Abdel-Kader, M. and Dugdale, D. (2001), Evaluating Investments in Advanced Manufacturing Technology: A Fuzzy Set Theory Approach, British Accounting Review, December, Vol. 33, No. 4, pp. 455-489. ISSN 0890-8389. doi:10.1006/bare.2001.0177. Definitive version available online at: http://www.sciencedirect.com/science/journal/08908389

Evaluating Investments in Advanced Manufacturing Technology: A Fuzzy Set Theory Approach

Magdy G. Abdel-Kader
University of Essex, Wivenhoe Park, Colchester, Essex CO4 3SQ, UK
(Corresponding author)
Email: Magdy.Kader@brunel.ac.uk

David Dugdale
University of the West of England, Bristol, Frenchay Campus, Bristol BS16 1QY, UK

Acknowledgement: The authors would like to thank the two anonymous reviewers, Professor John Pointon, participants at the BAA conference (Manchester, March 1998), participants at the second conference on new directions in management accounting: innovations in practice and research (Brussels, December 2000) for their valuable comments on earlier drafts of this paper.

ABSTRACT

In this paper, a model for the evaluation of investments in advanced manufacturing technology is developed. Many authors have called for an integration of financial and non-financial factors in such evaluations and this paper demonstrates that it is conceptually possible to do this using the mathematics of the analytic hierarchy process and fuzzy set theory. The development of the model has certain distinguishing features. First, it is based on a conceptual framework that combines the three dimensions of risk, financial return and non-financial factors. The empirical basis for this has been investigated and previously reported by the authors. Second, models previously developed and reported in the literature are shown to suffer from certain flaws relating to the use of linguistic scales, the ranking of fuzzy performance indicators and partiality in the treatment of investment decision variables. These issues are addressed through the development of simpler linguistic scales based on the analytic hierarchy, a revised procedure for ranking fuzzy numbers and an attempt to build a comprehensive model through the three dimensions described above. Triangular fuzzy numbers are used throughout in order to make the mathematics tractable and relatively easy to understand and to facilitate presentation of a worked example. However, so that the reader is not misled, attention is drawn to some of the complexities in fuzzy arithmetic, especially the important distinction between subtraction/division and deconvolution of fuzzy numbers.

INTRODUCTION

The use of traditional investment models based on return on investment (ROI) or cash flow analysis, payback, net present value, internal rate of return, for Advanced Manufacturing Technology (AMT) projects has been criticised as failing to capture all relevant information. Arguably these models emphasise quantitative, financial analysis but fail to capture many of the 'intangible' benefits that should flow from AMT investments such as greater manufacturing flexibility, improved product quality and better employee morale (see for example, Abdel-Kader, 1997; Chen & Small, 1996; Dugdale & Jones, 1995; Accola, 1994; Cheung, 1993; Lavelle & Liggett, 1992; Naik & Chakravarty, 1992; Azzone, Bertele & Masella, 1992; Rayburn, 1989; Park & Son, 1988; Srinivasan & Millen, 1986; Kaplan, 1986; ACARD, 1983; Knott & Getto, 1982).

It is also argued that the high risk inherent in new technologies often leads to the use of arbitrarily high hurdle discount rates (Accola, 1994; Canada & Sullivan, 1990; Kaplan & Atkinson, 1989; Kaplan, 1986). This disadvantages long-term projects with large cash flows in the later part of their lives, and because there are many different determinants of risk, it is difficult to capture them all through a single modification of the discount rate (Ronen & Sorter, 1972). Also, adjustments to the discount rate are affected by managers' attitudes toward risk rather than by an explicit representation of the risks inherent in the investment alternatives (Accola, 1994).

Lefley (1996, p347) concluded: *'there is a need for a more sophisticated approach to the appraisal of AMT projects*, one that will take into account the strategic nature and the full benefits from such investments. ' (*emphasis added*). This echoed Currie (1994, p. viii) who argued for: '… *a new method of evaluating AMT should be developed which includes a wider array of financial and non-financial benefits*. This would improve managements' understanding of some of the key advantages of AMT and, in the process, supplement traditional management accounting techniques (DCF, NPV, payback) by considering the benefits of quality, organisational learning, training and process improvement and innovation' (*emphasis added*). Slagmulder, Bruggeman & Wassenhove (1995) summarised: '… more and more authors are convinced that good investment appraisal requires that strategic and financial considerations be reconciled and integrated.'

In response to this need 'integrated' models that can accept both quantitative and qualitative factors have been suggested. Simpler models are based on a weighted combination of attribute scores (Meredith & Suresh, 1986; Nelson, 1986; Parsaei & Wilhelm, 1989) while more sophisticated models are often based on the Analytic Hierarchy Process (AHP) (Naik & Chakravarty, 1992; O'Brien & Smith, 1993; Srinivasan & Millen, 1986; Putrus, 1990; Accola, 1994; Angelis & Lee, 1996). However, even these relatively sophisticated models can be criticised because the use of precise values does not reflect the qualitative and subjective nature of many factors.

To overcome this criticism an AMT investment model based on the mathematics of fuzzy set theory (Zadeh, 1965) and hierarchical structure analysis (Saaty, 1980) is developed. The model is designed to permit estimated 'fuzzy' values and to provide a consistent method of accounting for non-financial factors. A basis for the comparison of competing alternatives is also provided.

The remainder of the paper is organised as follows. The next section provides a brief discussion of the basic elements of fuzzy set theory. This is followed by a review of previous fuzzy investment models. The proposed model is then developed followed by an illustrative example. The last section sets out some conclusions.

FUZZY SET THEORY

In 1965 Zadeh proposed fuzzy set theory as a device for modelling fuzzy variables in a mathematical domain. The term 'fuzzy' refers to the situation where ambiguity and vagueness exist (Bellman & Zadeh, 1970; Zebda, 1989). Mathematically, let $U = \{ x_1 \dots\dots\dots .x_n\}$ be a universal set of objects x, then a fuzzy set A in U is defined by ordered pairs:

$$A = \{ x, \mu_A(x) \},\ \forall x \in U \tag{1}$$

where $\mu_A(x)$ is called a membership grade of x in A, and $\mu_A: U \rightarrow M$ is a function from U to a space M. Usually M takes a real number in the closed interval [0,1] where 0 and 1 refer to full membership and non-membership respectively. When M contains only two points, 0 and 1, A is non-fuzzy and its membership function is identical with the characteristic function of a classical (crisp) set (Bellman & Zadeh, 1970).

