Management and
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THESIS EXTRACT

Tamás Nagy

Real option decision model applications for a European Union Emissions Trading System participant gas-fueled power generator

Ph.D. Dissertation

Supervisor:

Dr. Sándor Kerekes
Full Professor, Doctor of HAS

Budapest, 2013

Department of Environmental Economics and Technology

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I. PREVIOUS RESEARCH AND TOPIC CHOICE

The developed countries ratifying the Kyoto Protocol have committed to reduce their greenhouse gas emissions. The European Union established the European Union Emissions Trading System (EU ETS) in order to reach its reduction target in a cost efficient way. In the system the maximal emission is fixed (capped); in the current phase of EU ETS, in every year the participants receive free of charge roughly 2 billion tonne carbon-dioxide emission quotas (allowance, EUA) from the authorities. The quotas are distributed between approximately ten thousand companies. The participants have to comply with the regulation in every year by giving back the same amount of emission quotas as their actual emissions. During the year the participants are free to trade allowances, which in case of low transaction costs results emission reduction at the minimal cost.

When receiving the free allocations, lots of the participants have faced with the following problems: how many emission quotas do they need to hold for covering their future emissions, how many quotas should be sold or purchased. In the spring of 2006 the market actors witnessed a huge price collapse, when EUA lost two thirds of its value in 8 days (24, April 2006: 29.43 EUR/ton, 2nd of May 2006: 10.90 EUR/ton). The reason of the huge devaluating was that the actual emission quantity data became public, and the overallocation proved to be more and more probable. The price decrease meant huge loss for the participants with surplus quotas, and big profit for the companies with short positions.

Compared to the financial sector, the most significant participants – the power generators – lack the effective risk management tools. The objective of Dissertation is to provide practical tools for the EU ETS participant power firms by melding together the environmental economic, corporate finance, real option and stochastic finance toolsets suitable for solving decisional, evaluational and financial risk management problems.

The answers for the following questions were looked for:

- Based on what method can the expected quantity of CO2 emissions and its probability density function be forecasted? How sensitive are the results for changing different technological and market factors?
- How much cost does the compliance cause for an EU ETS participant? How many emission quotas are needed to be held by the generator for minimizing its

compliance risk, with other words what is the optimal quantity of quotas at a given time point?
- How sensitive is the financial value of the power generator for changing different technological and market factors? What type of hedging process should be applied to minimize the market risks?
- How can a thermal efficiency improvement investment be evaluated? What are the most important market factors changing the value of this investment?
- Why and how big loss will a power generator suffer if it fulfills its long term production contracts by constant operation?
- How can the MNPB (Marginal Net Private Benefit) and the private demand function be calculated regarding emission quota? The earlier has importancy from environmental economic theoretical viewpoint, the latter has potential role when forming the auction strategy of a power generator for the emission quotas.

The relevant literature is connected to the emissions trading system. Among the Hungarian authors, Dobos (2002) analyzed the effect of tradable permits with a comparative statical model for a standard microeconomical, profit-maximising company. In their dissertation Lesi and Pál (2004) analyzed the efficiency of emissions trading system and its effect for Hungarian power generators. Fazekas (2009) researched the effect of EU ETS to the Hungarian participants based on interviews.

Publications, with subjects being closer to the Dissertation, model the price of emission quota as a stochastic variable. Daskalakis et al. (2009) tested the fitting of different price models based on price data from the first phase (2005-2007). Seifert, Uhrig-Homburg and Wagner (2008) analyzed the matureness of the emissions market by using an equilibrium stochastic model. Reilly and Paltsev (2005) estimated the expected quota price with the EPPA-EURO (Emissions Prediction and Policy Analysis) model. Alberola et al. (2008) analyzed the First Phase by using econometrical method. They stated that the price of electricity, the extreme weather events and the political and policy decisions are the most significant factors influencing the price of EUA. Benz and Trück (2009) divided the price influencing factors into two separate categories: policy factors affecting the price in longer term and fundamental factors influencing the price in shorter time horizon. Mansanet-Bataller et al. (2007) analyzed the effect of financial and weather factors to the price of emission quota. Oberndorfer (2009) and Veith et al. (2009) examined the correlation between carbon-dioxide price and share prices of European power generators. Kanen (2006) analyzed the connection between prices of emission quota and energy resources. He concluded that the oil determined the price of natural gas, and this latter influenced both of the electricity and emission quota prices. Convery et al. (2007) have found that the carbon-dioxide price is mainly driven by the price of energy resources.

Number of authors used real options[1] when modeling decision of power generators. Laurikka (2006) has created a stochastic simulation model with real options, and analyzed the effect of EU ETS for an integrated gasification combined cycle power generator. The author concluded that in case of EU ETS the traditional DCF formula is not appropriate for evaluation, because EU ETS has lots of risk factors and real option situations. Hlouskova et al. (2005) applied a real option model for a power generator operating in liberalized energy market. They used the model for evaluation of generator, and for determining the distribution of profit and loss. Their model did not contain the cost of emission permits, but they have taken into account the technological constraints (e.g. minimum operating time, capacity constraints, starting and breaking costs). Herbelot (1994) modeled the decisions of power generators. The author used binomial model for calculating the value of changing the burning fuel to a lower sulfur content coal, the value of an end-of-pipe scrubber installation, and a gasification block. He analyzed the effect of different factors to the value of real options. Abadie and Chamorro (2008) analyzed a coal burning power generator with the possibility to invest into carbon capture and storage (CCS) technology. The two stochastic variables were the price of emission quota and electricity. The authors used two-dimensional binomial model to detect the optimal investment decision. Cragg et al. (2011) used three underlying model for an emissions trading participant company. They showed that the power generator could reduce its risk significantly, if emission quota was added to the traditionally two instrumental (electricity and gas) hedging process. As a result, the standard deviation of the profit was decreased significantly.

