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Estimation of failure probability in braced excavation using Bayesian networks with integrated model updating

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Abstract

A probabilistic model is proposed that uses observation data to estimate failure probabilities during excavations. The model integrates a Bayesian network and distanced-based Bayesian model updating. In the network, the movement of a retaining wall is selected as the indicator of failure, and the observed ground surface settlement is used to update the soil parameters. The responses of wall deflection and ground surface settlement are accurately predicted using finite element analysis. An artificial neural network is employed to construct the response surface relationship using the aforementioned input factors. The proposed model effectively estimates the uncertainty of influential factors. A case study of a braced excavation is presented to demonstrate the feasibility of the proposed approach. The update results facilitate accurate estimates according to the target value, from which the corresponding probabilities of failure are obtained. The proposed model enables failure probabilities to be determined with real-time result updating.

Keywords: Failure probability; Braced excavation; Bayesian networks; Stochastic model updating; Sensitivity analysis

1 Introduction

As urban construction activities increase, so does foundation pit excavation, as this is the first step of most construction projects. However, this activity often has unfavorable consequences in urban areas. During the excavation process, deformation of the diaphragm wall and ground surface elevation can occur, which can cause collapse of the adjacent building structures and sometimes results in human casualties. Many numerical models have been proposed to compute maximum wall displacement and maximum ground surface settlement. Do, Ou, and Chen (2016) studied the failure mechanism of excavation in soft clay using finite element (FE) analysis. They showed that the FE method could effectively estimate excavation stability. Kung, Juang, Hsiao, and Hashash (2007) proposed a simplified semi-empirical model, named the KJHH (for Kung–Juang–Hsiao–Hashash) model, to estimate the deformation behavior of a braced excavation. The KJHH model incorporated three models that assessed wall deflection, ground surface settlement, and deformation rate. These studies provide empirical or semi-empirical methods to predict the response values of deformation and failure threshold using various input parameters.

Furthermore, FE analysis is a popular approach for addressing problems associated with sophisticated excavations, wherein two-dimensional (2D) plane strain problems are utilized to predict the stability of excavation with the purpose of simplification. However, three-dimensional

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(3D) effects are more suitable and more accurate for the analysis of certain geotechnical problems (such as tunnel excavations) in practical situations. Several 3D FE analyses have been studied (Ou, Chiou, & Wu, 1996; Zdravkovic, Potts, & John, 2005; Lee, Hong, Gu, & Zhao, 2011). Additionally, Janin et al. (2015) compared the relative ability of both 2D and 3D approaches. They found that the 3D approach enabled representation of the reinforcements, ground reaction, and the 3D phenomenon of tunnel excavations, while the 2D simulation failed to represent these complex effects fully. Therefore, given the case studied in this paper, the 3D FE method was applied using the software package ABAQUS 6.13. This software package was selected because it has been proven effective for 3D FE analyses (Lee et al., 2011; Liu, Lee, Quek, Chen, & Yi, 2015; Li & Gang, 2018).

Owing to the unavoidable model errors associated with insufficient knowledge of the reality of a situation and its complex excavation conditions, discrepancies between design parameters and observation parameters are quantified using an updating method. In situations in which field measurements are provided, it is common practice to update geotechnical parameters with back analysis or inverse analysis based on modeling functions (Ledesma & Alonso, 1996; Finno & Calvello, 2005; Calvello & Finno, 2004; Hashash, Levasseur, Osouli, Finno, & Malecot, 2010; Zhang, Zhang, Zhang, & Tang, 2011). For trial-and-error calibration methods, inverse model algorithms are initially considered. For supported excavations, Finno and Calvello (2005) handled design prediction updates using UCODE (a computer code) to assign identification numbers to physical objects. They were able to minimize model errors using their proposed numerical procedure. Juang, Luo, Atamturktur, and Huang (2013), attempted to present the maximum likelihood method for updating soil parameters in a stage-by-stage manner. The likelihood function was obtained according to the bias factor of the KJHH model, wherein the bias factor was often assumed to have a normal or lognormal distribution based on expert knowledge. It is considered impractical to implement updating for wall and ground surface deformation prediction using back analysis by way of implicit probability distribution, mass functions of the model, or black-box methods.

Approximate Bayesian computation (ABC) (Beaumont, Zhang, & Balding, 2002; Turner & Zandt, 2012) can reveal discrepancies by checking the distance, rather than likelihood function. Bi, Broggi, and Beer (2018) developed an ABC model updating framework that considered both Euclidian and Bhattacharyya in quantifying uncertainty. The study revealed an efficient and capable metric for stochastic model updating. In geotechnical updating problems, when the likelihood function is intractable or cannot be approached in a closed form (as a likelihood-free method), an ABC approach is typically used. Nonetheless, there remains scant research on updating geotechnical materials using distance-based ABC approaches. Accordingly, the method presented in this paper incorporates distance metrics for material parameter updating in the supported excavation.