Fuzzy numbers

Buckley (1987) showed how fuzzy mathematics can be applied to financial problems by developing expressions for present value and annuities. However, general fuzzy solutions can involve extensive computations and, here, the mathematics will be rendered more tractable (and perhaps more practically useful) by the use of triangular fuzzy numbers (Laarhoven & Pedrycz, 1983).

A fuzzy number M in $\Re$ is a triangular number if its membership function $\mu_M: \Re \rightarrow [0,1]$ is:

$$\mu_M(x) = \begin{cases} 0 & x \le a \\ (x-a)/(b-a) & a \le x \le b \\ (c-x)/(c-b) & b \le x \le c \\ 0 & x \ge c \end{cases} \quad (2)$$

where $a, b, c \in \Re$ and $a \le b \le c$ (see figure 1).

A triangular fuzzy number can be written as follows:

$$M = (a, b, c) \quad (3)$$

Operations on triangular fuzzy numbers can be carried out as follows (Laarhoven & Pedrycz, 1983; Kaufmann & Gupta, 1991; Chen, Hwang & Hwang, 1992). Given two triangular fuzzy numbers *M* and *N* defined as:

$M = (a, b, c)$

$N = (e, f, g)$

and an ordinary number $k \in \Re$

Image of N

$$-N = (-g, -f, -e) \quad (4)$$

Inverse of N

$$N^{-1} = (1/g, 1/f, 1/e) \qquad 0 \notin [e,g] \quad (5)$$

Addition

$$M (+) N = (a + e, b + f, c + g) \quad (6)$$

Subtraction

$$M (-) N = (a - g, b - f, c - e) \quad (7)$$

Scalar Multiplication

$$\forall k > 0, k \in \Re: k (\bullet) M = (ka, kb, kc) \quad (8)$$

$$\forall k < 0, k \in \Re: k (\bullet) M = (kc, kb, ka) \quad (9)$$

Multiplication[1]

$$M (\bullet) N = (ae, bf, cg) \qquad a \ge 0, \ \ e \ge 0 \quad (10)$$

Division

$$M (\div) N = (a/g, b/f, c/e) \qquad a \ge 0, \ \ e > 0 \quad (11)$$

Triangular fuzzy numbers are easy to use and to interpret. For example, 'approximately equal to 1000' can be represented by (990, 1000, 1010) and the non-fuzzy number 500 can be represented by (500, 500, 500). However, the algebra of fuzzy numbers is more subtle than it may appear. For example, subtracting a fuzzy number (3, 5, 7) from the same fuzzy number using equation (7) gives a fuzzy zero (-4, 0, 4). Surprising perhaps but consistent with fuzzy literature because there is no reason to suppose the two numbers represent identical quantities. A problem arises when these two fuzzy numbers *are* equal because subtraction 'ought' to result in the crisp number zero. The operation that would result in the solution zero, is known as the deconvolution of fuzzy numbers and this should not be confused with subtraction.

To illustrate the deconvolution of fuzzy numbers we follow Kauffman and Gupta (1985), Let us define two triangular fuzzy numbers A = (a_1, a_2, a_3) and B = (b_1, b_2, b_3). If C is defined as:

$$C = (c_1, c_2, c_3) = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

---

[1] It should be noted that, strictly, multiplication of two triangular fuzzy numbers yields a fuzzy number with a non-linear membership function (See, Kauffman and Gupta, 1985, pp 25 – 31). The definition presented in equation (10) yields a triangular fuzzy number with linear membership functions and is an approximation to the exact product. Chiu and Park (1994) examined the reliability of this approximation when calculating fuzzy present value. Based on simulation results, they showed that the difference between the membership function of the present value in its exact and approximate forms is not significant. Accordingly, to simplify the computation and make it more manageable, we use the approximate definition as presented in equation 10. As division is multiplication by the inverse, a similar argument applies for equation (11).

The issue of deconvolution arises if A and C are known and the equation is to be solved for B.

In order to solve for B the operation of *deconvolution* must be applied and this would result in the fuzzy number B:

$$\text{Deconvolution of C and A} = (c_1 - a_1, c_2 - a_2, c_3 - c_3) = B$$

Deconvolution can be undertaken only if the following condition is satisfied:

$$(c_3 - a_3) \geq (c_2 - a_2) \geq (c_1 - a_1)$$

Note that subtraction:

$$C - A = (a_1 + b_1 - a_3, b_2, a_3 + b_3 - a_1)$$

does not solve the equation for B unless $a_1 = a_3$ and this would only be true if A were a crisp number.

The deconvolution problem applies to the division operation as well. Consider the equation:

$$A\ (\bullet)\ B = C \qquad a_1 \geq 0,\ b_1 > 0,\ c_1 \geq 0 \qquad (12)$$

The necessary and sufficient condition for the existence of a fuzzy number B satisfying the above equation is:

$$(c_3 / a_3) \geq (c_2 / a_2) \geq (c_1 / a_1)$$

If this condition is satisfied, the solution exists, and it is unique (Kauffman and Gupta, 1985) and is given by:

$$B = (c_1 / a_1, c_2 / a_2, c_3 / a_3) \qquad (13)$$

It may seem, to the reader, that deconvolution is, intuitively, what would be expected when one fuzzy number is subtracted from or divided by another. This is not the case and this digression is intended to draw attention to this, possibly counter-intuitive, aspect of the arithmetic of fuzzy numbers.

Linguistic variables

Linguistic variables were developed by Zadeh (1972; 1973; 1975a; 1975b; 1975c) to provide a systematic means of characterising approximate, complex, ill-defined phenomena. A linguistic variable is defined by Zadeh (1973, p. 28) as 'a variable whose values are sentences in a natural or artificial language'. For example, 'high' is a linguistic variable if its values are 'tall', 'very tall', 'not tall', etc. Each value is defined by a fuzzy subset in which the membership function represents the compatibility of each element, in the universe of discourse (base values), with the label of the fuzzy subset.