The Dissertation would like to contribute to the fields of real option publications in connection with EU ETS. The new results are partly concentrated on the deduction of probability density function of quantity of emission, and the forecasting of compliance cost. In addition to this the value of power generator is calculated based on a four underlying instruments, containing the off-peak and peak electricity prices. The

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[1] About real option models in generally see Dixit és Pindyck (1994) and Bélyácz (2011)

advantage of using intraday power prices is that the model becomes closer to the real situation, in what the gas turbine dominantly operates in the peak hours and rest in off-peak ones. In connection with the generator evaluation, the value of a theoretical 5 percentage point thermal efficiency improvement investment was calculated and the effect of different technological and market factors was also analyzed. Based on the real option model the potential loss caused by constant production of generators was examined. The actuality of this subject is that a number of Hungarian gas fueled generators operate according to a fixed schedule. By having long term production contracts with potentially high selling price, the constant operation seems to be not just risk minimizing but very profitable for the operators. But in this case the generator looses the option value of flexibility. Finally, by using the real option model, the MNPB (Marginal Net Private Benefit) function was deduced, which has a key role in environmental economics. In connection with this, the demand curve of emission quota was also deduced, which can serve as a basic tool for determining the efficient auction strategy for quota. The actuality of this is that from 2013 the currently mostly free allocation will gradually be replaced by auctions, resulting a more complicated decision environment for the power generators.

II. METHODS USED

II.1. The real option decision model

In the Dissertation, a real option model is used for forecasting decisions, emissions, and the realized profit. This type of model should be applied when the outcomes are stochastic, when the probabilities of different outcomes are given or can be calculated and/or when the future decisions are linked together. In these cases the traditional DCF (discounted cash flow) method fails to provide appropriate value.

The power generators are conditional conversion assets, which – depending on the production decision – operate and transform gas and emission quota into electricity conditionally. From the viewpoint of short term profit maximizing, the fix costs are not relevant. The variable costs are divided into three parts, and the spread (margin) can be calculated as follows:

Spread = Revenue of produced electricity – Costs of burning fuel – Cost of emission quota – Other variable costs 1.

The η thermal efficiency of power generator shows the ratio of output and input energy. Its value is between 0% and 100% (the higher number indicates the more efficient generator). δ is the carbon intensity of burning fuel, indicating the quantity of carbon-dioxide (tCO2/MWh) emitted by the burning of the given fuel. If we denote S_{pow} (EUR/MWh$_{out}$) the electricity price, S_{gas} (EUR/MWh$_{in}$) the gas price, S_{eua} (EUR/tCO2) the emission quota price and v the other variable cost, then the spread of 1 MWh unit of produced electricity can be calculated as follows:

$$spread = S_{pow} - S_{gas}/\eta - S_{eua} \cdot \delta/\eta - v \qquad 2.$$

In the formula, the fuel cost of one MWh output energy is calculated as the gas price divided by the thermal efficiency. In case of emission quota cost, the δ/η multiplier shows the quantity of CO_2 emission of 1 MWh produced energy.

The formula of spread is very similar to the so called *clean sparks spread* (about different sort of spreads see details in (Alberola – Chevallier – Cheze, 2008)), with such differencies as the given technological factors (thermal efficiency, carbon intensity of fuel burning) of the modeled generator, and that the formula contains also an other variable cost.

The profit maximizing company operates and emits only if the spread is positive. If it is negative, then the resting of capacity is rational. The π profit per 1 MWh unit of produced electricity can be calculated by the following conditional formula:

$$\pi = max(\,spread, 0) = max\left(S_{pow} - S_{gas}/\eta - S_{eua} \cdot \delta/\eta - v, 0\right) \qquad 3.$$

Thesis 1: For an EU ETS participant power generator, the conditional value of spread per unit of electricity produced corresponds to the payoff function of a three underlying spread option with exercise price of v. By using the option analogy we can apply stochastic finance toolset for solving different decision, valuation, and modeling problems.

In reality the electricity cannot be stored efficiently. In practice, the production is adjusted to the actual and expected demand, resulting different electricity prices for every hour. Strong seasonality can be detected in price process of electricity (Marossy, 2011). In a given day, the price of electricity varies according to the economic activity: in daytime the high demand causes high price, in more calm night hours the low level of activity causes low electricity prices. For the better approximation of a real power generator, the three-underlying model (using power, gas and emission quota) was widened to four-instrument model, in which the total day was divided into two separate parts: the peak hours covers time between 8:00 and 20:00, the off-peak hours covers the low demand time between 20:00 and 8:00. The advantage of four instruments model is that we can approximate the real life behavior of the gas turbine better, in which the generator dominantly operates in peak hours, and rests in off-peak hours.

In the model a 100MW open cycle gas turbine was supposed, with thermal efficiency (ratio of output and input energy) of 38% (Commission of the European Communities, 2008). The natural gas used as a burning fuel had carbon intensity of 0.2014 tCO2/MWh, the other variable cost was 3 EUR/MWh. The technological constraints (minimum up- and down time, etc) were neglected.