Regardless of whether it is from the perspective of design considerations or risk management, prediction of excavation stability enables the implementation of crucial pre-failure controls. The risks and causes of potential failures in the excavation process are complicated, various, and interactional. Numerous dynamic factors such as soil structure and strength, excavation width, and workmanship affect surface settlement and the movement of braced walls. As a diagnostic tool, Bayesian networks possess the powerful capability of being able to analyze multiple causal failures. The flexibility of network structures contributes to their application in many fields for risk analysis, risk management, and decision analysis.

Zhang, Wu, Ding, Skibniewski, and Yan (2013) presented a decision support Bayesian network (BN) model to predict ground settlement for safety control. Influential factors in this network were all defined by discrete nodes with three states. A dynamic BN model (Spackova & Straub, 2013) was utilized to assess the risk of human factors and other external events in the tunnel construction process. Zhou, Li, Zhou, and Luo (2018) used a BN for the analysis of risk classification for diaphragm wall deflection based on field data. Their model combined the field data of the diaphragm wall with other data as evidence input to validate the predicted results. In these application of BNs, the prior probabilities of each node were highly dependent on collected data and expert opinion.

The primary objective of this study is to present a real-time probabilistic model that will use updated information to predict the possibility of collapse during the excavation process. To accomplish this, we focus on capturing the uncertainty of material parameters, and on characterizing their effect on a diaphragm wall using a BN model. To overcome the difficulty of monitoring wall deformation in tandem with ground surface settlement, ground settlement is accounted for as a field observation. Hence, it is incorporated into the BN model as an input used to update the material parameters. The proposed method combines Bayesian networks with a model updating approach. Not only does the proposed model incorporate information based on expert judgment and limited data from direct and indirect factors, it also captures the propagation of uncertainty throughout the network components. Thus, the proposed model characterizes the relationship between uncertain parameters and the safety states of the excavation process, and identifies the influence of induced-factors on the stability of the excavation.

2 The Bayesian network of excavation evaluation by integrating field observation

2.1 Structure of Bayesian networks

A BN is a graphical statistical model that realizes powerful probability theory. In this directed acyclic graph, a visible cause-effect relationship can be shown with a set of variables linked by an arc that shows their conditional dependency, while the indirect edges indicate the independent conditional relations among the nodes. A detailed overview of BNs can be found in Pearl (1988) and Jensen (2001). Bayesian updating for braced excavation is usually conducted in stages. The use of a BN model addresses the complexities of stepped excavations for multiple layers while providing estimates of the real-time failure state of the excavation.

Additionally, BNs are well suited to capturing uncertainty propagation. Because of this, a general network for braced excavations by steps is constructed by considering the material parameters X_i (as root nodes in the model) and the parent nodes of deformation parameters D_i of the different layers. The discrete nodes Y_i present the states of safety or failure for diverse materials of each layer, while Y_{total} represents the final failure events. Additionally, monitor parameters M_i: ($i = 1, 2, \cdots, n$) in the excavation process are integrated into the network. Per Fig. 1, a BN is built considering two material layers and one monitor parameter.

According to the chain rule, the factorization of partial variables in this network is written as

$$f(D_2, X_2, X_3, X_4, M_1) = f(X_2)f(X_3)f(X_4)f(D_2|X_2, X_3, X_4)f(M_1|X_3, X_4), \quad (1)$$

where $f(D_2, X_2, X_3, X_4, M_1)$ is a joint probability distribution, and the probability of node D_2 can be obtained with the marginal computation of Eq. (1). The probability of excavation failure Y_2 is given by

$$\Pr(Y_2|D_2) = \int_{\Omega_{D_2}} f(D_2|X_2, X_3, X_4)\mathrm{d}(D_2), \quad (2)$$

where the domain Ω_{D_2} of variable Y_2 is divided by safe and failure domain. Likewise, the conditional probability distribution (CPD) of node D_1 and the conditional probability of node Y_1 also can be computed. Furthermore, the events of Y_1 and Y_2 are independent and both exist in a binary state. The event Y_{total} is the joint event of Y_1 and Y_2.

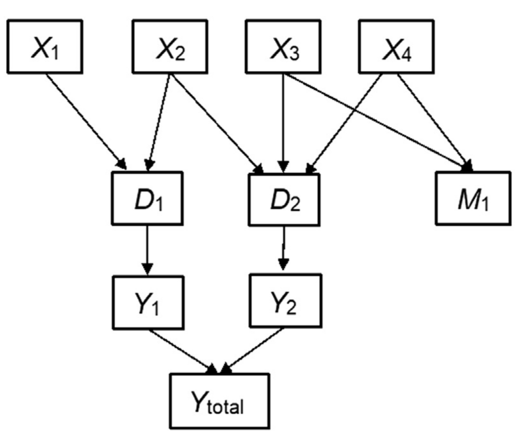


Fig. 1. Bayesian network (BN) incorporating field measurements.

