presumably high LGD as well). Either the leasing object has been sold, then we assume the disposal value after deducting for the residual value to be the sole

[4]The definition of MLP is based on the International Accounting Standard. At the time, when the dataset is collected, it was the IAS 17.

recovery, or the leasing object has not been sold, then we estimate the expected LGD conditional on the default duration. The expected conditional LGD is calculated with the following formula

$$\mathbb{E}[LGD_i|T(i)] = \sum_{t \geq T(i)} \mathbb{E}[LGD_i|T_r(i) \in \tau_t] \cdot \mathbb{P}(T_r(i) \in \tau_t),$$

where $T(i)$ and $T_r(i)$ are defined as the time period where the contract i has been unresolved since the default began and the time period until the resolution respectively. τ_t is predefined time intervals in months: $\tau_1 = [0,3], \tau_2 = (3,12], \tau_3 = (12,24], \tau_4 = (24,36], \tau_5 = (36,48]$, and $\tau_6 = (48,60]$. The purpose is to estimate the LGD of unresolved defaults without an asset sale. This technique is based on the method used to estimate the loss in Deloitte (2013b). The population of resolved defaults without an asset sale is the only appropriate choice for this estimation because the unresolved defaults without an asset sale can be assumed to behave similarly as the resolved defaults without an asset sale. Outliers in the data may cause a biased estimation, but since the goal is to investigate unexpected losses, we only exclude the extreme outliers. We only consider defaults with calculated LGDs within a $[-10.000\%; 10.000\%]$-interval for our calculation.

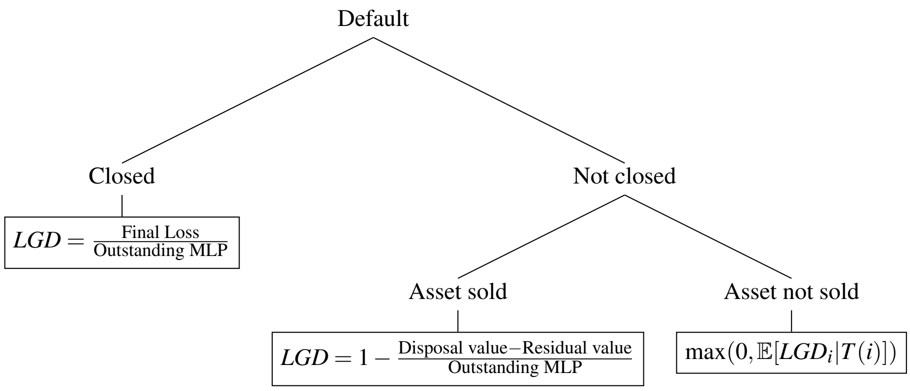

Figure 2.2: Tree diagram of LGD calculation/estimation based on contract's default and sale status

Table 2.10 shows the average LGD of the resolved defaults without asset sale, which is applied as the estimated LGD for unresolved defaults without asset sale. In the overall population, LGD estimates over 140% are excluded in the further calculations and negative losses (profit)

Table 2.10: LGD estimates applied to unresolved defaults without asset sale as proxy. The calculation is based of resolved defaults with asset sale.

$T_r(i) \in$	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6
All asset types	2.43%	3.64%	7.63%	20.00%	24.94%	7.34%
Commercial Vehicles	2.95%	4.04%	9.64%	28.04%	45.42%	62.84%
Passenger Cars	5.16%	8.91%	20.90%	45.03%	55.04%	1.44%
Machinery & Industrial Equipment	2.18%	3.19%	6.28%	13.04%	11.55%	23.62%
Computers & Business Machines	2.02%	2.71%	3.43%	10.36%	2.83%	0.00%
Other and Unknown	1.39%	2.62%	3.45%	1.29%	0.00%	0.17%

Table 2.11: Average LGD broken down by contract begin and asset type.

	2007	2008	2009	2010	2011	All years
All asset types	27.09%	27.41%	27.02%	20.14%	10.77%	21.93%
Commercial Vehicles	23.45%	28.59%	24.53%	16.11%	9.96%	19.98%
Passenger Cars	26.61%	23.10%	32.48%	16.49%	11.40%	22.26%
Machinery & Industrial Equipment	22.88%	21.53%	18.75%	15.70%	7.02%	16.22%
Computers & Business Machines	34.44%	33.96%	38.02%	34.51%	14.87%	30.64%
Other and Unknown	22.91%	24.61%	19.62%	16.89%	8.82%	17.72%

are replaced by zero losses. Note that negative LGDs are not rare for leases, as observed by Hartmann-Wendels et al. (2014); Miller and Töws (2018). The average LGD broken down by the asset type over the year corrected for resolution bias is shown in table 2.11. The calculated LGDs produce a bimodal distribution. LGD from leases in other works, such as Laurent and Schmit (2005); Hartmann-Wendels et al. (2014), also exhibit a bimodal or multimodal distribution.

Although our dataset is comparatively bigger than datasets used in the literature with a similar topic (see table 2.1), the long-run average LGD as defined in EBA/GL/2019/03 cannot be calculated, which requires at least a five-year period of data and various economic circumstances. We estimate the *long-run average* LGD using the average LGD over all previous years. However, if the current realised LGD is higher than the long-run average, institutions are required to use the higher LGD instead, as stated in EBA/GL/2019/03. There is no appealing reason to assume that the years before 2007 are worse (LGD-wise) for leasing exposures than an average year.

However, the long-run average LGD, as shown in table 2.12, is not yet adequate as input

Table 2.12: Long-run average LGD broken down by contract begin and asset type.

	2007	2008	2009	2010	2011
All asset types	27.09%	27.41%	27.14%	24.90%	21.93%
Commercial Vehicles	23.45%	28.59%	25.46%	22.46%	19.98%
Passenger Cars	26.61%	24.55%	32.48%	24.76%	22.26%
Machinery & Industrial Equipment	28.88%	21.96%	20.14%	18.59%	16.22%
Computers & Business Machines	34.44%	34.13%	38.02%	35.54%	30.64%
Other and Unknown	22.91%	24.61%	21.69%	20.19%	17.60%

for the IRBA. The IRBA is designed to calculate EL during a downturn period. Therefore, a downturn LGD is also required. Both the EBA/CP/2018/07 and the EBA/CP/2018/08 set the guidelines and requirements for a downturn LGD calculation. In summary, the downturn LGD is defined as the (either observed or estimated) LGD in a period where various selections of relevant macroeconomic factors have been most severe over the last 20 years. Typically, banks will identify 2008/2009 as a downturn period using the GDP growth (and other proxies) as identifiers and assign the respective LGD as the downturn LGD. As shown in table 2.11 and 2.12, it is clear that the LGDs over the years show a peak in the years 2008 and 2009. Given the available dataset, using the maximum LGD from table 2.12 per asset type is adequately conservative. In order to outset the lost effect from missing discounting rate and workout costs, an additional 3% is added in the downturn LGD.

Similarly to PDs, there are also input floors for the calculated LGDs (in this case, the downturn LGDs). As per Basel Committee on Banking Supervision (2017a), the input floor is 15% for a secured exposure with physical collateral and 25% or 30% for an unsecured exposure. In contrast to the SA, leasing objects can be used as collaterals in the IRBA, which reduces the RWs through the LGD input. Even with a 40% haircut, it is clear that the LGD parameter floor lies below the downturn LGD in our dataset, so a further adjustment for the LGD floors is not necessary.

Although the LGD is a parameter required as an input in the IRBA, it is more natural to discuss the recovery rate (RR) for leasing objects. A leasing exposure is tied closely to its leased object. Thus, it makes more sense to analyse the recovery from selling the leased asset. In

Table 2.13: Direct Comparison of asset recovery rate and 1-LGD from table 2.11 by asset type.

	Asset Recovery Rate	1-LGD
Commercial Vehicles	78.13%	80.02%
Passenger Cars	72.49%	77.74%
Machinery & Industrial Equipment	81.67%	83.78%
Computers & Business Machines	65.66%	69.36%
Other	78.21%	82.28%

general, the recovery rate is typically defined as (1-LGD) or vice versa. In terms of the asset recovery, we can calculate the recovery rate as the quotient of the disposal value (after deducting the residual value) and the initial outstanding. The residual value risk is more prominent for leasing exposures than for secured loans. If the lessor misestimates the residual value, it will be reflected in the asset recovery directly. Assuming the asset is sold, the disposal value should be available. In the other case, there are two possible outcomes. The default is either closed or not closed in the dataset. Resolved defaults without an asset sale can occur due to many reasons, including the lessor writes off the remaining exposure, the default is cured, or the leased asset is worthless. In this case, we calculate the recovery rate by using the recorded final loss (since the default is closed). If the default is not closed and the asset is not sold, then we use the estimate from table 2.10.

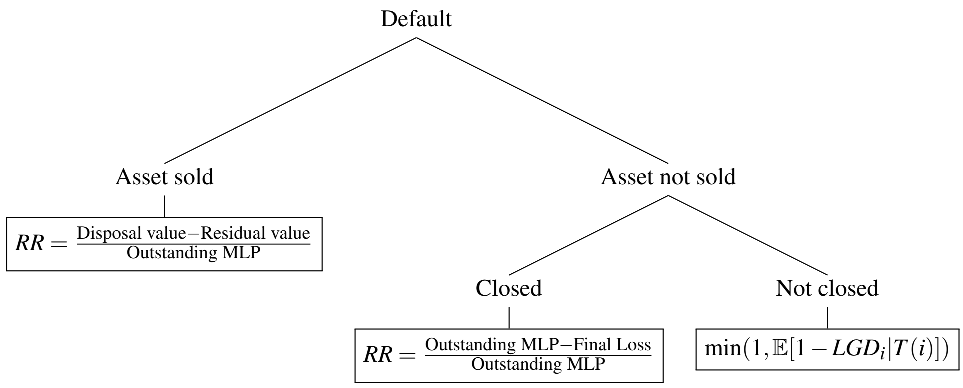

Figure 2.3: Tree diagram of RR calculation/estimation based on contract's default and sale status

Table 2.13 shows a direct comparison between the asset recovery rate and the parameter (1-LGD). It is not surprising that the latter is often bigger because the recovery typically not only consists of the sale value. The fact that they are close to one another highlights the importance of the asset's role in the recovery process of defaulted leasing contracts.

2.4.2 Regulatory Risk Weights

There are currently three possible approaches under the credit risk framework to calculate the regulatory capital requirements: the SA, the foundation IRBA (F-IRBA), and the advanced IRBA (A-IRBA). These approaches serve mainly as a guideline to assign an RW to a given exposure. While the SA is quite rigid, the IRBA is designed to be flexible.

The SA sets the RW based on the exposure's asset classification and its credit quality. At best, the RW can be 0%, e. g. for sovereign exposures with the highest credit quality, and at worst 150% for exposures with the lowest credit quality. Since lessees are often of an SME or a retail type, it is in many cases not possible to acquire an external credit rating for these obligors. In the dataset, such information is also not available. It is realistic to assume that most of the time lessors do not have a reliable source to classify SME lessees to the given credit quality groups. Only some of the financial collateral types are eligible for risk mitigation under the SA, while physical collateral types, i. e. leased assets, are not eligible. This treatment renders a leasing contract similar to an unsecured exposure. This restriction holds as well in the revision of the Basel framework. Other changes include the introduction of new asset classes.

The dataset gives clear information on the exposure type for all leasing contracts. We assume all exposures are unrated, except the sovereign class. The assigned RWs are, therefore:

- **Corporate (CRR Art.122)**: 100% for unrated corporates and since none of the exposure's countries has higher RW,
- **Institutions (CRR Art.120-121)**: to be treated similarly to sovereign exposures,
- **Retail (CRR Art.123)**: flat 75%,
- **Sovereign (CRR Art.114)**: 0%-100% based on the worst rating out of Standard & Poor's or Moody's ratings in the given period,
- **Unknown**: 100% conservatively treated as an unrated corporate exposure,

- **SME (CRR Art.501)**: apply the SME supporting factor of 0.7619 if the exposure is not defaulted,
- **Defaulted (CRR Art.114, 120-123)**: flat 150%.

The revision of Basel III introduces different classifications for the SA. A summary of the changes can be found in the Basel Committee on Banking Supervision (2017b). One of the significant changes, which have impacts on our calculation, is the introduction of an RW of 85% for corporate SME exposures specifically. Due to the RW reduction for SME exposures, it is also discussed whether the SME supporting factor will be removed. Although it is not yet certain, the removal is recommended by the EBA/OP/2019/09a, since the supporting factor has not yet fulfilled its intended purpose as reported by the EBA/OP/2016/04. Overall, the regulatory capital requirement of an average lessor using the SA will most likely experience a slight increase if its portfolio composition does not change.

Both the F-IRBA and the A-IRBA require a number of input parameters to calculate the RWs. For our purposes, the most relevant ones are the PD and the LGD as well as the total yearly revenue for corporates. While institutions with the A-IRBA have the flexibility to model and calculate their own LGD, those with the F-IRBA do not. The LGD used for the F-IRBA is predetermined at 40% for senior secured exposures (by physical assets with a 140% ratio of collateral to exposure values) and 45% for senior unsecured exposures. These values are generally conservative, which lead institutions as well as leasing companies to prefer A-IRBA to F-IRBA. However, the finalisation of the Basel III Accord limits some asset classes to use only the F-IRBA or the SA, e. g. for corporates with consolidated revenues over €500m. The finalisation also reduces the predetermined LGD for senior secured exposures with physical collateral to 25% and for unsecured exposures on corporates or banks to 40% or 45%, respectively. For the A-IRBA, an input floor of 15% (for secured exposures), 25% (for unsecured corporate exposures), and 30% (for unsecured retail exposures) are introduced. For leasing exposure, where the collateral value and the exposure value are equal without taking profit into account, only 71.43% ($\approx$ 100%/140%) of the outstanding amount can be recognised as collateralised and the remaining

28.57% as uncollateralised. It implies an effective LGD of

$$LGD_F = 45\% \cdot \frac{40\%}{140\%} + 40\% \cdot \frac{100\%}{140\%} = 41.43\%.$$

Note that the effective LGD would be 30.71% for banks or 29.29% for corporate exposures in the new framework. The effective LGD floor for the A-IRBA would be 17.86% for corporate exposures (if the A-IRBA is allowed) and 19.29% for retail exposures, assuming there is no additional collateral. In this essay, the total yearly revenue, which affects the correlation coefficient, is assumed to be €10m.

For the sake of comparability between the regulatory approaches, we assume that retail exposures are also allowed to be treated with the F-IRBA. If we exclude retail exposures in the F-IRBA evaluation, any difference in the result between the A-IRBA and the F-IRBA may inherently come from the portfolio composition and not necessarily from the regulatory treatments. Since the main purpose is to evaluate the different regulatory approaches, we decide to apply a pseudo-F-IRBA for retails, i. e. an effective LGD of 41.43% is used for non-defaulted exposures.

With all the components, the RW for non-defaulted exposures is calculated as follows

$$\begin{aligned} RW = \Big(& \underbrace{LGD \cdot N\Big(\frac{1}{1-R} \cdot G(PD) + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\Big)}_{\approx \text{VaR}} - \underbrace{LGD \cdot PD}_{\approx \text{EL}}\Big) \cdot \\ & M_F \cdot 12.5 \cdot 1.06, \end{aligned}$$

where for corporate, institution, and sovereign exposures:

$$\begin{aligned}
R &= 0.12 \cdot \frac{1-e^{-50 \cdot PD}}{e^{-50}} + 0.24 \cdot \left(1 - \frac{1-e^{-50 \cdot PD}}{e^{-50}}\right) \\
&\quad - 0.04 \cdot \left(1 - \frac{\min\{\max\{5, S\}, 50\} - 5}{45}\right) \\
M_F &= \frac{1 + (M - 2.5) \cdot b}{1 - 1.5 \cdot b}, \text{ where} \\
M &= \max\left\{1, \min\left\{\frac{\sum_t t \cdot CF_t}{\sum_t CF_t}, 5\right\}\right\} \text{ for A-IRBA or } M = 2.5 \text{ for F-IRBA, and} \\
b &= (0.11852 - 0.05478 \cdot \ln(PD))^2
\end{aligned}$$

and for retail exposures:

$$R = 0.03 \cdot \frac{1-e^{-35 \cdot PD}}{e^{-35}} + 0.16 \cdot \left(1 - \frac{1-e^{-35 \cdot PD}}{e^{-35}}\right) \text{ and } M_F = 1.$$

N and G denote the cumulative distribution function of a standard normal random distribution and its inverse. S represents the total annual group revenue. Given a constant monthly lease payment, a simple algebraic transformation leads to

$$M = \max\left\{1, \min\left\{\frac{T+1}{12 \cdot 2}, 5\right\}\right\},$$

where T is the remaining leasing term in months. The factor 1.06 is omitted in the revised Basel.

Defaulted exposures have RWs of 0% under the F-IRBA, and $RW = \max\{0, 12.5 \cdot (LGD - ELBE)\}$ under the A-IRBA, where ELBE stands for the Expected Loss Best Estimate. Despite the name, the LGD in the IRBA formula is an input parameter for the downturn LGD and is not the average LGD. Thus, the difference (LGD-ELBE) is the loss ratio, which is not yet covered by the loss loan provisions. We set the LGD as the downturn LGD explained in section 2.4.1 and ELBE as the long-run-average LGD (table 2.12), assuming leasing companies adequately predict their average LGD for the year and set the loss provisions accordingly.

The SME supporting factor of 0.7619 is multiplied with the calculated RW if the contract does not default. The classification of SME is given in the dataset. In practice, the burden of proof for this lies on the institutions. Since some may refuse or are not able to give the information on their revenues, there might be some SME exposures to which the SME supporting factor cannot be applied.

Although most of the predetermined LGDs for the F-IRBA are lower in the revision and other changes hints to less strict regulation, some new rules are introduced such as input and output floors or limitations of which approaches to use for a particular asset class. Overall, a slight increase in portfolio RW can be expected as reported by the European Banking Authority (2019).

2.4.3 Portfolio Simulation

The main goal is to estimate the UL of a leasing portfolio. As explained at the beginning of this section, the EL conditional on the latent factor at 99.9% level (what the IRBA is modelled for) and the 99.9% loss percentile are different objects. Their equivalence is only guaranteed asymptotically under some assumptions.

The simplest way to estimate the 99.9% loss percentile is by approximating the loss distribution. The losses can be closely replicated by simulating randomly i.i.d. portfolios of a representative lessor. It is easy to calculate the portfolio loss, given its exact composition, since losses are observed/estimated based on section 2.4.1. As the number of drawn contracts in a simulated portfolio grows to infinity, the resulted loss distribution converges to the systematic loss distribution due to the asymptotic equivalence (see Gordy (2003)). However, lessors with infinitely fine-grained portfolio do not exist. By overdrawing in the simulation, there is an issue of over-diversification and the fact that the drawing pot does not have infinitely many contracts are good arguments to limit the number of drawn contracts to an appropriate amount.

To avoid those issues, the portfolio size is assumed conservatively to be 40,000 leasing contracts, which is less than the average portfolio size (vary from years to years). Considering the fact that the participating companies are big leasing companies or belong to a big group, se-

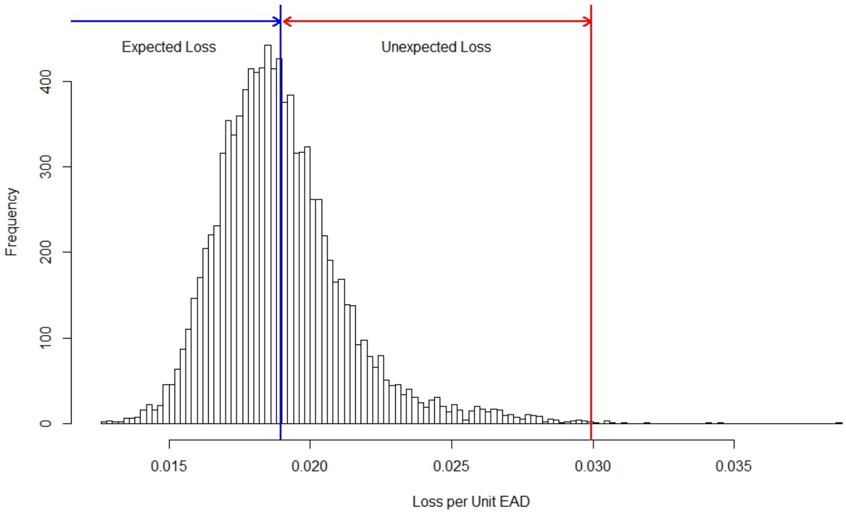

Figure 2.4: Histogram of portfolio loss simulation

lecting a relatively lower number of contracts than the average portfolio size is a conservative assumption for the UL estimation. The choice of portfolio size can affect the 99.9% loss percentile. A small portfolio will have a higher loss percentile, which in return implies a higher UL (see Pirotte and Vaessen (2008)). It might be even more conservative in choosing for a portfolio size less than 40,000 contracts, but assuming a small number of contracts violates the fine-granularity assumption. Thus the asymptotic equivalence does no longer apply. In other words, the UL estimate cannot be compared to the IRBA capital requirements. Repeating this process 10,000 times ensures the convergence of the sample distribution towards the real loss distribution of a representative leasing portfolio due to the law of large numbers, and in extension also towards the systematic loss distribution due to Gordy (2003).

In each random portfolio, there are a number of defaulted contracts. The number of defaults in each simulation can be controlled by simulating the default rates. This step can e. g. be done by using the Poisson distribution to generate a random number of defaults and draw a number of defaulted and non-defaulted contracts accordingly. Such a method can be found e. g. in Deloitte (2013a). While this step ensures the number of defaults match with the statistics, there is a mismatch between the proportion of defaulted contracts in a portfolio and the default rate. Although leasing companies generally try to minimise the workout duration, the overall process may take longer than one year. During this duration, the defaulted contracts stay in the portfolio and raise the proportion of defaulted contracts over the observed default rates. For the calculation of the capital requirement, it is necessary to take long durations into account, since these defaults have to be covered by capital as well over their workout process. Thereby, we decide on a simulation, which does not draw defaulted and non-defaulted contracts separately.

Given the simulated random portfolio losses $P_1, \ldots, P_{10,000}$ and let the $P_{(0.999)}$ be the 99.9%-th biggest loss among the 10,000 simulated losses and $\overline{P}$ the mean of the simulated random portfolio losses, the UL can be calculated by

$$\widehat{UL} = P_{(0.999)} - \overline{P}\,.$$

For each simulation type, 10,000 random portfolios (each with 40,000 leasing contracts) will be drawn. Various simulation types will be considered, e. g. year-specific, asset-specific, exposure-specific, etc.

As argued before that leasing is more asset-based, a similar analysis can also be done using recovery rates instead of losses. Let us consider an asset pool where seized collateral items are collected for further liquidation or re-lease. In particular, we are interested in the one-tailed 1% quantile of the pool value, which is oriented on the risk mitigation technique for the SA if the collaterals are eligible (CRR Art.225). Physical assets are in general ineligible as collaterals for the SA. Consequently, this limitation belongs to the reasons why asset-based finance institutions (such as leasing companies) prefer the IRBA to the SA, simply due to the fact that physical assets can be used to mitigate the risk in the IRBA.

Physical assets are generally depreciating assets, so their values depend significantly on their age. Although, the information on the asset's age as well as the depreciation rate is not available. However, typically a leased asset is often new at the beginning of the contract, so the contract

Table 2.14: Number of defaults broken down by asset type and contract age at default in years. Default definition is based on the Basel framework.

	$t < 1$	$1 \leq t < 2$	$2 \leq t < 3$	$3 \leq t < 4$	$t \geq 4$	Total
Commercial Vehicles	5,383	8,037	7,544	5,277	2,948	29,189
Passenger Cars	4,016	5,049	4,779	3,037	865	17,746
Machinery & Industrial Equipment	5,754	7,510	6,984	4,993	4,032	29,273
Computers & Business Machines	7,569	8,419	6,577	3,729	1,753	28,047
Other and Unknown	2,130	2,340	1,814	1,119	861	8,264

Table 2.15: Average asset recovery rates broken down by asset type and contract age at default in years. Default definition is based on the Basel framework.

	$t < 1$	$1 \leq t < 2$	$2 \leq t < 3$	$3 \leq t < 4$	$t \geq 4$	Total
Commercial Vehicles	72.63%	74.34%	79.05%	81.37%	81.08%	78.13%
Passenger Cars	72.90%	69.93%	71.75%	76.66%	73.36%	72.49%
Machinery & Industrial Equipment	83.87%	79.36%	80.43%	83.93%	81.83%	81.67%
Computers & Business Machines	69.19%	64.35%	63.44%	65.47%	66.58%	65.66%
Other and Unknown	80.24%	77.62%	77.78%	80.38%	76.14%	79.40%

age can act as a proxy. For a defaulted contract, there are generally four available timestamps: the contract begin, the default date, the sale date, and the resolution date. In the context of depreciating assets, the duration between the contract begin and the sale event should be the best choice to be a determinant for their values. Not only does the sale event not necessarily exist in all cases but both the sale event and the resolution event are also post-default information. If lessors know the exact sale and resolution dates, then there is less need for volatility analysis. Henceforth, we use the duration between contract begin and default event (t) to determine the trend of the recovery rate. Since most of the defaults occur under three or four years in our dataset, we define five time-buckets: $t < 1$; $1 \leq t < 2$; $2 \leq t < 3$; $3 \leq t < 4$; and $t \geq 4$ years.

Similarly to the previous analysis, the procedures are done with 10,000 repetitions for each asset type and time bucket. The asset pool size has an impact on the convergence rate of the loss/recovery distribution. A large pool ensures convergence but may inhibit an over-diversification effect, while a small pool can be extremely volatile. The appropriate choice for the representativeness would be the average asset pool size of lessors. We decide on 1,000 assets for each time-bucket and asset type. In some rare cases, in which the dataset within a time-bucket is too small (see table 2.14), only about half from the available dataset is drawn randomly.

Table 2.16: Regulatory capital requirements under three approaches (Standardised Approach, Foundation and Advanced Internal Ratings-Based Approach) for each EAD unit and the simulated unexpected loss broken down by year. The calculation assumes a solvency ratio of 10.5%.

	2007	2008	2009	2010	2011	All years
SA	8.17%	7.97%	8.33%	8.44%	8.50%	8.31%
F-IRBA	5.29%	5.41%	5.92%	5.94%	5.92%	5.76%
A-IRBA	4.03%	4.57%	5.56%	6.12%	6.50%	5.55%
UL	1.03%	1.45%	1.29%	0.65%	0.52%	1.09%

2.5 Simulation Result

This section presents the simulation results in two parts: 1) a comparison of the regulatory RWs and the ULs; and 2) a recovery analysis of the leased assets.

To calculate the regulatory capital requirements of an exposure, the calculated exposure's RW is multiplied by its exposure amount at default (EAD). In the portfolio, the total of the risk-weighted exposure amounts is then multiplied with the capital adequacy ratio of 10.5%. Table 2.16 shows high ULs in the pre-crisis and crisis periods, but low ULs in the post-crisis periods. This result, in particular, represents a total of 60,000 randomly drawn leasing portfolios (10,000 each for every year and 10,000 for the global simulation). Both the IRBA responds sensitively with the crisis. As the input parameters, the PD and the LGD, soar high during the downturn periods, so do the IRBA regulatory capital requirements. These effects typically will not vanish for the following years afterwards because of the use of long-average default rates. The RWs under the SA are quite rigid across the years compared to those under the IRBA. In contrast to the IRBA, the only risk factor that can drive the SA capital requirement up is the default composition of the portfolio (150% RW for defaulted exposures compared to 75%-100% for other asset classes). If the regulatory capital requirements are only weakly risk-sensitive (as observed in table 2.16 for the SA), then there is only a weak incentive for institutions to reduce their risk profile and to offer better risk management.

In general, we expect the SA capital requirements to be the highest, followed by the F-IRBA capital requirements and then the A-IRBA capital requirements. The fact that the F-IRBA capital requirements fall below the A-IRBA capital requirements after the financial crisis (2010-2011) is contra-intuitive and mostly caused by the framework design itself. The A-IRBA capital requirements are not only sensitive towards the PD and the LGD inputs but also towards the number of defaults remaining in the portfolio. After the financial crisis, the default composition in the portfolio increases, which does not necessarily imply a higher workout duration compared to the pre-crisis period. While the A-IRBA sets a high capitalisation rate for defaulted exposures, the F-IRBA sets it to zero, which altogether explains the lower percentages for the F-IRBA capital requirements in 2010-2011 compared to the A-IRBA capital requirements.