II.2. Stochastic model of underlying instruments

In the real option model the future spread is stochastic, the four underlying instruments are assumed to follow geometric Ornstein-Uhlenbeck process (also known as one factor Schwartz model (Schwartz, 1997)):

$$dS = \lambda(\theta - lnS)Sdt + \sigma Sdz \qquad 4.$$

The stochastic model was fitted to market prices originating from EEX. The analyzed time period was 28/02/2008 – 31/05/2012. Only those days were taken into account, on which all four underlying instruments had price. Since emission quota has prices for workdays, therefore totally 1010 observation days were taken into account, and the time between days was supposed to be $\delta = 1/252$ year. Further smaller corrections had to be made because of using a log-model: 4 days with negative prices were skipped. The parameters of fitted model were the following:

	Off-peak	**Peak**	**Gas**	**EUA**
R^2	0.3574	0.5303	0.9934	0.9929
S(0)	38.8167	67.6667	23.4700	6.2600
exp(μ)	46.5685	68.0518	21.7829	6.8357
λ	129.6231	79.9205	0.8251	0.2804
σ	5.3291	4.1001	0.4545	0.4375

Table 1: The regressed parameters of geometric Ornstein-Uhlenbeck process.

The determination coefficients of regression (R^2) regarding off-peak and peak electricity prices are low, but in case of inputs (gas and emission quota) they are significantly high. By assuming geometric Ornstein-Uhlenbeck process, it was possible to use analytical approximation formulas resulting faster calculations.

The correlation coefficients between the four underlying instruments and the result of their hypothesis testing[2] were the following:

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[2] Null hypothesis stated that correlation coefficient was zero; alternative hypothesis was that the coefficients were not equal to zero.

Correlation	**Off-peak**	**Peak**	**Gas**	**EUA**
Off-peak	1.0000	0.4830	0.0190	-0.0192
Peak	0.4830	1.0000	0.0275	-0.0051
Gas	0.0190	0.0275	1.0000	0.1655
EUA	-0.0192	-0.0051	0.1655	1.0000

p-value	**Off-peak**	**Peak**	**Gas**	**EUA**
Off-peak	1.0000	0.0000	0.5481	0.5439
Peak	0.0000	1.0000	0.3845	0.8717
Gas	0.5481	0.3845	1.0000	0.0000
EUA	0.5439	0.8717	0.0000	1.0000

Table 2: The correlation coefficients between residuals and the p-values of hypothesis tests for correlation.

There was a strong correlation between off-peak and peak electricity prices (0.48, the p-value was 0, the coefficient was significant). The reason behind positive correlation could be, that few demand and supply factors of electricity belonging to a given day affect both electricity prices parallel. The absence of perfect correlation showed the importance of intraday factors. Between gas and emission quota there was a still significant, but lower positive correlation (0.17). The reason behind could be, that in case of gas price increasing, the share of power generator using higher carbon-intensity fuel (such as coal) was risen causing raise of required emission quantity and price. The other correlation coefficients were not significantly different from zero, therefore their interpretation was less reliable.

II.3. Pricing of spread options

Calculation of expected emission and value of power generator requires pricing of European spread options, which is more complex than pricing of plain vanilla options (about vanilla calls and puts see in details (Hull (1999, pp. 301-303), Benedek (1999) and Száz – Király (2005)). The biggest problem in pricing spread options is that the sum of lognormals are not lognormally distributed and cannot be described by closed analytical formulas. The problem of sum of lognormals is significantly researched by mathematicians, engineers, financial academics and experts for more than 50 years (Fenton, 1960). It is notable that the Modern Portfolio Theory (Markowitz, 1952) approximates the resulted distribution by normal density function, which can be far from the real distribution.

Closed formula for the price of European spread option exists only in case of two underlying instruments with zero exercise price (Margrabe, 1978), for more general cases we lack the exact analytical solution. Kirk's formula (1995) approximates price for a European spread option of two underlyings with non-zero exercise price. Carmona and

Durrleman (2003) invented a precise, but hard to implement pricing algorithm. Milevsky and Posner (1995) approximated the density function of a portfolio by reciprocal gamma distribution. Borovkova, Permana and Weide (2007) used negatively shifted lognormal density function for pricing European basket options, which allows negative portfolio values, therefore it can also be used for pricing options on spread.

The problem with the early solutions is that they provide result for spreads of two underlyings in very special cases and they are inaccurate for baskets containing few assets. Deng et al. (2008) deduced an analytical solution for multi-asset European spread option providing accurate and fast result. It the Dissertation their method was used for calculating option prices and expected payoffs.

In the real option model the risk neutral pricing of option is not relevant, because the market is not complete: the quantity of emission in the future is not traded instrument; and we cannot buy or sell arbitrary portion of a given power generator[3] with low transaction costs. Therefore in the evaluation not the risk neutral but the physical measure was used[4].

Besides analytical approximation, Monte Carlo simulation was carried out when analytical calculation could not be used. The probability density function of emission quantity could not be deducted by using analytical pricing formulas (which calculate present values of expected payoffs), it required simulation. In the background of multidimensional Monte Carlo simulation stands the generation of multidimensional normal distributions with a given correlation structure (see in details Glasserman (2003, p. 65) and Nagy (2011a)).

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[3] There are electricity producer companies traded in the exchange, but they hold more than one power generator, and/or there are also other assets in their balance sheets.

[4] About considerations of risk neutral and physical measure, see in details Medvegyev (2009).

III. RESULTS OF THE THESIS

III.1. Estimating of CO_2 emission

By neglecting the technological constraints (e.g. minimum up- and down time), the profit maximizing power generator operates only if the spread (calculated based on the prompt prices in the future) is positive. While the emission is a linear consequence of production, therefore the emission can also be deducted based on the spread. Let us use a Bernoulli binary (0/1) variable (Λ) for denoting production or resting of capacity. In case of positive spread the generator realizes profit if it operates; as a consequence the power plant is turned on ($\Lambda = 1$) and carbon-dioxide is emitted. In case of negative spread the operation would cause loss, the generator is turned off ($\Lambda = 0$) and it does not emit any emission.