2.2 Quantitative components of a Bayesian network

In a BN, each node should be defined by the corresponding prior probabilities. In this work, we employ a neural network to quantify the inter-relationship among the main parameters. Artificial neural networks (ANN) are an efficient tool to simulate the response of output associated with input variables (Anjum, Tasadduq, & AlSultan, 1997; Yuan & Bai, 2011; Mia & Dhar, 2016). Hashash, Jung, and Ghaboussi (2004) demonstrated complex stress–strain behavior of engineering materials could be effectively captured using an ANN.

A neural network is designed using three layers: input, hidden, and output. Two response relationships comprise the required input to provide a single output. The displacement of the wall is taken as an indicator parameter for detecting failure in the excavation process. As previously mentioned, the response of ground surface settlement is selected as observation data. Furthermore, the response of wall deflection has four material variable inputs, while two variable inputs represent ground surface settlement. For the training and test data, we adopted the simulated values from a dataset calculated using the FE method in the software package ABAQUS 6.13. The amount of training was defined by full factorial designs. The ANN computation was run in the MATLAB R2017 ‘nnstart’ toolbox, and the network was trained using Bayesian regularization (MacKay, 1992). The performance of the training was evaluated by mean square error (MSE) as given in Eq. (3).

$$\mathrm{MSE} = \frac{1}{2}\sum(\mathrm{Actual} - \mathrm{Predicted})^2. \quad (3)$$

Note that the response surface built via an ANN is a black-box. In the next step, we integrate this black-box with the BN model. From this point forward in the process, information updating and sensitivity analysis are executed based on this integrated model.

2.3 Bayesian updating

Soil parameters will vary as the excavation is conducted, which makes direct measurement intractable in current practice. Therefore, soil parameters are generally updated with monitor parameters. This article considers ABC as a likelihood-free method. The recently developed ABC updating framework utilizing various statistical distances (Bi et al., 2018) is employed to update the soil parameters in the proposed approach. Given the problem addressed by this paper, this model updating approach is applied to update the key soil parameters: a Bayesian updating framework is presented with the distance-based ABC approach, including Euclidian and Bhattacharyya distances. The entire update process will be briefly presented in the next section.

Let θ be the uncertain parameters, and D be the observed data. Then, from Bayes’ theorem, the computation of the posterior probability density function (PDF): $f(\theta|D)$ improves the accuracy of the predictions of the model given the data D; and is expressed as

$$f(\theta|D) = \frac{f(D|\theta)f(\theta)}{\int f(D|\theta)f(\theta)\mathrm{d}\theta}, \tag{4}$$

where $f(\theta)$ is the priori distribution and $f(D|\theta)$ is equal to the likelihood distribution; $\int f(D|\theta)f(\theta)\mathrm{d}\theta$ represents normalization and should be constant, but can be intractable to compute because of the high-dimension of the parameter space or multimodal distribution. In this regard, the transitional Markov Chain Monto Carlo (TMCMC) method (Ching & Chen, 2007) is proposed to overcome the difficulty in evaluating the target PDF.

Briefly, the TMCMC method is an effective simulation to support sampling from a set of the intermediate PDFs and converge to the target PDF. Generally, the sampling from the posterior distribution with the TMCMC method is estimated based on the following Eq. (5).

$$f(\theta|D) \propto f(D|\theta) \times f(\theta). \tag{5}$$

Accordingly, these intermediate PDFs are constructed:

$$f_j \propto f(D|\theta)^{P_j} \times f(\theta), \tag{6}$$

where j is the stage number, and P_j denotes the exponent of the likelihood, ranging from 0 to 1 (Ching & Chen, 2007). In the updating framework, $f(D|\theta)$ is estimated by approximate distance-based likelihood based on the Gaussian function,

$$f(D|\theta) \propto e^{\left(-\frac{d^2}{\varepsilon^2}\right)}, \tag{7}$$

where d is the distance metric, which can be either the Euclidian distance or the Bhattacharyya distance; $\varepsilon$is the width factor with a range between 0.001 and 0.1. The smaller the value of ε, the more likely that the result converges to the true value, but the increasing likelihood brings a corresponding requirement for more calculation (Bi et al., 2018).