PREVIOUS FUZZY MODELS

Two approaches have been proposed which use the concepts of fuzzy set theory in evaluating capital investment projects. The first extends traditional discounted cash flow analysis into fuzzy cash flow analysis (Ward, 1985; 1989; Chiu & Park, 1994). The second is based on the concept of linguistic variables (Wilhelm & Parsaei, 1988; 1991).

Ward (1985; 1989) developed fuzzy present value analysis in which the cash flows are modelled as trapezoidal fuzzy numbers. However, derivation of the membership function of fuzzy present value results in complex non-linear representations and requires tedious computational effort. To overcome this problem and so make fuzzy cash flow analysis more manageable, Chiu & Park (1994) developed an approximate form of the 'complex' fuzzy present value formula based on triangular (rather than trapezoidal) fuzzy numbers.

The second use of fuzzy set theory, based on the linguistic approach. Wilhelm & Parsaei (1988; 1991) defined two fuzzy linguistic variables 'importance' and 'capability'. Then they developed a heuristic algorithm to characterise the capability of available advanced technologies and so select the technology that best meets company goals according to their importance.

Wilhelm & Parsaei's (1988; 1991) work was seminal in its application of fuzzy set theory to AMT investment decisions. However, they noted that: 'this approach is still very much in the preliminary stages of development' and we identify three issues in their work which need to be addressed. First, the computations in constructing relationships between linguistic variables seem complex and, perhaps because of this complexity, some relationships between linguistic values are counter-intuitive and even inconsistent. For example, there are two values with the same meaning: 'unimportant' and 'not important' but each has a separate membership function. Second, the operations of concentration and dilation are used in order to give effect to the qualifiers ‘very’ and ‘more-or-less’ so that, for example, the fuzzy variables for ‘important’ and ‘very important’ appear as in figure (2)[2]. This treatment is consistent with fuzzy literature but, here, we find it counter intuitive. We would expect the right-side of ‘very important’ to be to the right-side of ‘important’. Finally, financial factors are ignored in Wilhelm & Parsaei’s model or, at most, treated as though they are intangibles.

A PROPOSED MODEL FOR AMT INVESTMENT DECISIONS

Figure (3) shows the framework of the proposed model, incorporating both financial and non-financial factors, and based on both survey and field research (Abdel-Kader & Dugdale, 1998; Dugdale & Abdel-Kader, 1998). AMT investment decision making can be divided into three sequential stages: examining consistency with overall strategy, measuring expected performance of projects and ranking of AMT investment projects.

Consistency with overall strategy

Field studies (e.g. Nixon, 1995; Slagmulder & Bruggeman, 1992; Slagmulder, 1997; Dugdale & Abdel-Kader, 1998) have identified the strategic fit of a proposed investment as critical. Typically, investment opportunities identified at lower levels must pass a ‘strategic test’ when put to more senior management. If a proposal is out of line with company strategy it will almost certainly be excluded from even indicative planning. The first step in the proposed model is therefore to recognise this ‘strategy test’. The remainder of the model becomes irrelevant if a project is rejected at this stage.

The expected performance of AMT investment projects

The expected performance of AMT investment projects can be based on three measures: financial return, intangible (strategic) benefits, and risk.

(i) The financial return measure

The financial measure reflects all the factors that can be expressed as financial cash flows, for example: investment(s), net cost savings, increased turnover, government grants and tax effects. Each factor is represented by a triangular fuzzy number specified as the lowest possible estimate, the best estimate, and the largest possible estimate. For example, the investment amount could be a value such as ‘approximately £2 million’ and this value may be interpreted as a triangular fuzzy number, for example, (1.9, 2, 2.1). An exact estimate would be represented by the triangular fuzzy number (2, 2, 2).

After estimating the fuzzy cash flows for each alternative investment project, one or more financial measure, net present value; internal rate of return; payback and/or return on investment, can be calculated.

The fuzzy net present value (FNPV) Let x_t be the net cash flow arising at the end of year t, r be the discount rate (assumed constant throughout the project life), I_0 be the initial investment at time 0, and n be the project’s life, then $FNPV$ is calculated as follows:

$$FNPV = \sum_{t=1}^{n} \frac{x_t}{(1+r)^t} - I_0 \qquad (14)$$

Because x_t, I_0, and r are triangular fuzzy numbers (x_{t1}, x_{t2}, x_{t3}), (I_{01}, I_{02}, I_{03}), *and* (r_1, r_2, r_3) respectively, the $FNPV$ is also a triangular fuzzy number ($FNPV_1, FNPV_2, FNPV_3$). Equation (14) can be extended as follows:

[2] The use of concentration and dilation when applied to linguistic variables has presumed that the qualifier ‘very’ can be equated with ‘more precise’ and the qualifier ‘more-or-less’ can be equated with ‘less precise’. With this interpretation it is logical to use the concentration and dilation of fuzzy variable to reflect the qualifiers. However, in the context of investment appraisal (and perhaps in other contexts) the qualifiers ‘very’ and ‘more-or-less’ are more appropriately reflected in the concepts ‘greater’ and ‘less than’ and operations of concentration and dilation are no longer applicable.

$$(FNPV_1, FNPV_2, FNPV_3) = \left(\sum_{t=1}^{n} \frac{x_{t1}}{(1+r_3)^t} - I_{03}, \sum_{t=1}^{n} \frac{x_{t2}}{(1+r_2)^t} - I_{02}, \sum_{t=1}^{n} \frac{x_{t3}}{(1+r_1)^t} - I_{01} \right) \qquad x_{t1} \geq o \qquad (15)$$

The membership function of *FNPV* is approximated by the triangular shape:

$$\mu_{FNPV}(y) = \begin{cases} (y - FNPV_1) / (FNPV_2 - FNPV_1) & FNPV_1 \leq y \leq FNPV_2 \\ (FNPV_3 - y) / (FNPV_3 - FNPV_2) & FNPV_2 \leq y \leq FNPV_3 \\ 0 & otherwise \end{cases} \qquad (16)$$

The derivation of fuzzy net present value in (15) is rather similar to that of Chiu & Park (1994). Here, the three parameters of triangular fuzzy numbers for the fuzzy estimates are used instead of the right and left representations used in Chiu & Park's derivation. By using equation (15), the calculation of fuzzy net present value is no more complex than the normal, crisp calculation; it is simply repeated three times.