Thesis 2.1.1: The conditional expected value of binary $\boldsymbol{\Lambda}$ production decision variable related to a future $\boldsymbol{\tau}$ day can correspond to the payoff function ($\boldsymbol{bno^{PO}}$) of a European binary spread option with $\boldsymbol{\tau}$ maturity and $\boldsymbol{v}$ exercise price.

If $\boldsymbol{S}(0)$ vector stores the initial prices of underlying instruments, $\boldsymbol{w}$ vector contains the weights, the formulas are the following:

$$\Lambda_{\text{peak}}(\tau) = bno_{peak}^{PO}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right) = \begin{Bmatrix} 1 & ha\ \boldsymbol{w'} \cdot \boldsymbol{S_{peak}}(\tau) > v \\ 0 & in\ other\ cases \end{Bmatrix}$$

$$\Lambda_{\text{off-peak}}(\tau) = bno_{off-peak}^{PO}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) =$$

$$= \begin{Bmatrix} 1 & ha\ \boldsymbol{w'} \cdot \boldsymbol{S_{off-peak}}(\tau) > v \\ 0 & in\ other\ cases \end{Bmatrix}$$

$$\boldsymbol{w} = \begin{bmatrix} 1 \\ -1/\eta \\ -\delta/\eta \end{bmatrix}, \boldsymbol{S_{peak}}(\tau) = \begin{bmatrix} S_{peak}(\tau) \\ S_{gas}(\tau) \\ S_{eua}(\tau) \end{bmatrix}, \quad \boldsymbol{S_{off-peak}}(\tau) = \begin{bmatrix} S_{off-peak}(\tau) \\ S_{gas}(\tau) \\ S_{eua}(\tau) \end{bmatrix} \qquad 5.$$

Thesis 2.1.2: The probability of the production in a future $\boldsymbol{\tau}$ day equals to the expected value of binary production decision variable $\boldsymbol{E[\Lambda(\tau)]}$, which corresponds to the expected payoff ($\boldsymbol{E[bno^{PO}]}$) of a binary spread option:

$$P\left(spread_{peak}(\tau) > 0\right) = P\left(spread_{peak}(\tau) > 0\right) \cdot 1 + P\left(spread_{peak}(\tau) \leq 0\right) \cdot 0 =$$

$$= \mathrm{E}\left[\Lambda_{\text{peak}}(\tau)\right] = \mathrm{E}\left[\mathrm{bno}_{\text{peak}}^{\text{PO}}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)\right]$$

$$P\left(spread_{off-peak}(\tau) > 0\right) =$$

$$= \mathrm{E}\left[\Lambda_{\text{off-peak}}(t)\right] = E\left[\mathrm{bno}_{\text{off-peak}}^{\text{PO}}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right)\right] \qquad 5.$$

If the daily maximal capacity is denoted as Γ, then the power generator emits carbon-dioxide at amount of $\Gamma \cdot \delta/\eta$ in case of production.

Thesis 2.1.3: The expected emission in a given future τ day is equal to the expected payoff of binary spread option multiplied by $\Gamma \cdot \delta / \eta$ daily maximal emission. In the four underlying model the expected $Q_c(0,T)$ cumulated emission for a longer time interval from the present to time point T can be calculated based on binary options as follows:

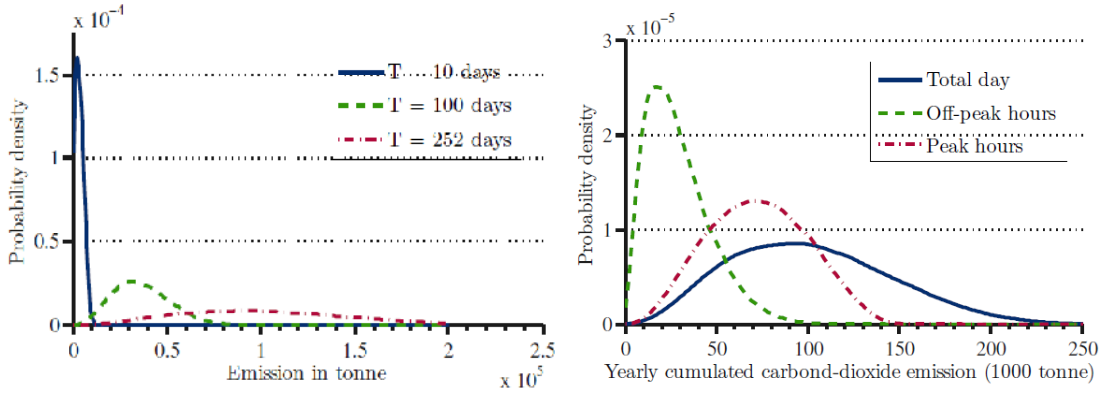
$$E[Q_c(0,T)] = \Gamma \cdot \delta/\eta \cdot \sum_{\tau=0}^{T} \frac{E[bno^{PO}(S_{off-peak}(0),w,v,\tau)]+E[bno^{PO}(S_{peak}(0),w,v,\tau)]}{2} \qquad 6.$$

The probability density function of emission

In addition to the expected emission, the probability density function of emission quantity can also be deduced from the real option model. We can calculate the Ω cumulated decision variable by cumulating realization of Λ decision variable.

Thesis 2.2.1: The probability density function of the cumulated emission from present to time point T can be approximated by a histogram of Ω cumulated production decision variable (calculated by cumulating the simulated Λ binary production decision variable from *0* to *T*) multiplied by the daily maximal potential emission.

