Cognitive Science

Mathematics for Cognitive Science

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Abstract:	That the state-of-affairs of cognitive science is not good is brought into figural salience in "What happened to cognitive science?" (Núñez et al., 2019). We extend their objective description of 'what's wrong' to a prescription of 'how to correct'. Cognitive science, in its quest to elucidate 'how we know', embraces a long list of subjects, while ignoring Mathematics (Fig. 1a, Núñez et al., 2019). Mathematics is known for making the unknown to be known (cf. solving for unknowns). This acknowledgement naturally raises the question: does mathematical knowing inform knowing in general? Here we show that drawing parallels to mathematical knowing can facilitate the advancement of cognitive science (Lawvere, 1994).

To,

Professor Richard P. Cooper

Executive Editor

Cognitive Science

Dear Professor Cooper,

I am herewith submitting my manuscript "Mathematics for Cognitive Science" to be considered for publication as a 'Letter To The Editor' in your journal Cognitive Science. My Letter was motivated by the unsatisfactory contemporary state-of-affairs of cognitive science highlighted in the recent study:

Núñez, R. et al. What happened to cognitive science? Nat. Hum. Behav. 3, 782-791 (2019).

Based on the kinship between mathematics [in particular and science in general] and cognition (cf. cognition is science writ small; Daedalus 135: 86, 2006), I make a case for mathematical abstraction as a pathway for the advancement of cognitive science. It may be noted here that the parallels between mathematical knowing and knowing in general, which I bring into focus, provide a means "to connect cognitive science theories to computational foundations" (Nat. Hum. Behav. 3: 782, 2019).

I outline the mathematics of calculating representation(s)—a fundamental notion in cognitive science. I felt that it is important to explicitly state that the notion of ‘representation’ figuring in the foundational tenet—“cognition is computation of representations” (Nat. Hum. Behav. 3, 782, 2019)—of cognitive science is indispensable in theorizing about cognition. However, we may need to replace computation with calculation [of representations] so as to move past the computer metaphor and bring the insights of functorial semantics to bear on cognitive science. Representation (or model), according to functorial semantics, is an interpretation of a theory into a background category (Reprints in Theory and Applications of Categories 5, 8-11, 2004). The category of all models of a theory T in a background B is a functor category B^T of all functorial interpretations T --> B of the theory T in the background B. Changing theories (T1 --> T2) induces contravariant changes in representations (B^T2 --> B^T1), while changing backgrounds (B1 --> B2) induces covariant changes in representations (B1^T --> B2^T). I discuss these mathematical insights into representation in a manner readily accessible to the multidisciplinary audience of your journal. This discussion can help insure against: throwing the baby (representation) with bathwater (hexagon; Fig 1a in Nat. Hum. Behav. 3, 782, 2019).

In closing, my manuscript brings out the reach of functorial semantics [of calculating representations] into sharper focus so as to facilitate ready recognition of the relevance of functorial semantics for the development of cognitive science.

If I may, the following may be considered for reviewing my manuscript since they are experts on cognitive science and category theory.

Professor Michael A. Arbib (arbib@usc.edu)

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I earnestly hope that you will find my manuscript suitable for publication in your journal Cognitive Science. I sincerely thank you for your kind consideration of my manuscript and I eagerly look forward to hearing from you.

Thanking you,

Yours truly,

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That the state-of-affairs of cognitive science is not good is brought into figural salience in "What happened to cognitive science?" (Núñez et al., 2019). We extend their objective description of 'what's wrong' to a prescription of 'how to correct'. Cognitive science, in its quest to elucidate 'how we know', embraces a long list of subjects, while ignoring Mathematics (Fig. 1a, Núñez et al., 2019). Mathematics is known for making the unknown to be known (cf. solving for unknowns). This acknowledgement naturally raises the question: does mathematical knowing inform knowing in general? Here we show that drawing parallels to mathematical knowing can facilitate the advancement of cognitive science (Lawvere, 1994).

What is cognition? One scientific approach is to answer: what is cognition good for? Science—reconstructing reality from appearances—is the signature product of human cognition. Since a product retains traces of the process that gave rise to the product, a declarative understanding [but not merely procedural knowledge] of the scientific reconstruction of reality from planned perception constitutes the foundations of the science of cognition. The mathematical basis of scientific reconstruction across disciplines is: comparison of the observed variation with constancy (Lawvere & Rosebrugh, 2003, pp. 125-126, 148-152). Reality is inferred from appearances by establishing an isomorphism between perceived (generalized points) and actual (points). The relationship between points and generalized points involved in scientific reconstruction is analogous to the relationship between stimuli and percepts: reconstruction of the causes that gave rise to sensation (Albright, 2015). The process of going from stimuli to percepts involves a two-step process of sensation followed by interpretation (Croner & Albright, 1999), which is reminiscent of the double-dualization involved in going from points to generalized points. Therefore, we can consider generalized points of scientific reconstruction of reality as an abstraction of conscious percepts of ordinary cognition (Lawvere, 2004a).