The fuzzy internal rate of return (FIRR) The internal rate of return is the rate of return that equates the *NPV* to zero. To calculate the *IRR*, it is necessary to solve the following equation for *r*:

$$0 = \sum_{t=1}^{n} \frac{x_t}{(1+r)^t} - I_0 \qquad (17)$$

where *r* is the *IRR*. It is usually solved through a number of trials. Substituting each variable in the above equation with its triangular fuzzy number, as in equation (15), the ***FIRR***, (r_1, r_2, r_3), can be calculated as follows:

$$(0,0,0) = \left(\sum_{t=1}^{n} \frac{x_{t1}}{(1+r_3)^t} - I_{03}, \sum_{t=1}^{n} \frac{x_{t2}}{(1+r_2)^t} - I_{02}, \sum_{t=1}^{n} \frac{x_{t3}}{(1+r_1)^t} - I_{01} \right) \qquad x_{t1} \geq o \qquad (18)$$

where (r_1, r_2, r_3) represents the *FIRR*, (IRR_1, IRR_2, IRR_3). The membership function of the *FIRR* is as follows:

$$\mu_{FIRR}(y) = \begin{cases} (y - FIRR_1) / (FIRR_2 - FIRR_1) & FIRR_1 \leq y \leq FIRR_2 \\ (FIRR_3 - y) / (FIRR_3 - FIRR_2) & FIRR_2 \leq y \leq FIRR_3 \\ 0 & otherwise \end{cases} \qquad (19)$$

The fuzzy payback period (FPB) The payback period is the time required to generate cash flows that recover the initial investment. If the initial investment or cash flows or both are fuzzy, the resultant payback period is also fuzzy. For example, suppose a four-year project with the estimated initial investment (I_0), in million, and cash flows (x_t), in million, (where t = 1,2,3,4) specified by triangular fuzzy numbers as I_0 = (£20, £25, £30), x_1 = (£7, £10, £12), x_2 = (£10, £12, £15), x_3 = (£10, £12, £15), x_4 = (£8, £10, £12).

Following the rules for subtraction of fuzzy numbers the first parameter in I_0 [£20] is recovered from the third parameter in x_t [£12, £15, £15, £12] (1.5 years), the second parameter in I_0 [£25] is recovered from the second parameter in x_t [£10, £12, £12, £10] (2.25 years), and the third parameter in I_0 [30] is recovered from the first parameter in x_t [£7, £10, £10, £8] (3.375 years). Assuming that x_t is realised at the end of year *t*, the fuzzy payback period (*FPB*), in years, is (1.5, 2.25, 3.375).

The fuzzy return on investment (FROI) One way to compute *ROI* is to divide the average accounting profit generated over the life of the project by the initial outlay (Lumby, 1995). Let *FROI* [($FROI_1$, $FROI_2$, $FROI_3$)] be the fuzzy return on investment, I_0 [(I_{01}, I_{02}, I_{03})] be the fuzzy initial outlay, D be the total depreciation over the project's life, x_t [(x_{t1}, x_{t2}, x_{t3})] be fuzzy net cash flows for year *t* (t = $1,2,\ldots,n$), and *s* [(s_1, s_2, s_3)] be the fuzzy scrap value at the end of the project. Based on the following assumptions: (1) the initial outlay minus scrap value is depreciable over the project's life, (2) depreciation is the only non-cash expense over the project's life, and (3) the straight-line method is used in calculating depreciation, the *FROI* is calculated as follows:

$$FROI = \frac{(\sum_{t=1}^{n} x_t - D) / n}{I_0} \qquad (20)$$

Substituting each variable by its triangular fuzzy number, the *FROI* is:

$$(FROI_1, FROI_2, FROI_3) = \frac{(\sum_{t=1}^{n}(x_{t1}, x_{t2}, x_{t3}) - [(I_{01}, I_{02}, I_{03}) - (s_1, s_2, s_3)])/n}{(I_{01}, I_{02}, I_{03})}$$

$$= \left(\frac{1}{nI_{03}}(\sum_{t=1}^{n} x_{t1} - I_{03} + s_1), \frac{1}{nI_{02}}(\sum_{t=1}^{n} x_{t2} - I_{02} + s_2), \frac{1}{nI_{01}}(\sum_{t=1}^{n} x_{t3} - I_{01} + s_3) \right) \quad (21)$$

The membership function of the *FROI* is as follows:

$$\mu_{FROI}(y) = \begin{cases} (y - FROI_1)/(FROI_2 - FROI_1) & FROI_1 \le y \le FROI_2 \\ (FROI_3 - y)/(FROI_3 - FROI_2) & FROI_2 \le y \le FROI_3 \\ 0 & otherwise \end{cases} \quad (22)$$

Representation of the financial return measure ($FRMA_K$) in the model As both NPV and IRR take account of the time value of money, both are theoretically preferred over payback and ROI but if one financial return measure is selected, fuzzy NPV is the theoretical recommendation[3].

In practice several measures of financial return are often calculated, sometimes because they are simpler (e.g. payback) or because of personal preferences, or because different measures provide extra information (e.g. payback provides a crude indication of liquidity, ROI provides some indication of the project impact on reported profit). The model proposed here is not, therefore, limited to a specific financial return measure and it would be possible to combine two or more financial measures into a single project score[4]. However, we recommend that only one financial return measure (either *FNPV* or *FIRR*) is used to represent the project's profitability. This is because the same data is used to produce all the financial return measures and, thus, representing the financial return measure in the model by more than one parameter may lead to duplication.

---

[3] The superiority of the NPV technique is nicely discussed in Clark et al. (1984, Ch. 6).