From the regulatory perspective, the expectation that the SA capital requirements are higher than the other approaches is not a desirable property. In practice, it is not surprising that IRBA institutions have a lower capital ratio compared to SA institutions because institutions need to actively seek for permission to be allowed to use the IRBA. Institutions which cannot profit from the IRBA will not actively seek for permission. However, given a particular exposure, the SA should not put SA institutions in disadvantages compared to IRBA institutions. Typically, IRBA institutions justify their lower capital ratio due to their extensive and more granular risk analysis. The results of the analysis, as presented in table 2.16, are not granular. This difference between the SA and the IRBA capital requirements for leasing gives evidence that lessors are at a disadvantage if they are not permitted to use the IRBA.

The results of the portfolio simulation in table 2.16 show significant differences between the regulatory capital requirements and the ULs. In summary, the regulatory capital requirements from all approaches can reach five to eight times as high as the UL in the same year. However, the UL alone is not sufficient to explain the loss potential of an exposure. This information needs to be paired with the information on the EL to understand the total credit risk profile of an exposure. A gap between the regulatory capital requirement and the estimated UL alone is not a surprise since the regulatory capital requirements are designed to be higher than the UL to accommodate for model/measurement error and margin of conservatism (MoC). An excessively high regulatory capital requirement may lead to negative effects, such as higher social/welfare costs (Van den Heuvel (2008); Mikkelsen and Pedersen (2017)) and higher WACC (Kashyap et al. (2010); Cosimano and Hakura (2011); Miles et al. (2013)), which ultimately leads to a hindrance in the economy.

Across all of the year-specific simulations and the global simulation, we examine whether there is any portfolio out of 60,000 simulated portfolios, in which the regulatory capital requirement cannot adequately cover the realised loss. Every single one of the 60,000 portfolios adequately covers the realised loss, although profits from healthy leasing contracts are not considered in this analysis. Based on the asymptotic equivalence between the VaR and the conditional loss as explained at the beginning of section 2.4, we would expect about 0.1% of the simulated portfolio losses to be higher than the regulatory capital requirement. The fact that we cannot observe any portfolio in which the regulatory capital requirement cannot cover the realised portfolio loss further supports the argument that the regulatory capital requirements may be too conservative.

We trace back the minimum amount of capital required to fulfil a given condition by using the reverse stress test method (see table 2.25). We choose the SA capital requirement as the base for this test[5]. The SA capital requirement is multiplied by a factor which is increased incrementally until a given condition is achieved, e. g. the UL is adequately covered. In other words, the solvency coefficient is reduced to a certain percentage, in which the simulation detects a breach, e. g. the UL exceeds the regulatory capital requirement. Surprisingly, a factor of merely 15% from the SA capital requirement is needed to cover the UL in the global simulation (in other words: the SA capital requirement has an 85% buffer) and at most of a 19% during the downturn period (2008) (which implies an 81% buffer). This analysis assumes that lessors estimated their ELs correctly and set their loss provision accordingly. If the condition is expanded for the coverage of both the EL and the UL (i. e. the VaR at 99.9%-level), the minimum percentage rises to a 37% and at most to a 52% of the SA capital requirement for the global simulation and the downturn period simulation (2008), respectively. After 42% and at most 58% of the SA

[5]The reason for this will be explained in details in section 2.6.

Table 2.17: Comparison of the regulatory capital requirements under three approaches (Standardised Approach, Foundation and Advanced Internal Ratings-Based Approach) for each EAD unit and the simulated UL broken down by year and asset type. The calculation assumes a solvency ratio of 10.5%.

		2007	2008	2009	2010	2011	All years
Commercial Vehicles	SA	8.15%	7.73%	8.19%	8.39%	8.54%	8.21%
	F-IRBA	5.29%	5.25%	5.62%	5.56%	5.51%	5.47%
	A-IRBA	4.06%	4.36%	5.56%	6.58%	7.44%	5.79%
	UL	0.46%	0.43%	0.38%	0.33%	0.24%	0.37%
Passenger Cars	SA	7.78%	7.81%	7.86%	7.88%	7.92%	7.86%
	F-IRBA	5.24%	5.47%	5.73%	5.89%	6.04%	5.74%
	A-IRBA	4.33%	4.71%	5.54%	5.76%	5.87%	5.37%
	UL	0.42%	1.08%	0.62%	0.34%	0.22%	0.76%
Machinery & Industrial Equipment	SA	7.96%	8.01%	8.40%	8.58%	8.69%	8.38%
	F-IRBA	5.15%	5.34%	5.84%	5.80%	5.73%	5.63%
	A-IRBA	3.41%	4.09%	5.05%	5.70%	6.16%	5.07%
	UL	0.94%	0.88%	0.81%	0.61%	0.48%	0.73%
Computers & Business Machines	SA	8.30%	7.71%	7.85%	7.81%	7.82%	7.85%
	F-IRBA	5.30%	5.36%	5.89%	6.05%	6.09%	5.79%
	A-IRBA	5.98%	6.13%	7.17%	7.19%	7.41%	6.87%
	UL	0.72%	1.30%	1.06%	0.98%	0.95%	1.01%
Other and Unknown	SA	9.14%	8.53%	8.79%	8.84%	8.67%	8.76%
	F-IRBA	5.84%	5.90%	6.54%	6.63%	6.48%	6.36%
	A-IRBA	4.21%	4.45%	5.51%	5.80%	5.76%	5.33%
	UL	0.33%	0.44%	0.38%	0.28%	0.19%	0.33%

capital requirement for the global simulation and the downturn period simulation, every single simulated portfolios' loss is covered by the reduced capital requirement.

For a specialised leasing company, which only leases a particular asset type, the results may differ from table 2.16. For this purpose, similar simulations are conducted by filtering for a particular asset type. Ships, Aircraft, Railway, and Rolling Stock are excluded. The simulated asset-specialised leasing portfolio can be interpreted as a random portfolio of a representative asset-specialised leasing company. This type is particularly popular among captives, as they seek to offer various financing possibilities for their customers. For comparison, the exact same bootstrap parameters were chosen (10,000 repetitions with 40,000 portfolio size).

Similar to table 2.16, the estimated ULs are significantly lower compared to the regulatory capital requirements under all approaches. The ULs of the asset-specific simulations (table 2.17) are consistently lower than the one from the global simulation (table 2.16). This observation can

Table 2.18: Comparison of the regulatory capital requirements under three approaches (Standardised Approach, Foundation and Advanced Internal Ratings-Based Approach) for each EAD unit and the simulated UL broken down by year and exposure type. The calculation assumes a solvency ratio of 10.5%.

		2007	2008	2009	2010	2011	All years
Retail	SA	6.13%	6.32%	6.67%	6.93%	7.07%	6.71%
	F-IRBA	4.16%	4.23%	4.29%	4.19%	4.08%	4.14%
	A-IRBA	3.36%	3.84%	4.47%	4.85%	4.73%	4.69%
	UL	0.70%	0.70%	0.58%	0.52%	0.47%	0.60%
Corporate	SA	9.60%	9.45%	9.73%	9.83%	9.84%	9.52%
	F-IRBA	6.04%	6.46%	7.23%	7.42%	7.45%	7.02%
	A-IRBA	4.76%	5.56%	6.52%	6.41%	5.92%	6.29%
	UL	0.54%	1.13%	1.07%	0.63%	0.49%	0.84%

be explained by the fact that randomly drawn contracts with the same asset types have a similar loss distribution. Hence, the deviation from the mean is lower compared to the global simulation. Note that the UL is the difference between the VaR (estimated by the 99,9% percentile of the loss distribution) and the EL (estimated by the mean of the loss distribution). Henceforth, the UL is proportional to the standard deviation.

Similarly, the simulation can also be done by segmenting over the exposure type. However, there are some exposure types of which the amount of data is not appropriate for a Monte Carlo simulation. A similar exclusion is also done for an asset type (Ships, Aircraft, Railway, Rolling Stock). For the exposure-specific simulation, we only use retail and corporate exposures with the same parameters as previous simulations. For the sake of comparability, we also calculate the F-IRBA capital requirement for retail-specific portfolios using the pseudo-F-IRBA, as explained in section 2.4.2.

The effect of the SME supporting factor can be clearly seen in table 2.18 since all of the retail exposures but only a fraction of the corporate exposures are considered SME exposures in the simulation. While the impact of the financial crisis can be distinctively identified for the corporate exposures, the ULs for the retail exposures interestingly are quite stable. Another interesting observation is the fact that the retail-specialised portfolios also show higher A-IRBA capital requirements than the F-IRBA capital requirements in the downturn periods. As men-

Table 2.19: Comparison of the regulatory capital requirements under three approaches (Standardised Approach, Foundation and Advanced Internal Ratings-Based Approach) for each EAD unit without SME supporting factor and the simulated UL broken down by year. The calculation assumes a solvency ratio of 10.5%.

		2007	2008	2009	2010	2011	All years
No Supporting Factor	SA	8.98%	8.75%	9.00%	9.06%	9.12%	9.62%
	F-IRBA	5.98%	6.12%	6.57%	6.54%	6.51%	6.76%
	A-IRBA	4.63%	5.37%	6.41%	6.46%	6.07%	5.93%
	UL	1.03%	1.45%	1.29%	0.65%	0.52%	1.10%

tioned before, the A-IRBA capital requirements are risk-sensitive and typically increase during the post-crisis period, mainly due to the increase in the PDs. If the F-IRBA capital requirements exceed the A-IRBA capital requirements, the proportion between defaulted and non-defaulted contracts must have been shifted. The fact that the retail-specialised portfolio does not follow this pattern indicates that the default proportion for the retail exposures is not changed significantly after the financial crisis.

Due to the recent recommendation for the removal of the SME supporting factor brought in EBA/OP/2019/09a, it is interesting to see how this particular change will impact the whole analysis. Note that the introduction of the SME supporting factor does not aim to ensure an appropriate risk evaluation of SME exposures, but rather to give incentive for banks to increase SME lending following a crisis, as stated in EBA/OP/2016/04, under the assumption that capital requirement is one of many determinants affecting lending decisions. Although table 2.19 shows a significant increase in the capital requirements, which widens the gap between the capital requirements and the ULs, it does not necessarily imply a stronger incentive to offer one financial instrument over another. A similar impact will most likely be observable within banks' portfolios with similar SME proportion. Note that the ULs from table 2.16 and table 2.19 are identical, since the SME supporting factor influence only the regulatory capital requirements.

Under the IFRS 16, lessors have to categorise a leasing contract into one of two types: finance lease or operating lease. Roughly speaking, the first one covers leasing contracts in which financing the leased asset is the primary purpose of the lease, while the latter one is associated

Table 2.20: Comparison of the regulatory capital requirements under three approaches (Standardised Approach, Foundation and Advanced Internal Ratings-Based Approach) for each EAD unit and the simulated UL broken down by year and contract type. The calculation assumes a solvency ratio of 10.5%.

		2007	2008	2009	2010	2011	All years
Finance Lease	SA	8.10%	7.89%	8.29%	8.44%	8.51%	8.28%
	F-IRBA	5.27%	5.39%	5.91%	5.91%	5.86%	5.74%
	A-IRBA	3.93%	4.45%	5.44%	6.09%	6.54%	5.52%
	UL	1.01%	1.44%	1.23%	0.71%	0.54%	1.23%
Operating Lease	SA	8.41%	8.49%	8.64%	8.55%	8.47%	8.52%
	F-IRBA	5.37%	5.59%	5.99%	6.25%	6.38%	5.93%
	A-IRBA	4.45%	5.34%	6.57%	6.34%	6.23%	5.82%
	UL	0.68%	0.71%	0.58%	0.49%	0.20%	0.62%

Table 2.21: 1% percentile of the asset pool's recovery rates distribution broken down by contract age and asset type.

	$t < 1$	$1 \leq t < 2$	$2 \leq t < 3$	$3 \leq t < 4$	$t \geq 4$	Total
Commercial Vehicles	72.84%	73.74%	74.99%	80.89%	75.30%	74.77%
Passenger Cars	62.41%	65.01%	66.45%	69.77%	67.08%	63.12%
Machinery & Industrial Equipment	72.82%	73.79%	78.89%	80.76%	75.90%	75.25%
Computers & Business Machines	68.70%	59.19%	54.96%	56.78%	62.09%	60.56%
Other and Unknown	68.86%	78.40%	75.12%	76.58%	62.84%	70.45%

with leasing contracts in which a temporary need for the asset for the business' operation is the core of the contract. The significant difference between those contract types is the residual value. Since we disregard the capital requirement for the residual value in this analysis, the difference between the finance lease and operating lease contains only information on the credit risk. Table 2.20 shows that the difference between the two contract types in the regulatory requirement is negligible. Although stated earlier that a UL comparison alone cannot say much without information on the ELs, lower ULs for operating leases may hint to operating leases being less susceptible to a downturn event compared to finance leases. The indirect influence due to portfolio composition cannot be excluded in this analysis, e. g. a finance lease is more popular for automotive, but an operating lease is more popular for office equipment, thus affects the risk profile explicitly.

The second part of this section presents the results of the recovery analysis of leased assets from the defaulted cases in the dataset. As stated previously, leasing is more asset-oriented, so it makes sense to expand the analysis to an asset pool simulation. From the regulatory point-of-view, this part can be interpreted as an investigation for the impact, should leased assets be an eligible collateral type for the SA. The method used for calculating the 1%-th one-tailed percentile of the asset pool recovery rates is similar to the portfolio simulation. We can interpret the resulted recovery rates as *downturn* recovery rates. The samples are divided depending on the duration between the contract begin and the default event (t) in years. Although the asset's book value depreciates linearly, the disposal value may inhibit a different pattern. To support this argument, we cannot observe a clear falling pattern in table 2.21, as it would be expected of the book value of a depreciating asset. The downturn recovery rates of commercial vehicles, machinery and industrial equipment are the highest at about 75% (the equivalent of a 25% LGD), while the downturn recovery rates of other types may reach 60% (the equivalent of a 40% LGD).

Although the analysis is designed to evaluate the asset recovery, the calculated recovery rate may include non-asset recoveries in some cases. This is especially the case if the default is not yet closed in the dataset. In practice, having the legal right to seize the asset may suffice for a high recovery rate without actually seizing the asset. Office equipment is a typical example of an asset type with a fast depreciation rate. However, defaulted firms will typically prioritise to pay their lease obligations during the default process since their priority is to recover. By neglecting the lease obligations, it will trigger an asset seizure, which may disable them to operate further.

In summary, we observe an overall low UL-level for leasing exposures. The required capital is conservative in the sense that it can cover the potential UL multiple times. This is only appropriate if we argue that the financial crisis is not an appropriate choice of downturn periods for leasing exposures. We sincerely doubt that a potential leasing-associated downturn period may have a much worse impact (than five- to eight-fold of the observed impact during the financial crisis) to justify for this wide gap. However, our portfolio analysis only compares the UL and the regulatory capital requirement because it is designed to cover the UL. In practice, lessors (and other financial institutions) do not only have to deal with the UL of their portfolio, but also the EL. The VaR (as the sum of the EL and the UL) may not be in a comparable magnitude as the

UL alone. Whether the EL is also comparably low depends primarily on the individual internal strategies of the lessors. If the trend persists that lessees as firms have in general low credit quality, as e. g. reported by Sharpe and Nguyen (1995), it may not be surprising to observe a high EL. The low level of the ULs in our portfolio analysis only supports the proposition that losses from leasing exposures are typically not unexpected. The asset recovery analysis confirms that the characteristics of leasing, such as legal ownership, internal expertise of the assets, etc. play in favour for the lessors. For some asset types, it may even reach an equivalent downturn LGD of 25%.

2.6 Lowest Bound on Risk Weight Reduction

The previous section not only confirms the adequateness but also the excessive conservativeness of the Basel capital requirement for leasing exposures, complementary to Schmit (2004, 2005); Pirotte and Vaessen (2008); Eisfeldt and Rampini (2009). This section concentrates on the other end of the discussion. For the relevant regulatory authorities, any change in the regulation will have some systematic effects which may have some negative impacts. In many respects, a finance lease is similar to a loan secured with a physical asset. Bayless and Diltz (1988) report that financial institutions do not differentiate capital leases (comparable to finance leases) and debts in terms of loan decision. It is debatable whether a change in the regulation regarding leases will trigger a behaviour shift from financial institutions.

A treatment change favouring leases will have either a positive or negative effect on the lease's RW. 0% RW reduction will not change any behaviour, but 100% RW reduction will. So, in-between there has to exist a critical boundary, in which a treatment change will still not trigger a change in behaviour. We look for this bound of which a treatment change will still be considered *neutral*, i. e. no incentive for institutions to favour offering leases over debts. The decision whether to offer leases or debts should not depend on the regulatory capital requirement, but rather the associated risk profile. If institutions start to favour one instrument due to its low capital requirement despite its (potentially high) risk, the regulatory framework may create an unintended portfolio shift in the system which ultimately can lead to an overall higher systematic risk. Regardless of how the treatment change takes form, the relevant authorities may evaluate the impact of their recommendation against the bounds from this essay. The two highlighted aspects (adequateness and neutrality) play a central role in the further analysis. For this purpose, this section consists of three parts: 1) an impact analysis on the eligibility of leased assets as collateral under the SA; 2) an impact analysis on the notion of neutral capital requirements; and 3) a robustness analysis by a reverse stress test on the previous results.

We first consider the possibility that physical assets are eligible for the SA. In particular, this implies that lessors are allowed to apply the risk mitigation techniques under the comprehensive approach for an eligible financial collateral type (CRR Art.223). If the exposure is secured and its collateral is eligible, independently from the fact whether it is a lease, then the institution can adjust the EAD amount by subtracting the estimated value of the security at the 1% percentile (distressed collateral value) from the initial EAD. At least for leasing exposures, there are some arguments defending the recognition of physical assets as eligible collaterals. A leasing asset's pool can arguably be considered high-quality security, not only due to the legal ownership or the higher seniority, but also the easier repossession process (Eisfeldt and Rampini (2009)), the high recovery rate (Schmit (2004); De Laurentis and Riani (2005)), the prepayments and the terminal options, which reduce the overall risk (Realdon (2006)), and many other reasons.

Table 2.21 serves as a foundation for the asset pool's value in the tail region of the value distribution. The volatility adjustment in the regulatory risk mitigation technique considers different liquidation periods (CRR Art.224). Since the sell time point is not known in our dataset, this cannot be investigated without further assumptions. The concern is that some asset types cannot be sold properly in a timely manner during a downturn period, which has been thoroughly discussed in the previous section. Seizing and selling the leased assets do not always have a high priority in the workout process. Lessors may yield a higher recovery rate by not seizing the leased assets. For our analysis, the results from table 2.21 can be used as a proxy for

Table 2.22: The asset-specific adjustment factors after 5% MoC based on the asset pool simulation.

	1% percentile recovery rates RR	Adjustment Factor $1-RR$	after 5% MoC
Commercial vehicles	0.7477	0.2523	0.27
Passenger cars	0.6312	0.3688	0.39
Machinery & industrial equipment	0.7525	0.2475	0.26
Computers & business machines	0.6056	0.3944	0.42
Other & unknown	0.7045	0.2955	0.32

a distressed collateral value (CRR Art.225). Assuming that the initial collateral value is equal to the initial exposure amount (which is mostly the case for leases), we can interpret the volatility adjustment as a haircut for the assets. The exposure after the haircut adjustment is defined as:

$$EAD^* = EAD \cdot (1 - RR_{1\%}).$$

To account for misspecified models or assumptions, inadequate data quality, or unaccounted bias, we adjust our result with a 5% MoC. Note that these values in table 2.22 are similar to common LGD values and can also be interpreted as downturn LGDs. If physical assets become eligible for the RW reduction, the appropriate haircut can at most be between 26% and 42% according to our analysis, as shown in table 2.22. On the other hand, if the physical asset type remains ineligible for the SA, any planned treatment change under the SA may have a reduction effect of at most these factors (otherwise it will not be adequate). However, this analysis alone only considers the collateral value but ignores the possibility of a higher default rate due to a downturn event. In the SA, this should be captured by the assigned RWs depending on the credit quality and the asset class.

Although the adjustment factors in table 2.22 cover the adequateness aspect, it is difficult to see whether it creates an incentive for institutions to favour offering leases over debts. In the second part of this section, we argue that the A-IRBA capital requirements can be considered *neutral* for the institutions. The rationale is that any arguments defending the low risk profile of leases should be reflected in the forms of either lower PD or lower LGD, which are the main

Table 2.23: Lowest bounds for the expected effect from a treatment change, in various segmentations, based on the portfolio simulation. Multiplying LB with the SA capital requirements will yield the A-IRBA capital requirements.

	2007	2008	2009	2010	2011	All years
Global	0.4941	0.5731	0.6683	0.7242	0.7649	0.6678
Retail	0.5220	0.5802	0.6675	0.7601	0.8232	0.6706
Corporate	0.4797	0.5658	0.6633	0.6964	0.7245	0.6260
Commercial Vehicles	0.4987	0.5640	0.6785	0.7840	0.8718	0.7054
Passenger Cars	0.5566	0.6032	0.7047	0.7300	0.7413	0.6833
Machinery & Industrial Equipment	0.4277	0.5100	0.6012	0.6645	0.7093	0.6048
Computers & Business Machines	0.7205	0.7954	0.9128	0.9205	0.9474	0.8755
Other and Unknown	0.4605	0.5218	0.6271	0.6560	0.6652	0.6089
Finance Lease	0.4852	0.5640	0.6562	0.7216	0.7685	0.6667
Operating Lease	0.5291	0.6290	0.7604	0.7415	0.7355	0.6831

components of the A-IRBA. Note that the results on tables 2.16-2.20 incidentally also confirm that the A-IRBA capital requirements are adequate to cover for the UL. So, any treatment change for the SA, which reduces the SA capital requirement to at most the A-IRBA capital requirement, can be considered as both adequate and neutral.

We set the lowest bound (LB) of the effect of a treatment change for leases as the ratio of the regulatory capital requirement of a leasing portfolio using the A-IRBA and the SA.

$$LB = \frac{C^{\%}_{A-IRBA}}{C^{\%}_{SA}},$$

where $C^{\%}$ denotes the regulatory capital requirement ratio for the respective approach. These bounds can also be calculated for more granular segmentation. Note that the choice of the solvency ratio does not matter since the same factor appears twice at both the numerator as well as the denominator.

The capital requirements based on the SA tend to be stable over time and independent from the economic environment. The reason is that the portfolio compositions in our simulation do not change drastically, and the SA capital requirements are based mostly on portfolio compositions. On the other hand, the A-IRBA capital requirements react sensitively towards the PD and LGD inputs. This characteristic explains the variation of the LB throughout time, as

Table 2.24: Global and asset-specific lowest bounds for expected effect from a favouring treatment change after 5% MoC. After multiplying these factors to the SA capital requirements, the resulted capital requirements remain adequate and neutral.

	LB all years	after 5% MoC
Global	0.6678	0.70
Commercial vehicles	0.7054	0.74
Passenger cars	0.6833	0.72
Machinery & industrial equipment	0.6048	0.64
Computers & business machines	0.8755	0.92
Other & unknown	0.6089	0.64

shown in table 2.23, which is exceptionally high during the financial crisis. Interestingly, the retail-specific LBs are overall higher than the corporate-specific one. It implies that the regulatory treatment for retail exposures using the SA is quite lenient (no substantial improvement by using the A-IRBA), but quite harsh for corporate exposures. The high LB for computers & business machines (almost 90%) indicates that the difference between both approaches is most likely negligible, while the other asset types seem to produce more consistent LBs, which are ca. 60%-70% globally. Even the contract type does not seem to contribute to the variation in LBs. For a lessor specialised only in office equipment, a different regulatory treatment to leasing may be appropriate. Compared to other specifications, the only specification in asset types gives a different level of LBs. Including 5% MoC to account for potential misspecification of model or assumptions, inadequate data quality, or unaccounted bias is appropriate, which results in the higher LBs, as shown in table 2.24. These values consider any unexpected increase in both PD or LGD during a downturn period. Any treatment change for leases should not have higher impacts to the point that the remaining RWs are reduced to below these values. To judge whether these values are substantial, the SME supporting factor of 0.7619 can be compared directly with the 0.70 LB after 5% MoC from table 2.24. A 70% LB means a 30% RW reduction for the relevant exposures.

To close this section, we finally confirm these LBs independently using the reverse stress test method. In short, we multiply the SA's capital requirement of an exposure by a given factor and investigate whether the portfolio meet various conditions: (UL) the SA capital requirement

Table 2.25: Reverse Stress Test based of three benchmarks: The required capital after linear reduction of risk weight is sufficient to cover for the Unexpected Loss (UL), the Value-at-Risk (VaR), and the simulated Maximum Portfolio Loss (MPL).

Benchmark	Simulation type	Minimum Factor
	Global simulation	0.15
UL	Max of all years	0.19 (2008)
	Max of asset type and years	0.17 (p.cars 2008)
	Global simulation	0.37
VaR	Max of all years	0.52 (2008)
	Max of asset type and years	0.58 (p.cars 2008)
	Global simulation	0.42
MPL	Max of all years	0.58 (2008)
	Max of asset type and years	0.63 (p.cars 2008)

covers the UL, (VaR) the SA capital requirement covers both the EL and the UL, and (MPL) the SA capital requirement covers the maximum of simulated portfolio losses. Using brute force, we look for the minimum factor which meets these conditions. The simulation type will be varied as well, either in the global dataset, a specific year, or a specific asset type. Compared to the previous analysis, the minimum factor in this reverse stress test assumes that the treatment change reduces the RW linearly. Any other risk weight reduction methods may give a different result.

We would like to highlight the fact that the calibrated LBs, in table 2.24 with the 5% MoC, are higher than the minimum factors shown in table 2.25, which confirms the adequateness aspect. The neutrality aspect is implied directly from the construction of the LB. Note that our simulation neither takes loss provision nor profit from non-defaulted contracts into account. In other words, we confirm the adequateness even if the lessor calculates its loss provision inadequately or cannot earn any profit due to a downturn event.

2.7 Conclusion

This essay reviews the adequateness of the existing Basel framework for leases while taking the finalisation of Basel III into account. In particular, we are interested in the UL of leasing exposures. To achieve this goal, we use a Monte Carlo based method to simulate a leasing portfolio and to quantify the associated UL, similar to the non-parametric methodology by Carey (1998); Schmit (2004, 2005). For this purpose, we use a dataset of 2.4 million lease contracts active during 2007-2011 from twelve major European leasing companies with contracts over 25 European countries. To our knowledge, this is the largest leasing dataset used for credit risk research in the literature to date.

Although a slight overestimation in the Basel capital requirement is to be expected by design, our result can no longer support the proposition that the framework is only slightly conservative. The simulation results show that the Basel capital requirements exceed five to eight-fold of the associated ULs. Based on the fact that the Basel framework is designed to cover for the UL, we conclude that the current regulatory requirement for leases is quite excessive. Furthermore, the SA capital requirement is only weakly risk-sensitive for leasing. SA lessors are at a disadvantage compared to IRBA lessors. Although our results are not new (compare with Schmit (2004, 2005); Pirotte and Vaessen (2008); Eisfeldt and Rampini (2009)), a change in regulatory treatment favouring lease exposures is unlikely without a further impact analysis. The regulatory framework should remain neutral in the sense that no incentives are created for institutions to favour offering leases over debts, which may have a (potentially negative) systematic effect in the overall lending market. We calculate the maximum reduction if any treatment change is planned under the adequateness and neutrality conditions. Our results show that any treatment change should not have a reduction effect of more than 30% to ensure both the adequateness and the neutrality aspect of the regulatory capital requirement. A reverse stress test ensures that such a reduction would not endanger the capability of lessors to cover for their ULs.

Chapter 3

Is the Regulatory Downturn LGD Adequate? Performance Analysis and Alternative Methods

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3.1 Introduction

After long discussions and consultation processes, the European Banking Authority (EBA) published the final regulatory technical standard (EBA/RTS/2018/04) and the final guidelines (EBA/GL/2019/03) on the appropriate estimation of downturn Loss Given Default (LGD)[1] under the Advanced Internal Rating-Based Approach (IRBA). The technical standard relies on the basic idea that downturn LGD estimates shall be based on macroeconomic proxies. During downturn periods, LGDs are expected to rise systematically, and this effect needs to be reflected in the capital requirement.