With conscious experiences as representations (Chalmers, 2006) in the foundational tenet of "cognition is computation of representations" (Núñez et al., 2019, p. 782), a comprehensive theory of cognition naturally subsumes conscious experiences. Consciousness, the totality of conscious experiences (Koch, 2018), can be construed as a mathematical category of conscious experiences along with their transformations (Lawvere & Schanuel, 2009, p. 21). Beginning with a theory of conscious experiences, say, 'conscious experience is an interpretation of sensation' (alluded to earlier) we obtain a category of two-sequential functions as the category of models of conscious experiences (Posina, Ghista & Roy, 2017). The theory 'conscious experience is an interpretation of sensation' is also a mathematical category consisting of three component structural objects (stimuli, neural codes, conscious percepts) and two component structural maps (sensation, interpretation; Lawvere & Schanuel, 2009, pp. 149-150). More broadly, for each abstract theory of conscious experiences we obtain a corresponding category of models. With every theory of the category of conscious experiences construed as a graph or a small category (Lawvere, 2016; Lawvere & Schanuel, 2009, p. 149, 199-203), we obtain a category of models, which is a functor category (a category whose objects are functorial interpretations of a theory category into a background category). The category of all functor categories subsumes every possible value of every property of each object of the category of conscious experiences, i.e. constitutes an adequate characterization of consciousness (Lawvere & Schanuel, 2009, pp. 370-371). This category of all functor categories is the space of all possible mathematical answers to the question: What is consciousness? (This is analogous to the scenario where the set $N = \{0, 1, 2\ldots\}$ of natural numbers can be thought of as the set of all answers to the question: What is the size of X, where X is any set?) Furthermore, since representations of a given category of particulars are functorial interpretations of a theory into a background category, representations of particulars vary contravariantly with changes in theory and covariantly with background changes (Lawvere, 1994, p. 46; Lawvere & Rosebrugh, 2003, pp. 120-122). In cognitive terminology, conscious experiences (representations) vary as a function of both mental concepts (theories) and intuitions (backgrounds); functorial semantics of mathematical knowing spells out how, i.e., provides a mathematical account of calculating representations of mathematical objects (Posina, Ghista & Roy, 2017; Lawvere, 2004b).

Returning to reality, if reality were the category of sets, then three-valued properties guarantee the recovery of “reality” from perception (Lawvere, 2004a). Of course, reality is not a set; it’s much more structured than the structure-less sets. For instance, reality is self-representing since its representations: collective science and individual perception are parts—reflective parts—of the reality (Lawvere & Schanuel, 2009, pp. 84-85). So, reality can be modeled as the self-representing mathematical category of categories, wherein a mathematical category (e.g. reflexive graphs) is represented in its discrete-and-constant subcategory (Lawvere, 2004b, p. 12; see also Lawvere, 2003, p. 215, 217). This correspondence suggests calculation of the basic types-of-knowing (objects analogous to the three-element set in the category of sets) in mathematical categories that are reflective of reality such as the category of categories. In doing so, we can address two related foundational questions:

1. What is the nature of reconstructable reality (the structure of reality that can be reconstructed from appearances)?

2. What is the nature of revealing appearances (the structure of appearances that is conducive for reconstructing reality)?

More fundamentally, reality is often analyzed into the categories of Being and Becoming, which is not satisfactory since reality consists, as noted above, of parts—individual cognition and collective science—reflective of the reality. Hence we need a mathematical category of Reflecting in addition to the categories of Being and Becoming in order to bridge the two categories of objective reality on the one hand and its subjective reflections on the other. A mathematical category of Reflecting can be objectified along the lines of the objectification of Being and Becoming as mathematical categories of reflexive graphs (exemplifying unity) and dynamical systems (change), respectively (Lawvere, 1991, 1992, 2007). The mathematical category of Reflecting makes room for the basis of science—human cognition—in the scientific representation of reality.

In closing, it must be noted that the foundational tenet—"cognition is computation of representations" (Núñez et al., 2019, p. 782)—is still viable provided we shift our focus from 'computation' to 'representation', move on from the computer metaphor along with its attendant conceptual baggage (cf. hardware vs. software), and build on the definitive mathematical understanding of calculating representations (Lawvere, 2004b). Furthermore, however exciting they may be, we also need to shift our focus away from the excitement of winning games such as Go, especially because of the anti-scientific faith that they demand (Editorial, 2016) and start focusing on the development of theory, as the Cognitive Science Society correctly decided: "greater effort must be made to connect cognitive science theories to computational foundations" (Núñez et al., 2019, p. 789). Here 'computational foundations' can be understood as 'foundations of calculating representations', i.e. functorial semantics (Lawvere, 2004b).

Summing it all, we suggest that functorial semantics is to cognition what calculus is to physics, i.e. the mathematics needed for the development of cognitive science (Lawvere, 1994, p. 43, 55; Lawvere, 1999, p. 412).

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