[4] In combining two or more financial return measures together, two problems should be addressed. (1) the indication of each measure must be in the same direction, in other words, if for some measures the alternative with the highest (largest) value is the best, then all measure results should indicate the same. For example, the alternative with the smallest payback period is the most desirable one while the alternative with the highest NPV is the most desirable one. In this case, one of these two measures should be transformed to convey the same direction in the results. (2) All combined measures should be represented by the same measurement unit. Because NPV gives values in pounds, IRR and ROI give results in percentages, and the payback period is represented by years, all these results should be transformed into a score in the interval [0,1]. One possible way to overcome these two problems is to convert the preference of the payback period direction to be the highest score is the better and then convert all original measures results into score in the interval [0,1] as follows:
For fuzzy payback measure:

$$FPBA_k^{\backslash} = (FPBA_k^{-1} / \sum_{i}^{k} FPBA_k^{-1}) / a$$

where:
$FPBA_k^{\backslash}$: the converted fuzzy payback period for the project A_k (k=1,2,...., N project),
$FPBA_k^{-1}$: the inverse of the payback period for the project i,
a : suitable constant so that all supports of $PBA_k\backslash$ is in the closed interval [0,1].
For other measures (for example *FNPV*):

$$FNPVA_k^{\backslash} = (FNPVA_k / \sum_{k=1}^{N} NPVA_k) / a$$

where:
$FNPVA_k^{\backslash}$: the converted *FNPV* for the project A_k (k =1,2,...., N project),
$FNPVA_k$: the *FNPV* for the project A_k,
a : suitable non-fuzzy constant so that all supports of $NPVA_k^{\backslash}$ are in the closed interval [0,1].
After converting all financial measures, they could be transformed into a single measure by weighting each measure and then combining them in the same way as for non-financial criteria measure.

(ii) The non-financial criteria measure
The non-financial criteria measure reflects all the strategic factors that cannot be translated into cash flows. To generate a measure for this group of factors, a three level hierarchical structure is suggested. The first two levels evaluate the fuzzy importance of the non-financial investment criteria and sub-criteria and the lowest level assigns ratings to alternative AMT investment projects. The hierarchical structure is shown in figure (4).

A large number of potential non-financial benefits of AMT have been identified in the literature and a survey by the authors identified which of these are considered important by practitioners. The development of an analytic hierarchy is always a subjective exercise and needs to be tailored to specific contexts. Nevertheless, the evidence suggests that many AMT projects generate benefits on three key dimensions: 'improving product quality', 'increasing process flexibility' and 'meeting customer requirements'. In order to avoid confusion between these criteria, 'product quality' is defined as *conformance to product specification* while 'customer requirements' is related to *development of new product features*. Additionally, process flexibility might be usefully divided into two sub-dimensions- relating to *increased product complexity* and *increased product volume*.

Developing a fuzzy linguistic scale A fuzzy linguistic scale is suggested to evaluate the fuzzy importance of the non-financial investment criteria and sub-criteria in the first two levels of the hierarchy [figure (4)] and to assign fuzzy ratings to the alternative AMT investment projects in the lowest level of the hierarchy. Previous studies (e.g. Rangone, 1997; Liang & Wang, 1991; Zebda, 1984), usually suggest generalised linguistic variables but, because the meaning (value) of any word (linguistic variable) is subjective, we suggest that the linguistic scale should be generated by decision makers themselves.

Decision makers are asked to make pair-wise comparisons between the linguistic values of non-financial criteria such as very important (*VI*), important (*I*), more-or-less important (*MI*), more-or-less unimportant (*MU*) and unimportant (*U*) expressing how many times a linguistic value exceeds or is less than every other linguistic value. Assuming that triangular fuzzy numbers are used in these pair-wise comparisons, the answers can be formulated in a positive reciprocal matrix as follows (the numerical values are for illustration):

$$\begin{array}{c} \\ VI \\ I \\ MI \\ MU \\ U \end{array}\begin{array}{c} \begin{array}{ccccc} VI & I & MI & MU & U \end{array} \\ \begin{bmatrix} (1,1,1) & (2,3,4) & (4,5,6) & (6,7,8) & (8,9,9) \\ (1/4,1/3,1/2) & (1,1,1) & (1,2,3) & (2,3,4) & (3,4,5) \\ (1/6,1/5,1/4) & (1/3,1/2,1) & (1,1,1) & (1,3/2,2) & (3/2,2,3) \\ (1/8,1/7,1/6) & (1/4,1/3,1/2) & (1/2,2/3,1) & (1,1,1) & (1,5/4,3/2) \\ (1/9,1/9,1/8) & (1/5,1/4,1/3) & (1/3,1/2,2/3) & (2/3,4/5,1) & (1,1,1) \end{bmatrix} \end{array}$$

Following Buckley (1985) the fuzzy number of each linguistic value is computed by the geometric mean method. Given the positive comparison matrix as:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix} \tag{23}$$

The geometric mean of each row is calculated as:

$$Q_i = \left[\prod_{j=1}^{n} a_{ij} \right]^{1/n} \tag{24}$$

The weight W_i is calculated as:

$$W_i = Q_i \,/\, (\sum_{i=1}^{n} Q_i) \tag{25}$$

Let $a_{ij} = (b_{ij}, c_{ij}, d_{ij})$, $Q_i = (e_i, f_i, g_i)$ and $W_i = (k_i, l_i, m_i)$ be a triangular fuzzy number, then the equations (24) and (25) can be written as:

$$Q_i = (e_i, f_i, g_i) = \left(\left[\prod_{j=1}^{n} b_{ij} \right]^{1/n}, \ \left[\prod_{j=1}^{n} c_{ij} \right]^{1/n}, \ \left[\prod_{j=1}^{n} d_{ij} \right]^{1/n} \right) \tag{26}$$

$$W_i = (k_i, l_i, m_i) = \left(\left[e_i \Big/ \sum_{i=1}^{n} g_i \right], \ \left[f_i \Big/ \sum_{i=1}^{n} f_i \right], \ \left[g_i \Big/ \sum_{i=1}^{n} e_i \right] \right) \tag{27}$$

Then, the triangular fuzzy number of each linguistic value is calculated as the accumulated sum of this linguistic value.

To illustrate these procedures, the previous pair-wise comparison matrix is used. Panel A in Table (1) gives the results of calculating the geometric mean (Q_i) [equation (27)] for each row in the pair-wise comparisons while panel B in table (1) gives the results of calculating the weight (W_i) for each linguistic value.