Shortly following the publication of EBA/RTS/2018/04, which specifies the definition of an economic downturn, the guideline EBA/GL/2019/03 is also published to set the appropri-

[1]LGD is defined as the fraction of loss to its exposure at the default time point. Loosely speaking, the downturn LGD is intended to be the expected LGD during downturn periods, i. e. during crises.

ate methodologies for estimating the downturn LGD. Since an economic downturn is already defined in the IRBA framework, there are two distinct downturn definitions. The IRBA relies on the calculation of the expected loss conditioned on a single systematic factor (also known as the latent variable X) and the definition of an economic downturn is traditionally defined as the event, where the latent variable takes a conservative value ($X = -\Phi^{-1}(0.999)$). In contrast, the macroeconomic based downturn definition is set to be the worst macroeconomic realisation over the last 20 years. This inconsistency in the downturn definition can potentially result in a risk underestimation. The required 99.9% confidence level will likely not be reached with the downturn LGD based on the aforementioned guideline.

The consistency itself was never a rigid technical requirement within the IRBA. However, any standards and practices potentially leading to a risk underestimation cannot be ignored, especially as regulatory policies. At the bare minimum, the macroeconomic based downturn definition should be at least as strict as the latent variable based one. To provide a quick judgement, one can directly compare the frequency of such a downturn period under a particular definition. The latent variable based downturn period occurs on average once every 1,000 years, while the macroeconomic based one occurs once every 20 years.

Existing conditional LGD estimation methods based on latent variables (see Frye (2013) for a summary of some existing models) usually focus on a market-based LGD dataset, i. e. the LGD values based on the price at which the defaulted debt instruments are traded shortly after default. For defaulted instruments with a long workout process (typically in years), many papers fail to see any significant effects of macroeconomic variables on realised LGDs (see section 3.2.4 for details). This fact suggests that a (downturn) LGD estimation based on systematic factors (either by a latent variable or macroeconomic proxies) will in general not perform well for workout LGD instruments. Finding systematic patterns in a workout LGD data can be challenging. Before constructing a latent variable based conditional LGD estimation method for the use in the IRBA framework, it is necessary to study the relationship between expected LGD and X. We believe the main issue lies in two key points: the time reference (vintage point) and the workout duration. For defaulted exposure, the final LGD is realised at the end of the workout process. During the workout period, the state of the economy may fluctuate and can impact the potential LGD systematically. Theoretically, more than one time-point may be needed to model the workout LGD. The central idea is to incorporate a latent variables time series in the LGD model (instead of only one latent variable referenced to a particular time point).

This essay can be divided into two main parts: 1) the sensitivity analysis between the latent variables time series and the expected LGD, and 2) the performance of some specific (latent variable based) downturn LGD methods compared to the downturn LGD from the guideline.

With the view that the LGD is influenced by a time series of latent variables, we analyse how sensitive the expected LGD is towards these latent variables. In the first part of this essay, we investigate the impact of the yearly systematic factors towards the LGD. This analysis is directly related to the popular discussion whether LGD models by vintage of default or vintage of recoveries are more appropriate. Based on a database containing 186,000 resolved default cases between 2000 and 2017, our results confirm that the LGD sensitivities towards the latent variables vary with the default age. This analysis reveals an interesting relationship between the latent variables and the (workout) LGD.

In the second part of this essay, we construct some basic latent variable based downturn LGD estimation methods and test their performance measured by two parameters: the survival chance of a bank and the LGD overestimation in case of survival (referred to as *waste*). We evaluate these various downturn LGD standards in terms of their ability to ensure the bank´s survival at a confidence level of 99.9%. The second performance measure is based on the idea that capital requirements should not be excessively high. The reasoning behind it is that increasing capital requirements brings not only benefits but also social costs. If banks' survival can be assured with a confidence level of 99.9%, the benefits do not outweigh the social costs any longer.

We measure the performance through simulations by randomly drawing a portfolio to represent a bank and estimate the downturn LGD of each exposure in its portfolio based only on past information. The bank survives if its downturn LGD is higher than its realised LGD on that year. Our alternative methods for downturn LGD can uphold the 99.9% required survival chance, while the advanced IRBA only shows an 81% survival chance. In comparison with the foundation IRBA, our methods pass the performance test by survival chance, but our methods are superior in terms of waste (our methods: 13-16%, foundation IRBA: 22%).

The remainder of this essay is structured as follows: section 3.2 discusses the theoretical foundation, the current standard, and empirical works, which give supporting evidence on the systematic dependence of LGD; section 3.3 derives our methods, both the theory and its calibration, including the data description; section 3.4 shows the result from the given model and its interpretation as well as the implication for the regulation; section 3.5 compares various latent variable based downturn LGD estimation methods with the foundation and the advanced IRBA by Monte Carlo simulations; and lastly, section 3.6 concludes this essay.

3.2 Background and Literature Review

This section starts with some background on the regulatory capital charge with LGD focus and highlights some of the theoretical arguments against the macroeconomic based downturn LGD estimation methods. Furthermore, we review the literature on latent variable models with the main purpose to estimate the downturn LGD. Aside from the theoretical works, many papers discuss the (in-)significance of macroeconomic variables in an LGD estimation. It seems that there are some convincing arguments against a macroeconomic based downturn LGD estimation.

3.2.1 Regulatory Capital Charge

One of the main purposes of regulatory capital requirements is to ensure that institutions have adequate capital to cover their losses, even in the case of a downturn period. In the credit risk context, the potential loss is generally split into the expected and the unexpected loss. The expected loss has to be covered by the loss loan provision, while the capital requirement covers the unexpected loss, i. e. the loss that exceeds the expected loss up to a 99.9% confidence level.

The unexpected loss (UL) can be calculated by subtracting the expected loss (EL) from the credit value-at-risk (VaR). Instead of calculating the VaR directly, the IRBA framework is constructed to estimate the conditional expected loss under a distressed value of a single systematic factor, i. e. the latent variable X. The asymptotic equivalence between both parameters is proven by Gordy (2003) under certain assumptions. Thus, the minimum capital charge (CC) should be at least as big as the difference between the two components.

$$\begin{aligned} CC \geq UL &= VaR_{99.9\%} - EL^* && \text{(E1)} \\ &\overset{\text{Gordy}}{=} \mathbb{E}[Loss_i | X = -\Phi^{-1}(0.999)] - EL^* \end{aligned}$$

Since EL is commonly defined as the product of exposure at default (EAD), PD, and LGD, the derivation of the conditional EL can be factorised to conditional expected EAD (or short: conditional EAD), conditional PD, and conditional expected LGD (or short: conditional LGD). In most cases, the (conditional) EAD is assumed to be constant for a given exposure.

In the IRBA, the conditional PD is given as a closed formula dependent on the latent variable X (which is set to $X = -\Phi^{-1}(0.999)$) to represent a downturn period. In contrast, the conditional LGD is to be determined separately (through the foundation IRBA or the advanced IRBA) independent from X. Note the difference between the conditional LGD (a theoretical object with an unknown form which describes the dependency of LGD from X) and the downturn LGD (the evaluated conditional LGD at a predefined downturn severity, including the way the regulation requires to estimate this value). Even though a closed formula for the conditional LGD does not exist in the IRBA framework yet, any conservative estimation for regulatory downturn LGD (estimated conditional LGD evaluated at $X = -\Phi^{-1}(0.999)$) should replace this parameter to ensure a sufficient loss coverage.

$$\begin{aligned} UL \leq & PD_X \cdot DLGD - PD \cdot DLGD \\ \text{where } PD_X := & \mathbb{E}[D_i | X = -\Phi^{-1}(0.999)], \\ DLGD \geq & \mathbb{E}[LGD_i | X = -\Phi^{-1}(0.999)], \\ \text{and } PD := & \mathbb{E}[D_i], \end{aligned} \tag{E2}$$

where D_i is a Bernoulli-distributed default indicator for borrower i and Φ is the distribution function of the standard normal distribution. The PD_X calculates the expected default rate under a distressed state of the economy. We refer DLGD as the regulatory defined downturn LGD and not to be confused with the conditional LGD evaluated at $X = -\Phi^{-1}(0.999)$ to replicate a downturn period. Regardless of what methods or standards are chosen for DLGD, the inequality in E2 needs to be fulfilled to reach the 99.9% confidence level.

3.2.2 Current Regulatory Downturn LGD Standard

With the requirement E2 in mind, the EBA recently published a regulatory technical standard on downturn EBA/RTS/2018/04 and shortly after, a downturn LGD guideline EBA/GL/2019/03. The downturn LGD is set to be the long-run average LGD with a downturn add-on. The institutions are required to estimate downturn LGDs for their exposures when using the advanced IRBA. For the specification of a downturn period, the EBA proposes the worst case of macroeconomic proxies in the latest 20 years span. Hence, this procedure can differentiate between different types of downturn periods e. g. a downturn period caused by a high unemployment rate or a downturn period caused by industry distress. In a simplified form, the DLGD is equal to $DLGD = \mathbb{E}[LGD_i|$ worst case of M over the 20 years span $]$. M denotes a variable vector of the relevant macroeconomic variables as proposed in EBA/RTS/2018/04. From this point, we shorten the conditional expectation with $\mathbb{E}[\,\cdot\,|X]$ or $\mathbb{E}[\,\cdot\,|M]$, where both conditions on X or M are meant to be the extreme cases to represent the (regulatory-defined) downturn periods.

Both X and M are intended to describe the state of the economy. However, the substitutability of X through a selection of macroeconomic variables M is questionable. A loss underestimation is unavoidable if the macroeconomic downturn definition is more lenient than the latent variable's downturn definition. The standard relies on the assumption that $DLGD := \mathbb{E}[LGD_i|M] \geq \mathbb{E}[LGD_i|X]$ is true (referring to E2). With the choice of 20 years span, $\mathbb{E}[LGD_i|M]$ should be approximately $\mathbb{E}[LGD_i|X = -\Phi(0.95)]$. Due to the monotonic nature of $\mathbb{E}[LGD_i|X = x]$ as a function of x, the asserted loss underestimation can be easily seen.

Another way to compare between M and X is simply to calculate their expected frequencies. The minimal requirement to reach the 99.9%-confidence level is to include the most severe realisation of the chosen macroeconomic variables within 1,000 years period. Of course, such a dataset do not exist.

The substitutability of X by macroeconomic proxies is not supported by empirical evidence. Koopman et al. (2011) report that over 100 macroeconomic covariates are not sufficient to replace the need for latent components. Another work by Betz et al. (2018) shows that macroeconomic variables, in general, are not suitable to capture the true systematic effects when modelling LGD.

Conclusively, it seems desirable that under the current regulatory framework the capital requirement is conditioned on the value of X:

$$CC \geq UL = \mathbb{E}[D_i|X] \cdot \mathbb{E}[LGD_i|X] - EL^*, \tag{E4}$$

where the condition X is to be understood as $X = -\Phi^{-1}(0.999)$. In other words, the capital requirement is calculated to cover for the most severe economic condition. The EL* in E4 represents the calculated expected loss, to differentiate it from the EL, which represents the true expected loss. It is important that the sum of UL and EL* should exceed the $\mathbb{E}[Loss_i|X = -\Phi^{-1}(0.999)]$. So a mismatch between EL and EL*, which typically occurs due to different methodologies between capital regulation and accounting standard (IRBA vs IFRS 9), does not influence our result.

3.2.3 Discussion on Existing Latent Variables Based LGD Models

As long as the IRBA rests on the conditional PD formula derived by Vasicek (1987), modelling the (downturn) LGD using a latent variable approach is preferable from a technical perspective. In this section, some of LGD models based on the latent variable approach in the literature are reviewed.

Early attempts to model the conditional LGD with latent variables can be found for example in Frye (2000a), Pykhtin and Dev (2002), and Pykhtin (2003). The central idea of the LGD modelling by the latent variable approach is that a common systematic factor drives both default events and the expected LGD. In their models, the LGD is influenced by a single-factor X. Aside from single-factor LGD models, many papers introduce multi-factor models to accommodate the demand for more model flexibility. These factors can be assumed to be independent, such as Pykhtin (2004), or with a particular dependence structure, such as Schönbucher (2001). The variations using a latent variables time-series may also assume a point-in-time dependency structure, as found in Bade et al. (2011), or an autoregressive process, as found in Betz et al. (2018).

While the conditional PD formula is derived from modelling an abstract asset value A_i of a borrower i, the conditional LGD can be modelled through an abstract collateral value C_i as well. Without loss of generality, the collateral value can be replaced by the general loan's capability of recovering a fraction of the outstanding debt value. In the plain vanilla model,

$$\begin{aligned} A_i &= p \cdot X + \sqrt{1-p^2} \cdot Z_i^A, \qquad && p \geq 0, Z_i^A \sim \mathcal{N}(0,1) \\ C_i &= q \cdot X + \sqrt{1-q^2} \cdot Z_i^C, \qquad && q \geq 0, Z_i^C \sim \mathcal{N}(0,1). \end{aligned} \tag{M1}$$

Z_i^A and Z_i^C denote the idiosyncratic or synonymously the (loan-)specific risk factors of the borrower i, which are independent of each other and the latent variable X. The parameter p itself is popularly known in its quadratic form p^2 (asset correlation). The scaling of the coefficients using the euclidean norm is solely to ensure A_i to be standard normally distributed as well, analogously for C_i. In this model, the default event is defined as an asset shortfall. Under this assumption, the asset correlation is closely related to the default correlation between two random borrowers. As a side note, Frye (2008) points out the potential difference between asset correlation and default correlation.

The borrower i defaults in the model if the value A_i drops below the threshold $\Phi^{-1}(PD_i)$. This also ensures that the default rate is exactly PD_i. Consequently, the conditional PD of the borrower i given X is $\mathbb{P}(A_i \leq \Phi^{-1}(PD_i)|X)$, which results directly in Vasicek's conditional PD formula:

$$\begin{aligned} PD_X =&\mathbb{P}(A_i \leq \Phi^{-1}(PD_i)|X) \\ =&\Phi\Big(\frac{\Phi^{-1}(PD_i) - p \cdot \Phi^{-1}(X)}{\sqrt{1-p^2}}\Big) =: g_A(X) \end{aligned} \tag{E5}$$

Note that the function g_A is invertible and differentiable in X, which is the sufficient conditions to identify the distribution of PD_X and guarantees its existence. The extreme cases $p \in \{0,1\}$ render the function g_A to be constant and therefore not invertible. Thus, they are ruled out.

One of the obstacles in modelling the systematic impact in LGD is to identify the relationship between LGD and X. An additional assumption on the connection between X (or C_i) and LGD is necessary to calculate the conditional LGD, i. e. the specification of the function $g_C(X) := \mathbb{E}[LGD_i|X]$. The simplest one is the linear relationship introduced by Frye (2000a), Frye (2000b), and Pykhtin and Dev (2002). The linearity implies the conditional LGD, $\mathbb{E}[LGD_i|X]$, to be normally distributed. Motivated by the restriction of $LGD_i \geq 0$, an exponential transformation can be used to ensure a log-normally distributed $\mathbb{E}[LGD_i|X]$, as can be found in Pykhtin (2003) and Barco (2007). Other suggestions include application of a beta distribution, such as work from Tasche (2004), or modelling LGD directly from PD, found in the work of Giese (2005), Hillebrand (2006), as well as Frye and Jacobs Jr (2012). Furthermore, Frye (2013) suggests that modelling the systematic risk in LGD can be replaced by modelling the default rates in LGD instead. We avoid selecting a particular g_C since it is solely the deciding factor to determine the distribution of $\mathbb{E}[LGD_i|X]$ and therefore the value $\mathbb{E}[LGD_i|X = -\Phi^{-1}(0.999)]$ as well.

3.2.4 Literature Review on the Systematic Dependency of LGD

This section reviews the empirical evidence on the systematic dependency of LGD in the literature. There is an impression that workout LGDs indeed behave differently than market-based LGDs. Evidence for a systematic dependency can be observed in papers showing that macroeconomic variables or latent variables are significant predictors for LGD. Here, we differentiate the results for the workout LGD and the market-based LGD to highlight the contrast.

The emerged pattern in the literature seems quite apparent. Systematic factors are generally good predictors to estimate the market-based LGD, but it is not so clear for the workout LGD. Specifically, we review 1) empirical papers for LGD estimations using macroeconomic covariates using a market-based LGD dataset or 2) using a workout LGD dataset; moreover, 3) papers dealing with the influence of systematic factors through latent variables on LGD.

The amount of published papers using a market-based LGD dataset, such as corporate bonds data, is overwhelming in comparison to papers using a workout LGD dataset. Varma and Cantor (2004) show the significant effect of macroeconomic variables for estimating the market-based recovery rates of North American corporate bonds. Bruche and González-Aguado (2010) use corporate bonds data to show the dependency of recovery rates on a selection of macroeconomic variables. Chava et al. (2011) find strong industry effects in their default and recovery models using the Moody's ultimate recovery database on bonds. Khieu et al. (2012) report a highly significant impact of GDP growth and industry distress in an OLS regression model for estimating 30-day post-default trading prices for bank loans. Jankowitsch et al. (2014) indicate the significance of market and industry default rates as well as the federal funds rate in the US corporate bond market. Leow et al. (2014) show how macroeconomic variables improve the

LGD estimate, which is based on forced sales amount of mortgage and final recovered amount on personal loans, in a two-stage model and an OLS regression model for UK retail loans data. Mora (2015) studies the macroeconomic dependence of recovery rates on defaulted debt securities, which are based on their post-default trading prices, and shows a high susceptibility of industry-related variables. Nazemi et al. (2017, 2018) use 104 macroeconomic covariates within a support vector machine-based regression model and a fuzzy decision fusion approach to improving corporate bonds recovery rate prediction. The significant macroeconomic effects on market-based LGD is undeniable considering the vast amount of empirical evidence, which implies a high systematic sensitivity of the market-based LGD.

In this segment, we review papers, which use workout LGD data (occasionally market-based LGD data might be included as well). Acharya et al. (2007) observe the industry distress effect in the recovery rates of bank loans, high-yield bonds, and other debt instruments. Still, macroeconomic variables such as GDP, S&P stock return, or bond market condition are not significant determinants of recoveries in the presence of industry variables. Caselli et al. (2008) use data on Italian bank loans for SME and real estate finance to verify the macroeconomic relation in LGD. They find that the GDP has less explanatory power than expected. Using German loan data, Grunert and Weber (2009) report that national and regional GDP growths do not show significant effects in their OLS model. Hartmann-Wendels and Honal (2010) analyse the LGDs of mobile lease contracts and find a macroeconomic dependency only in the vehicles segment. Bellotti and Crook (2012) show significant impacts of bank interest rates and unemployment level in their OLS model using recovery rates from credit cards data, which are calculated based on the sum of repayments made 1-year post-default. Tobback et al. (2014) report that the influence of macroeconomic variables on LGD depends on the model selection for a dataset containing revolving credit lines secured by real estates or corporate loans. Krüger and Rösch (2017) show the variation of macroeconomic effects to US corporate loans LGD in different quantile regions. Yao et al. (2017) apply a supply vector machine methodology for estimating the credit card's LGD, which is fitted using 24-months post-default accrued interests and overdue fees. They find a high relevance of the macroeconomic variables for the estimation accuracy. Apart from credit cards, significant effects of macroeconomic covariates are rarely observed in the workout LGD.

Lastly, we discuss papers, which identify systematic effects through means other than macroeconomic proxies, typically through the latent variable approach. To our knowledge, the earliest paper discussing the systematic sensitivity of LGD is Frye (2000b). With his method, one can calculate the correlation between the LGD and the latent variable implied from the single risk factor model, which is comparable to the parameter q from the model M1. Using US corporate bonds data, Frye (2000b) estimates $\hat{p} = 23\%$ and $\hat{q} = 17\%$. Düllmann and Trapp (2004) use a database, which contains bonds, corporate loans, and debt instruments in the US and they report $\hat{p} \approx 20\%$ and a $2\% - 3\%$ recovery rates' systematic sensitivity depending on the distribution assumptions. Betz et al. (2018) adapt random effects using a Bayesian finite mixture model to measure the latent variables. However, their results are not directly comparable with the parameters p or q from the model M1. Nonetheless, they claim that the latent variable approach measures the true systematic effects in LGD better than a selection of macroeconomic variables.

While papers studying LGD's systematic effects using macroeconomic proxies are abundant, there is still a severe need for further research in the LGD's systematic effect based on latent variables. As explained in section 3.2.1, estimating the downturn LGD using macroeconomic proxies instead of latent variables is flawed. However, studies on the relationship between latent variables and LGD are uncommon.

3.3 Methods and Data

This section introduces an expansion of the traditional single-risk factor model. The central idea is that a defaulted loan with a high workout duration gets influence from the systematic factor throughout its workout process. During this period, the systematic factor affects the potential LGD (or the recovery capability in general) as long as the default is unresolved.

3.3.1 Theoretical Framework

3.3.1.1 Expanded Single-Risk Factor Model

In this model, the recovery capability C_{i,t_d} is set to be a function of the latent variables X_{t_d} (the state of economy at the default year), $X_{t_d+1},\ldots,$ and X_{t_d+T} (the state of economy at the resolution year), where T denotes the workout duration. However, the impact of each latent variable towards the recovery capability C_{i,t_d} is unknown.

The issue is that the LGD is observable only at the end of the workout process. In the course of the workout process, costs and recoveries can be realised, but most of the components remain uncertain until the resolution time, such as recoveries from unsold collaterals or future legal fees. Thus, the final LGD reflects the accumulated impact of the systematic factor during the whole workout duration. To isolate the systematic influence towards the LGD for each workout year is a difficult task, especially if only the final LGD is observable. The simplest possible model which incorporates the time-series of latent variables, would be

$$\begin{aligned} A_{i,t_d} &= p \cdot X_{t_d} + \sqrt{1-p^2} \cdot Z_i^A, \\ C_{i,t_d} &= q_{t_d} \cdot X_{t_d} + \ldots + q_{t_d+T} \cdot X_{t_d+T} + \sqrt{1-||q||_2^2} \cdot Z_i^C, \\ &\text{where } 0 < p < 1 \text{ and } Z_i^A, Z_i^C \sim \mathcal{N}(0,1). \end{aligned} \tag{M2}$$

The coefficient $q = (q_{t_d},\ldots,q_{t_d+T})$ is an element inside the $(T+1)$-dimensional unit circle excluding the origin. The vanilla model M1 is a special case of the expanded model M2. The specific model M1 would be an LGD model by vintage of default, in particular for $q = (1,0,\ldots,0)$. Unlike other latent variable based models in the literature (see section 3.2.3), we do not impose any assumption on the dependence structure of the latent variable time series $(X_t)_{t\in\mathbb{N}}$, i. e. how X_t and X_s are dependent to each other for any given year t and s. Note that this model still relies on a single-factor, since $(X_t)_{t\in\mathbb{N}}$ describes a time-series of one systematic risk factor.

The coefficient p describes the sensitivity of the asset value A_{i,t_d} towards X_{t_d}. The restriction for p to be positive is economically necessary to reflect the fact that financial distress causes a higher default rate. Similarly, the coefficients $q_{t_d}, \ldots, q_{t_d+T}$ describe the sensitivity of C_{i,t_d} towards the latent variables during the workout years. The restriction of q_t to be positive (for each t) is not necessary from a technical point of view. A negative q_t only implies negative correlation between X_t and C_{i,t_d}. This case may be rational from an economical perspective for a large gap between t and t_d. The signs and magnitudes of $q_{t_d}, \ldots, q_{t_d+T}$ give hints on which vintage models have the most explanatory power.

There are economic arguments supporting a high q_{t_d} as well as those supporting high q_{t_d+T}. Loosely speaking, the coefficient q gives information, which year within the workout duration is the most "responsible" for the systematic effects in the realised LGD. It is not clear beforehand, how the coefficients q will behave when the model is fed with workout LGD data. Different arguments supporting different propositions exist:

1. **High systematic sensitivity at the default year** (in line with the vintage of default). The empirical evidence on a high PD-LGD correlation (such as Frye (2003); Altman et al. (2004)) ties a loan's LGD to its default time rather than to the rest of the workout periods. The fact that the default occurred in a downturn year contributes to the low market value of the collateral and the low cure chance of the defaulted obligor. This translates directly into a high q_{t_d}. This proposition is related closely to the plain vanilla model M1, which performs well for the market-based LGD.
2. **High systematic sensitivity at the resolution year** (in line with the vintage of resolution/recovery). The largest portion of a typical bank loan portfolio consists of secured loans. The collateral usually accounts for the predominant share of the recoveries and the workout process often stops soon after the collateral is sold. This implies that the LGD is tied mostly to the resolution time, which means a high q_{t_d+T}.
3. **High systematic sensitivity near the default year** (a weaker version of the vintage of default). Alternatively, one may argue that cash inflows (but also outflows) are relevant factors for calculating LGD. These transactions occur most often in the first years after the default event, which implies a high q_{t_d} or q_{t_d+1}. Eventually, this parameter continues

to decrease as the default gets older.

The model developed in this essay can principally cover each conceivable time pattern of recoveries. The previous three recovery structures are likely to be the most common ones. We emphasise that the model does not exclude the market-based LGD. The model is intended to cover a typical bank, which may have a mixed portfolio containing exposures with workout LGDs and exposures with market-based LGDs.

3.3.1.2 Estimation Techniques

It is necessary to find out the latent variables impact on the LGD values to design an adequate downturn LGD estimation based on a latent variable approach. The coefficient q decodes in which workout year the LGDs are particularly sensitive towards the latent variables. There are two central issues regarding any estimation technique of q: 1) the uncertainty about the dependence structure of $(X_t)_{t\in\mathbb{N}}$, and 2) the uncertainty about the relationship between X and LGD (or equivalently between C_{i,t_d} and LGD). In this essay, both uncertainties will remain open to avoid any unintentional influence on the result.

The maximum likelihood method, similar to Frye (2000b), is applied to estimate p. According to Gordy and Heitfield (2002), the maximum likelihood method produces less bias (coming from a lack of data) than the method of moments. The likelihood function can be derived through the theoretical distribution of the conditional PD, i. e. the distribution of $g_A(X_{t_d}) := \mathbb{E}[D_i|X_{t_d}]$ (see E5). We have established that the function g_A is invertible and differentiable in X. The change-of-variable technique produces the density function of $PD_X = \mathbb{E}[D_i|X_{t_d}]$, which is

$$\begin{aligned} f_{PD_X}(y) =& f_X(g_A^{-1}(y)) \cdot \Big|\frac{dg_A^{-1}(y)}{dy}\Big| \\ =& \varphi\Big(\frac{\Phi^{-1}(PD_i) - \sqrt{1-p^2}\Phi^{-1}(y)}{p}\Big) \cdot \frac{\sqrt{1-p^2}}{p} \cdot \Big|\frac{d\Phi^{-1}(y)}{dy}\Big|, \end{aligned} \tag{E6}$$

where φ is the density function of the standard normal distribution. If the parameter PD_i is known, the estimation of p is reduced to a one-dimensional problem. The maximum of the likelihood function can be approximated numerically, which yields the estimated p. By applying the estimated p in the equation E5, the implied X_t can be calculated for each year t.

A similar approach to estimate q would require the information on the joint distribution of $(X_t)_{t\in\mathbb{N}}$, referring to the first uncertainty mentioned above. It seems unrealistic and overly simplified to assume that the latent variables are intertemporally independent or follow a particular Markovian process, i. e. today's state is only influenced by that of yesterdays. A non-Markovian behaviour of the $(X_t)_{t\in\mathbb{N}}$ seems to be more realistic. However, specifying one can be challenging and testing whether it is true even more difficult.

By using the realisations of $\mathbb{E}[LGD_i|X_{t_d},\dots,X_{t_d+T}]$ and $(X_t)_{t\in\mathbb{N}}$, the coefficient q can be estimated via a regression methodology. At this point, we are confronted with the second uncertainty mentioned above. In contrast to the function g_A, the exact form of the function $g_C(X_{t_d},\dots,X_{t_d+T}) := \mathbb{E}[LGD_i|X_{t_d},\dots,X_{t_d+T}]$ is not known[2]. Choosing a particular form of g_C seems arbitrary. With different asset and exposure types, various jurisdictions, or even the institution's internal strategy, the correct form is most likely complex.

Even though g_C remains unknown, we can safely assume that this function is locally smooth, i. e. (at least one time) partially differentiable, at a chosen value $x := (x_{t_d},\dots,x_{t_d+T})$. The idea is to construct its Taylor series representation at the chosen value x. This value x can serve both as an evaluation point as well as a conservative value representing a downturn event. Two possibilities emerge on the function behaviour at the evaluation point: either 1) the function g_C is linear (or similar to one) or 2) the function is substantially different from a linear function. Avoiding this step by taking an assumption may substantially simplify many things, but it puts our analysis in the same bucket as the currently existing models in the literature. Since this essay aims to offer an alternative to the current downturn LGD methodology, which potentially results in a fatal LGD underestimation, this amount of thoroughness is necessary.