The formula for calculating the Euclidian distance between two n-dimension vectors $\boldsymbol{x}$ (predicted data) and $\boldsymbol{y}$ (observed data) is delineated in Eq. (8).

$$d = \sum_{i=1}^{n} (x_i - y_i)^2. \tag{8}$$

The Bhattacharyya distance is defined in Eq. (9).

$$d = -\log\left[\int_n p_{\mathrm{pre}}(x)p_{\mathrm{obs}}(x)\mathrm{d}x\right]. \tag{9}$$

In the Bhattacharyya distance, $p_{\mathrm{pre}}(x)$ and $p_{\mathrm{obs}}(x)$ are the PDF of the prediction and observation samples, respectively. This stochastic distance metric is especially suitable for measuring the overlap of the sample set. However, without an overlap situation, it is insensitive to the center of mass of the sets. Therefore, as described in the framework depicted in Fig. 2, we first conduct the updating with Euclidian distance to measure the likelihood and then use the results as the new prior distribution of θ and execute the next update using the Bhattacharyya distance-based ABC.

2.4 Moment-independent sensitivity analysis using Monte Carlo simulation

The sensitivity analysis of a BN contributes to the process of building the BN from data, and is especially useful for large and complex networks. Several types of methods have been studied to determine how causal nodes influence the target node in traditional BNs (Laskey, 1995; Chan & Darwiche, 2004). In these approaches, the evidence is inserted by querying the different states of each variable. The characteristic of the sensitivity is then estimated based on the changes of the posterior probabilities for the target node. However, this is not suitable for complex networks. Given the problem addressed in this paper, a sensitivity approach for identifying the critical inputs before network updating is proposed.

The variance-based sensitivity analysis method is a summary measure of sensitivity that studies how the variance of the output changes when an input variable is fixed. Li and Mahadevan (2017) applied the first-order Sobol' index to BNs to analyze the sensitivity and proposed an approximated algorithm to reduce the computational cost. For problems of risk analysis, the robust sensitivity measure for a BN should enable to capture the entire distribution of an output node, rather than only a single moment.

Moment-independent sensitivity analysis enables capture of the entire distribution of output referring to varying input parameters. Therefore, in this paper, we employ the PDF-based sensitivity approach to measure the contribution of the uncertain parameters in the deformation of a retaining wall. Using this method, we attempt to select the key factors to update. Generally, the moment-

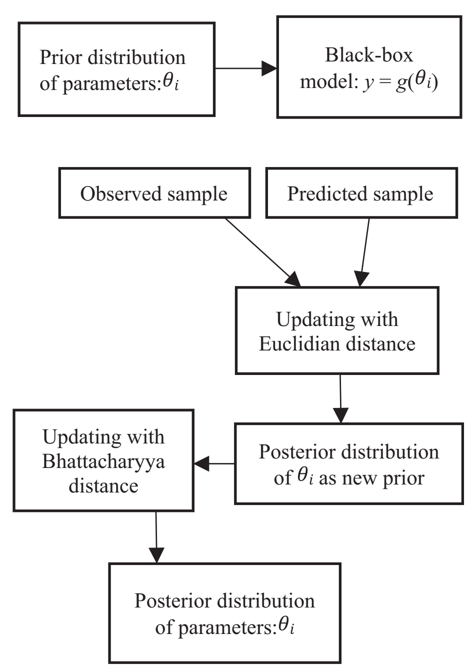


Fig. 2. Proposed update framework with distance-based approximate Bayesian computation (ABC) method.