In principle, the weights (W_i) computed in table (1) can be used as fuzzy meanings for their equivalent linguistic values in developing measures for non-financial criteria. However, we prefer to transfer these weights into a rating scale. This can be done for each linguistic value by adding all lowest weights to the weight of this linguistic value. Using the results provided in table (1) the rating scale could be as follows:

U = (0.04, 0.06, 0.08),
MU = (0.04, 0.06, 0.08) (+) (0.05, 0.07, 0.11) = (0.09, 0.13, 0.19),
MI = (0.09, 0.13, 0.19) (+) (0.07, 0.11, 0.18) = (0.16, 0.24, 0.37),
I = (0.16, 0.24, 0.37) (+) (0.14, 0.21, 0.33) = (0.30, 0.45, 0.70),
VI = (0.30, 0.45, 0.70) (+) (0.39, 0.55, 0.73) = (0.69, 1.00, 1.00*)

Accordingly, the fuzzy linguistic scale (*LS*) is:

LS = {U, MU, MI, I, VI} = {(0.04, 0.06, 0.08), (0.09, 0.13, .19), (0.16, 0.24, .37), (0.30, 0.45, 0.70), (0.69, 1.00, 1.00)}

The membership functions of each linguistic value in *LS* are shown in figure (5) and, in contrast to the linguistic scale developed by Wilhelm & Parsaei (1988; 1991), this scale is intuitively attractive.

Developing a fuzzy non-financial criteria measure The importance weight of non-financial criteria can be assessed by weighing the non-financial criteria against each other using the fuzzy linguistic scale (*LS*). This importance weight is denoted (C_i). The use of linguistic ratings facilitates qualitative assessment of the importance of different non-financial criteria without the pressure of being precise (Naik & Chakravarty, 1992).

The decision maker is then asked to rate each alternative with respect to each non-financial criterion using the fuzzy linguistic scale (*LS*). Similar linguistic scales can be used for non-financial criteria and rating alternatives. However, the terms used to express the points on each scale need to be suitably modified. Table (2) presents possible interpretations of the linguistic terms used in the previous example.

Suppose A_k denotes an alternative k (k= 1, 2, … , m alternatives) and LSA_{ki} denotes the fuzzy linguistic rating for alternative A_k with respect to a non-financial criterion i (i = 1, 2, … , n non-financial criteria). Then, the fuzzy non-financial measure of alternative A_k is denoted $FNFMA_k$ and can be computed as follows:

$$FNFMA_k = \frac{1}{n}\left(\sum_{i=1}^{n} (C_i)(LSA_{ki}) \right) \tag{28}$$

$C_i = (V_{i1}, V_{i2}, V_{i3})$, $LSA_{ki} = (G_{ki}, H_{ki}, I_{ki})$ and $FNFMA_k = (W_{k1}, W_{k2}, W_{k3})$, are all traingular fuzzy numbers. Equation (32) can be written as:

* The actual value is 1.43 and it should be approximated to the highest value in the scale (1.00) since any value more than the upper bound of the scale is redundant.

$$FNFMA_k = (W_{k1}, W_{k2}, W_{k3})$$
$$= \left(\left[\frac{1}{n} \left(\sum_{i=1}^{n} (V_{i1})(G_{ki}) \right) \right], \ \left[\frac{1}{n} \left(\sum_{i=1}^{n} (V_{i2})(H_{ki}) \right) \right], \ \left[\frac{1}{n} \left(\sum_{i=1}^{n} (V_{i3})(I_{ki}) \right) \right] \right) \quad (29)$$

The membership function of the fuzzy non-financial criteria measure ($FNFMA_k$) is as follows:

$$\mu_{FNFMA_k}(y) = \begin{cases} (y - W_{k1}) / (W_{k2} - W_{k1}) & W_{k1} \le y \le W_{k2} \\ (W_{k3} - y) / (W_{k3} - W_{k2}) & W_{k2} \le y \le W_{k3} \\ 0 & otherwise \end{cases} \quad (30)$$

(iii) The risk measure

The usual first step in analysing project risk is to conduct sensitivity analysis in order to identify the most sensitive variables. Then, possibilities of reducing project risk can be searched and every effort made to minimise or eliminate obvious risk in the sensitive variables. Finally, the residual risk, which cannot be minimised or eliminated, can be evaluated. Accola (1994, p. 21) argued that: '… Models used should enable managers to measure and evaluate expected cash flows separately from the risks associated with them' and, in the model suggested here, risk is treated as a separate dimension from financial return. Two types of AMT investment project risk were identified in a field study conducted by Abdel-Kader & Dugdale (1996): risk related to the market and risk related to technology. Each type of risk can be represented by a sub-dimension and a hierarchical representation for project risks is shown in figure (6).

The decision maker is asked to weigh each risk using a fuzzy linguistic scale. The weight assigned to a risk factor is denoted (WR_j) (where j =1, 2, …m risk factors for each type of risk). Then, the decision maker has to evaluate each alternative AMT project with respect to each risk factor. This evaluation could be done qualitatively (using the linguistic scale) or quantitatively (using proxy measures). For example, proven technology can be expressed qualitatively as 'very high' or quantitatively as how many times this type of technology has been applied in the industry and/or how long this type of technology has been in use. If quantitative evaluation is preferred, this evaluation should first be converted into a score in the closed interval [0,1] by normalising its values. Suppose a decision maker evaluates a project A_k (k = 1, 2, … N projects) with respect to a risk factor R_j (j = 1, 2, … m risk factors) as $RA\grave{}_{kj}$, the converted value is denoted as RA_{kj} and computed as follows:

$$RA_{kj} = (RA\grave{}_{kj} \ / \sum_{i=1}^{k} RA\grave{}_{kj}) / a \quad (31)$$

where: a is a suitable non-fuzzy constant so that all supports of RA_{kj} are in the closed interval [0,1].

If the decision maker prefers to evaluate the riskiness of alternative projects qualitatively using the linguistic rating scale, the corresponding fuzzy number is used in the following procedures. RA_{kj} is a fuzzy number denoting the riskiness of a project A_k with respect to risk factor j whether it is a converted value or the corresponding value on the linguistic value.