The resulted probability density functions for given time periods and for different part of the days are the folowings:

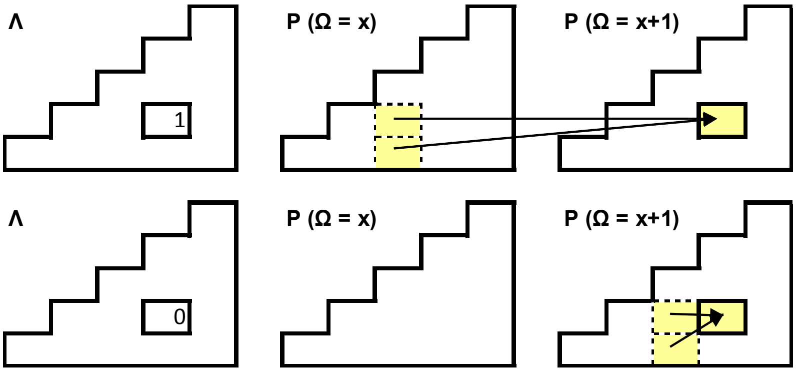


1. Figure: The Kernel probability density functions of cumulated emission belonging to different lengths of time (left) and for different parts of the day (right).

The distribution of cumulated emission at the beginning of modeling period is skewed right: the probability of zero emission is exceptionally high. The reason for skewness is that the emissions of consecutive days are highly correlated. The initial spread is negative, the power generator does not operate at the starting day, and it is very probable that the spread will be negative on the consecutive few days, resulting absence of production. As the time goes by, the expected number of days with production increases, the probability density function becomes more symmetric and flatter. Based on the graph showing probability density function (pdf) of yearly emission for a given part of the day (peak/off-peak hours) it can be stated, that on off-peak hours the low emission level is probable, the pdf is skewed right. In higher demand peak hours the electricity prices and the resulted spread is higher, therefore the power generator operates more frequently, the resulted pdf becomes more symmetric.

In the Dissertation an alternative method for deducting the pdf of cumulated emission is also presented. The quantity of cumulated emission is path dependent: in addition to the given value of spread, the paths of historical spreads are also important. The usual binomial tree methods generally calculate backwardian the present value of cash flows. Here the conditional probabilities (related to a given cumulated emission level) of the derived emission process should be calculated for a future time point.

Thesis 2.2.2: If the spread is modeled by a one dimensional process, the probability density function of cumulated emission can be deducted by using interconnected binomial trees. In the method, the probability tree of spread process is split into sub-trees related to the given cumulated emission levels. The probabilities in sub-trees are interconnected in a special way: if the spread value at a given state implies production, then the probability for this state in a given sub-tree have to be calculated based on probabilities of previous states from the sub-tree one cumulated emission level bellow. If the spread of given state implies zero emission (resting of capacity) then the probability has to be calculated based on the previous probabilities from the same sub-tree. The following figure illustrate the connection between probability sub-trees:



2. Figure: The interconnection between the sub-probability trees in two different cases (up: Λ=1, down: Λ=0).

The probability density function can be deducted by summing the probabilities of last steps in every sub-tree and mapping the resulted values to the given cumulated emission level.

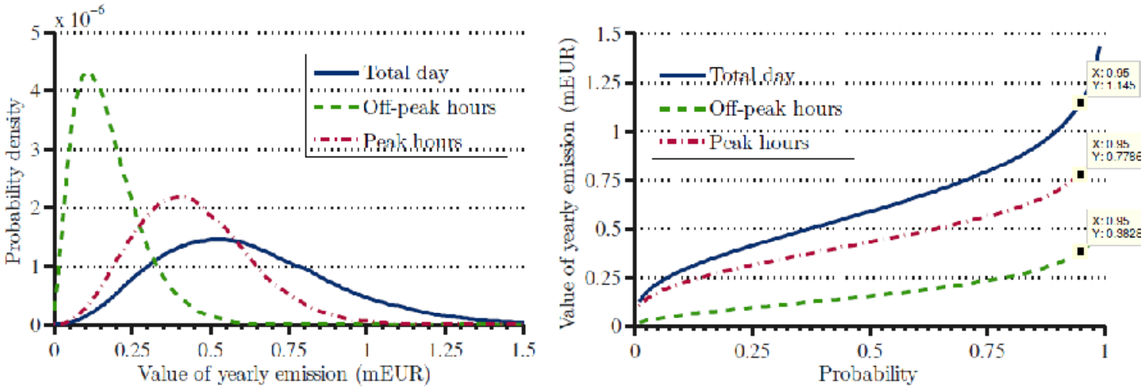
In the appendix of Dissertation the method is explained in details through an example.

Determining the cost of compliance

The power generator has to comply with the EU ETS rules: the same amount of emission quotas equal to the yearly actual emission are needed to be given back to the authority.

Thesis 2.3: The value of quotas necessary to cover the future emission − the cost of compliance − can be determined based on the real option model. Its value depends on the quantity of emission and the price evolution of emission quota.

Based on the probability distribution function of compliance cost, the value at risk (VaR (Jorion, 1999, p. 97)) can be determined showing the maximal cost required to cover the emission by quotas at a given confidence level:



3. Figure: The probability density function of compliance cost, its inverse distribution function, and the points belonging to the 95% probability.