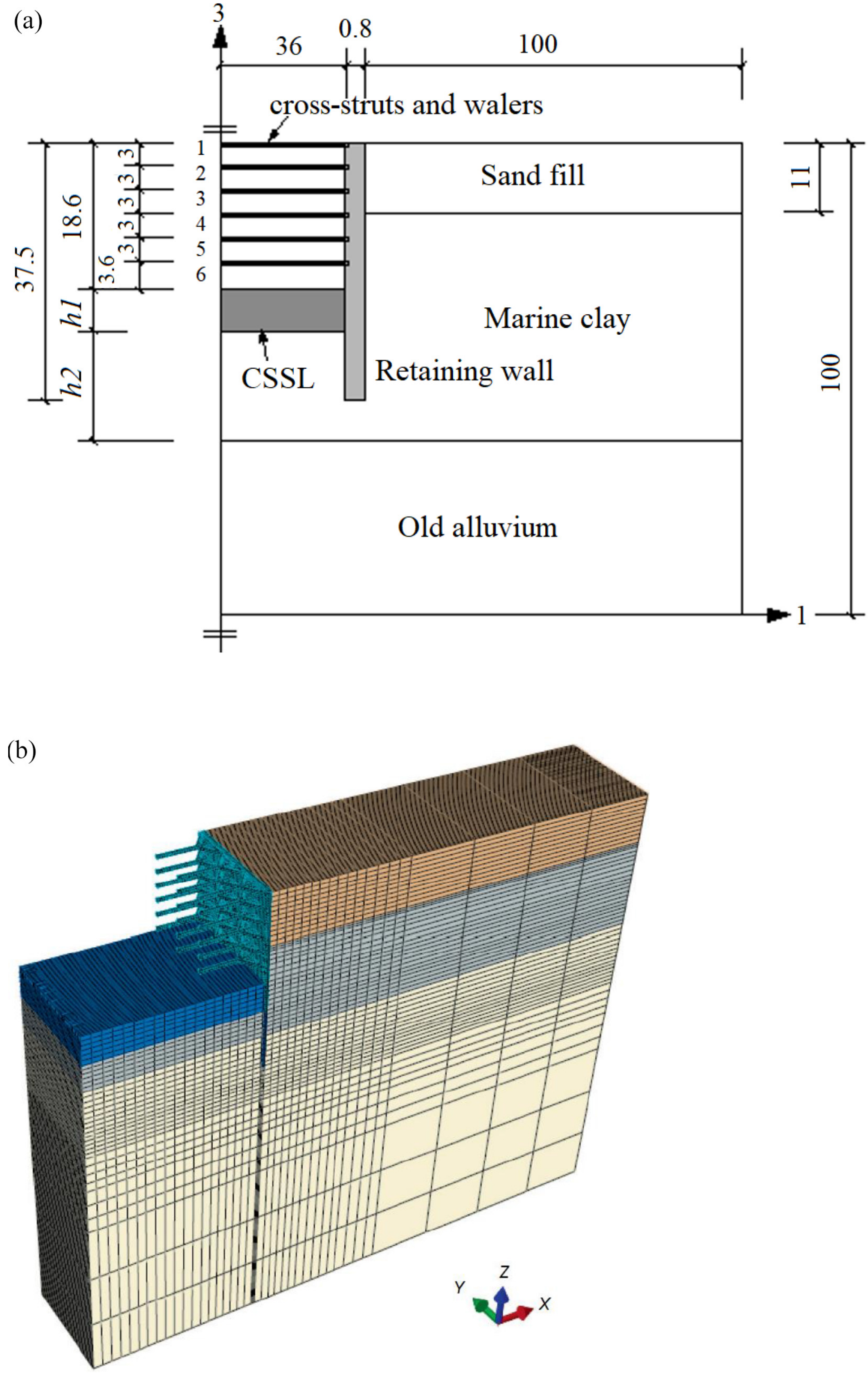


Fig. 3. Lateral cross section of basement excavation in Singapore.

independent sensitivity index δ_i is evaluated to identify the effect of any of an input X_i on the PDF of model output Y. According to the definition of the delta index (Borgonovo, 2007), the formulation of the computation is written as

$$\delta_i = \frac{1}{2}E_{X_i}[s(X_i)] = \frac{1}{2}\int s(X_i)f_{X_i}(x_i)\mathrm{d}x_i, \tag{10}$$

where $s(X_i)$ denotes the area difference between the unconditional PDF of output Y and its conditional distribution given the individual input X_i,

$$s(X_i) = \int \left|f_Y(y) - f_{Y|X_i}(y)\right|\mathrm{d}y, \tag{11}$$

where $f(\cdot)$ denotes the PDF. Note that $\delta_i \in [0,1]$, where 0 means input X_i has no effect on the PDF of Y and the contribution of all the inputs are the same to the PDF of Y when $\delta_i = 1$. Additionally, the computation of the delta index using the single-loop Monte Carlo simulation (Wei, Zheng, & Yuan, 2013) is employed. Based on Eq. (11), the form of δ_i can be further considered by

$$\begin{aligned}\delta_i &= \frac{1}{2}\left|f_{X_i}(x_i)f_Y(y) - f_{Y,X_i}(y,x_i)\right|\mathrm{d}y\mathrm{d}x_i \\ &= \frac{1}{2}E_{Y,X_i}\left(\frac{f_{X_i}(x_i)f_Y(y)}{f_{Y,X_i}(y,x_i)} - 1\right).\end{aligned} \tag{12}$$

So, for a group of input parameters $R = (X_{i1}, X_{i2}, \cdots, X_{in})$, similarly, we can obtain

$$\delta_i = \frac{1}{2}E_{Y,R}\left(\frac{f_R(x_{i1},x_{i2},\cdots,x_{in})f_Y(y)}{f_{Y,R}(y,x_{i1},x_{i2},\cdots,x_{in})} - 1\right). \tag{13}$$

In this paper, the PDF of output $f_Y(y)$is estimated with the kernel density estimator (KDE) method (Botev, Grotowski, & Kroese, 2010), while a bivariate KDE toolbox (Botev, 2015) is used to achieve the joint PDF of Y and R. Thus, the sensitivity index $\delta_i$is easily calculated by means of Monte Carlo simulation.