It should be noted that not all risk factors indicate the same preference. For example, a linguistic value of 'very high' for 'volatility of demand' means higher risk than a linguistic value of 'high' while the corresponding linguistic values mean the opposite for 'life of market'. So, for consistency, the corresponding fuzzy numbers in the fuzzy linguistic scale should take into account this mixed meaning for the same linguistic value. Another fuzzy linguistic scale can be developed in the same way as stated earlier for the non-financial criteria. Example fuzzy numbers associated with the fuzzy linguistic scale for each risk factor are presented in table (3).

Then, the fuzzy risk measure can be represented by $FRMA_k$ and calculated as follows:

$$FRMA_k = \frac{1}{m} \left(\sum_{j=1}^{m} (WR_j)(RA_{jk}) \right) \quad (32)$$

Because WR_j and RA_{jk} are triangular fuzzy numbers (WR_{j1}, WR_{j2}, Wr_{j3}) and (RA_{jk1}, RA_{jk2}, RA_{jk3}) respectively, then $FRMA_k$ is also a triangular fuzzy number ($FRMA_{k1}$, $FRMA_{k2}$, $FRMA_{k3}$). Hence, equation (32) can be written as follows:

$$FRMA_k = (FRMA_{k1}, FRMA_{k2}, FRMA_{k3}) = \left(\left[\frac{1}{m}\left(\sum_{j=1}^{m}(WR_{j1})(RA_{jk1})\right)\right],\left[\frac{1}{m}\left(\sum_{j=1}^{m}(WR_{j2})(RA_{jk2})\right)\right],\left[\frac{1}{m}\left(\sum_{j=1}^{m}(WR_{j3})(RA_{jk3})\right)\right]\right) \quad (33)$$

The membership function of the fuzzy risk measure ($FRMA_k$) is as follows:

$$\mu_{FRMA_k}(y) = \begin{cases} (y - FRMA_{k1})/(FRMA_{k2} - FRMA_{k1}) & FRMA_{k1} \le y \le FRMA_{k2} \\ (FRMA_{k3} - y)/(FRMA_{k3} - FRMA_{k2}) & FRMA_{k2} \le y \le FRMA_{k3} \\ 0 & Otherwise \end{cases} \quad (34)$$

Ranking AMT investment projects

After developing the three fuzzy measures of financial return, non-financial criteria and risk for each AMT project under consideration, the question is how can the decision maker(s) choose the 'best' AMT project? This requires: (1) the reduction of each fuzzy measure to an equivalent crisp measure so that relative differences among AMT projects for each measure can be identified, and (2) a rule for selecting the 'best' AMT project. Each of these is illustrated as follows.

(i) Reducing fuzzy measures into crisp measures

Because each fuzzy measure is represented by a triangular fuzzy number, the decision maker may find it difficult to rank AMT alternatives. This difficulty arises because each fuzzy number is represented by imprecise (multiple) quantities and there may be an overlap between these quantities across the fuzzy numbers. This problem is widely recognised in the literature and a number of ranking methods have been suggested to reduce each fuzzy number to an equivalent crisp value (e.g. Jain, 1976; 1977; Chen, 1985; Kim & Park, 1990; Chen et al., 1992). However, because each method utilises only partial information for each fuzzy number, different methods can lead to different orders for the same fuzzy numbers. A fuzzy number can be divided into three parts: (1) full memberships, (2) partial memberships located in the right-hand side, and (3) partial memberships located in the left-hand side. Existing ranking methods either reflect the membership functions of the left-hand side or both sides. In order to reflect all three parts of a fuzzy number in the ranking process, a new ranking method is proposed[5].

Let FMA_k be a triangular fuzzy measure (FMA_{k1}, FMA_{k2}, FMA_{k3}) for A_k project (k =1, 2, … , N), the reduced value for project A_k (RVA_k) can be computed as:

$$RV(A_k) = (FMA_{k2})\{(O)\left[(FMA_{k3} - x_{\min})/(x_{\max} - x_{\min} + FMA_{k3} - FMA_{k2})\right] + (1-O)[1 - (x_{\max} - FMA_{k1})/(x_{\max} - x_{\min} + FMA_{k2} - FMA_{k1}]\} \quad (35)$$

where:

$$x_{min} = inf\, S \quad (36)$$

$$x_{max} = sup\, S \quad (37)$$

$$S = \bigcup_{k=1}^{n} S_k \quad (38)$$

$$S_k = \left\{FMA_{k1}, FMA_{k2}, FMA_{k3} \middle| \forall\ k = 1,2,\cdots N\right\} \quad (39)$$

and

O : an index of optimism in the closed interval [0,1]

(ii) Selecting the 'best' AMT project

Using the equation (35) with the three fuzzy measures should produce three reduced values for each AMT project. Because the fuzzy measures of financial return and non-financial criteria were formulated as 'higher value is better', the reduced value retains the same feature that the higher value of financial return measure or non-financial criteria measure is the better. For the risk measure, the lower value is the better as it means less associated risk. In principle it would be possible to combine the three major dimensions of financial return, non-financial criteria and risk into a single project score. However, this combination would mean a significant loss of information (Accola, 1994).

[5] See Abdel-Kader, Dugdale & Taylor (1998, pp. 224-229) for details of this method.

Instead, decision makers are provided with more information regarding each dimension for the AMT investment projects being evaluated. Thus, the ‘efficient plane’ suggested by Accola (1994) is used. The efficient plane is represented by all alternative AMT investment projects that dominate other alternatives below the plane. In figure (7), the shaded area represents an efficient plane bounded by four alternatives A, B, C and D. Any alternative located on the efficient plane dominates all alternatives below the plane. For example, alternative B dominates alternative F because both alternatives have the same levels of risk and intangible benefits but the financial return of B is higher than F. The selection of an alternative from the set of alternatives on the efficient plane depends on the relative preferences of decision maker(s) for risk, non-financial benefits and financial return.

An algorithm which summarises the suggested model for AMT investment decisions is shown in table (4).

AN ILLUSTRATIVE EXAMPLE

A large engineering company produces a number of products sold mainly to other industrial companies in the UK and overseas. The current manufacturing system is based on an operator-controlled NC machine cell. In order to maintain its competitive position, the company is considering an investment in an AMT system. After an initial screening, three alternative projects I, II and III in addition to the current manufacturing system (CS), are to be evaluated. Project I consists of multi-machine CNC, operator-assisted MHS, manual tool changeover and manual off-line inspection. Project II consists of integrated CNC machine, computer controlled MHS, automated tool delivery and automatic on-line inspection. Project III is a fully integrated manufacturing system which consists of integrated CNC machine, integrated MHS with AS/RS, computer controlled tool migration, feedback for automatic process control and CAD/CAM. The steps in the suggested procedure are as follows.