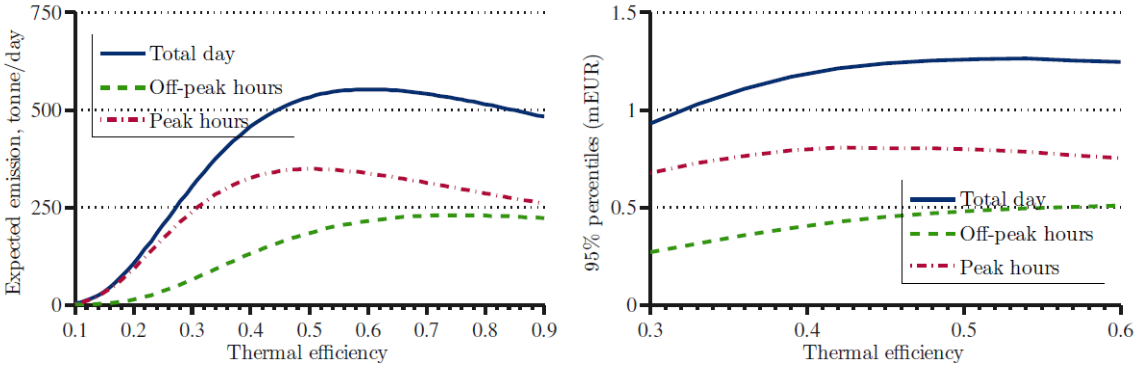
For the power generator the cost of covering the yearly emission requires less than 1.15 million Euro at significance level of 95%.

Sensitivity analyses for emission

The real option model helps to analyze the effect of different technological and market factors to the emission level.

Thesis 2.4: The daily maximal emission ($\Gamma \cdot \delta/\eta$) is inversely proportional to to the thermal efficiency if the Γ daily maximal capacity is fixed. The improvement of thermal efficiency apparently decreases the emission. Based on the real option model a reverse effect can be detected: the increasing thermal efficiency raises the spread by decreasing the necessary quantity of inputs, therefore the probability of production increases, resulting higher emission.

The changing of thermal efficiency modifies the expected quantity of emission as follows:



4. Figure: The expected emission (left) and 95% VaR of compliance cost covering the yearly emission (right) as a function of thermal efficiency.

In addition to the thermal efficiency, effects of different market factors were also analyzed such as: initial prices of underlying instruments, their long term averages, volatilities and correlation coefficients.

Thesis 2.5: Based on the real option model it can be stated that the initial price of electricity had no major effect for the emission level and compliance cost. The reason is that the fitted model volatility and mean reverting rate of electricity was very high, therefore the effect of initial price changing vanished quickly. The initial value of less volatile and slower mean reverting gas price had profound effect for

emission: in case of low gas price the expected emission gets its maximum level (showing constant production), high gas price resulted reduced expected emission. The changing of emission quotas' price had fifth the effect to the spread as the gas price: in the formula of spread, both of gas and emission quota price are divided by thermal efficiency, but the price of emission quota is further multiplied by carbon intensity of gas (0.2014 $tC0_2/MWh_{in}$). The level of emission was not really sensible to the changing of initial price of quota, but the compliance cost changed significantly: in case of high quota price the compliance cost was high, low quota prices resulted decreasing in cost of covering the emission. The compliance cost was very high in such a case when gas price was low, and the emission quota price was high. This situation can happen when the prices of burning fuels drop, and the authorities significantly reduce the total emission cap.

III.2. Evaluation of power generator

The financial value of power generator can also be deducted by real option model. The conditional value of π profit for the peak and off-peak hours of a given τ day in the future can be approximated by the spo^{PO} payoff function of a European spread option where $\boldsymbol{S}(0)$ is the vector of initial prices, $\boldsymbol{w}$ is the weight vector (in which the first instrument has positive, the others have negative weights), v is the exercise price and τ is the maturity:

$$\pi_{off-peak}(\tau) = spo^{PO}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right)$$
$$\pi_{peak}(\tau) = spo^{PO}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right) \qquad 7.$$

If spo^{Pr} denotes the price of the spread option, then present value of $E[\Pi(\tau)]$ expected profit for a τ future day can be calculated by the arithmetic average of two option prices multiplied by Γ daily capacity:

$$PV\left[E[\Pi(\tau)]\right] = PV\left[E\left[\Gamma \cdot \frac{spo^{PO}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + spo^{PO}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)}{2}\right]\right] =$$

$$= \Gamma \cdot \frac{spo^{Pr}(S_{off-peak}(0),w,v,\tau)+spo^{Pr}(S_{peak}(0),w,v,\tau)}{2} \qquad 8.$$

Thesis 3.1: By using option analogy, we can deduct the financial value of a power generator. Its value equals to the sum of present values of the expected realized spreads in its *T* lifetime; which can be calculated in the four instrument model based on the cumulated sum of spread option values multiplied by half of the daily capacity:

$$\boldsymbol{V = \frac{\Gamma}{2} \cdot \sum_{\tau=0}^{T}\left[spo^{Pr}\left(S_{off-peak}(0), w, v, \tau\right) + spo^{Pr}\left(S_{peak}(0), w, v, \tau\right)\right]} \qquad \mathbf{9.}$$

In the evaluation 30 years long lifetime was supposed for the power generator, and that the maintenance cost and repairing investments were contained by v other variable cost (which is the exercise price of options), and the effects of taxes were neglected. The model value of the power generator was 97.3 million Euros.

The spread options used in deduction of power generator value are sensitive to changing of different technological and market factors. Therefore their effects were analyzed for detecting the most relevant factors for the value of spread options and power generator. The *ceteris paribus* changing of thermal efficiency, initial prices and long term means of underlying instruments, volatilities and correlation coefficient between gas and emission quota were analyzed.

Thesis 3.2: Based on the real option model, it can be concluded that the increasing of thermal efficiency raises the value of power generator roughly linearly. 1 percentage point change increases the value of the modeled generator by approximately 5 million Euros. In the model the generators with 37-40% thermal efficiencies are the most sensitive to the efficiency improvement.