In the first case, the Taylor series representation only contains the first partial derivative and

[2]A simple linear relationship or restrictions on possible LGD values are the typical assumptions in the literature (section 3.2.3).

the conditional LGD can be written as

$$
\begin{aligned}
\mathbb{E}[LGD_i|X_{t_d},\dots,X_{t_d+T}] &= g_C(x_{t_d},\dots,x_{t_d+T}) + \sum_{s=t_d}^{t_d+T} \frac{\partial g_C(x_{t_d},\dots,x_{t_d+T})}{\partial X_s}(X_s - x_s) \quad \text{and} \\
\mathbb{E}[LGD_i|X_{t_d},\dots,X_{t_d+T}] &= \mu - \sigma\Big(\underbrace{q_{t_d}X_{t_d} + \ldots + q_{t_d+T}X_{t_d+T} + \sqrt{1-||q||_2^2}Z_i^C}_{C_{i,t_d}}\Big).
\end{aligned} \tag{E7.1}
$$

Both parameters μ and σ are intended to be the sum of the constant terms and a scaling factor to fulfil the unit circle requirement of q. Note that $\mathbb{E}[Z_i^C|X_{t_d},\dots,X_{t_d+T}] = \mathbb{E}[Z_i^C] = 0$. The mean and standard deviation of the portfolio LGDs may serve as estimators for both μ and σ. However, this should not be confused with the LGDs at the loan level.

By looking at the structure of both equations in E7.1, the OLS regression method would require data samples, in which the idiosyncratic factor Z_C is zero or minimal. The OLS method consequently produces an indirect estimation of q, which is $\widehat{\sigma q_t}$ for all t. This requirement can be achieved by constructing samples of $\mathbb{E}[LGD_i|X_{t_d},\dots,X_{t_d+T}]$ from a large portfolio. In a large (fine-grained) portfolio, the idiosyncratic risk converges to zero and is intertemporally uncorrelated. Thus, the property that the OLS estimator is unbiased (from σq_t) is guaranteed by the Gauss-Markov theorem.

In the other case, where the function g_C is believed to be non-linear in at least one of its parameter at the evaluation point, then the Taylor series representation would produce a non-zero rest term $R(X_{t_d},\dots,X_{t_d+T})$.

$$
\begin{aligned}
\mathbb{E}[LGD_i|X_{t_d},\dots,X_{t_d+T}] =& g_C(x_{t_d},\dots,x_{t_d+T}) + \sum_{s=t_d}^{t_d+T} \frac{\partial g_C(x_{t_d},\dots,x_{t_d+T})}{\partial X_s}(X_s - x_s) + \\
& R(X_{t_d},\dots,X_{t_d+T})
\end{aligned} \tag{E7.2}
$$

The information on the rest term $R(X_{t_d},\dots,X_{t_d+T})$ (in short: R) should reside in the OLS residuals and the intercept. A problem occurs if the downturn impact on R is far stronger than the linear effect, then the coefficients q do not hold much information weight for the conditional LGD. In this particular case, the coefficient q may be biased and the result (in form of a downturn LGD estimation) will likely to fail the performance tests (see section 3.5).

3.3.2 Parameter Estimation

The data should consist of observed default rates as well as observed LGDs by year and rating. The methodology estimates the parameter p, then the implied latent variables X, and then the parameter q, in the exact order.

3.3.2.1 Estimating p

By using the observed default rates of a given rating segment $r \in \mathcal{R}$ and a given year $t \in \mathcal{T}$, samples of $\mathbb{E}[D_i|X_t]$ can be generated by the arithmetic average of default dummies of a given rating r in a given year t, denoted by $(Y_{r,t})_{r\in\mathcal{R},t\in\mathcal{T}}$. The sets $\mathcal{R}$ and $\mathcal{T}$ denote the set of available ratings and years in the data with $n = |\mathcal{R} \times \mathcal{T}|$. A generated sample $Y_{r,t}$ is non-representative and biased if there are too few samples in a given (r,t)-segment. Therefore, only samples with at least 100 resolved defaults (chosen arbitrarily) in each segment are generated. The density function of $Y_{r,t}$ is theoretically known from E6, so the log-likelihood function is

$$\begin{aligned} l(p) =& \log\Big(\prod_{r,t\in\mathcal{R}\times\mathcal{T}} f_{Y_{r,t}}(PD_r, p)\Big) \\ =& -\frac{n}{2}\log(2\pi) - \frac{1}{2p^2}\sum_{r,t\in\mathcal{R}\times\mathcal{T}}\big(\Phi^{-1}(PD_r) - \sqrt{1-p^2}\,\Phi^{-1}(Y_{r,t})\big)^2 \\ &+ \frac{n}{2}\log(1-p^2) - n\log(p) + n\log\big(\frac{d}{dy}\Phi^{-1}(Y_{r,t})\big). \end{aligned}$$

The maximum likelihood estimator for p is the solution of $\hat{p} = \arg\max_{p\in(0,1)} l(p)$, which can be solved numerically. We set $PD_r = \frac{1}{|\mathcal{T}|}\sum_{t\in\mathcal{T}} Y_{r,t}$ to simplify the problem into a one-dimensional numerical task.

3.3.2.2 Estimating X_t

Given $\hat{p}$, we can estimate $(X_t)_{t\in\mathcal{T}}$ by using the equation E5. For a given rating $r \in \mathcal{R}$ and a given year $t \in \mathcal{T}$, $\widehat{X_{r,t}}$ is defined as the solution of

$$Y_{r,t} = g_A(X_{r,t}) = \Phi\Big(\frac{\Phi^{-1}(PD_r) - \hat{p}\cdot\Phi^{-1}(X_{r,t})}{\sqrt{1-\hat{p}^2}}\Big) \text{ in } X_{r,t}.$$

Since g_A is invertible and therefore a bijective function, there is only one single solution to the equation above for a given PD_r and $\hat{p}$, which is

$$\widehat{X_{r,t}} = \Phi\Big(\frac{\Phi^{-1}(PD_r) - \sqrt{1-\hat{p}^2}\cdot\Phi^{-1}(Y_{r,t})}{\hat{p}}\Big).$$

The solution $\widehat{X_{r,t}}$ represents the implied latent variable of a system at year t for an asset class with the rating r. The global latent variable X_t, independent from the rating, can be represented by an arithmetic average or a weighted average (weighted by the number of obligors with rating r in the data) of $X_{r,t}$. However, a weighted average would under-represent loans from bad ratings since there are typically not many of them. Hence, $\widehat{X_t} = \frac{1}{|\mathcal{R}|}\sum_{r\in\mathcal{R}}\widehat{X_{r,t}}$.

3.3.2.3 Estimating q

The samples of $\mathbb{E}[LGD_i|X_{t_d}\dots,X_{t_d+T}]$ are required to have minimal idiosyncratic effect as explained in section 3.3.1.2. The idiosyncratic risk factor converges to zero as the number of loans increases. This is achieved by bundling defaulted loans from the population, which are defaulted in t_d and resolved in t_d+T, to calculate the arithmetic average of the realised LGD, which will be denoted by $(L_{t_d,t_d+T})_{t_d,t_d+T\in\mathcal{T}}$. For a given pair (t_d,t_d+T), the sample L_{t_d,t_d+T} is generated in our analysis only if there are at least 100 default cases in this category.

In general, LGD is defined as the quotient of the realised loss and the outstanding amount at default. We consider four different LGD definitions in our analysis, which are pre-calculated in the dataset: 1) discounted LGD with the principal advance (additional loan typically to help the obligor's recovery) as loss, 2) discounted LGD with the principal advance in the recovered amount, 3) nominal LGD with the principal advance as loss, and 4) nominal LGD with the principal advance in the recovered amount. The 3-Months EURIBOR rate at the default date is chosen for the discounting factor. To avoid extreme outliers, LGD is capped within the $[-200\%, 300\%]$-interval (smaller intervals do not change the result). The results are quite robust, regardless of LGD definition choice.

The OLS coefficients estimate $\widehat{\sigma q_t}$ in the linear case E7.1. In the non-linear case E7.2, the coefficients sum up the linear effects of both σq_t and R. The remaining non-linear effects reside in the OLS residuals and its intercept. We denote the linear effect R_L and the non-linear effect (both from the intercept and residual parts) $R_{NL} := R_{NL,I} + R_{NL,R}$ that are originated from R. The OLS regression is tasked to minimise the remaining idiosyncratic risk Z^C and R_{NL}.

$$\begin{aligned}
&\forall t_d, t_d + T \in \mathcal{T}: \\
&L_{t_d,t_d+T} = (\mu + R_{NL,I}) + \sum_{s=t_d}^{t_d+T} \big(\underbrace{-\sigma q_s + R_{L,s}}_{\beta_s}\big) X_s + \big(\underbrace{-\sigma\sqrt{1-||q||_2^2}Z^C + R_{NL,R}}_{\varepsilon}\big).
\end{aligned} \tag{E8}$$

In the linear case ($R_L = R_{NL} = 0$), E8 assumes $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2 = \sigma^2 - \sigma^2||q||_2^2)$. Estimating μ and β_s is equivalent to solving coefficients of the OLS regression model, which allows us to solve σ and therefore q algebraically using the residuals' standard error.

$$\begin{aligned}
\hat{\sigma} &= \sqrt{\sigma_\varepsilon^2 + \sum_{s=t_d}^{t_d+T} \beta_s^2} \\
\forall t: \hat{q}_t &= -\frac{\beta_t}{\hat{\sigma}}.
\end{aligned}$$

In the non-linear case, the estimate $\hat{q}_t$ contains the information of R_L as well. Without knowing the form of the function g_C, its extraction from $\hat{q}_t$ is not possible. However, there is no compelling reason to extract $R_{L,s}$ from β_s. The more important concern is whether R_{NL} is significantly different from zero. Without overcomplicating the issue, it is sufficient to test for the required 99.9% survivability of the resulted downturn LGD methodologies in the context of this essay (see section 3.5) regardless whether the rest term R is substantial.

For robustness purposes, we introduce the reduced model as well from E8 to

$$\begin{aligned}
&\forall t_d, t_r \in \mathcal{T}: \\
&\quad L_{t_d,t_r} = (\mu + R_{NL,I}) + \big(\underbrace{-\sigma q_{t_d} + R_{L,t_d}}_{\beta_{t_d}}\big)X_{t_d} + \big(\underbrace{-\sigma q_{t_r} + R_{L,t_r}}_{\beta_{t_r}}\big)X_{t_r} + \\
&\qquad \underbrace{\big(-\sigma\sqrt{1 - q_{t_d}^2 - q_{t_r}^2}Z^C + R_{NL,R}\big)}_{\varepsilon}.
\end{aligned} \tag{E9}$$

This model only includes the default and resolution time. Note that OLS regression methodologies deliver a non-robust result if the explanatory variables are highly correlated. By reducing the model to E9, observing a high correlation of the explanatory variables is unlikely. Solving for q_{t_d} and q_{t_r} can be done similarly

$$\begin{aligned}
\hat{\sigma} &= \sqrt{\sigma_\varepsilon^2 + \beta_{t_d}^2 + \beta_{t_r}^2} \\
\hat{q_{t_d}} &= -\frac{\beta_{t_d}}{\hat{\sigma}} \quad \wedge \quad \hat{q_{t_r}} = -\frac{\beta_{t_r}}{\hat{\sigma}}.
\end{aligned}$$

The estimated coefficients $\hat{q}$ from E8 and E9 should not lead to two different conclusions. If they do, it confirms the high dependency between the explanatory variables, i. e. the latent variables are intertemporally dependent.

3.3.3 Data and Descriptive Statistics

We obtain two databases: 1) the PD and Rating Platform and 2) the LGD and EAD Platform, from Global Credit Data (GCD)[3]. They contain the observed number of defaults counted by banks within a predefined segment and any information related to credit failures in contract

[3]GCD is a non-profit association owned by its member banks from around the world and active in data-pooling for historical credit data. As of 2020, it has 55 members across Europe, Africa, North America, Asia, and Australia. For details: `https://www.globalcreditdata.org`

level leading to LGD and EAD. All participating banks are obliged to specify default and loss in the same way to ensure data comparability within the sample.

3.3.3.1 PD and Rating Platform

In the PD and Rating Platform, the numbers of defaulted and non-defaulted loans for defined segments from 1994 until 2017 are reported. These numbers are low before 2000, so the data from these early years may not as representative as the following years. Starting from 2000, the yearly number of loans rises to over 50,000 and reaches its peak to over 710,000 in 2014. The dataset composition on rating, asset class, or industry segment fluctuates yearly.

The platform contains pooled numbers of defaulted as well as non-defaulted loans in various segments. In total, the dataset contains over 6.2 million non-defaulted loan-years distributed over the 18 years interval. Assuming that the typical duration to maturity or default time is about two years, the dataset contains information on over 3 million different loans internationally. Three of the most represented asset classes are: SME (50.71%), Large Corporate (22.14%), and Banks and Financial Companies (14.67%) and three of the most represented industries are: Finance and Insurance (15,33%), Real Estate, Rental, and Leasing (14,88%), and Wholesale and Retail Trade (13,62%)[4].

Figure 3.1 shows the observed default rates in the dataset throughout the years between 2000 and 2016. Global Credit Data (2019b) has a dedicated report regarding this dataset. The dataset classifies every default into defined segments of

- **asset class**: SME, Large Corporate, Banks and Financial Companies, Ship Finance, Aircraft Finance, Real Estate Finance, Project Finance, Commodities Finance, Sovereigns, Public Services, Private Banking;
- **industry**: Agriculture, Hunting and Forestry, Fishing and its Products, Mining, Manufacturing, Utilities, Construction, Wholesale and Retail, Hotels and Restaurants, Trans-

[4] Counted in loan-years. Assuming the typical duration to maturity or default time is similar throughout the segments, the composition remains unchanged when counted in number of loans.

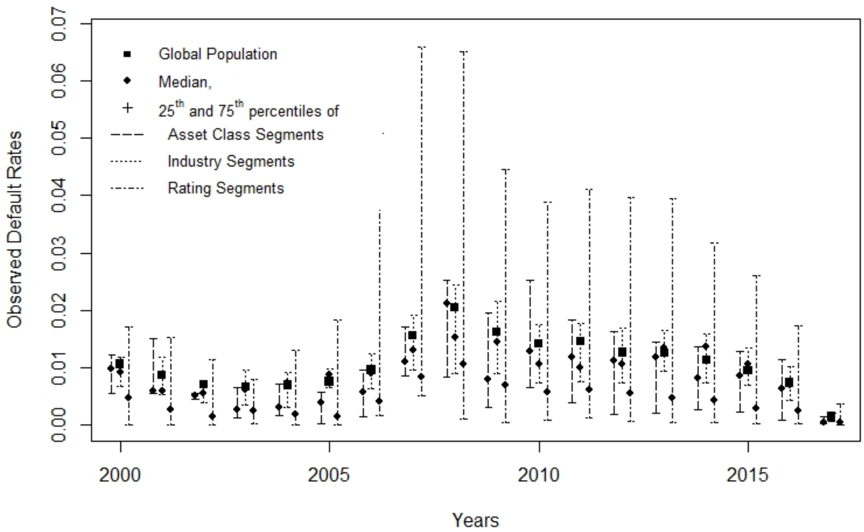

Figure 3.1: Observed default rates in the global population and segmented by asset class, industry, and rating

portation and Storage, Communications, Finance and Insurance, Real Estate, Rental and Leasing, Professional, Scientific and Technical Services, Public Administration and Defence, Education, Health and Social Services, Community, Social and Personal Services, Private Sector Services, Extra-territorial Services, and Individual; and

- **rating**: mapped to S&P rating categories (from AAA to C), as well as defaulted.

In each category, figure 3.1 shows the observed default rates of the 25% best and the 25% worst segment, as well as the median (only if there are at least 100 loans in the particular subcategory).

3.3.3.2 LGD and EAD Platform

The LGD and EAD platform contains extensive information about credit failures on loan level for non-retail exposures. The LGD is pre-calculated based on the realised loss per outstanding unit applying a variation of LGD definitions (discounting the recovery cash flows or by including/excluding principal advances). Variation in the LGD definition does not affect the results significantly.

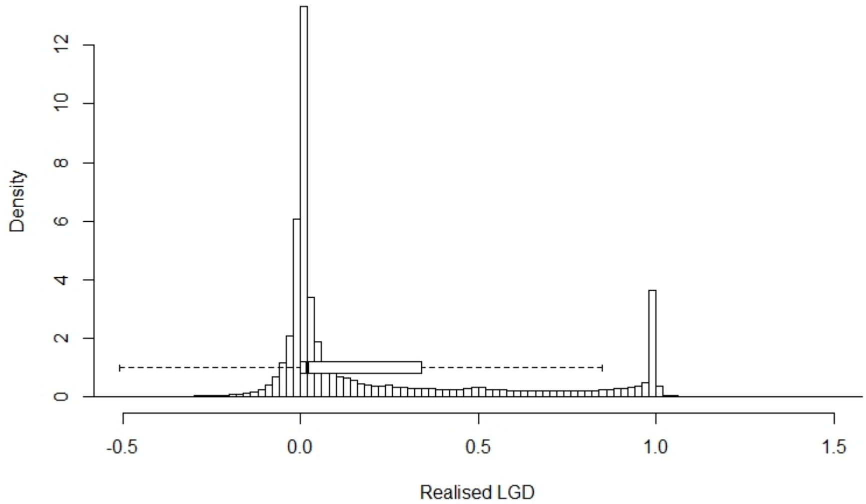

Figure 3.2: Histogram and Boxplot of the realised LGD

The dataset contains over 186,000 defaulted loans after 2000, both resolved (92.5%) and unresolved cases (7.5%). The number of resolved loans between 2000 and 2017 with non-zero exposure is 161,365 from 93,775 different obligors with an average EAD of €3 million. Three of the most represented asset classes are SME (62.57%), Large Corporate (16.61%), and Real Estate Financing (12.42%). Three of the most represented industries are Manufacturing (16.44%), Real Estate, Rental, and Leasing (15.94%) and Wholesale and Retail Trade (14.01%).

LGD samples outside the $[0,1]$-interval are possible and are not rare for workout LGDs. Typically, the realised workout LGDs in loan level inhibit the bimodal distribution, as also shown in figure 3.2 for our dataset. The mean of realised LGDs (referenced by default year) are highly correlated with the observed default rate as shown by figure 3.3. However, some of the defaults in the dataset are not resolved yet, resulting in low average realised LGDs in the last five years. A report regarding the LGD distribution of Large Corporate exposures in this dataset and a study for a downturn effect in the LGD pattern are conducted by the Global Credit Data (2017, 2019a).

A long workout duration is often associated with a high average LGD. As figure 3.4 shows,

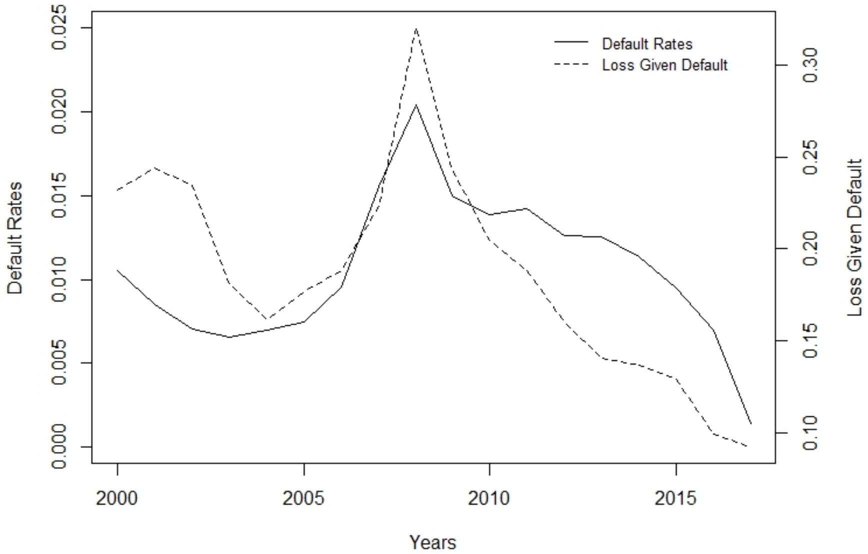

Figure 3.3: Observed default rates and realised loss given default for each year

this is not only true for the LGD's mean, but also its deviation from average. This figure depicts an increasing LGD's mean when it is categorised by its workout duration in years (rounded to one decimal place).

3.4 Result

With the observed default rates, the maximum likelihood method gives an estimated $\hat{p} = 27.95\%$ (equivalent to an asset correlation of $\hat{p}^2 = 7.81\%$). For a comparison, Frye (2000b) reports a p of 23% (for bonds) and Düllmann and Trapp (2004) report a p to be ca. 20% (for bonds and loans). Within the EU capital regulation CRR, p^2 is equivalent to R under the Art.153-154 of the CRR with predefined values between 3% and 24% depending on asset classes and the historical PD.

Figure 3.5 depicts the implied latent variables time-series. One of the important aspects of a latent variables approach is that it measures the change of default rates relative to the mean rather than the default rate itself. During the global financial crisis, which started in 2007, the

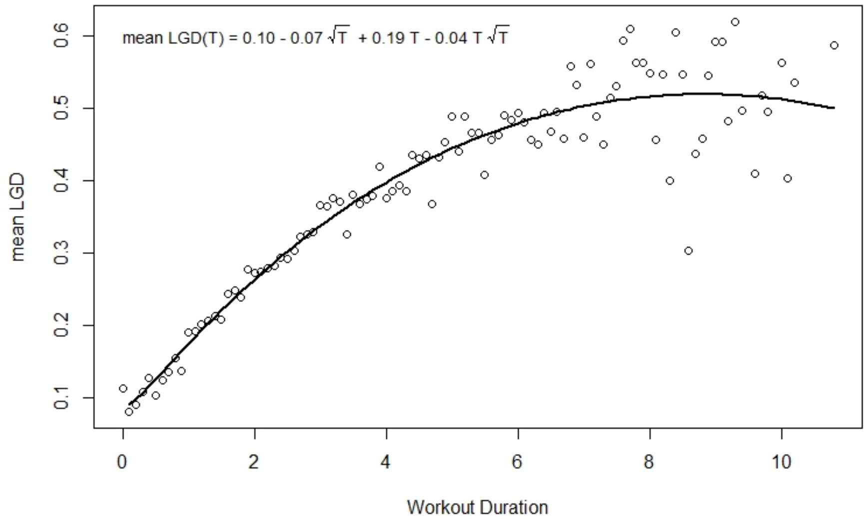

Figure 3.4: Mean LGD trend based on the workout duration

downturn effect is observed soon after the Lehmann fall in 2008. The implied latent variables in both years are at the lowest points compared to others. In comparison, the implied latent variable in 2017 is high due to the low observed default rate in 2017, as shown in figure 3.1.

3.4.1 LGD's Systematic Sensitivity

Our method for estimating q is designed such that for every pair $(t_d, t_r) \in \mathcal{R}^2$, a sample is generated. A potential issue may occur for defaulted loans with an extraordinarily long workout duration because such cases are rare compared to defaults with one or two years workout duration. To avoid a potential bias originated from these extreme cases, samples with excessively long duration are excluded. About 95% of the resolved defaults in the database have workout durations less than six years, and we choose this to be the cutting point. We successively extend the maximum workout duration length in the analysis to replicate a portfolio of a random financial institution. Due to the model design, the result is to be interpreted as a portfolio rather than a single exposure.

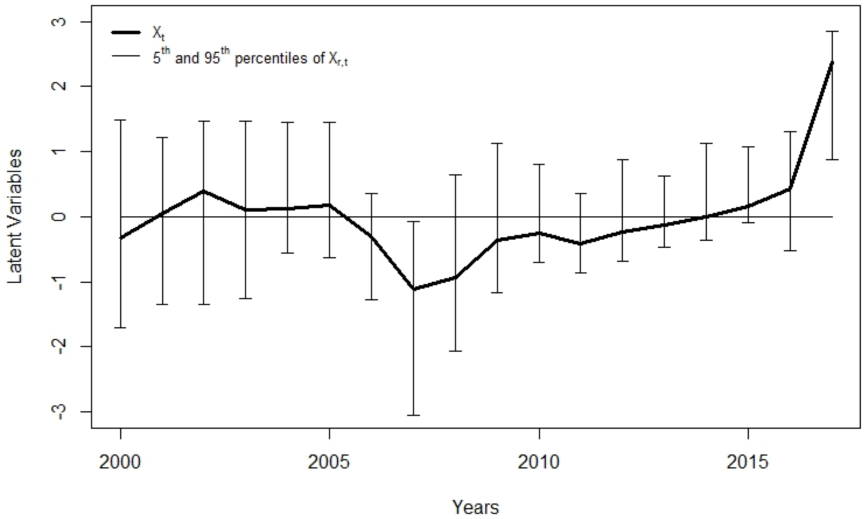

Figure 3.5: Estimated latent variables

The results presented in table 3.1 show a systematic sensitivity of the expected LGD towards a particular default year during its workout duration. The discussion of whether LGD is to be analysed by vintage of default or vintage of recovery can be answered. Acquiring a sensitivity coefficient of $q = (1, 0, \dots, 0)$ is an argument for vintage of default, while a coefficient of $q = (0, \dots, 0, 1)$ or $(0, \dots, 0, -1)$ speaks for vintage of resolution. None of the patterns emerges, which suggests that a single vintage point may not be adequate to explain the expected LGD. In particular, the expected LGD is highly sensitive towards the systematic factor soon after its default date, but this sensitivity mostly diminishes with increasing default age. Note that these values stand for the sensitivity towards the systematic factor and not for the expected LGD itself. They may hold some explanatory power on the downturn add-on for LGD, but not on the downturn LGD itself. The results confirm that loans, which are defaulted during a downturn period, will be expected to perform worse (LGD-wise) than loans, which are resolved during a downturn period given a similar workout duration. Thus, the financial crisis has different impacts on the LGD depending on the default age of the exposures. Table 3.2 shows that there is some

Table 3.1: LGD's systematic sensitivity coefficient towards the latent variables from different years during the workout period. q_t is the sensitivity of expected LGD towards X_t and σ_ε^2 is the variance of the residuals. T represents the workout duration, so each row can be understood as banks' limit strategies for workout duration.

	q_{t_d}	q_{t_d+1}	q_{t_d+2}	q_{t_d+3}	q_{t_d+4}	q_{t_d+5}	σ_ε^2
$T=0$	0.3472						0.0005
$T\leq 1$	0.3834	0.0779					0.0012
$T\leq 2$	0.3796	0.2745	-0.2418				0.0035
$T\leq 3$	0.4093	0.3347	0.0702	-0.1045			0.0065
$T\leq 4$	0.4040	0.3288	0.1101	0.0895	-0.1817		0.0080
$T\leq 5$	0.4344	0.3188	0.1345	0.0839	-0.0370	0.1934	0.0094

slight variation to our results depending on how LGD is calculated. However, this difference is negligible, and it ultimately leads to the same conclusion.

The systematic sensitivity at the first default year q_{t_d} should approximately range between 34% to 44%, based on our results. Interestingly, it is higher than the estimated $p = 27.95\%$. Both parameters p and q can be compared directly when analysing the systematic sensitivities of A_{i,t_d} and C_{i,t_d}. However, the systematic sensitivities of PD and LGD also depend on the functions g_A and g_C. Without specifying the function g_C, the systematic sensitivities of PD and LGD cannot be compared.

Nevertheless, the fact that the estimated value of $\hat{q}_{t_d}$ is possibly higher than $\hat{p}$ is alarming. Assuming the function g_C behaves similarly as g_A as a function of X_{t_d}, it can be concluded that an economic shock would have a more severe effect on the LGD than the PD. In less technical words, the downturn impact at the default year towards the LGD is expected to be more severe than towards the PD.

As explained in section 3.2.1, a downturn LGD estimation needs to be latent variable based to be consistent with the conditional PD under the IRBA. The results show that $\mathbb{E}[LGD_i|X]$ is not only sensitive towards the latent variables at its default time (X_{t_d}) but also to latent variables during its whole course of the workout process ($X_{t_d+1}, \dots, X_{t_r}$). In the regulatory context, two downturn LGD methodologies are required: 1) downturn LGD estimation for non-defaulted exposures and 2) for unresolved defaulted exposures.

Table 3.2: LGD's systematic sensitivity coefficient towards the latent variables from different years during the whole workout period, using various other LGD definitions. q_t is the sensitivity of expected LGD towards X_t and σ^2_ε is the variance of the residuals. T represents the workout duration, so each row can be understood as banks' limit strategies for workout duration.