3 Example application

The braced excavation for the tunnel was conducted in the Marina Bay area of Singapore. The pit excavation, depicted in Fig. 3(a), consisted of three layers of material: sand fill, marine clay, and Old Alluvium. The marine clay was modelled using the Cam Clay model, while the sand fill and Old Alluvium were modelled as Mohr–Coulomb materials with effective stress parameters. The model shown in Fig. 3(b) is 24 m width along the Y direction. At the two surfaces, i.e., $Y = 0$ and $Y = 24$, vertical rollers were set, and only vertical movement was allowed.

In this study, the cement-treated soil was modelled as a Mohr–Coulomb material. The properties of the soil layers and cement stabilized soil layer (CSSL) used in this analysis are listed in Table 1. The height and width of the pit excavation were 100 m. The excavation had a total of six stages using the top-down construction method. The final excavation depth was 18.6 m. A retaining wall with a thickness of 0.8 m was supported by cross-struts and walers. The maximum movement of the retaining wall was 162.2 mm. The diaphragm wall was modelled as an elastic material with an equivalent Young's modulus of 10.5 GPa and Poisson's ratio of 0.2. The cross-struts and walers supporting the retaining wall were simplified to a rectangular section (400 mm×400 mm), with equivalent bending stiffness. This equivalent was derived by equating the product of Young's modulus and cross-sectional area for struts. The Young's modulus and Poisson's ratio for the walers were 47.5 GPa and 0.2, respectively. The groundwater table was assumed stable and located 1 m below the ground surface. The magnitude of the wall movement, as well as ground surface settlement, was measured with the FE package ABAQUS 6.13.

According to the conditions described above, a BN model, shown in Fig. 4, was constructed to analyze the failure state, i.e., safe or failure, during the excavation process. In this paper, we only considered the potential risk of the excavation process in the layer of marine clay, as this is where potential failure is most likely to occur. On this basis, we selected three key material parameters: k, M, and λ; where λ is the logarithmic hardening constant defined for the clay plasticity material behaviour, k is the logarithmic bulk modulus of the material defined for the porous elastic material behavior, and M is the ratio of the shear stress. These factors exert substantial influence on the displacement of the wall. The proposed network also considered the effect of different excavation depths. In the BN model, node H represented the varying height of the pit excavation from the ground surface, mainly from

Table 1
Properties of soils and cement stabilized soil layer (CSSL) (Lee et al., 2011).

Soil type	Marine clay	Sand fill	Old alluvium	CSSL
Bulk unit weight (kN/m^3)	16	19	20	16
Isotropic swelling index	0.093	–	–	–
Isotropic compression index	0.27	–	–	–
Critical state friction coefficient	0.87	–	–	–
Effective Poisson's ratio	0.3	0.3	0.3	0.2
Earth pressure coefficient	0.7	0.5	1.0	0.7
Coefficient of permeability	1×10^{-9}	1×10^{-6}	1×10^{-7}	1×10^{-10}
Void ratio	1.9	–	–	–
Friction angle (°)	–	30	37	41
Effective Young's modulus	–	10	130	272
Angle of dilation (°)	–	0	10	0
Effective cohesion	–	2	20	400

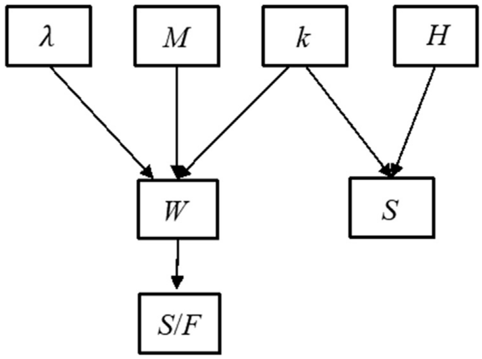


Fig. 4. The Bayesian network for braced excavation.

stages 4 to 6 of excavation in the marine clay, approximately 11.0–18.6 m, as this is a crucial factor in the stability of excavation. Given the difficulty of monitoring the wall deformation, ground surface settlement was incorporated as a field observation used to update the material parameters. Furthermore, the sensitivity analysis in the BN was used to identify the key objects to update.

This paper regards the whole process of excavation as a continuous process. We assumed that node H followed the uniform distribution between [11, 18.6] and other parameters were also defined with the known distributions, per Table 2. Based on the sensitivity analysis, k and H have the greatest effect on the deformation of retaining wall. So, only parameter k was selected as the update target used herein to demonstrate the proposed method. Moreover, the initial values of mean μ_k and standard variation σ_k for variable k were initially estimated with an interval from the limited information, though the true value is provided as a reference to validate the credibility of the outcome. Hence, they were assumed to be known with the exact values of 0.1 and 0.03, respectively, and were then compared with the updated results of μ_k and σ_k.