The initial prices of both peak and off-peak electricity had no profound effect for the generator's value, because their high volatilities and the fast mean reversion rates attenuate the effect of initial prices quickly. The increasing of initial prices of inputs (and particularly the gas) decreases significantly the value of the power generator. Assuming the given technological parameters, the effect of gas price is roughly five times higher than the effect of emission quota.

The long term means of prices have significant effect on the value of power generator. In case of electricity, the increasing of long term averages raises the value, in case of inputs, increasing of their means decreases the value of power generator. The increasing of volatility raises the value of European spread options and the value of power generator, most significantly in case of gas price.

In the fitted model the first significant correlation was the one between off-peak and peak electricity prices. It has no effect to value of generator, because the two electricity prices are existing in different spread options. The other significant correlation was the one between gas and emission quota price, which theoretically influences the value of power generator because they are existing in the same option pricing formula. Based on the model it can be stated that the correlation between gas and emission quota has no significant effect to the value of power generator.

Hedging of the power generator, the optimal quota position

If we do not want to expose the power generator to unnecessary market price risk, then it should be hedged against price movements. This can be done by applying delta hedge, trying to zero out all four delta parameters by having hedging positions. While the volatilities of electricity prices are especially high, and the most significant initial price is the gas price, it is advisable to hedge at least against the gas price changing.

Thesis 3.3: The four–instruments dynamic delta hedging strategy provides solution for the problem of optimal emission quota quantity: in a given future time point the delta parameter of emission quota for the rest of the year indicates the amount of EUA needs to be hold for covering the future emission. By considering the emission of total year, the optimal emission quota position equals to sum of the past emission and sum of emission delta covering the future emissions of the year.

Value of a thermal efficiency improvement investment

Based on the evaluation model, the V_{inv} value of a theoretical 5 percentage point thermal efficiency improvement investment of the modeled generator can be calculated as the difference between values of generator with $\eta = 43\%$ and $\eta = 38\%$:

$$V_{inv} = V_{gen}(\eta = 43\%) - V_{gen}(\eta = 38\%) \qquad 10.$$

The model value of the investment was 28.2 million Euros. By using the real option model the sensitivity for different factors can be analyzed.

In the formula of spread, the parameter of thermal efficiency exists in connection with input prices. Therefore mainly the input prices influence the value of efficiency improvement. The effect of electricity price is: in case of higher efficiency, the higher prices will result positive spread at higher probabilities.

Thesis 4: Among initial prices, the increasing of gas and emission quota price decreases the theoretical value of efficiency improvement, while the raising of long term electricity prices increases the value.

Constant production by holding long term production contracts

In lots of cases the power generator is not operated in a profit maximizing way based on the actual market prices, but by having long term production contract it is constantly turned on. Considering the assumptions of the model (e.g. neglecting technological constraints), the power generator looses part of its optional value.

Thesis 5.1: The value of power generator operating constantly can be expressed as sum of future swaps for the spread. The realized loss per one unit of produced electricity (lcp) in a future τ day can be estimated as difference of spread options (spo) and spread swaps (ssw):

$$lcp(S(0),w,v,\tau) = spo^{Pr}(S(0),w,v,\tau) - ssw^{Pr}(S(0),w,v,\tau) \qquad 11.$$

If the generator is operated constantly for a longer term until time T, then the present value of total loss can be estimated as the following:

$$\frac{\Gamma}{2} \cdot \sum_{\tau=1}^{T} \left[lcp\left(S_{off-peak}(0),w,v,\tau\right) + lcp\left(S_{peak}(0),w,v,\tau\right) \right] \qquad 12.$$

The European option worth more than the swap with the same exercise price, therefore the power generator always loses money in case of constant production. The loss evolves from the fact, that in case of negative spread it is better to rest the capacity and fulfils the production contract by market buying, than by producing the required electricity by its own capacity. It is important to emphasize that this argument is also true if the generator has production contract with an especially high selling price: the supplying contract has its own value, which is independent from the production decision. The value of supplying contract can also be realized if the generator is operated conditionally, according to the market prices. It is always possible to freely choose between different ways of fulfilling the contract: by production or by market buying.

The calculated losses resulting by constant production for three different time lengths (1 year, 3 years, 5 years) are the followings:

Values in million Euros	**1 yr**	**3 yr**	**5 yr**
Option value (spo)	3.34	12.09	21.04
Value of swaps (ssw)	-7.05	-16.17	-22.79
Loss caused by constant production (lcp)	**10.39**	**28.26**	**43.82**

Table 3: Loss caused by constant production for three different time lengths.

Based on the model, the sensitivities to different technological and market factors were analyzed.

Thesis 5.2: In case of efficiency improvement, the loss of constant production decreases. The increased efficiency decreases the probability of negative spreads, it becomes rarer when the contractual supplying obligations have to be fulfilled by market buying.

Based on the real option decision model it can be stated, that the increase in initial price of slower mean reverting input prices increase the loss. The increasing resource price (particularly gas) raises the probability of negative spreads; hence the ratio of those days becomes higher, when the market buying is rational instead of production. The increasing of long term mean of electricity price decreases, the increasing of inputs' long term price increases the loss caused by constant production. The increase of volatility of electricity raises the loss of constant production, changing of volatilities of gas and emission quotas in a narrower interval does not change the loss significantly. The effect of correlation coefficient between gas and emission quota is not significant.

III.3. The MNPB and private demand function for emission quota

The MNPB (Marginal Net Private Benefit) function (Kerekes–Szlávik, 2003, pp. 92–93) has a central role in environmental economics. Its value shows the amount of profit increasing in case of one additional unit of production or pollution. With MEC (Marginal External Cost) function the optimal pollution level can be determined. In the followings MNPB is considered as the expected spread without emission covering cost per unit of pollution.