Discounted LGD, Principal Advance in Recovered Amount

	q_{t_d}	q_{t_d+1}	q_{t_d+2}	q_{t_d+3}	q_{t_d+4}	q_{t_d+5}	σ^2_ε
$T=0$	0.2769						0.0006
$T\leq 1$	0.3906	-0.0495					0.0015
$T\leq 2$	0.3967	0.2447	-0.3156				0.0047
$T\leq 3$	0.4387	0.3273	0.0228	-0.1421			0.0091
$T\leq 4$	0.4287	0.3293	0.0835	0.0777	-0.2662		0.0116
$T\leq 5$	0.4720	0.3379	0.1170	0.0994	-0.0741	-0.0078	0.0132

Nominal LGD, Principal Advance in Recovered Amount

	q_{t_d}	q_{t_d+1}	q_{t_d+2}	q_{t_d+3}	q_{t_d+4}	q_{t_d+5}	σ^2_ε
$T=0$	0.2198						0.0005
$T\leq 1$	0.3842	-0.1447					0.0013
$T\leq 2$	0.3940	0.2067	-0.3836				0.0045
$T\leq 3$	0.4430	0.3100	-0.0083	-0.2045			0.0088
$T\leq 4$	0.4241	0.3214	0.0903	0.0380	-0.3352		0.0115
$T\leq 5$	0.4578	0.3443	0.1547	0.0726	-0.1276	-0.1021	0.0134

Nominal LGD, Principal Advance as Loss

	q_{t_d}	q_{t_d+1}	q_{t_d+2}	q_{t_d+3}	q_{t_d+4}	q_{t_d+5}	σ^2_ε
$T=0$	0.2749						0.0004
$T\leq 1$	0.3765	-0.0222					0.0011
$T\leq 2$	0.3774	0.2382	-0.3268				0.0033
$T\leq 3$	0.4216	0.3287	0.0362	-0.1584			0.0062
$T\leq 4$	0.4088	0.3312	0.1187	0.0559	-0.2466		0.0077
$T\leq 5$	0.4373	0.3431	0.1815	0.0626	0.0841	0.0803	0.0087

For non-defaulted exposures, the CC at year t should generally be

$$CC \geq \mathbb{E}[D_i|X_t]\cdot\mathbb{E}[LGD_i|X_t,\ldots,X_{t+T}] - EL^*,$$

with a random workout duration $T+1$ (note that $\mathbb{E}[D_i|X_t] = \mathbb{E}[D_i|X_t,\ldots,X_{t+T}]$ because D_i is a point-in-time variable). The formula assumes that the year t is a downturn period with an expected default rate of $\mathbb{E}[D_i|X_t]$ and an expected LGD of $\mathbb{E}[LGD_i|X_t,\ldots,X_{t+T}]$. While X_t is typically assumed to have a conservative value, the variables $X_{t+1},\ldots,X_{t_d+T}$ and T are not observed yet.

Table 3.3: LGD's systematic sensitivity coefficient towards the latent variables only on default and resolution years. q_t is the sensitivity of expected LGD towards X_t and σ_ε^2 is the variance of the residuals. T represents the workout duration, so each row can be understood as banks' limit strategies for workout duration.

	q_{t_d}	q_{t_r}	σ_ε^2
$T = 0$	0.3472		0.0005
$T \leq 1$	0.3834	0.0779	0.0012
$T \leq 2$	0.4734	-0.1567	0.0036
$T \leq 3$	0.5408	-0.0320	0.0067
$T \leq 4$	0.5534	-0.1078	0.0083
$T \leq 5$	0.5770	0.2069	0.0092

For unresolved defaults, the CC has far less unknown variables

$$CC \geq \mathbb{E}[LGD_i | X_{t_d}, \ldots, X_t, \ldots, X_{t+T}] - ELBE^*,$$

given its default year t_d with a remaining random workout duration $T+1$ and a calculated loan loss provision per exposure unit ELBE*. The past latent variables $(X_{t_d}, \ldots, X_{t-1})$ lie in the past and are highly relevant for the LGD$_i$. The future latent variables $(X_{t+1}, \ldots, X_{t+T})$ as well as the workout duration T remain unobserved.

The multiple latent variables approach for the regulatory capital requirement has additional merits. Such an approach can capture various stress scenarios, e. g. a downturn event lasting for two or three years ($X_{t+1} = X_{t+2} = -\Phi^{-1}(0.999)$) or a volatile state of economy ($X_{s \geq t} \sim \mathcal{N}(0, \sigma^2 \geq c)$ for a given positive constant $c > 1$). Analysing the appropriateness of these assumptions on the latent variable time series is on its own an interesting topic.

3.4.2 Robustness Analysis

The estimated coefficients $\hat{q}$ from E8 and E9 should not lead to different conclusions. Otherwise, it is evidence for a non-robust estimate $\hat{q}$ mainly due to highly correlated explanatory variables. Thus, we expect a relatively high valued q_{t_d} and a comparably low valued q_{t_r} to maintain the same conclusion, as concluded from the previous analysis.

The pattern found in table 3.3 is a confirmation to the robustness of the previous results.

While the systematic sensitivities at the default year t_d are high regardless of the cutting point, the sensitivities at the resolution year are relatively low in comparison.

3.5 Alternative Downturn LGD Estimations

The results obtained from the previous section suggest that incorporating multiple latent variables from several workout years for a downturn LGD estimation can have more explanatory power. While the early years after default are shown to be more relevant than the later years, it is not yet clear whether incorporating only the early default years is sufficient to reach a conservatism level of 99.9%. In a simulation, this requirement is equivalent to 99.9% survivability, i. e. only 0.1% chance of an LGD underestimation in expectation.

Four basic downturn LGD estimation procedures based on a latent variable approach are chosen for the performance analysis. A Monte Carlo simulation is applied to measure their performance and compare them with the current regulatory downturn LGD method. This simulation is constructed using the LGD and EAD platform, which covers defaults from 2000 to 2017. Within this time interval, banks will most likely identify 2008 or 2009 as the worst downturn period under the EBA/RTS/2018/04. The idiosyncratic risk will not be evaluated since the focus of downturn LGD methods is to measure the systematic risk. It is necessary to measure the performance parameters independent from any bias correction methods (if any bias in the data exists). Thus, neither bias correction nor margin of conservatism will be applied.

3.5.1 Various Approaches for Downturn LGD

The performance of the following downturn LGD estimations is compared for the year t (we refer t as *today*). Up to the time point t, the latent variables are assumed to be available. We are only interested in the exposures, which will be resolved today. Other exposures do not contribute to the portfolio's LGD for the year t. In practice, it is unknown whether a defaulted exposure will be resolved today. For regulatory purposes, this specification will be generalised later. The general idea underlying these estimations is to estimate LGDs using the past latent variables. The latent variable for today will be stressed, i. e. a downturn period is assumed for t ($X_t := -\Phi^{-1}(0.999)$). The downturn LGD estimations are defined as follows:

1. **A forward-looking single-factor estimation** (in line with the vintage of resolution). This procedure assumes that the expected LGD depends only on the (fully-weighted) today's latent variable, which will be stressed (set to $-\Phi^{-1}(0.999)$). The conditional PD is derived with this assumption in mind.

$$\mathbb{E}[LGD_i|X_t = -\Phi^{-1}(0.999)] = \hat{\mu} - \hat{\sigma}\Big(1 \cdot X_t + \underbrace{0 \cdot Z_C}_{\text{loan-specific risk is set to zero}}\Big). \tag{A1}$$

2. **A backward-looking single-factor estimation** (in line with the vintage of default). This procedure assumes that the expected LGD depends on the (fully-weighted) default year's latent variable. The latent variable is stressed only if the default year is t.

$$\mathbb{E}[LGD_i|X_{t_d}, \text{ and if } t = t_d : X_{t_d} = -\Phi^{-1}(0.999)] = \hat{\mu} - \hat{\sigma}\Big(1 \cdot X_{t_d} + 0 \cdot Z_C\Big). \tag{A2}$$

3. **A three-years-factors estimation** (a mixture of the vintage of default and the vintage of recovery). Compared to the previous methods, this estimation method incorporates multiple latent variables. The result shown in the previous section supports the proposition that the expected LGD is most sensitive towards the latent variables in the first three default years. If the default age is shorter than three years, the last latent variable will be stressed. Otherwise, only the first three realised latent variables after default are included

in the estimation. This procedure weights the relevant latent variables equally[5].

$$\mathbb{E}[LGD_i|X_{t_d},\ldots,X_{t_d+2},\ \text{and if}\ t\leq t_d+2: X_t=-\Phi^{-1}(0.999)] =$$
$$\hat{\mu}-\hat{\sigma}\Big(\sum_{s=t_d}^{\min(t_d+2,t)}\underbrace{\sqrt{\frac{1}{\min(3,t-t_d+1)}}}_{\text{equal weight on each relevant workout year}}\cdot X_s+0\cdot Z_C\Big). \tag{A3}$$

4. **A complete-history based estimation** (no particular vintage point). Different from the previous method, this approach incorporates the complete history of past latent variables within the workout duration and stresses only today's latent variable. All latent variables are equally weighted.

$$\mathbb{E}[LGD_i|X_{t_d},\ldots,X_{t-1},\ \text{and}\ X_t=-\Phi^{-1}(0.999)] =$$
$$\hat{\mu}-\hat{\sigma}\Big(\sum_{s=t_d}^{t}\underbrace{\sqrt{\frac{1}{t-t_d+1}}}_{\text{equal weight on each workout year}}\cdot X_s+0\cdot Z_C\Big). \tag{A4}$$

The constraint of only using right-censored data has to be accounted for in the simulation, i. e. only information up to t can be used for estimations. Both the required parameters $\hat{\mu}$ and $\hat{\sigma}$ can be estimated by the expected value and the standard deviation of the institutions' portfolio LGD, which can be estimated solely from past information. In practice, right-censored data will be heavily influenced by resolution bias, which generally leads to an underestimation.

The proposed downturn LGD procedures assume a linear or semilinear relationship between $\mathbb{E}[LGD|X]$ and X. If the function g_C is indeed non-linear, then it would imply that the rest term R in E7.2 has a substantial effect and it will be reflected in the performance of these procedures in the simulation. As a side-note, the true nature of the function g_C only plays a secondary role. The 99.9% survivability is the necessary requirement for the use of such a downturn LGD procedure regardless of the form of g_C. Conversely, we do not argue that a good performance is sufficient evidence for the linearity of the function g_C.

[5]Using the estimated values of q from the previous section might induce too much overfitting.

It is not difficult to give first estimate whether these methods are conservative. We can look at the number of exposures in a random portfolio which are affected by the stressed latent variable varies in these proposed methods. A1 is the most conservative (likely to be borderline excessive) and is also in line with the IRBA's conditional PD. A2 is the least conservative because it stresses the latent variable only if the loan defaults in the current year. Based on the results in table **??**, we argue that A3 can cover most of the necessary information to construct an adequate downturn LGD estimate. The last procedure A4 includes the remaining latent variables as well. From its degree of conservativeness, A2 is the least conservative, then A3, followed by A4, with A1 to be the most conservative one.

It is important to mention that there is a flaw in all latent variable based methods. A latent variable is an abstract mathematical object, and its economic interpretation is not rigid. In the ASRF model, the latent variable could be represented by the global economic situation. However, this is quite vague. A large part of section 3.2 is dedicated to explaining the issues related to the conflict between the macroeconomic based approach and the latent variable-based approach to outline the difference. Although it might be not satisfying only to have a vague interpretation, a latent variable based approach is unavoidable for consistency with the IRBA.

3.5.2 Institution's Survival Chance and Waste

This essay introduces two concepts for performance measurement: the institution's *survival chance* and *waste*. What survivability represents and why it is chosen is obvious. With waste, we want to measure the LGD overestimation, since an overly conservative downturn LGD implies a higher capital requirement. From the banks' perspective, higher capital requirements are associated with higher weighted average capital costs (WACC), as empirically shown by Kashyap et al. (2010); Cosimano and Hakura (2011); Miles et al. (2013). Van den Heuvel (2008) shows a surprisingly high welfare cost of capital requirements. Mikkelsen and Pedersen (2017) argue that increasing capital requirements is a trade-off between benefits and social costs. In the case of downturn LGD methodologies, a trade-off between survival chance and waste occurs. At some point, the bank's survival chance is sufficiently large that being even more conservative only increases social costs without the benefits.

Per definition, an institution *survives* the year t if the regulatory CC at t is at least as high as the realised loss at t. In the LGD context, an institution *survives LGD-wise* if the (regulatory) downturn LGD at t given the portfolio composition at t is at least as high as the realised LGD for all defaults resolved at t on the portfolio level. If the calculated downturn LGD is lower than the realised one, we refer to this event as a *failure*. A high survival chance is compulsory for a well designed regulatory rule. For the use in the IRBA, an overall survival rate of 99.9% is required to avoid any risk underestimation.

For a particular method to estimate downturn LGD (or any loss in general), its *waste* describes the degree of the LGD overestimation. Per definition, the downturn LGD should always be higher than the expected LGD. Thus, some degree of overestimation from realised LGD is not surprising and theoretically necessary to act as a margin of conservatism. In particular, we define the waste of a survived institution as the difference of the calculated downturn LGD mean and the realised LGD mean for a particular portfolio at time t.

$$\text{waste}_t = \min(0, \overline{\text{Downturn LGD}_t} - \overline{\text{Realised LGD}_t}).$$

An over-conservative rule would obviously produce higher downturn LGD estimations than the realised LGD in expectation, e. g. setting the downturn LGD to be equal 100% for all case would ensure high survivability but a high waste as well.

3.5.3 Performance Test

As downturn events occur unexpectedly, institutions are required to anticipate a downturn event anytime. For each t, the parameters μ and σ can only be estimated using available data up to t. The realised LGDs from resolved defaults build the basis of the simulation. For each year before t, we calculate the portfolio LGD's mean and standard deviation to acquire $\hat{\mu}$ and $\hat{\sigma}$. This information is available for institutions at the time t. In our view, a loss database of five years would be the absolute minimum to calculate any decent estimate. Therefore, we evaluate the performance test only from 2005 onwards.

For each iteration, 1,000 default cases are randomly drawn from the defaults population[6], which are resolved in the year t. The process is repeated 10,000 times. One iteration can be interpreted as the LGD realisation of a default portfolio of a random institution in the year t. The simulated institution does not survive in the year t LGD-wise, if its estimated downturn LGD for the year t (e. g. by applying A1-A4) is lower than the average realised LGD of its portfolio. Subsequently, if the simulated institution survives, the difference between the downturn LGD mean and the realised LGD mean for its portfolio will be the associated waste in the year t.

The average of the survival rates is calculated as a geometric mean, while the average of the waste is an arithmetic mean, due to the nature of each parameter. Considering that any loan-specific information is omitted and the models have to deal with right-censored data with an expected underestimation, the performance of methods A3 and A4 are on average extraordinarily high. Except for A2, the survival chance is overall comparable. Note that the IRBA requires 99.9% survivability, which renders any method with a lower survival rate to be worthless.

According to table 3.4, the complete-history based estimation method A4 performs best and is even sufficient for a large portfolio without any additional loan-specific information. Remarkably, this procedure ensures high survivability even during the financial crisis and the post-crisis periods. The sufficiency of the first three years after default is supported by the 97% survival chance of the three-years-factor estimation method A3. Although this performance is not adequate in terms of 99.9% survivability, it is remarkable to see that two additional years of latent variables have almost reached the goal.

As shown in table 3.4, the realised portfolio's LGD at t does not necessarily follow a systematic pattern. The long-run average LGD ($\hat{\mu}$) does not reach its peak in 2008-2009, but somewhat

[6]The number of randomly drawn default cases has to be large enough to isolate the systematic effect. However, not too large that a non-trivial granularity add-on might be needed later. The validity for a smaller portfolio will also be analysed in section 3.5.5.

Table 3.4: Survival chance and waste of different downturn LGD estimation models in % for each year from 2005 until 2017. $\hat{\mu}$ and $\hat{\sigma}$ are the estimated portfolio's realised LGD mean and standard deviation based on historical data up to each year. A1 is the forward-looking single-factor estimation, A2 is the backward-looking single-factor estimation, A3 is the three-years-factor estimation, and A4 is the complete-history based estimation.

	$\hat{\mu}$ ($\hat{\sigma}$)	Survival Chance (Waste)			
		A1	A2	A3	A4
2005	16.09 (4.48)	100 (12.90)	99.98 (3.64)	100 (7.15)	100 (8.62)
2006	16.55 (4.35)	100 (12.35)	99.64 (2.58)	100 (6.01)	100 (7.69)
2007	17.00 (4.36)	100 (13.20)	99.98 (3.91)	100 (8.65)	100 (9.83)
2008	16.98 (5.17)	100 (14.10)	100 (6.21)	100 (11.58)	100 (13.22)
2009	17.46 (4.23)	100 (10.23)	100 (5.42)	100 (9.35)	100 (10.16)
2010	18.09 (3.57)	100 (8.49)	95.64 (2.10)	100 (6.56)	100 (7.25)
2011	18.56 (3.81)	100 (9.83)	97.05 (2.40)	100 (6.79)	100 (7.84)
2012	18.67 (4.73)	100 (12.18)	99.15 (3.13)	100 (7.55)	100 (9.79)
2013	18.72 (5.29)	100 (14.69)	99.81 (3.91)	100 (8.76)	100 (11.43)
2014	18.56 (5.93)	100 (11.72)	9.32 (0.69)	99.09 (3.41)	100 (6.93)
2015	18.85 (5.81)	100 (11.55)	7.56 (0.60)	93.96 (2.38)	100 (5.87)
2016	18.76 (6.17)	100 (11.03)	0.09 (0.33)	73.05 (1.30)	99.98 (4.04)
2017	18.76 (6.01)	100 (15.38)	6.97 (0.45)	100 (4.23)	100 (6.60)
Average		100 (12.13)	32.23 (2.72)	97.08 (6.44)	100 (8.41)

lagged likely due to workout duration and other unknown factors, as shown in the second column of the table. An LGD time-series by vintage of default typically has a high peak in 2008, but the peak of an LGD time-series by vintage of resolution wanders off in a later period. The portfolio composition and the bank's workout strategies play a significant role when the peak is.

Among the procedures with a high survival rate, the results report less average waste by the three-years-factors methods A3 (6%) and the complete-history based estimation A4 (8%)

than the forward-looking single-factor method A1 (12%). The waste value can be interpreted as the average amount of LGD overestimation in an average year, provided failure does not occur. Lastly, the unsatisfactory performance of the method A2 confirms that it is not the systematic factor of the default year specifically, which influences the LGD, but rather the whole workout duration.

3.5.3.1 Comparison to the Foundation IRBA

The performance as seen in table 3.4 does not mean much if it is not compared to a benchmark and the results cannot offer a meaningful contribution to answer the question whether a change in regulation will worth the cost and time. This section concentrates on the comparison towards the foundation IRBA while considering the recent change in the Basel Accord as well. The finalised Basel III, as suggested by the Basel Committee on Banking Supervision (2017a), forbids the use of advanced IRBA for some asset classes. This section highlights the overall better performance of our method compared to the foundation IRBA within the relevant asset classes. By limiting the simulation to a particular asset class, we indirectly also show that the methods also work well for specialised banks. In particular, the performance of A1-A4 is compared with the downturn LGD assigned using the foundation IRBA, considering collaterals and haircuts in the finalised Basel III document as suggested by Basel Committee on Banking Supervision (2017a).

The relevant asset classes for the foundation IRBA, which are available in the dataset, are Large Corporate, Banks and Financial Companies, Sovereigns, and Private Banking. All required assumptions are taken generously, including collaterals are assumed to be always eligible and financial collateral's haircut is assumed to be 0%. With generous assumptions, the simulation should produce the least amount of waste for the foundation IRBA. Nonetheless, both A3 and A4 still produce significantly lower waste than the regulatory downturn LGD under the foundation IRBA, while maintaining a similar survival chance.

It is not surprising that the foundation IRBA will achieve the 99.9% survival rate on average since it is generally a very conservative approach. However, even under generous assumptions,

Table 3.5: Comparison of survival chance and waste of different downturn LGD estimation models with the foundation IRBA for relevant asset classes in % for each year from 2005 until 2017. $\hat{\mu}$ and $\hat{\sigma}$ are the estimated portfolio's realised LGD mean and standard deviation based on historical data up to each year. A1 is the forward-looking single-factor estimation, A2 is the backward-looking single-factor estimation, A3 is the three-years-factor estimation, A4 is the complete-history based estimation, and F-IRBA is the LGD assigned according to the foundation IRBA.

	$\hat{\mu}$ ($\hat{\sigma}$)	Survival Chance (Waste)				
		A1	A2	A3	A4	F-IRBA
2005	21.23 (7.66)	100 (23.80)	100 (5.02)	100 (10.49)	100 (14.55)	100 (22.10)
2006	20.24 (8.23)	100 (25.90)	100 (4.01)	100 (7.83)	100 (14.04)	100 (18.92)
2007	19.68 (8.66)	100 (34.14)	100 (14.87)	100 (23.58)	100 (26.55)	100 (31.08)
2008	17.96 (9.54)	100 (26.30)	100 (15.24)	100 (21.89)	100 (24.67)	100 (23.34)
2009	19.09 (7.79)	100 (21.39)	100 (12.76)	100 (19.59)	100 (21.12)	100 (23.39)
2010	20.01 (6.96)	100 (19.76)	100 (6.24)	100 (15.84)	100 (17.03)	100 (22.02)
2011	20.76 (6.60)	100 (22.47)	100 (9.45)	100 (17.07)	100 (18.94)	100 (25.52)
2012	20.13 (7.07)	100 (17.82)	100 (4.12)	100 (10.99)	100 (14.21)	100 (19.90)
2013	20.61 (6.40)	100 (18.83)	100 (6.42)	100 (12.25)	100 (15.12)	100 (21.78)
2014	20.21 (7.08)	100 (17.13)	92.64 (1.55)	100 (8.21)	100 (11.74)	100 (18.02)
2015	20.68 (6.36)	100 (12.60)	0.27 (0.27)	99.26 (2.39)	100 (6.31)	100 (13.74)
2016	20.68 (6.22)	100 (12.44)	2.96 (0.34)	99.63 (2.43)	100 (5.66)	100 (14.87)
2017	20.81 (6.15)	100 (22.96)	100 (5.90)	100 (13.01)	100 (14.65)	100 (27.70)
Average		100 (21.19)	48.11 (6.63)	99.91 (12.74)	100 (15.74)	100 (21.72)

table 3.5 confirms that the LGD based on the foundation IRBA is wasteful compared to other methods. In fact, its performance is similar to A1 on average, both the survival chance and the waste. However, it is also worth noting that A1 is adapting to the economic situation as more information is available. It can be observed that its waste values are only higher than the foundation approach's waste values only during 2005-2008. It may suggest a more accurate

estimation by A1 if more data is available compared to the foundation IRBA. In summary, the LGD estimation is overestimated by about 22% under the current foundation IRBA, whereas the method A3 by approximately 13% and the method A4 by about 16% while maintaining high survivability of 99.9% as required in the IRBA.

3.5.3.2 Comparison to the Advanced IRBA

Similarly to the previous simulation, we compare our proposed methods with the advanced IRBA. The finalised Basel III Accord largely limits the use of the advanced IRBA. According to the Basel Committee on Banking Supervision (2017a), supervisors may permit the use of the advanced IRBA to the corporate asset class with a consolidated revenue of less than €500 million. In our dataset, the relevant asset class would be the SME. We assume that each SME exposure in the dataset fulfils the corresponding requirement for the use of the advanced IRBA.

Although the EBA has spent a lot of effort to harmonise the calculation of capital requirements under the advanced IRBA, institutions still have some leeway in determining their downturn LGD estimates. Choosing a particular downturn LGD model appropriate for the advanced IRBA may reduce the representativeness of our analysis. Therefore, we choose three methods, each representing a different level of conservatism. A low level (L) represents the regulatory minimum for the downturn LGD estimation per guideline. A mid level (M) stands for the downturn LGD estimates that fully implement the intent of the EBA's downturn LGD guideline. A high level (H) represents the most conservative downturn LGD estimate that the EBA has proposed, especially when an (M)-type estimation is not possible. The simulation will be done with the application of the LGD input floors (as proposed by the Basel Committee on Banking Supervision (2017a)) and without them.

Beside the downturn LGD method, the guideline EBA/GL/2019/03 offers a reference value as well. Even though this value is legally non-binding, it arguably serves as a guide for the supervisors. Any institution with a strong intention to minimise its capital requirement will try to produce a downturn LGD estimate near to this reference value. Anything significantly below this value will be a red light for the supervisors and it might induce stricter supervision. Thus, the reference value can be seen as the regulatory minimum for the downturn LGD estimation. This value is calculated by the arithmetic average of the yearly realised LGD mean (by vintage of default) in the two most severe downturn years of all available loss data. We assume that the LGDs in the two most severe downturn years are also the two maximum LGDs observed. Let $LGD_{s,t}$ be the realised LGD average of all exposures defaulted in year s and resolved up to year t (today), $LGD_{\max,t}$ the maximum and $LGD_{\max_2,t}$ the second maximum LGD over all possible $s \leq t$.

$$DLGD := \frac{LGD_{\max,t} + LGD_{\max_2,t}}{2} \tag{A-IRBA-L}$$

In general, IRBA banks are required to analyse the relationship between LGD and some other external factors, especially the downturn impact on the LGD. The (M)-level downturn LGD estimation in this essay is represented by the maximum realised LGDs without an additional margin of conservatism in the whole SME population, since this margin seems to be arbitrary and calibration segments are chosen individually. Overall, this is a simplification of the proposed downturn LGD estimation in the EBA/GL/2019/03. The logic behind this estimate is that most banks will most likely identify 2008 as a downturn period for the SME calibration segment, given our dataset.

$$DLGD := LGD_{\max,t} \tag{A-IRBA-M}$$

The guideline also sets a conservative limit which serves as a backstop downturn LGD, when everything else fails. In the worst case, where downturn impacts cannot be observed nor estimated, the institution has to set the downturn LGD estimate in the concerned calibration segment to be at least as conservative as the long-run average LGD with an add-on of 15%

capped at 105%. This estimate is often viewed as extremely conservative.

$$DLGD := \min\left(105\%, \frac{\sum_{s\leq t} LGD_{s,t}}{|\{s|s\leq t\}|} + 15\%\right) \tag{A-IRBA-H}$$

The results shown in table 3.6 are alarming. Briefly, the downturn LGD methods as proposed in EBA/GL/2019/03 (A-IRBA-L and A-IRBA-M) are inadequate to ensure the 99.9% survival chance and are even lower than the theoretical 95% survival rate. Most of the failures happen in the post-crisis period. Most likely, banks will identify 2008/2009 as the downturn periods in compliance with EBA/RTS/2018/04. Even though institutions are required for thorough impact assessment in compliance to EBA/GL/2019/03, the information on the downturn impact associated with the financial crisis is simply not available in 2010. Unfortunately, the impacts observed from the past are not as severe as the financial crisis. Even the A-IRBA-M estimate only shows a survival rate of 81% without input floors mostly due to failures during post-crisis periods.

The inclusion of input floors brings the survival chance to 99.4%. On the one hand, this is a reassurance that the advanced IRBA will work with input floors as intended, assuming banks will at least adopt an (M)-type estimate. On the other hand, this high survivability is only guaranteed by some rudimentary constants. It is likely that the input floors, as introduced in the Basel III reforms, will be outdated in the future. Moreover, input floors generally give the wrong incentive for low-risk banks to increase their risks.

In comparison, the method A4 gives a stunning precision towards the required 99.9% survivability, without any input floors. As expected, the A-IRBA-H is sufficiently conservative as well, but with a 10% LGD overestimation compared to a 5% waste from the A4.

3.5.4 Generalisation under Workout Duration Uncertainty

Each defaulted exposure needs an estimated downturn LGD in the IRBA. The method A4 requires latent variable components $X_{t_d},\ldots,X_{t_r}$ as inputs. In practice, the resolution time t_r is unknown. Therefore, some of the required latent variable components are not available. By

Table 3.6: Comparison of survival chance and waste of different downturn LGD estimation models with Advanced IRBA for SME asset class in % for each year from 2005 until 2017. $\hat{\mu}$ and $\hat{\sigma}$ are the estimated portfolio's realised LGD mean and standard deviation based on historical data up to each year. A1 is the forward-looking single-factor estimation, A2 is the backward-looking single-factor estimation, A3 is the three-years-factor estimation, A4 is the complete-history based estimation, A-IRBA-L is the downturn LGD calculated according to the advanced IRBA with a low level of conservatism, A-IRBA-M with a medium level of conservatism (which is the likely representation of most banks), and A-IRBA-H with a high level of conservatism. The displayed results are the simulations without the application of LGD input floors, but the averages for the simulations with the application of LGD input floors are presented in the last row.