Given the preceding factors, the observed samples of ground subsidence were generated by Monte Carlo sampling from the input variables H and k in the built input-output model. The distribution parameters of variable k used the target mean and standard deviation listed in the 3rd column of Table 2. The size of the observation sample was set to 100.

4 Results

Before updating, the sensitivity indices of input parameters of wall movement were computed to identify the key factors. As shown in the last column of Table 2, the ranking of importance was $H > k > M > \lambda$. Given their uncertain influence on wall movement, we mainly consider k and H for the purpose of parameter updating. The dependency relation of nodes W and S with the causal nodes were quantified with the FE method, respectively. Then, the response of wall movement and ground subsidence on uncertain input parameters were predicted using the ANN method.

Figure 5 shows the regression plots of the retaining wall and ground surface settlement, and the results of the ANN prediction are detailed in Table 3. We note that both exhibit strong linear relationships with the input parameters. The associated mean square error for each parameter was also calculated. The model results indicate that the ANN models can be employed to predict wall defection and ground surface settlement. The next step used the input-output black-boxes, and then estimated the uncertainty quantification in the BN.

As the depth of excavation H is a random variable with aleatory uncertainty, only two statistic parameters of k as

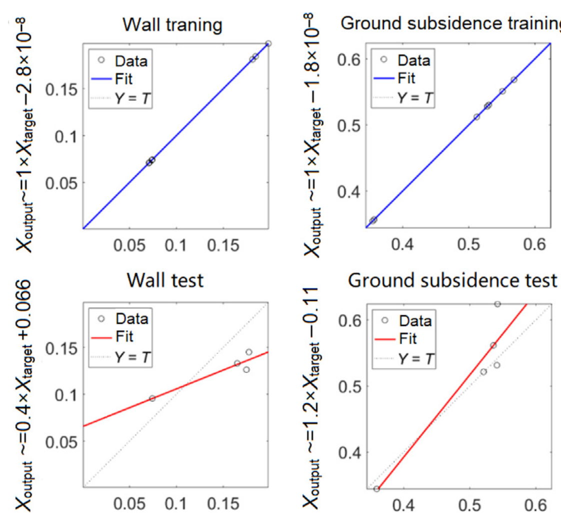


Fig. 5. Neural network training regression.

Table 3
The results of the prediction with the artificial neural networks (ANN).

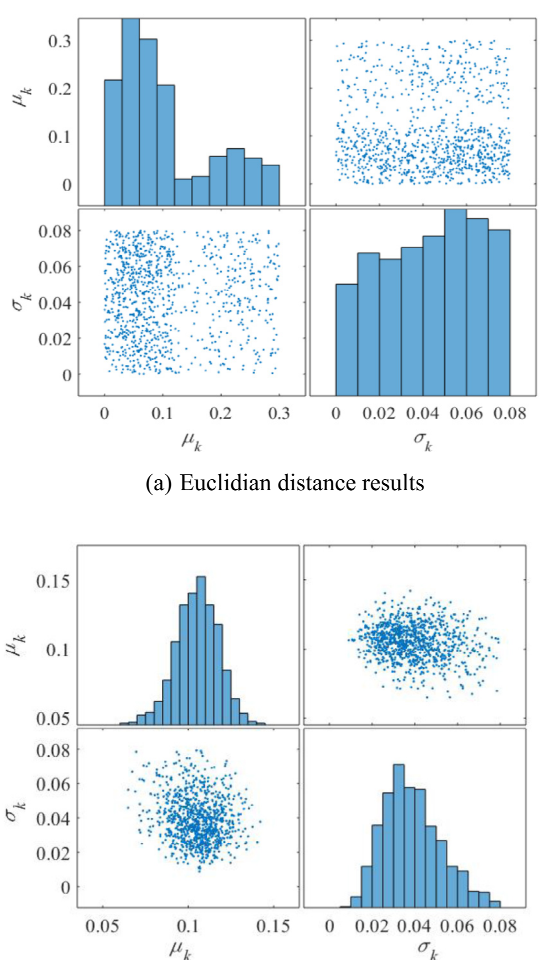
Parameter	MSE	R-square (%)
$W_{training}$	3.84×10^{-15}	99.99
$W_{testing}$	1.45×10^{-3}	93.85
$S_{training}$	2.71×10^{-15}	99.99
$S_{testing}$	1.54×10^{-3}	94.32

Table 2
Inputs of the parameters for the Bayesian network (BN) model.