Thesis 6.1: The reservation price[5] of emission quota for the generator can be deducted by two-underlyings spread option prices $spo2^{Pr}$ as follows:

$$\mathrm{P}_{\text{off-peak}}^{\text{EUA,reservation}} = \eta/\delta \cdot spo2_{off-peak}^{Pr}\left(S_{off-peak}(0), w, v, \tau\right)$$

$$\mathrm{P}_{\text{peak}}^{\text{EUA,reservation}} = \eta/\delta \cdot spo2_{peak}^{Pr}\left(S_{peak}(0), w, v, \tau\right)$$

where

$$w = \begin{bmatrix} 1 \\ -1/\eta \end{bmatrix}, S_{off-peak}(0) = \begin{bmatrix} S_{off-peak}(0) \\ S_{gas}(0) \end{bmatrix}, \quad S_{peak}(0) = \begin{bmatrix} S_{peak}(0) \\ S_{gas}(0) \end{bmatrix} \qquad 13.$$

The option prices have to be multiplied by η/δ to get the value per unit of pollution instead of per unit of production.

Based on the reservation prices, the MNPB function can be calculated, which has an important role in environmental economics.

Thesis 6.2: The MNPB function without cost of emission quota for a given period can be deducted, if the prices of two underlying spread options for the days of given period are multiplied by η/δ, and sorted the values in a descendent order and plot with the cumulated quantity of emission.

From the third phase (2013-2020) the predominantly free allocation of quotas will gradually be replaced by auctions. For having an efficient bidding strategy, the private demand function for emission quota is essential. This can be calculated by integrating the MNPB function and dividing it by cumulated emission quantity. The resulted MNPB and private demand functions are the following:

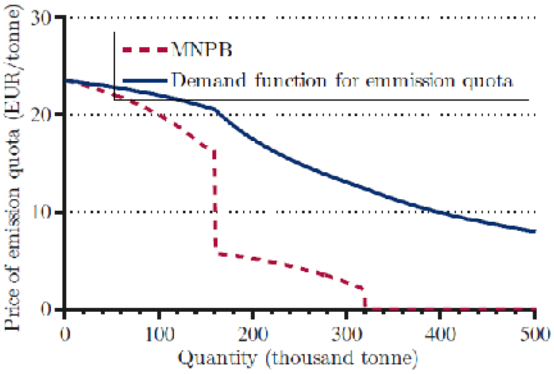


Figure 5: MNPB function and private demand function of emission quota considering a 1 year long period.

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[5] In the model of reservation price, the selling possibility of superfluous quotas was not considered

The private demand function starts from roughly 23 Euro/ton price, originating from the most valuable spread option related to peak hours. The function has two steps, the higher contains the more valuable spread options for peak hours, the lower step contains the less pricy ones for off-peak hours. The maximal emission for a year is 321 thousands ton; above this level the marginal benefit is zero: the generator is not able to produce electricity above its maximal capacity. Based on the model it can be determined how these functions behave in case of changing the model parameters.The +/- 5 percentage point changing of thermal efficiency modifies the functions as the following:

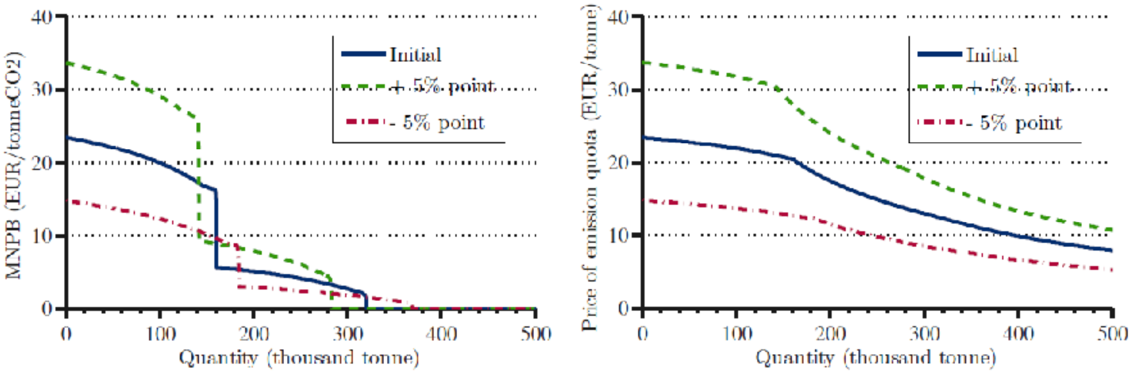

Figure 6: The changing of MNPB function and private demand function of emission quota in terms of thermal efficiency.

Thesis 6.3: The improvement of thermal efficiency increases the steepness of MNPB function and lowers the intersection point on horizontal axis; the private demand function is shifted upward.

The increasing of long term mean of electricity shifts both function upward. The price change of off-peak electricity modifies the lower step of MNPB functions containing the option prices for lower demand hours; the changing in electricity prices for peak hours modifies the upper step of MNPB.

The increase of gas price shifts the functions downward, the effect of price decrease is reverted. The raising of initial price of gas increases the steepness of MNPB. The effect of changing the long term gas price mean has the same direction, but its characteristic is reverted: the price increasing lowers the steepness of the function.

Based on the applications presented in Dissertation it can be concluded that for a gas fueled power generator operating in a liberalized market, the modern risk management tools are essential. The four underlying real option model provides practical applications for forecasting the optimal decisions and their environmental and corporate financial effects.

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