	$\hat{\mu}$ ($\hat{\sigma}$)	Survival Chance (Waste)						
		A1	A2	A3	A4	A-IRBA-L	A-IRBA-M	A-IRBA-H
2005	14.54	100	99.80	100	100	90.98	93.43	100
	(3.60)	(9.60)	(2.72)	(5.52)	(6.49)	(1.36)	(1.47)	(13.48)
2006	15.33	100	86.66	100	100	89.78	92.20	100
	(3.18)	(7.79)	(1.31)	(3.90)	(4.77)	(1.37)	(1.48)	(12.97)
2007	16.14	100	73.21	99.99	100	84.13	89.02	100
	(2.94)	(6.81)	(1.09)	(3.94)	(4.64)	(1.27)	(1.42)	(12.71)
2008	16.62	100	100	100	100	99.21	99.33	100
	(4.02)	(10.69)	(4.02)	(8.81)	(10.06)	(2.27)	(2.34)	(13.26)
2009	16.86	100	99.99	100	100	86.78	88.17	100
	(3.43)	(8.00)	(4.00)	(7.31)	(7.99)	(1.47)	(1.54)	(12.41)
2010	17.27	100	64.81	99.98	100	62.01	62.68	100
	(2.86)	(5.70)	(1.12)	(4.06)	(4.74)	(1.07)	(1.08)	(11.85)
2011	17.56	100	16.23	99.12	99.97	13.52	15.23	100
	(3.66)	(6.14)	(0.66)	(3.15)	(4.20)	(0.62)	(0.64)	(9.82)
2012	18.36	100	42.08	99.11	100	70.72	98.57	100
	(3.73)	(7.09)	(0.96)	(3.22)	(5.14)	(1.33)	(2.90)	(10.57)
2013	18.48	100	65.08	99.99	100	93.26	100	100
	(4.86)	(10.67)	(1.33)	(4.95)	(7.60)	(2.25)	(5.10)	(10.65)
2014	18.61	100	1.52	85.86	99.99	59.83	99.78	100
	(5.15)	(8.79)	(0.49)	(1.86)	(4.63)	(1.24)	(3.70)	(7.88)
2015	18.79	100	6.39	93.05	100	97.75	100	100
	(5.37)	(10.43)	(0.57)	(2.16)	(5.29)	(2.63)	(5.53)	(8.83)
2016	18.69	100	1.78	98.81	100	99.94	100	100
	(5.82)	(11.80)	(0.38)	(2.50)	(5.31)	(3.31)	(6.22)	(8.82)
2017	18.47	100	0	33.36	99.60	99.88	100	100
	(6.01)	(11.23)	(-)	(0.57)	(2.25)	(2.63)	(5.54)	(7.65)
Average		100	31.63	90.12	99.97	73.75	80.85	100
		(8.83)	(1.56)	(4.00)	(5.62)	(1.76)	(3.00)	(10.84)
Average with input floors						94.62	99.40	100
						(4.12)	(5.25)	(10.84)

assuming t_r is today, A4 can be calculated. If the resolution time is indeed today, then the 99.9% survivability is ensured according to the result in the previous section. If the resolution time lies in the future, then the loss is not realised yet.

3.5.5 Validity for Medium-sized Banks

In the theoretical model, fine granularity is generally assumed. Per definition, an infinitely fine-grained portfolio does not carry any idiosyncratic risk. The closed formula for the conditional PD under the IRBA is intended for such a portfolio. In practice, portfolios from small or medium-sized banks violate this requirement easily. The concept of granularity add-on under the IRBA is also discussed in the literature (see Gordy and Lütkebohmert (2013)), and a similar concept is introduced as a part of the previous version of the IRBA under the Basel II Accord. This issue applies to downturn LGD models as well. Although a granularity add-on to our LGD estimation methods is not the scope of this essay, a benchmark is required to get a first grasp, whether our LGD methods may contain a severe granularity issue.

Based on the results presented in table 3.4 and 3.5, only the methods A3 and A4 have the potentials to be used for regulatory purposes. To represent medium-sized banks, we study how some arbitrarily low amounts of resolved cases per year (n) affect the average survival chance and the average waste.

Both methods A3 and A4 assume the linearity or semi-linearity of the function g_C. An estimation method directly implied from the true form of g_C would ensure a faster convergence rate with minimum waste. Although finding the true form of g_C is an interesting research topic, the results show that there might be only a little benefit. In fact, the results in table 3.7 suggest that the method A4 reaches the 99.9%-confidence level for a fairly small n. The regulatory downturn LGDs, both from the foundation and the advanced IRBA with input floors, show a comparable convergence rate. Especially for the foundation IRBA, the waste values do not change much. Compared to other methods, the foundation approach does not give an incentive for large banks to diversify their portfolio.

Based on these results, we argue that even medium-sized banks can sufficiently cover their potential downturn LGD with our methods according to the simulations. Furthermore, banks

Table 3.7: Average survival chance and average waste in % when only n default cases are resolved per year. A3 is the three-years-factor estimation, A4 is the complete-history based estimation, F-IRBA is the LGD assigned according to the foundation IRBA, A-IRBA-L is the downturn LGD calculated according to the advanced IRBA with a low level of conservatism and LGD input floors, A-IRBA-M with a medium level of conservatism and LGD input floors, and A-IRBA-H with a high level of conservatism and LGD input floors. (I) is calculated in the global population, (II) only for relevant asset classes in the foundation IRBA, and (III) only for SME asset class.

n	Average Survival Chance (Average Waste)									
	A3 (I)	A4 (I)	A3 (II)	A4 (II)	F-IRBA (II)	A3 (III)	A4 (III)	A-IRBA-L (III)	A-IRBA-M (III)	A-IRBA-H (III)
100	88.40 (7.19)	96.34 (8.72)	94.57 (13.09)	98.73 (15.84)	99.99 (21.72)	78.70 (5.30)	89.83 (6.39)	79.98 (5.36)	88.38 (6.09)	99.02 (10.95)
200	92.32 (6.77)	98.88 (8.50)	96.51 (12.91)	99.71 (15.77)	100 (21.73)	83.81 (4.63)	95.45 (5.89)	85.21 (4.73)	93.78 (5.58)	99.91 (10.86)
300	93.70 (6.63)	99.54 (8.44)	97.48 (12.82)	99.92 (15.73)	100 (21.72)	86.42 (4.37)	97.58 (5.75)	88.17 (4.49)	96.07 (5.43)	99.98 (10.85)
400	94.65 (6.56)	99.77 (8.42)	98.21 (12.79)	99.99 (15.73)	100 (21.71)	87.72 (4.24)	98.65 (5.68)	90.11 (4.35)	97.21 (5.35)	100 (10.84)
500	95.43 (6.52)	99.90 (8.42)	98.77 (12.77)	99.99 (15.74)	100 (21.72)	88.79 (4.15)	99.15 (5.65)	91.29 (4.27)	97.96 (5.31)	100 (10.84)
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1000	97.08 (6.44)	100 (8.41)	99.91 (12.74)	100 (15.74)	100 (21.72)	90.12 (4.00)	99.97 (5.62)	94.62 (4.12)	99.40 (5.25)	100 (10.84)

suffering from a high default rate after a downturn period will most likely have higher n, ensuring a near-convergence state. When there is no downturn period, n will most likely decrease, which results in a lower overall potential loss.

3.6 Conclusion

A regulatory technical standard (EBA/RTS/2018/04) and a guideline (EBA/GL/2019/03) are published by EBA to guide institutions on how to estimate the downturn LGD. The downturn definition relies on signales from macroeconomic proxies. As argued in section 3.2.2, the current downturn LGD standard cannot theoretically reach the conservatism degree as traditionally required under the IRBA due to a mismatch in the downturn definitions. This inconsistency potentially results in a capital requirement below the VaR at 99.9% confidence level. Even though estimating downturn LGD using macroeconomic proxies is easy to understand and appears to be logical, the guarantee of loss coverage with a 99.9%-level of confidence would require a historical data of 1,000 years, which is practically impossible to acquire.

The goal of this essay is not only to confirm our hypothesis that the downturn LGD proposed by the guideline is inadequate but also to provide an alternative. A downturn LGD estimation based on latent variables would address the problem. Since workout LGDs behave differently compared to market-based LGDs, we argue that the systematic influence on the expected LGD cannot be referenced to a single vintage point. Our analysis confirms that a random downturn event has a different impact degree towards the potential LGD of a defaulted instrument depending on the default age. We observe a decreasing pattern in the systematic sensitivity of the expected LGD, as the default gets older.

The message captured from the preliminary analysis is that the downturn LGD estimation should include latent variables from the past as well. This discovery helps us construct some downturn LGD estimation alternatives which are based on latent variables: forward-looking single-factor (A1), backward-looking single-factor (A2), three-years-factor (A3), and complete-history based estimation (A4). These methods are compared directly with the foundation and advanced IRBA, taking the Basel III reforms into account. We test with a Monte Carlo simulation whether an average bank will be able to survive from 2005 until 2017 LGD-wise (required downturn LGD is greater than realised LGD).

Without the LGD input floors, the survival rate of the advanced IRBA in the SME asset class is worryingly only 73%-81%. With the LGD input floors, it can reach 94%-99%. Apart from the fact that LGD input floor values will likely be outdated after some years and are prone to create wrong incentives, they almost reach the required 99.9% as traditionally required in the IRBA. In the global population, the three-years-factor estimation method (A3) achieves 97% survivability with a ca. 6% LGD overestimation. The complete-history based estimation method (A4) achieves the 99.9% survivability in a portfolio containing only defaulted exposures with a ca. 8% LGD overestimation. For asset classes relevant for the foundation IRBA, both the three-years-factor estimation method (A3) and the complete-history based estimation method (A4) are on par with the foundation approach in term of survivability. However, both of our methods show a significantly lower LGD overestimation (12-16%) compared to the foundation approach (22%).

For an adequate public policy regarding capital requirements, regulators need to revise the aforementioned standard and guideline. More research with focus on a latent variable based downturn LGD estimation is needed. The unpopularity of latent variable based approach lies in the fact that the interpretation of latent variables is often vague. This essay shows that there is at least an alternative and the risk underestimation issue can be solved.

Chapter 4

How A Credit Run Affects Asset Correlation and Financial Stability

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4.1 Introduction

Widespread concern about the solvency and liquidity of the banking sector has led firms to increase their liquidity. Ivashina and Scharfstein (2010) describe this phenomenon as a "run" due to its similarities to a traditional bank run. Likewise, Gorton and Metrick (2012) refer to it as a run on repos. If the intention is solely to get adequate liquidity to bridge a period of uncertainties, the type of financial instruments should only play a secondary role. In fact, higher demand for various types of financial instruments can be observed: asset-backed commercial essays (Acharya and Schnabl (2010)), revolving lines (Ivashina and Scharfstein (2010)), term loans and credit lines (Cornett et al. (2011)), repurchase agreements (Gorton and Metrick (2012)), etc. Whether the borrowers' motivation has purely originated from the uncertainty in the money market is not essential in this essay. The primary concern is the consequence which is the systematic shift of banks' portfolio compositions. In return, this structural shift may destabilise the financial system.

We are particularly interested in the link with the Basel III framework, especially the Internal Ratings-Based Approach (IRBA). The underlying foundation of this approach is the asymptotic single-risk-factor (ASRF) model, which models the asset value movements of obligors. An asset correlation describes the co-movement of asset values between two random obligors, which is assumed to be constant over time in the ASRF model. The current IRBA applies a predetermined correlation coefficient for this parameter ranging from 3% to 24% depending on the asset class and its default rate. The fact that this coefficient has not been recalibrated since its introduction with the Basel II Accord is concerning. It means that the current correlation coefficient was not determined with a dataset covering the financial crisis. If asset correlations increase due to a credit run, there might be a need to introduce the concept of a *downturn* asset correlation. However, the framework needs to be the ASRF model to ensure applicability in the IRBA.

Research essays in the literature covering this topic are often incompatible with the ASRF model. As an example, Lee et al. (2012) use the CAPM's β as a proxy for the asset correlation. Both parameters describe the systematic risk, so it can be argued that the proxy is an appropriate substitute. In their analysis, a significant increase in asset correlations can be observed in 2007, supporting our hypothesis. The applicability of this result to justify a higher correlation coefficient in the IRBA context is questionable. Other essays in the literature with various methodologies show inconsistent estimations (see Chernih et al. (2006); Hashimoto (2009) for literature surveys). A large body of essays even contradicts the underlying relationship between the asset correlation and the default rate/firm size (see section 4.3).

The economic question in this essay is whether the financial crisis has had any impact on the asset values' co-movement in the financial system, while the technical question is whether a change in banks' portfolio composition (due to a credit run) can cause an increase in the asset correlation parameter which in return implies an underestimation of the IRBA capital requirement. A time-dependent asset correlation is difficult to estimate if we take into account that the methodology choices may produce inconsistent estimations in the literature. On the other end, only a limited amount of methodologies can be applied if the methodology has to be directly compatible with the ASRF model to ensure a possible application in the IRBA.

This essay proposes *adjusted* default rates, i. e. default rates weighted by the exposure amount of the loans. Theoretically, its use in the ASRF model forces the exposure amount to play a role in the estimation. Some corporates may max out their existing credit lines, while others may take new loans or resort to other financial instruments. Within the adjusted model, the estimated asset correlation is referred to as the *adjusted* asset correlation as well to differentiate from the traditional version. This simple adjustment does not substantially change the interpretation of the parameter itself. We use an aggregated default rates database as well as a credit loss database from over 50 international large banks provided by Global Credit Data (GCD). The same analysis can also be conducted with default rates datasets from external rating agencies, such as Moody's or Standard & Poor's, which may offer a more transparent and objective result. However, a dataset acquired from mostly IRBA banks can be seen as an advantage within this analysis[1]. Since the primary goal is the revision of the IRBA, it is only appropriate to use a dataset based on portfolios of (typical) IRBA institutions as well.

The increased adjusted asset correlation due to a credit run can be confirmed in the model. Interestingly, the adjusted asset correlation of a high-rated SME asset class is less susceptible to a credit run, but this asset class inhibits a rather high adjusted asset correlation compared to other asset classes. Depending on the severity and duration of the credit run, an (absolute) difference of up to 2% in the adjusted asset correlation can be observed. 2% may not sound much, but this difference means a 50% relative increase in some cases. To control our analysis, we simulate a credit run during an average year. The increase in the adjusted asset correlation is no longer significant. An important factor driving the increase in the adjusted asset correlation is the ratio of average initial exposure of healthy and non-healthy debts. As demand in the money market rises, it causes a higher concentration of unhealthy exposures which drives the adjusted asset correlation up.

[1]Most GCD members are large banks with more incentives to apply for the IRBA.

To sum up, the key contributions are two-fold: 1) a theoretical foundation to adjust the ASRF model accommodating the systematic shift in the portfolio's composition and 2) confirmation that a systematic change of behaviour influences the asset correlation as the IRBA's input parameter. This essay is structured as follows: section 4.2 deals with the background on the credit run and the asset correlation, section 4.3 reviews the literature on asset correlation research in the relevant context, section 4.4 introduces the ASRF model and the proposed adjustment followed by the methodology on the confidence interval for an asset correlation, section 4.5 discusses the dataset used and the pre-calibration process to acquire all the necessary input parameters, section 4.6 shows the results and analyses the impact magnitude, and section 4.8 concludes this essay.

4.2 Background

4.2.1 Credit Run

Many studies report irregularities in lending behaviours in the pre-crisis period. Acharya and Schnabl (2010) show a constant increase in the total outstanding amount of asset-backed commercial essays since 2004 (shortly after the announcement of new accounting rules), which reached its peak at the end of 2007 (marking the start of the subprime mortgage crisis). This peak can also be observed for other financial instruments. Cornett et al. (2011) report a peak in the dollar amount of new term loans and new credit lines at the beginning of 2008. They argue that this liquidity hoarding behaviour was motivated by concern about the liquidity of loans and securitised assets. Ivashina and Scharfstein (2010) study US firms' statements on revolving lines draw-downs and report that firms drew on their credit lines to ensure that they had access to funds when there was widespread concern about the solvency and liquidity of the banking sector. The effect of this rising demand for lending shortly before the financial crisis on the asset correlation parameter remains ignored. Corporates tend to take new loans or max out their existing credit lines to ensure sufficient liquidity. Such behaviour dries up the lender's liquidity and is detrimental to the market. Ivashina and Scharfstein (2010) argue that these additional credit line draw-downs were part of the "run" on banks which is, unlike old-style bank runs, instigated by short-term creditors, counterparties, and borrowers, who are concerned about the liquidity and the solvency of the banking sector. Gorton and Metrick (2012) describe this run as a run on repos since it took place within the securitised banking system and was driven by the withdrawal of repos. The term *credit run* is aimed at generalising the phenomenon to be independent of the financial instrument type. In the pre-crisis period, a credit run may be difficult to differentiate from a credit boom. Both describe soaring demand for liquidity (through loans or other financial instruments), but the difference lays in the borrower's intention. As observed during the financial crisis, this behaviour change puts the money market in distress, which in return negatively influences financial stability.

The change in the borrowers' behaviour generally leads to a systematic composition shift of the lenders' portfolios. Firms with steady revenues independent from the system may not have the urgent need to engage in credit run behaviour, while other corporates with questionable liquidity or with a low credit rating are more likely to engage in such behaviour. Thus, during a credit run, potential borrowers are more likely to be those who desperately need the liquidity. Notwithstanding the foregoing, reacting to money market uncertainty by taking the necessary precautionary measures to survive should never be seen as a decision only poor firms take nor a hint of low creditworthiness. Thus, there is no small chance that the asset mix on the macro level structurally shifts with the behaviour change on the borrowing decision. It is simply more likely for a bank's potential customer to have a lower credit quality (than their current rating may suggest) following a credit run. Consequently, institutions' credit portfolios are proportionally riskier compared to previous years, despite the institutions' unchanged internal risk strategy. The clustering of borrowers with a low credit risk profile in the portfolio directly impacts the default risk of the portfolio. If this high concentration of a particular borrower type becomes a trend, it leads to higher systematic risk.

4.2.2 Asset Correlation

The asset correlation within the banking regulation context describes the co-movement of asset values between obligors. Beside the probability of default (PD), the loss given default (LGD), and the exposure amount at default (EAD), the asset correlation is one of the main parameters of the IRBA with a direct impact on the calculated capital requirement. The asset correlation is often used to measure the systematic risk as well. A high asset correlation, i. e. a high probability of co-movement between borrowers' asset values, can be associated with a high probability of a co-default, which in return can be associated with a high probability of mass default. This parameter, as the name suggests, is a correlation from a technical perspective. Hence, all properties and technical issues related to a correlation are also inherited for an asset correlation. For a given time interval, only one estimate can be produced. Without further assumptions, a time-dependent asset correlation would theoretically need the moving time-window procedure. Unfortunately, such a procedure only produces non-robust estimates if data is scarce.

The ASRF model, which is based on Merton's debt valuation model (Merton (1974)), mainly assumes that the asset correlation is constant over time. There is a good argument that a macroeconomic event may have a direct influence on the asset correlation of an asset class within a financial ecosystem, which is motivated by the changing asset composition in the ecosystem. Therefore, a credit run can be linked to an increase in the asset correlation, but not necessarily the other way around, an asset correlation increase is not always caused by a credit run. Relaxing the assumption within the ASRF model is challenging, considering that asset correlations from empirical studies show some inconsistencies which may have originated from the model itself.

In contrast to other risk parameters, e. g. default rates or recovery rates, asset correlation cannot be directly observed as a variable over time. In the earliest version of the IRBA, the Basel Committee on Banking Supervision (2001) set the asset correlation to a flat 20%. In the current version, as stated by the Basel Committee on Banking Supervision (2017a), the predetermined asset correlation can range from 3% to 24% depending on PD, asset class, and some other factors. Henneke and Trück (2006) give a chronological evolution of the asset correlation until

Basel II, from which point the rule has not been updated until now. The negative relationship between PD and asset correlation is embedded in the asset correlation formula in the IRBA. However, the number of empirical studies with contradicting results grows, not only concerning the dependency with some factors but also regarding the estimated values (see section 4.3).

Independent from the ASRF model as the foundation of the IRBA formula, the estimation methods for asset correlations are numerous across the literature. Some require additional assumptions or models, e. g. by an indirect proxy or an underlying distribution assumption. In most cases, the compatibility with the ASRF model is questionable. Düllmann et al. (2007) use equity value correlation towards the index value as a proxy for the asset correlation parameter. This approach has drawn some criticism, such as De Servigny and Renault (2002), who argue that an equity value-based methodology is not sufficient to model a default correlation. Other methods include Lee et al. (2012), who take the CAPM's β as a proxy for the parameter, or Curcio et al. (2011), who fit losses to a particular distribution (in this case, to a beta distribution) and match these losses with the IRBA formula to acquire an implied asset correlation. However, the assumption on the loss distribution defeats the purpose of the IRBA. The ASRF model is initially built using an asset correlation to estimate (unknown) losses. With the compatibility of any alternative models or proxies for the asset correlation in the literature being questionable (especially as an application for capital regulation), using these results as a foundation for an asset correlation recalibration may not be adequate.

With these difficulties in mind, this essay does not aim to offer a better and more robust estimate or a time-dependent asset correlation within the boundary of the ASRF model, but rather a technical solution to detect an asset correlation shift following a credit run. For the sake of usability in practice, we investigate the magnitude of this effect, which ultimately may give clues as to how much the downturn add-on should be and if it is indispensable. The aim is to ensure that the increase in the asset correlation during a downturn period is sufficiently captured, under the presumption that the link is true.

4.3 Literature Review

It is important to understand that many uncertainties shroud asset correlation research. Plenty of essays suggest that the asset correlation in the IRBA formula needs a recalibration. The number of empirical studies that cannot confirm the negative relationship between PD and asset correlation grows, directly contradicting the predetermined formula for the asset correlation parameter in the IRBA. As the number of empirical studies on asset correlation grows, the uncertainty towards the one which was calibrated for the IRBA also grows. Not only are the aforementioned relationships problematic, but surveys of estimated asset correlations from various empirical studies also show that the estimated asset correlation values are spread very broadly (see Chernih et al. (2006); Hashimoto (2009)), which only makes the whole issue worse.

There are two prominent factors which are known to have a direct relationship with the asset correlation: PD and firm size. While there are some essays with the same conclusion, there are many others with contradicting results. Negative dependency between asset correlation and PD (mostly measured by credit rating) are shown in Lopez (2004); Chernih et al. (2006). An unclear or mixed relationship between asset correlation and PD can be observed in Gordy and Heitfield (2002); Düllmann and Scheule (2003); Frye (2008); Bams et al. (2012); Haddad (2013); Düllmann and Koziol (2014); Bams et al. (2019). Both Dietsch and Petey (2004); Vozella and Gabbi (2010) report a positive relationship between asset correlation and PD. Others argue that the relation might be U-shaped, such as reported by Hamerle et al. (2003); Hashimoto (2009); Lee et al. (2012). Curcio et al. (2011) provide evidence from Italian data that certain conditions may cause the inverse relationship between PD and asset correlation to dissipate.

The relation to firm size is contradicting as well. The IRBA applies a lower asset correlation for SMEs and a higher one for other asset classes, implying a positive relationship between asset correlation and firm size. There is some variation on the definition of firm size in the literature (asset size, sales revenue, turnover, or number of employees). Several essays, such as Düllmann and Scheule (2003); Lopez (2004); Hahnenstein (2004); Chernih et al. (2006); Hashimoto

(2009); Lee et al. (2012); Düllmann and Koziol (2014), report a positive or mostly positive relationship between asset correlation and size. Dietsch and Petey (2004) report a decreasing pattern for asset correlation towards firm size, but observe a high asset correlation for large SMEs (with a turnover of €7-40 million). Similarly, Bams et al. (2019) argue that the asset correlation of SMEs is lower than required in the IRBA. Vozella and Gabbi (2010) report a monotonic negative relationship without exception and show the least amount of asset correlation for large SMEs (a turnover of €10-25 million). As observed in Bams et al. (2012); Haddad (2013), there is no clear pattern of asset correlation when grouped by size. Further, Düllmann and Scheule (2003) argue that positive dependency with firm size can be rationalised with the more flexible diversification opportunity of large firms compared to SMEs.

Literature surveys, such as e. g. Chernih et al. (2006); Hashimoto (2009), report a substantial variation in the estimated values for asset correlations. Some estimated asset correlation values can be close to zero, while others report an estimated asset correlation of over 50%. The origin of the dataset, its size, ans the methodology choice may contribute to the high variance. There are two common choices for the dataset: asset value-oriented datasets or default rates-oriented datasets. Some prominent examples of papers using asset values as input are Lopez (2004); Hamerle et al. (2004); Düllmann et al. (2007); Vozella and Gabbi (2010); while others use historical default rates or migration matrices such as Gordy and Heitfield (2002); Düllmann and Scheule (2003); Frye (2008); Hashimoto (2009); Bams et al. (2019). The compatibility of using asset values to represent a bank's portfolio is questionable. De Servigny and Renault (2002) argue that an equity value model is not sufficient to estimate the asset correlation parameter.

The methodology choices also play a big role. As the asset correlation comes from the ASRF model, it serves as a main component in the model and can be derived by exploiting the theoretical density function of the conditional PD. Frye (2000b) introduces the maximum likelihood (ML) estimation method as he points out that the density function of the conditional PD can be derived. Another statistical method, such as the method of moments (MM), can replace the ML method in most cases. However, Gordy and Heitfield (2002) show that a persistent downward bias for asset correlation estimations by ML or MM exists. In their simulations, they show that the bias by MM is, in general, slightly worse than by ML in scarce datasets. There are also approaches exploiting regression techniques as found in Hamerle et al. (2003, 2004). Others rely on commercial risk management software such as KMV (Lopez (2004)), CreditRisk+ (Dietsch and Petey (2004)), or CreditMetrics (Hahnenstein (2004)). A copula-based technique can also be found, such as in Vozella and Gabbi (2010), by assuming a Gauss copula, or using the CAPM to derive asset correlations from the firm's β, as shown in Lee et al. (2012). Given sufficient data points, the correlation of asset values (mostly equity values) between firms can also be calculated, as found in Chernih et al. (2006); Düllmann et al. (2007). The methodology choice may not matter if the resulting asset correlations are primarily used for understanding the systematic risk. However, the methodology choice may be limited if the intention is a sound and consistent application in the IRBA. Although the compatibility is questionable, the estimation results in Lee et al. (2012) exhibit a significant increase in the asset correlation of 2007 compared to the average, signalising a potential impact from a credit run.

4.4 Methodology

The main idea behind the IRBA is to calculate the expected loss of a position given a single systematic factor, which is supposed to be approximately equal to the value-at-risk with certain assumptions (see Gordy (2003)). Loss as a random variable can be factorised into a default indicator (D), remaining exposure, and loss fraction from the remaining exposure. These parameters are directly associated with the PD, the EAD, and the LGD.

$$\begin{aligned} \mathbb{E}[\text{Loss}|X] &= \mathbb{E}[D \cdot \text{Remaining exposure} \cdot \text{Loss fraction}|X] \\ &\overset{(*)}{=} \mathbb{P}(D=1|X) \cdot EAD \cdot \mathbb{E}[LGD|X] \end{aligned} \tag{4.1}$$

For $(*)$ in (4.1) to be true, two conditions need to be met: $(*.1)$ EAD has to be constant (therefore independent from any factor) and $(*.2)$ both PD and LGD are independent for a given value of the systematic factor (X). In the IRBA, the term $\mathbb{P}(D = 1|X)$ is calculated by the conditional PD formula based on the ASRF model, and the term $\mathbb{E}[LGD|X]$ is the downturn LGD based on the institution's internal model. The conditional PD formula requires the asset correlation as an input parameter. In short, it defines how the asset values of two firms correlate to each other, which affects the probability of co-defaults. Although the asset correlation is exclusively calculated for calibrating the IRBA, it is also often used as a measure of the systematic risk. The reason is that co-defaults lead to mass defaults, which is a primary component of a financial crisis.

The rationale behind the assumption $(*.1)$ is that a loan's remaining exposure is in most cases considered a constant amount from the micro perspective (e. g. a single loan). From the macro perspective (the bank's portfolio or the debt mix in a financial ecosystem), the amount of total debt is certainly not independent from the systematic factor. Since (4.1) is the foundation of the IRBA, this is the first motivation leading to the hypothesis that a variation of the total amount of debt may have an impact on the asset correlation.