Parameter	Prior distribution*	Target value of epistemic input	Sensitivity index, δ_i
λ	Normal, $\mu = 0.27, \sigma = 0.04$	–	0.1398
M	Normal, $\mu = 0.87, \sigma = 0.07$	–	0.1694
K	Normal, $\mu \in [0, 0.3], \sigma \in [0, 0.05]$	$\mu = 0.1, \sigma = 0.03$	0.3286
H	Uniform, $H \in [11, 18.6]$	–	0.3575

* μ and σ denote the distribution parameter mean and standard deviation, respectively.

inputs were used in the distance-based updating procedure. The observed values of variable S were defined as previously described, and used as monitoring data for variable S. In this example, the prior distributions of distribution parameters μ_k and σ_k were set with an interval, following the uniform distributions. Based on each distribution parameter, a group of prior values of variable S can be obtained by the model evaluation and comprise the predicted samples seen in Fig. 2. Subsequently, the distance metric for ABC updating can be computed based on Eqs. (8) and (9). After executing the first step of updating using the Euclidian distance metric, the posterior distributions of variable k's mean and standard deviation are shown in Fig. 6(a), where the proper width coefficient ε was set as 0.0015, and 13 TMCMC iterations were executed to reach the convergence. Table 4 shows the mean values of their posterior distributions. Comparing with the prior uniform distribution, the posterior distribution of μ_k accurately converged to the target value, where the updated value of μ_k was 0.1067.



(a) Euclidian distance results

(b) Bhattacharyya distance results

Fig. 6. The posterior distribution of distribution parameters in variable k after updating with distance-based approximate Bayesian computation (ABC).

Table 4
Values of posterior distributions.

Parameter	μ_k	σ_k
Target value	0.1	0.03
Updated with Euclidian distance	0.1067	–
Updated with Euclidian distance	0.1052	0.0381

From the histogram plot of σ_k in Fig. 6(a), we observe that the posterior distribution of $\sigma_k$was still nearly uniform. This indicates that the standard deviation of variable k was incapable of updating using the Euclidian distance metric. Therefore, further updating based on Bhattacharyya distance must be conducted.

In the second update step, six TMCMC iterations were implemented with a width coefficient of $\varepsilon = 0.08$ in the ABC update process. As seen in Fig. 6(b), the posterior distributions of μ_k and σ_k are dramatically more peaked than those in Fig. 6(a), while both remained close to their respective targets ($\mu_k = 0.1$ and $\sigma_k = 0.003$). In the last row of Table 4, the updated values μ_k and σ_k are 0.1052 and 0.0381, respectively. Thus, the second update step reduced the discrepancy between the initial sample and the observation sample. This also demonstrated that the Bhattacharyya distance improved prediction accuracy, where the update results from the Euclidian distance were used as the prior distribution of input in the second update step. Thus, the distributions of μ_k and $\sigma_k$were distinctly more centralized to the target values. Using the soil parameter update results, the corresponding failure probability of the excavation was then obtained accordingly in the network. The failure probability of the braced excavation can be obtained in real-time within a few seconds using the BN software. In this paper, OpenCossan (Edoardo, Broggi, Tolo, & Sadeghi, 2017) software was used to execute the computation. The failure probability of the examined excavation was determined to be 95.02%. The updated failure state was provided to the appropriate decision-makers.

5 Summary

This paper introduced a novel framework for incorporating a Bayesian network with a distance-based ABC real-time update method. The proposed framework estimates real-time failure probabilities using ground surface settlement as observation data collected during the excavation process. The distanced-based ABC approach updates the soil parameters for the model using an input-output black-box. This update method overcomes the limitation of the complex likelihood function and proved effective in reducing the discrepancy between the updated soil parameters and pre-design. Both Euclidian and Bhattacharyya distance-based ABC update methods are computed in the example. The results in the example demonstrate that the mean value of the distribution of a soil parameter approximates the true value using the distance-based ABC updating. However, for the variance of this soil parameter, only the updated value that used Bhattacharyya distance-based ABC was close to the target value. That being said, the prior distribution is also a key factor in the accuracy of this update approach, and must be defined reasonably.

Given the reduced number of calculation points, moment-independent sensitivity analysis applied prior to updating can provide the vital information about which factors exert the greatest influence on the safety or failure state of the excavation. Furthermore, the sensitivity analysis approach captures the dependency relations amongst the nodes in the network. Moreover, it is especially suitable for estimating the variables in the large structure of the BN. With the ranking information, the key parameters can be selected to link with the monitor parameters in the BN model. Based on the updated soil parameters, the uncertainty of the induced-factors is then captured by the BN model. The real-time updated probabilities of the failure in the excavation process can then be estimated, the results of which provide valuable information to support decision-makers.

Conflict of interest

The authors declare that there is no conflict interest.

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