So, what exactly is the consequence if the assumption $(*.1)$ is wrong? From a technical perspective, the three components in (4.1) are not easily factorisable even if $(*.2)$ is true. The correlation effect from EAD is therefore lost, which implies a risk underestimation in the current IRBA. During a credit run, the outstanding amount in banks' portfolios increases and, therefore, their capital requirements as well. However, the amount of debts in banks' portfolios are artificially increased, although there are not many new obligors in the system. If the total debt amount and the number of obligors do not change proportionally, the asset correlation will change. To illustrate this, a small and isolated financial ecosystem consisting of only one borrower with many loans would have a 100% co-default probability and also a 100% asset correlation (since there is only one single obligor). Since the current IRBA's asset correlation has not been updated since the IRBA introduction, there is a mismatch between the predetermined regulatory asset correlation and the real asset correlation during a credit run. Although the question of how to calculate the *current* asset correlation or a *downturn* asset correlation is also non-trivial in itself.

To accommodate the effect of a credit run in an asset correlation estimation, we propose an adjustment to the model. The main idea is to redefine the PD, ensuring the adjusted model only has two distinct factors (adjusted PD and LGD), rather than including all three factors (PD, LGD, and EAD). Let us assume that banks are only allowed to issue new debts with an initial outstanding of €1, which means a borrower would need to get 100 new loans to lend €100. This adjustment is only a new perspective on looking at the default rate parameter without changing the risk profile of the debt. Looking back at (4.1), the conditional expected loss would be:

$$\begin{aligned}\mathbb{E}[\text{Loss}|X] =&\ \mathbb{E}[D \cdot \text{Remaining exposure} \cdot \text{Loss fraction}|X] \\ \overset{(*.2)}{=}&\ \mathbb{P}(D=1|X)\cdot 1 \cdot \mathbb{E}[LGD|X]\end{aligned} \tag{4.2}$$

In this particular case, both the conditional PD and the PD have to be defined in the 1-Euro loan context. From the technical point-of-view, adjusting the ASRF model in the 1-Euro loan context allows fluctuation in the exposure amount over time to affect the PD time series and therefore the asset correlation. From this point, the word *adjusted* is used to refer to the 1-Euro loan context, thereby setting it apart from the traditional context.

The conditional PD is derived from the ASRF model by looking at the asset value's return $A_{i,t}$ of a firm i at year t, which is affected by a systematic factor X_t and an idiosyncratic factor $Z_{i,t}$. Per definition, the systematic factor and the idiosyncratic factor are independent. In the model, both factors are traditionally assumed to be of standard normal distribution.

$$A_{i,t} = p \cdot X_t + \sqrt{1-p^2} \cdot Z_{i,t}.$$

Per definition, the firm i defaults if and only if $A_{i,t}$ falls below a certain threshold $\Phi^{-1}(PD_i)$. The threshold is deliberately chosen to be $\Phi^{-1}(PD_i)$ to ensure that the PD of firm i is equal to PD_i. In the 1-Euro loan context, the adjusted PD and the PD are in general not equal if there is any dependency between creditworthiness and initial credit amount. Regardless of the context, the correlation of $A_{i,t}$ and X_t is p. So, for two homogeneous random firms, their asset correlation is p^2. The asset correlation is mostly denoted by ρ in the literature and by R in the IRBA. The use of p instead of ρ $(= p^2)$ in the model has the slight advantage that it can identify the rare case of an anti-systematic pattern (negative p) in some sectors. This slight modification in the ASRF model can also be found in Frye (2000b); Bams et al. (2012, 2019); Haddad (2013). For a given $X_t = x$, the conditional PD can be derived as follows:

$$\begin{aligned} \mathbb{P}(D_{i,t} = 1|X_t = x) &= \mathbb{P}(A_{i,t} < \Phi^{-1}(PD_i)|X_t = x) \\ &= \mathbb{P}(p \cdot X_t + \sqrt{1-p^2} \cdot Z_{i,t} < \Phi^{-1}(PD_i)|X_t = x) \\ &= \mathbb{P}\Big(Z_{i,t} < \frac{\Phi^{-1}(PD_i) - p \cdot x}{\sqrt{1-p^2}}\Big) \\ &= \Phi\Big(\frac{\Phi^{-1}(PD_i) - p \cdot x}{\sqrt{1-p^2}}\Big) =: g(x), \end{aligned} \tag{4.3}$$

where Φ is the distribution function of the standard normal distribution.

As pointed out by Frye (2000b), the function $g(x)$ in (4.3) is invertible with respect to x. Therefore, its density can be calculated by the change-of-variable technique.

$$f_{PD|X}(x) = \varphi(g^{-1}(x)) \cdot \Big| - \frac{\sqrt{1-p^2}}{p} \cdot \frac{d\Phi^{-1}(x)}{dx} \Big|,$$

where φ is the density function of the standard normal distribution. Therefore, given this density and samples (either in the form of historical default rates or the number of defaulted loans), the ML method can determine the unknown parameters, which are the PD and the parameter p. To avoid a high-dimensional and overly complex model which may cause unreliable estimations in ML methods as, e. g. addressed by Sur and Candès (2019), PD as well as its adjusted variant will be assumed to be known in this essay. However, compared to other methods, the ML method still generates smaller biases for estimating asset correlations than other statistical approaches, as pointed out by Gordy and Heitfield (2002). The curse of dimensionality may play a role in why asset correlations estimated in the literature are so different from each other (see section

4.3).

Let $PD_{r,a,t}$ denote the observed default rate of rating $r \in \mathcal{R}$, asset class $a \in \mathcal{A}$, and year $t \in \mathcal{T}$ and $PD_{r,a}$ the average default rate of rating r of a particular asset class a, which serves as a proxy for the probability of default PD_i of firm i[2]. The amount of available data (observed default rates) within a given rating r and asset class a is denoted by $n_{r,a}$, which sums up to N. The parameters $\mathbb{p} = (p_{1,1}, \dots, p_{n_r,n_a})$ can be acquired through the optimisation of the log-likelihood function. The result of the optimisation problem is the square root of the asset correlation for each rating category and asset class. Note that the calculation only requires the PDs of each category as inputs.

$$
\begin{aligned}
l(\mathbb{p}) =& \log\Big(\prod_{r,a,t} f_{PD|X}(PD_{r,a,t})\Big) \\
=& -\frac{N}{2}\log(2\pi) - \sum_{r,a}\Big(\frac{1}{p_{r,a}^2}\sum_t(\Phi^{-1}(PD_{r,a}) - \sqrt{1-p_{r,a}^2}\Phi^{-1}(PD_{r,a,t}))^2 \\
& - \frac{n_{r,a}}{2}\log(1-p_{r,a}^2) + n_{r,a}\log(p_{r,a})\Big) + N\sum_{r,a,t}\log\Big(\Big|\frac{d\Phi^{-1}(x)}{dx}\Big|_{x=PD_{r,a,t}}\Big)
\end{aligned}
\tag{4.4}
$$

The minimum of the log-likelihood function $l(\mathbb{p})$ can be calculated numerically. By applying the historical default rates in $PD_{r,a,t}$, the calculation yields the asset correlation p^2 and by applying the adjusted default rates, the calculation yields the adjusted asset correlation p^2_{adj}.

There is not a substantial difference in the interpretation, which sets p^2_{adj} from p^2 apart. The crucial element which differentiates the adjusted asset correlation from the traditional version is whether the initial outstanding amount is accommodated in the model or is assumed to be constant. A default of a €2 loan will be counted once in the PD calculation, but twice in the adjusted PD. Nevertheless, the adjusted asset correlation is also associated with an asset co-movement, which influences co-default and mass defaults, and therefore the financial stability.

Confidence intervals for (adjusted) asset correlations can be acquired by bootstrap-based methods, such as in Cassart et al. (2007). Unfortunately, bootstrap-based confidence intervals

[2]Indexing through all the available asset classes and ratings has its reasons. The asset correlation is known to be dependent on firm size and PD. See section 4.3 for more information.

depend on the choice of sample size and iteration size, which produces an non-robust confidence interval. However, the ML method is known to be consistent, i. e. an ML estimator always converges to the true parameter in probability, and the ML estimators are asymptotic normal. In our context, it means

$$\hat{\mathbb{p}} \to \mathcal{N}(\mathbb{p}, \frac{1}{N \cdot I(\mathbb{p})}),$$

where $\hat{\mathbb{p}}$ denotes the ML estimator of $\mathbb{p}$ (in either traditional or adjusted version) and $I(\mathbb{p})$ denotes the Fisher information, i. e. the variance of the score function. Note that, given the distribution, a confidence interval can be easily constructed. However, we cannot ignore the fact that it requires the true asset correlation $\mathbb{p}$ as an input, which is unknown. Typically, $\hat{\mathbb{p}}$ can be used as a proxy in practice.

By looking at the structure of the log-likelihood function in (4.4), we can see that for each rating r and asset class a the variables $p_{r,a}$ are separated from another rating and asset class. This fact ensures that the Hessian matrix $\mathcal{H}$, i. e. the second partial derivative of $l(\mathbb{p})$, is a diagonal matrix, which simplifies a majority of the derivation. The diagonal component of the Fisher information (of a given r and a) is derived as follow:

$$\begin{aligned} I(\hat{\mathbb{p}})_{r,a} = -\mathcal{H}_{r,a} = n_{r,a}\Big(&\frac{1}{1-\hat{p}_{r,a}^2} + \frac{\hat{p}_{r,a}^2}{(1-\hat{p}_{r,a}^2)^2} - \frac{1}{\hat{p}_{r,a}^2}\Big) \\ &+ \sum_t \Big(\frac{3\beta_{r,a}^2}{\hat{p}_{r,a}^4} - \frac{3\beta_{r,a}\Phi^{-1}(PD_{r,a,t})}{\hat{p}_{r,a}^2\sqrt{1-\hat{p}_{r,a}^2}} + \frac{\beta_{r,a}\Phi^{-1}(PD_{r,a,t})}{\big(\sqrt{1-\hat{p}_{r,a}^2}\big)^3} + \frac{\Phi^{-1}(PD_{r,a,t})^2}{1-\hat{p}_{r,a}^2}\Big) \\ \text{for } \beta_{r,a} &:= \Phi^{-1}(PD_{r,a}) - \sqrt{1-\hat{p}_{r,a}^2} \cdot \Phi^{-1}(PD_{r,a,t}). \end{aligned}$$

Due to the fact that the Fisher information is a diagonal matrix, a more robust confidence interval can be constructed by replacing N with the respective population number of each segment $n_{r,a}$. Thus, the confidence interval of $\hat{\mathbb{p}}$ with a confidence level α for a given rating r and asset class a is

$$\hat{p}_{r,a} \pm \frac{t_{n_{r,a}-1;1-\frac{\alpha}{2}}}{\sqrt{n_{r,a} I(\hat{\mathbb{p}})_{r,a}}}.$$

Respectively, the confidence interval for the asset correlation is its square, which is

$$\hat{p}_{r,a}^2 + \frac{t^2_{n_{r,a}-1;1-\frac{\alpha}{2}}}{n_{r,a} I(\hat{\mathbb{p}})_{r,a}} \pm \frac{2\hat{p}_{r,a} t_{n_{r,a}-1;1-\frac{\alpha}{2}}}{\sqrt{n_{r,a} I(\hat{\mathbb{p}})_{r,a}}}. \tag{4.5}$$

However, the resulted confidence interval should be understood only as an estimate. Note that the distribution used to derive (4.5) is an asymptotic distribution. Whether this distribution can be used to estimate the downturn asset correlation is not a trivial question. One may have the idea of applying $\alpha = 99.9\%$ into one-side of the formula in (4.5). In comparison to the PD and the LGD, there is no guarantee that the asset correlation and the systematic factor have a monotonic relationship. In non-technical words, it is unknown whether the asset correlation will on average increase/decrease as the systematic factor gets better/worse. There is no guarantee that the *worst* asset correlation only occurs in the *worst* downturn period. Nevertheless, there is much need for research to be done on the asset correlation and how the systematic factor influences it.

In summary, there are two methodologies introduced in this section: 1) the method to calculate the adjusted asset correlation and 2) the ML-based confidence interval for the asset correlation. The adjusted asset correlation is represented by p^2_{adj}, which can be acquired from solving the optimisation problem in (4.4). The confidence interval is the given interval in (4.5).

4.5 Data and Calibration

To calculate the adjusted asset correlation, information on defaulted as well as on non-defaulted loans are necessary. The adjusted PD calculation requires information on the composition of defaulted and non-defaulted loans in the financial ecosystem. Although datasets on defaulted loans are difficult to acquire, they are still available. In contrast, (detailed) datasets on non-defaulted loans are very sensitive. Banks that offer their datasets on defaulted loans do not necessarily offer information on their whole portfolio as well. As explained, it is more appropriate to use default data from IRBA banks. Such datasets are more representative in the context of our analysis. The credit databases from GCD fulfil this requirement[3].

GCD provides two sets of databases: 1) the PD and Rating Platform and 2) the LGD and EAD Platform. The first one gives information on historical default rates of particular asset classes. The second one gives detailed information on defaulted loans. All participating banks are obliged to specify default and loss similarly, ensuring data comparability within the sample. Complementary to the following section, dedicated reports regarding these datasets are also provided by Global Credit Data (2019a,b).

4.5.1 Data Statistics

Banks are required to classify their exposures in predefined facility asset classes as well as to map them into S&P ratings. In this essay, we only consider SMEs and large corporates (LC) as well as financial institutions (FI). Other asset classes are excluded due to their small size. Thereby, the dimension of the parameters is minimised to limit a potential bias. The ratings are summarised into three larger categories: **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C). The country classification is extensive and contains hundreds of different entries. The most represented regions are North and West Europe as well as North America. Although there are some arguments to restrict the datasets only to a particular region, finding out the exact regional asset correlation is not the scope of this essay. Furthermore, there are hints in the literature, such as found in Bams et al. (2019), suggesting that segmenting the dataset into too many categories may produce "jumps" in the asset correlations of neighbouring segments.

The number of GCD members may have an indirect influence on the number of loans in the dataset, but the demand and supply of the money market should be the main factor influencing the documented number of loans. Table 4.1 displays the amount of non-defaulted and recently defaulted loans (less than one year) in the PD and Rating platform. The remaining defaulted

[3]GCD is a non-profit association owned by its member banks from around the world and active in data-pooling for historical credit data. As of 2020, it has 55 members across Europe, Africa, North America, Asia, and Australia. For details: `https://www.globalcreditdata.org`

Table 4.1: Number of loans in the PD and Rating platform. Only non-defaulted loans and recently defaulted loans are included. Not-recent defaults are excluded to avoid double counting. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

	SMEs			Sum	LCs and FIs			Sum
	A	**B**	**C**		**A**	**B**	**C**	
2000	308	29,492	657	30,457	2,458	11,135	333	13,926
2001	606	32,261	476	33,343	2,776	10,555	458	13,789
2002	419	28,203	432	29,054	2,818	9,180	434	12,432
2003	386	28,578	452	29,416	4,221	9,216	265	13,702
2004	622	28,989	607	30,218	6,109	10,605	261	16,975
2005	715	26,454	648	27,817	8,755	16,520	419	25,694
2006	9,553	79,750	2,842	92,145	18,409	53,295	1,849	73,553
2007	8,568	166,091	4,970	179,629	27,068	91,431	2,173	120,672
2008	14,825	209,070	9,199	233,094	29,468	100,519	3,427	133,414
2009	15,087	245,157	16,631	276,875	33,031	154,113	6,211	193,355
2010	16,188	274,051	19,396	309,635	36,872	140,610	7,990	185,472
2011	12,929	268,032	17,853	298,814	39,270	156,320	7,066	202,656
2012	12,548	256,733	14,736	284,017	91,877	158,897	6,015	256,789
2013	13,948	225,490	15,604	255,042	95,696	155,765	5,868	257,329
2014	26,278	271,455	21,352	319,085	105,925	167,396	5,619	278,940
2015	26,273	270,866	20,431	317,570	99,285	156,980	5,137	261,402
2016	25,853	178,657	8,266	212,776	26,469	97,856	3,476	127,801
2017	17,958	165,943	6,635	190,536	21,408	65,857	1,353	88,618

loans population (older than one year) is already counted in the year before, so they are excluded to avoid double-counting. In practice, non-recent defaulted loans in the workout process will still be part of the portfolio waiting to be resolved or cured. However, these older defaulted loans do not directly influence default rates. Note that the fluctuation in the number of loans in table 4.1 is not necessarily a hint for an increase of loan demand in general, but for an increase of loan issuance (or buyouts) by IRBA banks.

Figure 4.1 depicts the historical default rates in the dataset. The peaks in the graph mark the financial crisis with a high default rate relative to other years. The year, the duration, and the severity of the financial crisis may vary depending on the asset class and the rating group. Note that figure 4.1 only shows default rates in the traditional sense. The adjusted default rates (in the 1-Euro loan context) are most likely quite different.

The LGD and EAD platform offers detailed information on defaulted loans during their

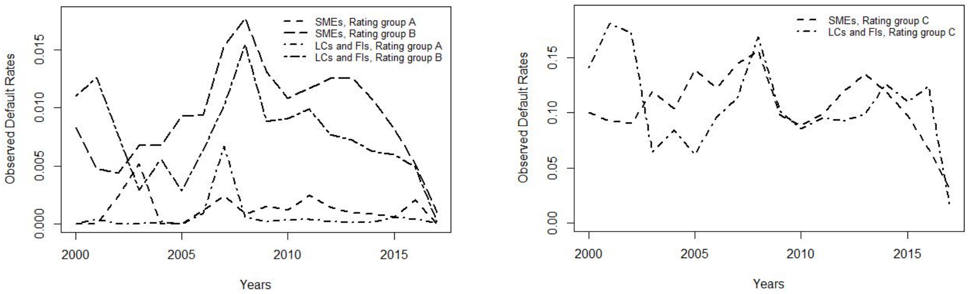

Figure 4.1: Observed default rates of SMEs, LCs and FIs, segmented by rating categories

workout processes. For the analysis, the proportion between the EADs of defaulted loans and the EADs of healthy loans is an essential factor to identify a credit run. Unfortunately, this platform only offers information about defaulted loans. The outstanding amount is defined as either the lender outstanding amount at the origination date or the lender limit[4]. In table 4.2, we can observe a higher outstanding amount in loans by obligors with a low rating. The default amount is defined as the nominal outstanding amount at the default date, including any additional drawn cash and mark-to-market adjustment. The default amount is on average not far from the initial outstanding amount. Given that these amounts are not discounted, we can say that the potential loss in present value can exceed near to the initial outstanding amount at origination date. In contrast, it is difficult to acquire similar information on the initial outstanding amount on healthy loans.

4.5.2 Calibration

We can assume that the initial outstanding amount is, in general, the obligor's choice, which is directly influenced by their need for liquidity and thus indirectly by their credit rating as well. For corporates, lending may also often be motivated by their strategy for the optimal capital structure. The total amount of debts outstanding circulating in a financial ecosystem is a central factor for the adjustment. In particular, adjusted default rates require an initial calibration to

[4]To avoid double-counting, the lender outstanding amount of a syndicated loan only consists of the exposure of the lender.

Table 4.2: Mean and standard deviation of log outstanding amount at origination date and log default amount at default date for defaulted loans. Outstanding amount is the initial amount of exposure in € at the issuance date. Default amount is the remaining exposure in € at default date including additional drawn cash and mark-to-market-adjustment. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

	Rating at Origination	Obs.	log(Outstanding Amount)	log(Default Amount)
	A	294	5.11	4.98
			(0.89)	(0.95)
SMEs	**B**	13,328	5.17	5.04
			(0.87)	(0.88)
	C	2,784	5.28	5.17
			(0.99)	(0.99)
	A	92	6.00	5.88
			(1.41)	(1.39)
LCs and FIs	**B**	4,431	5.95	5.81
			(1.18)	(1.19)
	C	1,078	6.20	6.04
			(1.20)	(1.21)

consider the total outstanding amount in a system.

For this purpose, we shall define ω_t as the ratio of average initial outstanding amounts of non-defaulted loans to average initial outstanding amounts of defaulted loans at year t and λ_t as the ratio of the total EAD to the total initial outstanding amount of defaulted loans at year t. Note that both parameters are ex post parameters, prior to the default events.

$$\omega_t = \frac{\overline{\text{Outstanding Amount}_{ND(t)}}}{\overline{\text{Outstanding Amount}_{D(t)}}}$$

$$\lambda_t = \frac{\sum_{D(t)} \text{Default Amount}}{\sum_{D(t)} \text{Outstanding Amount}},$$

where $D(t)$ and $ND(t)$ denote the set of defaulted and non-defaulted loans in a portfolio of a particular year t respectively.

The importance of ω_t is clear which is to explain the risk when the total exposure to unhealthy borrowers unintentionally gets bigger due to the loan demand-supply dynamics in the financial ecosystem. This parameter should be understood at the macroeconomic level. It does not describe the growth of an exposure of a particular borrower, but rather that borrowers from a particular asset class category tend to ask for more loans.

While LGD describes the realised loss percentage from the EAD, the variable λ_t describes the potential loss for each invested euro, given the exposure is defaulted at year t. Thus, $(1-\lambda_t)$ represents the fraction of the exposure which has been paid (or written off) up to the default date. Both variables, ω_t and λ_t, are typically not present in risk models (yet). By considering both, the adjusted PD can be estimated. Subsequently, the potential impact in the asset correlation is observable.

Let's assume there are in total $n(\neq 0)$ loans in both $D(t)$ and $ND(t)$. Then, the adjusted PD in the 1-Euro loan context, given a particular asset class and a credit rating, can be approximated as follows:

$$\begin{aligned}
PD_{adj,t} &= \frac{\text{Total Default Amount}}{\text{Total Outstanding Amount}} \\
&= \frac{0+\sum_{D(t)} \text{Default Amount}}{\sum_{ND(t)} \text{Outstanding Amount} + \sum_{D(t)} \text{Outstanding Amount}} \\
&= \frac{\lambda_t}{\left(\frac{\sum_{ND(t)} \text{Outstanding Amount}}{\sum_{D(t)} \text{Outstanding Amount}} + 1\right)} \\
&\approx \frac{\lambda_t}{\left(\frac{n\cdot(1-PD_t)}{n\cdot PD_t} \cdot \omega_t + 1\right)} \\
&= \frac{\lambda_t \cdot PD_t}{\omega_t - \omega_t \cdot PD_t + PD_t}.
\end{aligned} \tag{4.6}$$

By design, a credit run should decrease the ω_t-value in the system. So, by varying the possible value of ω_t, we can study whether the variation of the ω_t-value influences the system's adjusted asset correlation and the magnitude of this impact. In the special case where the outstanding amount is not affected by the borrower's PD, ω_t should be equal to 1 for all t. This most elementary case simplifies the approximation (4.6) to $PD_{adj,t} \approx \lambda_t \cdot PD_t$.

Historically, the dataset shows that λ_t is slightly under 1 in most of the years, as depicted in table 4.3. Defaults are rare in the high rating category. As a consequence, we can observe

Table 4.3: Observed λ_t in the PD and Rating platform. λ_t denotes the ratio of total defaulted amount to total initial outstanding amount of defaulted loans at year t. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

Default	SMEs			LCs and FIs		
Year	**A**	**B**	**C**	**A**	**B**	**C**
2000	-	1.0000	1.0000	-	-	1.0000
2001	-	0.9868	0.3713	-	0.9791	0.4371
2002	-	0.8003	0.9192	-	0.9461	0.9086
2003	-	0.8695	0.9369	-	0.6844	0.8419
2004	-	0.8342	1.0316	1.0000	0.9424	0.8769
2005	1.0546	0.5862	1.0462	0.8000	0.6476	1.3577
2006	0.9176	0.8368	0.6520	-	0.7751	0.9640
2007	0.8590	0.8497	0.8145	0.0526	0.6670	0.4986
2008	0.8396	0.8558	0.6927	1.1398	0.7058	0.8493
2009	0.5962	0.8116	0.7602	0.9035	0.8322	0.8313
2010	0.8640	0.7599	0.8505	0.9968	0.7260	0.8565
2011	0.8158	0.7835	0.9425	1.0690	0.8214	0.5827
2012	0.6604	0.8462	0.8187	0.7488	0.6812	0.6157
2013	0.5765	0.7016	0.8393	0.8292	0.9038	0.7809
2014	0.9551	0.8129	0.8916	1.3454	0.6949	0.8789
2015	1.6776	0.7815	0.8043	0.6185	0.7631	0.7265
2016	2.3781	0.7868	0.7632	0.3630	0.7834	0.6459
2017	0.9153	0.7063	0.5668	0.2251	0.7973	0.5504

a high variation in λ_t. For some years, λ_t is higher than 1 which implies a higher total default amount than the total invested amount for those loans. Although such cases are rare, it is not surprising that it can happen because default amounts are often conservatively estimated. For the case $\omega_t = 1$ across all years, the associated adjusted default rates are depicted in figure 4.2.

The properties of the variable ω_t as well as its typical value are unknown, as banks' internal strategies may have a direct influence. Banks with proper risk management strategies should have some limitations on the allowed initial loan amount depending on the obligor's credit rating. Banks may additionally adopt some counter-strategies when the obligor's creditworthiness worsens, such as decreasing the credit line's limit or even terminating the loan agreement. These strategies limit the exposure of potentially unhealthy loans, thus increasing the value of ω_t. It is most likely the case that ω_t is individual from the bank's perspective, but may also be affected to some extent by external factors.

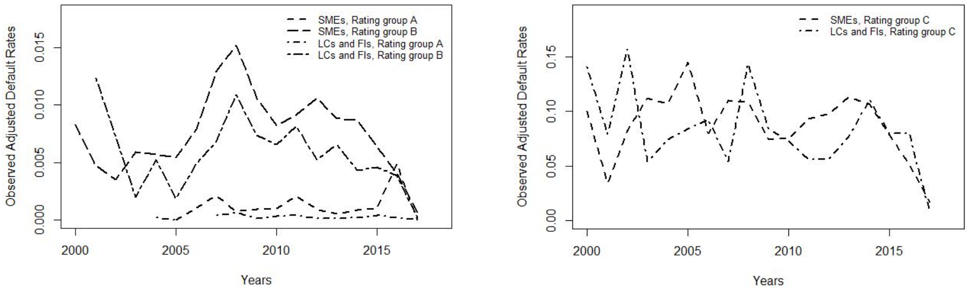

Figure 4.2: Adjusted default rates of SMEs, LCs and FIs, segmented by rating categories

4.6 Result

4.6.1 The Adjusted Model

In this section, we first compare the estimated asset correlation in the traditional version with its adjusted version in the 1-Euro loan context. The (traditional) asset correlation can be compared directly with asset correlations in the literature as well as from the IRBA. The asset correlations reported in the literature seem to be scattered, as reported by Chernih et al. (2006); Hashimoto (2009). It may reach values as low as $< 1\%$ and as high as $> 50\%$. Section 4.3 emphasises that the negative relationship between PD and asset correlation, and the positive relationship between firm size and asset correlation are not as clear as claimed by regulators. In the end, it is not surprising to see a similar observation in this essay.

Table 4.4 shows a negative dependency between the asset correlation and the credit rating, which implies a negative dependency between the asset correlation and the PD. However, the asset correlations of rating categories **B** and **C** are not significantly different. The positive dependency between the asset correlation and the firm size cannot be confirmed in our result, especially for the rating category **A**.

In some of the categories, the adjustment has a significant impact, which is positive for the rating category **C** but negative for the rating category **A**. Note that it can be observed from figure 4.2 that this asset category has far fewer data points compared to the data points needed in the traditional model, as shown in figure 4.1. As less data implies a high variance, this may be the

Table 4.4: The coefficient p in the traditional and adjusted version and its 95% confidence interval. The asset correlation is the square of p. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

	SMEs			LCs and FIs		
	A	**B**	**C**	**A**	**B**	**C**
$\hat{p}$	0.3364	0.2150	0.1869	0.3209	0.2734	0.2700
95% CI	[0.3052,0.3675]	[0.1992,0.1973]	[0.1764,0.1973]	[0.2426,0.3953]	[0.2544,0.2924]	[0.2513,0.2887]
$\hat{p}_{adj}$	0.2809	0.2163	0.2387	0.1533	0.2773	0.2836
95% CI	[0.2546,0.3072]	[0.2003,0.2323]	[0.2205,0.2570]	[0.1380,0.1715]	[0.2571,0.2976]	[0.2627,0.3044]
$\hat{p}^2 = \rho$	0.1132	0.0462	0.0349	0.1030	0.0747	0.0729
95% CI	[0.0931,0.1351]	[0.0397,0.0389]	[0.0311,0.0389]	[0.0589,0.1563]	[0.0647,0.0855]	[0.0632,0.0833]
$\hat{p}^2_{adj}$	0.0789	0.0468	0.0570	0.0235	0.0769	0.0804
95% CI	[0.0648,0.0944]	[0.0401,0.0540]	[0.0486,0.0660]	[0.0190,0.0294]	[0.0661,0.0886]	[0.0690,0.0927]

reason for this deviation.

4.6.2 Credit Run Scenarios

The goal is to answer the question on how a credit run affects the systematic risk (and thus influences the financial stability) in the context of the IRBA. To replicate a credit run in the model, the adjusted model comes into play. As a reminder, the parameter ω_t stands for the ratio of the average outstanding amount from non-defaulted loans to the average outstanding amount from defaulted loans. During a credit run, the portfolio compositions are systematically shifted due to a change in the borrowing and lending behaviour. In the system, there are more (potentially) unhealthy obligors engaged in a credit run, which can be simulated by increasing the denominator of the ω_t during this period. Sufficiently capitalised corporates do not necessarily need to increase their lending in periods with a liquidity bottleneck, while insufficiently capitalised corporates do not have the same luxury. Thus, the numerator of the ω_t can remain constant during this period.

Although banks would be interested in measuring their individual ω_t, this parameter should be understood for a whole financial ecosystem in the context of this essay. In the context of the equation (4.1), this parameter captures the missing influence of the systematic factor in the EAD. How the value of ω_t fluctuated historically is challenging to investigate within our dataset. In a credit run, we should expect a fall in the ω_t value temporarily (the denominator increases while the numerator stays constant). We set five different scenarios to capture the effect of a credit run: 1) constant ω_t over time, 2) ω_t falls abruptly once before the financial crisis (2008), 3) slow decrease of ω_t before the financial crisis (2008) and slow stabilisation of ω_t after the crisis, 4) ω_t decreases slowly before the financial crisis (2008) but neutralises quickly afterwards, and 5) a control scenario which is an abrupt fall in an average year (2015).

The first scenario is motivated by the notion that ω_t is likely to be constant over the year. From a bank's perspective, this is not unusual and is likely to be the case. All other scenarios assume a non-constant ω_t, which is equal to 1 in most of the time but can change in value due to a systematic event, such as a credit run. In this essay, we assume the year 2008 to represent the financial crisis. However, the duration and the severity of the credit run during this period may vary for different financial instruments. The second, third, and fourth scenarios are a selection of various possible cases that could occur. The difference between these three scenarios is the duration of the credit run. Note that before the crisis, a credit run can be perceived as a credit boom, which is not necessarily a negative signal for banks. This wrong interpretation of the current economic situation may prolong the duration of the credit run. The same can also be asserted for the normalising rate, i. e. the duration it takes for ω_t to stabilise back to its previous value. No doubt surviving banks have reacted appropriately and put stricter risk management in place as a reaction to the financial crisis, which is an argument for a fast normalising rate. However, an argument can be made that supports a slow normalising rate since any change in risk strategy mostly only applies for new loans. Additionally, the last scenario investigates the credit run effect in an average year with the sole purpose of being a control case for our result. The choice for 2015 is influenced by the argument that any effects coming from the financial crisis, the European sovereign crisis, or the introduction of Basel II have to be avoided. There is also an argument against choosing an upturn year such as 2017 in the dataset as there are only

Table 4.5: Evolution of adjusted p with scenario 1, where ω_t is constant throughout the years. To acquire the adjusted asset correlation, the coefficient p needs to be squared. ω_t can be understood as the ratio of healthy and unhealthy exposures. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

Scenario 1: constant ω_t						
ω_t	SMEs			LCs and FIs		
	A	**B**	**C**	**A**	**B**	**C**
1:0.50	0.226875	0.202986	0.217032	0.103267	0.255500	0.259945
1:0.75	0.258088	0.210509	0.229316	0.137463	0.270660	0.273406
1:1.00	0.280910	0.216297	0.238743	0.154768	0.277341	0.283584
1:1.25	0.298870	0.221064	0.246447	0.167934	0.282828	0.291791
1:1.50	0.313672	0.225155	0.252976	0.179803	0.287524	0.298669
1:1.75	0.326266	0.228753	0.258648	0.190443	0.291647	0.304586
1:2.00	0.337226	0.231977	0.263664	0.200021	0.295340	0.309771
1:2.25	0.346932	0.234905	0.268160	0.208696	0.298690	0.314383
1:2.50	0.355642	0.237593	0.272234	0.216608	0.301764	0.318533
1:2.75	0.363543	0.240081	0.275959	0.223870	0.304609	0.322301
1:3.00	0.370776	0.242398	0.279390	0.230576	0.307259	0.325753

a few defaults, rendering the parameter λ_t to have a higher deviation and therefore non-robust results.

In the first scenario, we assume that there is no change in ω_t, i. e. if there is any difference in the choice of initial outstanding between unhealthy loans (those which default during its term) and healthy ones (those which do not default), its ratio is assumed to be constant over time. At the moment of the origination date, banks do not know whether loans will default, so a direct control over ω_t is not possible. Either most banks put a strict exposure limit on the borrowers with a lower rating (thus there is a tendency for $\omega_t > 1$), or we can assume that borrowers with a lower rating have more need on capital in expectation (thus $\omega_t < 1$ may occur naturally). The case $\omega_t = 0.5$ $(= 1 : 2.00)$ means that the average initial outstanding of loans, which are later defaulted, is twice as much as the average initial outstanding of healthy loans. A low ω_t value is more interesting than a high one since a credit run implies a low ω_t.

The adjusted p in table 4.5 first needs to be squared to acquire the adjusted asset correlation. Table 4.5 shows a slow but steady increase in adjusted p as ω_t decreases. The change rate is

Table 4.6: Evolution of adjusted p with scenario 2, where ω_t falls/rises abruptly in 2008, otherwise $\omega_t = 1$. To acquire adjusted asset correlation, the coefficient p needs to be squared. ω_t can be understood as the ratio of healthy and unhealthy exposures. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

Scenario 2: abrupt ω_t change in 2008						
ω_{2008}	SMEs			LCs and FIs		
	A	**B**	**C**	**A**	**B**	**C**
1:0.50	0.287031	0.204823	0.239179	0.132065	0.269155	0.271588
1:0.75	0.282642	0.209617	0.236325	0.143392	0.272589	0.276134
1:1.00	0.280910	0.216297	0.238743	0.154768	0.277341	0.283584
1:1.25	0.280475	0.223424	0.243301	0.165431	0.282461	0.291762
1:1.50	0.280766	0.230530	0.248720	0.175331	0.287629	0.299916
1:1.75	0.281496	0.237451	0.254428	0.184549	0.292719	0.307756
1:2.00	0.282510	0.244127	0.260133	0.193173	0.297684	0.315178
1:2.25	0.283707	0.250545	0.265699	0.201280	0.302502	0.322159
1:2.50	0.285027	0.256707	0.271057	0.208934	0.307172	0.328708
1:2.75	0.286430	0.262628	0.276177	0.216187	0.311695	0.334845
1:3.00	0.287890	0.268320	0.281049	0.223083	0.316075	0.340599

also different throughout the rating categories. It seems that the high rating categories are more susceptible to a change in p as the ω_t decreases in value. Since the choice of the initial exposure amount determines the value of ω_t, the exposure amount (especially the EAD used in the IRBA) is consequently an important factor as well. So together with the result, it is confirmed that the exposure amount can affect the calibration of the asset correlation substantially. However, the current IRBA sets values for asset correlations which were calibrated before the financial crisis. Therefore, it is easy to conclude that the IRBA underestimates the correlation effect between EAD and the systematic factor (see section 4.4). Moreover, since the estimated adjusted p moves continuously with the variation of the value ω_t, it implies that a minimum p should exist. In other words, there exists an *optimal* value of ω_t, at which the (adjusted) p is minimal.

In the second scenario, we start to evaluate how a credit run affects the adjusted p over time. This particular case assumes that $\omega_t = 1$ is normative, but it falls rapidly in 2008 due to the financial crisis and neutralises in 2009. The motivation for this particular case is to simulate a sudden and quick credit run. The comparability to the first case is limited. A constant low $\omega_t < 1$

over a long period, as observed in the first scenario, can be perceived to be far worse than the short-term credit run in 2008 of the second scenario. On the other hand, a sudden change in ω_t over t can be seen as a shock which may further amplify the systematic effect.

The aggressive increasing pattern on the adjusted p as the ω_t decreases for high-rated SMEs, as shown in the first scenario, can no longer be observed in table 4.6. For other classes, the results are comparable to the first scenario. Furthermore, it seems that there also exists a minimum of possible asset correlations for the SME asset class, given a variation of ω_t. From here onward, this *optimal* ratio of healthy and unhealthy exposures is referred to as the optimal value of ω_t. Similarly, optimal values should also exist for LC and FI asset classes as well, but they are probably higher than 2 (the maximum value of ω_t in our calculation, since a higher value is less plausible).

The optimal value only refers to the lowest systematic risk measured by the adjusted asset correlation, but it does not say anything about other risk types. In this perspective, the optimal value is only optimal for the financial system, but not necessarily for each bank as a part of the system. From the micro perspective, the optimal ω_t should always be infinite, i. e. the outstanding amount of unhealthy loans should be minimal. The individual best possible strategy does not necessarily bring the best outcome for the system. From the macro perspective, the sudden increase in demand in the money market is mostly unavoidable. The best option is to reduce any exposure cluster (if possible), e. g. by allocating these (potentially) unhealthy loans uniformly through all of the system's members. Depending on the value of the λ_t (the loss fraction from each invested euro), the optimal value of the ω_t will most likely be near 1 (or >1), as observed in the second scenario. Only then the systematic risk is the lowest.

The third scenario depicts a credit run event with a slow transition as well as a slow normalising rate. While the fall of the ω_t value in the second scenario is prompt, the third one assumes a transitional year before and after 2008. In 2007 and 2009, ω_t is assumed to be in the middle of the chosen value of ω_{2008} and 1. The idea is that a behaviour change in the lending and borrowing strategies may not happen overnight. The increase in demand for liquidity by

Table 4.7: Evolution of adjusted asset correlation with scenario 3, where ω_t falls/rises slowly in 2007-2008 and neutralises slowly in 2009, otherwise $\omega_t = 1$. To acquire adjusted asset correlation, the coefficient p needs to be squared. ω_t can be understood as the ratio of healthy and unhealthy exposures. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

Scenario 3: slow ω_t change during 2007-2009						
ω_{2008}	SMEs			LCs and FIs		
	A	**B**	**C**	**A**	**B**	**C**
1:0.50	0.281647	0.192811	0.238616	0.130476	0.264291	0.278352
1:0.75	0.280001	0.204279	0.234743	0.142827	0.270178	0.277760
1:1.00	0.280910	0.216297	0.238743	0.154768	0.277341	0.283584
1:1.25	0.282651	0.227349	0.245249	0.166154	0.284406	0.291319
1:1.50	0.284689	0.237326	0.252427	0.176635	0.291076	0.299451
1:1.75	0.286811	0.246356	0.259591	0.186254	0.297320	0.307416
1:2.00	0.288932	0.254594	0.266472	0.195121	0.303167	0.315006
1:2.25	0.291021	0.262174	0.272976	0.203346	0.308660	0.322152
1:2.50	0.293060	0.269201	0.279081	0.211022	0.313843	0.328849
1:2.75	0.295048	0.275761	0.284799	0.218224	0.318756	0.335118
1:3.00	0.296979	0.281919	0.290145	0.225013	0.323421	0.340976

unhealthy borrowers may occur gradually. It is to be expected that they must try not only to take new loans but also explore other options of financial instruments to satisfy their liquidity needs, which overall requires time. Even though such an event will not likely be perceived as a shock or at least only a mild shock, this scenario may be perceived to be worse overall than the quick sudden fall in ω_t. The reason lies in the duration of this anomaly.

The difference between the adjusted p from this scenario in table 4.7 and the previous scenario in table 4.6 is quite small. The adjusted p grows slightly faster compared to the second scenario, as the value of ω_t falls. The optimal value of ω_t also shifts slightly from the one in the second scenario. From the perspective of the results, there is only a small difference in both of the scenarios. The most difference can only be seen in the SME asset class. To get a bigger picture, the result of the next scenario needs to be compared directly to the other credit run scenarios as well. However, the result gives the first hint that the duration of a credit run may not be a crucial factor for the asset correlation. Regardless, it should not be underestimated.

The combination of the aforementioned two scenarios is the essence of the next scenario.

Table 4.8: Evolution of adjusted asset correlation with scenario 4, where ω_t falls/rises slowly in 2007-2008, otherwise $\omega_t = 1$. To acquire adjusted asset correlation, the coefficient p needs to be squared. ω_t can be understood as the ratio of healthy and unhealthy exposures. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

Scenario 4: slow ω_t change in 2007-2008 then fastly neutralises in 2009						
ω_{2008}	SMEs			LCs and FIs		
	A	**B**	**C**	**A**	**B**	**C**
1:0.50	0.280853	0.197532	0.233990	0.125366	0.267205	0.278071
1:0.75	0.279890	0.206403	0.233513	0.140597	0.271543	0.278144
1:1.00	0.280910	0.216297	0.238743	0.154768	0.277341	0.283584
1:1.25	0.282607	0.225792	0.245683	0.167325	0.283348	0.290749
1:1.50	0.284555	0.234638	0.252980	0.178509	0.289219	0.298348
1:1.75	0.286591	0.242844	0.260137	0.188595	0.294858	0.305867
1:2.00	0.288637	0.250479	0.266965	0.197800	0.300247	0.313096
1:2.25	0.290664	0.257614	0.273400	0.206284	0.305392	0.319958
1:2.50	0.292659	0.264317	0.279436	0.214168	0.310312	0.326427
1:2.75	0.294612	0.270640	0.285087	0.221545	0.315025	0.332513
1:3.00	0.296522	0.276631	0.290379	0.228485	0.319547	0.338235

The combination of a slow transition of ω_t in 2007 (similar to scenario 3) but a sharp normalising rate in 2009 (similar to scenario 2) motivates the fourth scenario. The rationale is that banks may not be aware of a credit run in 2007. However, as soon as the financial crisis occurs, the (surviving) banks are likely to use an aggressive strategy and limit their exposures to low-rated obligors immediately. From the duration perspective, the credit run is shorter in this case than the third scenario. The overall comparability to other cases is limited. The result in table 4.8 shows almost no difference to the results in table 4.7. This highlights the fact that the strategies to reduce the exposure to unhealthy loans are ineffective (in the systematic risk context) if they are implemented after the financial crisis. In this particular scenario, the optimal values of ω_t can be observed to persist in the region close to 1 or higher than 1, similar to the previous two scenarios.

All in all, it could be the case that the adjustment in the model itself causes higher asset correlations in general, not necessarily triggered by the credit run scenarios. A conclusion that a higher asset correlation is caused by the parallel combination of a credit run and a financial

Table 4.9: Evolution of adjusted asset correlation with the control scenario, where ω_t falls/rises abruptly in an average year (2015), otherwise $\omega_t = 1$. To acquire adjusted asset correlation, the coefficient p needs to be squared. ω_t can be understood as the ratio of healthy and unhealthy exposures. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

Scenario 5: abrupt ω_t change in 2015						
ω_{2015}	SMEs			LCs and FIs		
	A	**B**	**C**	**A**	**B**	**C**
1:0.50	0.284149	0.223041	0.250439	0.148226	0.281879	0.290016
1:0.75	0.281336	0.217770	0.241417	0.150189	0.278294	0.284424
1:1.00	0.280910	0.216297	0.238743	0.154768	0.277341	0.283584
1:1.25	0.281593	0.216644	0.239207	0.160142	0.277646	0.284959
1:1.50	0.282866	0.217981	0.241367	0.165700	0.278633	0.287475
1:1.75	0.284472	0.219889	0.244481	0.171208	0.280019	0.290591
1:2.00	0.286273	0.222138	0.248132	0.176574	0.281649	0.294022
1:2.25	0.288194	0.224594	0.252073	0.181766	0.283430	0.297593
1:2.50	0.290178	0.227171	0.256152	0.186776	0.285306	0.301209
1:2.75	0.292201	0.229817	0.260271	0.191609	0.287238	0.304804
1:3.00	0.294239	0.232495	0.264374	0.196274	0.289204	0.308341

crisis may be flawed. Without a control scenario, a credit run alone may be the only cause for the increase in the asset correlation regardless of the economic situation. To test this, the last scenario simulates a credit run during an average year. If the result shows an increase in the adjusted p, it is then safe to conclude that the system's portfolio composition (but not necessarily the increase in the default rates) is responsible for the increase in the asset correlation. The last scenario assumes an abrupt change of ω_t without any transition in a year which is neiter a non-crisis year nor in an upturn year. The year 2015 is chosen to represent an average year without any significant influence from either the financial crisis, the European sovereign crisis, or the introduction of Basel II.

The result in table 4.9 shows a significantly weaker effect of an abrupt increase of demand in the loan market to the adjusted p for most of the asset segments compared to the previous scenarios[5]. Conclusively, the presence of a higher default rate coupled with a lower value of ω_t affects the increase in the adjusted p, but not the fall of the ω_t value alone. Interestingly, the

[5] Technically, it is no longer a credit run.

Table 4.10: The adjusted asset correlation and its absolute difference between the case of $\omega_t = 1:1$ and $\omega_t = 1:2$. ω_t can be understood as the ratio of healthy and unhealthy exposures. SME stands for Small and Medium sized Enterprise, LC stands for Large Corporate, and FI stands for Financial Institution. LC and FI are combined similar to how these asset classes are treated in the IRBA. Rating categories **A** (includes AAA to A-), **B** (includes BBB+ to B-), and **C** (includes CCC+ to C) are simplified S&P rating categories.

	SMEs			LCs and FIs		
	A	**B**	**C**	**A**	**B**	**C**
Scenario 1: constant ω_t						
$\omega_t = 1:1$	0.078910	0.046785	0.056998	0.023953	0.076918	0.080420
$\omega_t = 1:2$	0.113721	0.053813	0.069519	0.040008	0.087226	0.095958
difference	0.034811	0.007029	0.012521	0.016055	0.010308	0.015538
Scenario 2: abrupt ω_t change in 2008						
$\omega_{2008} = 1:1$	0.078910	0.046785	0.056998	0.023953	0.076918	0.080420
$\omega_{2008} = 1:2$	0.079812	0.059598	0.067669	0.037316	0.088616	0.099337
difference	0.000902	0.012814	0.010671	0.013363	0.011698	0.018917
Scenario 3: slow ω_t change in 2007-2009						
$\omega_{2008} = 1:1$	0.078910	0.046785	0.056998	0.023953	0.076918	0.080420
$\omega_{2008} = 1:2$	0.083482	0.064818	0.071007	0.038072	0.091910	0.099229
difference	0.004572	0.018034	0.014009	0.014119	0.014992	0.018809
Scenario 4: slow ω_t change in 2007-2008 then fastly neutralises in 2009						
$\omega_{2008} = 1:1$	0.078910	0.046785	0.056998	0.023953	0.076918	0.080420
$\omega_{2008} = 1:2$	0.083312	0.062740	0.071270	0.039125	0.090148	0.098029
difference	0.004401	0.015955	0.014272	0.015172	0.013230	0.017610
Scenario 5: abrupt ω_t change in 2015						
$\omega_{2015} = 1:1$	0.078910	0.046785	0.056998	0.023953	0.076918	0.080420
$\omega_{2015} = 1:2$	0.081953	0.049346	0.061569	0.031178	0.079326	0.086449
difference	0.003042	0.002561	0.004571	0.007225	0.002408	0.006029

optimal value of ω_t is 1 for almost all asset classes. In other words, an equal ratio of healthy and unhealthy exposures in an average year is optimal, in the sense of a minimum amount of systematic risk. The same cannot be asserted for a credit run during a downturn period.

The overall increase in the adjusted asset correlation is most likely caused by the fact that a low ω_t value tends to increase the volatility of the adjusted PD. Thus, it magnifies the systematic effect even further than what may be observed in the PD time series (figure 4.1). It is clear that the current regulation has been neglecting the asset correlation in the IRBA, and our results show how sensitive this parameter in the adjusted model can be towards the increasing demand for liquidity. However, the asset correlation jumps only if the increasing demand for loans is coupled with a higher PD as well. Our results confirm that a change of composition shortly before the financial crisis drives the asset correlation up and thus accelerates the occurrence of the financial crisis. The addressed risk underestimation does not originate from the institutions or their miscalculation, but rather from the core of the model currently used in the IRBA and its calibrated correlation coefficient.

The results confirm that the adjusted asset correlation increases with a credit run. The severity of a credit run may depend on various external factors, which makes it difficult to choose a particular severity for a downturn asset correlation. For a direct comparison, we choose to compare the cases $\omega_t = 1:1$ and $\omega_t = 1:2$ (chosen arbitrarily) for each scenario. We are interested in the magnitude of the credit run effect towards the adjusted asset correlation. The second, third, and fourth scenarios are especially important for this purpose. The first and last scenarios serve only as control scenarios. Table 4.10 sums up the effect of a credit run for each scenario and shows that the increase is similar (0.1% to 1.9%) for all asset classes in the credit run scenarios. This increase is to be understood as an absolute difference[6]. There seems to be no pattern in the relative difference. When the value of ω_t falls during an average year (the control scenario), the difference in the adjusted asset correlation is lower than 1%, which is negligible. Apart from the first case, there is no evidence that there is any substantial increase for high-rated SMEs. Thus, this asset class can be classified as the most secure in the event of a credit run in the systematic risk context.

4.7 Discussion

The concept of a *downturn asset correlation* is as necessary as the concept of a downturn LGD within the IRBA. Everything comes back to equation (4.1) which requires any following calculation to be conditioned to the systematic factor. Since the IRBA calculates the expected loss conditional on the 99.9%-percentile of the systematic factor, the LGD and the asset correlation should be calibrated accordingly. Based on our previous analysis, an absolute increase of 2%

[6]The term *difference* (including increase or add-on) in this context will be understood as an absolute difference without reference of the term *absolute*.

should be adequate to cover the downturn cases. However, we cannot ignore the fact that this add-on is derived from the adjusted model, while the IRBA uses the traditional model with the constant asset correlation assumption. The transferability between both versions of the models is not yet obvious. Moreover, applying an add-on of 2% to the existing correlation coefficient in the IRBA assumes that the current correlation coefficient in the IRBA is correct, which is also questionable, as explained in section 4.3.

This 2% increase in the asset correlation can be easily translated to the increase in the conditional PD because the IRBA ties the PD and the asset correlation directly. In this impact analysis, we assume that the result in this essay is to some extent also representative for the retail asset class, although our dataset does not contain retail exposures which are not classified as SME exposures. Figure 4.3 depicts the impact of the 2% add-on towards the conditional PD, relative to the PD. It shows that there is an absolute increase with a range from 0.3% to 3.5% in the conditional PD. Interestingly, the impact is independent of the asset class, and it is lowest for exposures with high creditworthiness and highest for exposures with low creditworthiness. An underestimation of this magnitude is quite significant for the capital requirement, which is of course also influenced by other factors within the formula. All in all, a revision for the IRBA is not just a matter of recalibration, as explained in section 4.4. The issue lies deep within the model itself. It is important to emphasise that the adjusted asset correlations suggested by the result are based on assumptions, which are depicted here as scenarios.

This essay outlines the observation that asset correlations fluctuate over time, depending on the demand in the money market. This observation contradicts directly the assumption in the ASRF model that the asset correlation stays constant over time. The behaviour change of lenders as well as of borrowers, especially in a credit run context, contributes to this fluctuation. The results of this essay confirm the scepticism of the banking supervision agencies towards the advanced approaches in the regulatory capital requirements. To restrict the impact of these advanced approaches towards the regulatory capital requirements, new tools have been introduced in the Basel III reforms, such as the input and output floors. However, the output floor is just

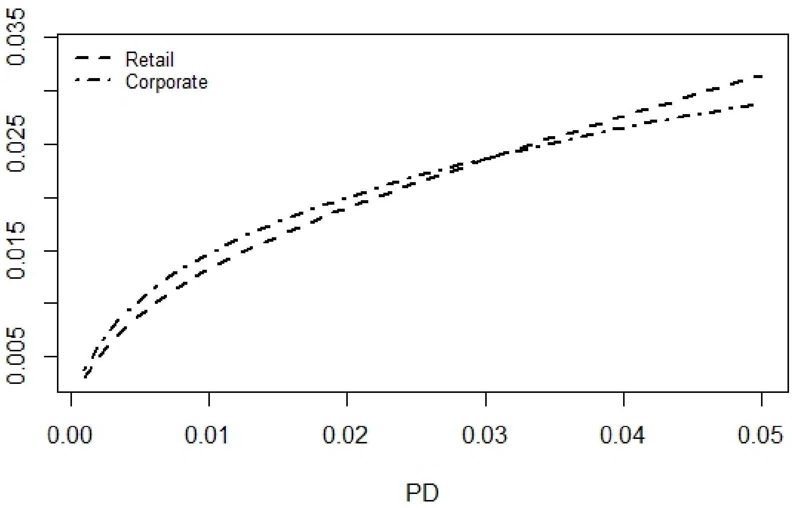

Figure 4.3: The absolute increase in the conditional PD using a 2% add-on for the correlation coefficient compared to without add-on

a rudimentary way to bind the IRBA capital requirement to the SA capital requirement. The suggested solution to apply an add-on of 2% on the current correlation coefficient acts only as a sticking plaster rather than a long-term solution. Addressing the asset correlation core issue as depicted in this essay continues to be irreplaceable to any short-term solution.

4.8 Conclusion

This essay addresses an issue regarding a particular risk parameter in the IRBA: the asset correlation. Unlike the PD and the LGD, the asset correlation is predetermined in the capital requirement regulation to particular values between 3% and 24%. These values have not been updated since the introduction of the IRBA in the Basel II Accord. So the financial crisis has not been taken into account with the calibration of these values. In the ASRF model, which serves as the foundation of the IRBA, the asset correlation parameter is assumed to be constant over time. However, there is evidence in the literature that the asset correlation may be volatile. For instance, Lee et al. (2012) show that asset correlations may vary over the years and, in particular, the asset correlation associated with the year 2007 is highest. To arrive at this conclusion, they use the CAPM's β as an indirect proxy. However, the compatibility of the CAPM to the ASRF model remains questionable.

Nonetheless, these results give the first hint that the asset correlation may be a dynamic parameter and can be influenced by the financial crisis. In the pre-crisis period, a systematic shift in the lending and borrowing behaviour can be observed. This change is mainly motivated by the worry of the solvency and liquidity of the banking sector. Ivashina and Scharfstein (2010) refer to the rising demand for liquidity shortly before the financial crisis as a "run" to describe the tendency of firms to take more loans than usual or max out their available credit lines, which is then referred to as a credit run in this essay to detach the concept from a particular financial instrument. A credit run influences the available amount of loan exposures in the market and indirectly changes the loan composition in banks' portfolios. Those who are insufficiently capitalised and possibly with a low credit rating tend to take more loans. Hence, banks' portfolios tend to be less diversified as the exposure of a particular asset class is more likely and, at the same time, the overall default rate of banks' portfolios increases. During this period, the banks' risk managements would not yet be able to observe the higher default rate as the defaults would occur in the future.

In the IRBA context, there are currently no methods in the literature to support detecting the effect of an external factor towards the asset correlation without taking strong assumptions. This essay suggests an adjustment in the ASRF model, which is practically the current model from a slightly different perspective. The ASRF model itself does not consider the dynamic of the loan market's composition as the asset correlation parameter is assumed to be constant. The main result is clear: a credit run increases the asset correlations within the adjusted model, which implies a risk underestimation in the IRBA. Although the impact and its magnitude in the adjusted model are clear, how this result should be translated into the old model is not. The magnitude may depend on the duration and severity of the credit run as well as on the PD shock.

We observe up to an absolute increase of up to 2% in the adjusted asset correlation throughout various credit run scenarios which may imply a relative increase over 50% in the adjusted asset correlation for some asset classes. Assuming this result is transferable to the old model, it would mean that the IRBA underestimates the conditional PD by 0.3 to 3.5 percentage points depending on the PD input.

The core issue seems to lie in the model itself and its assumptions. Although the banking supervision authorities introduced revisions of the Basel Accord which address their doubt on the IRBA, some of these revisions are more of a short-term solution rather than a long-term one. The results of this essay reinforce this doubt even further. In particular, it should be discussed whether the concept of a downturn asset correlation may be a necessity. For now, a flat 2% absolute increase in the correlation coefficient for the current version of the IRBA may solve this issue (assuming the previous calibration is correct). However, it does not replace the need for a long-term solution, and we implore more research to be done in a non-Merton-based alternative to the IRBA.

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Christopher Paulus Imanto