Step 1. A committee of managers of the finance, production and marketing departments is formed to evaluate the available alternatives. The company uses NPV and payback period as financial return measures. The non-financial criteria include ‘product quality’, ‘process flexibility’ and ‘customer requirements’. Both market and technology risks will be measured.

Step 2. The company adopts an overall strategy of ‘innovation and product proliferation’. The committee believes that the CS is not consistent with this strategy and should be omitted. Only the other three alternatives are considered.

Step 3. The planning horizon (n) is 7 years and the fuzzy discount rate (r) is (7%, 8%, 10%). The initial investments (I_o) and net cash flows after tax (x_t) at end of year t to the planned horizon are as in table (5).

Using equation (15) the FNPV for project I can be calculated as follows:

$$FNPV_1 = \left(\left[\frac{.33}{(1+.10)^1} + \frac{.33}{(1+.10)^2} + \frac{.33}{(1+.10)^3} + \frac{.33}{(1+.10)^4} + \frac{.33}{(1+.10)^5} + \frac{.31}{(1+.10)^6} + \frac{.28}{(1+.10)^7} \right] - 1.1 \right) = 0.47$$

$$FNPV_2 = \left(\left[\frac{.35}{(1+.08)^1} + \frac{.35}{(1+.08)^2} + \frac{.35}{(1+.08)^3} + \frac{.35}{(1+.08)^4} + \frac{.35}{(1+.08)^5} + \frac{.33}{(1+.08)^6} + \frac{.30}{(1+.08)^7} \right] - 1.0 \right) = 0.78$$

$$FNPV_3 = \left(\left[\frac{.37}{(1+.07)^1} + \frac{.37}{(1+.07)^2} + \frac{.37}{(1+.07)^3} + \frac{.37}{(1+.07)^4} + \frac{.37}{(1+.07)^5} + \frac{.35}{(1+.07)^6} + \frac{.31}{(1+.07)^7} \right] - 0.8 \right) = 1.14$$

$\therefore$ $FNPV_{project\ I}$ = (0.47, 0.78, 1.14)

Similarly $FNPV_{project\ II}$ and $FNPV_{project\ III}$ are (0.22, 0.65, 1.00) and (0.08, 0.38, 0.71) respectively.

The fuzzy payback period (in years) for the three projects are (3, 3, 4), (3, 4, 5) and (4, 5, 5) respectively.

Step 4. Sensitivity analysis is performed for three variables: expected demand, project life and discount rate. See tables (6), (7) and (8).

Step 5. The non-financial criteria are classified into three groups ‘product quality’, ‘process flexibility’ and ‘customer requirements’ similar to that shown in figure (4).

Step 6. The decision makers employ the ‘*importance*’ fuzzy linguistic scale of table (2).

Step 7. The importance of each non-financial criterion is determined qualitatively and converted into its equivalent fuzzy number in table (9).

Step 8. The ratings assigned to each alternative with respect to the non-financial criteria are determined qualitatively and converted into their numerical equivalent in table (10).

Step 9. Using equation (29), the fuzzy non-financial measure for project I ($FNFM_{project\ I}$) can be calculated as follows:

$W_{project\ I1}$ = 1/6 [(.69)(.00) + (.69)(.15) + (.69)(.15) + (.69)(.15) + (.3)(.15) + (.16)(.15)]
= 0.06

$W_{project\ I2}$ = 1/6 [(1)(.10) + (1)(.25) + (1)(.25) + (1)(.25) + (.45)(.25) + (.24)(.25)] = 0.17

$W_{project\ I3}$ = 1/6 [(1)(.19) + (1)(.40) + (1) (.40) + (1)(.40) + (.7)(.40) + (.37)(.40)] = 0.30

$\therefore\ FNFM_{project\ I}$ = (0.06, 0.17, 0.30)

Similarly, $FNFM_{project\ II}$ and $FNFM_{project\ III}$ are (0.25, 0.56, 0.69) and (0.38, 0.78, 0.85) respectively.

Step 10. The risk factors are classified into two groups 'market risk' and 'technology risk' similar to that shown in figure (6).

Step 11. Weighting the importance of risk factors can be undertaken using the fuzzy linguistic scale set out in table (2) while the fuzzy linguistic scale in table (3) can be used to evaluate each alternative with respect to each risk factor. Hence, the weight of the importance of each risk factor is determined qualitatively and its equivalent fuzzy number is shown in table (11). The evaluation of each alternative with respect to each risk factor are determined qualitatively and their equivalent fuzzy numbers in table (12).

Step 12. Using equation (33), the fuzzy risk measure for project I ($FRM_{project\ I}$) can be calculated as follows:

$FRM_{project\ I1}$ = 1/5 [(.16)(.30) + (.3)(.16) + (.69)(.30) + (.69)(.09) + (.09)(0)] = 0.07

$FRM_{project\ I2}$ = 1/5 [(.24)(.45) + (.45)(.24) + (1)(.45) + (1)(.11) + (.11)(.06)] = 0.16

$FRM_{project\ I3}$ = 1/5 [(.37)(.70) + (.7)(.37) + (1)(.70) + (1) (.19) + (.19)(.08)] = 0.28

$\therefore\ FRM_{project\ I}$ = (0.07, 0.16, 0.28)

Similarly, $FRM_{project\ I2}$ and $FRM_{project\ I3}$ are (0.04, 0.09, 0.15) and (0.03, 0.06, 0.11) respectively.

Step 13. The summary of the three fuzzy measures computed for the three projects are presented in table (13). Also, the membership function of the three measures are depicted in figures (8a-c).

Using equations (35) - (39) and assuming O = 0.5, the ranked values (RV_{FNPV}), (RV_{FNFM}) and (RV_{FRM}) for $FNPV$, $FNFM$ and FRM are calculated and shown in table (14). For example, the calculation of (RV_{FNPV}) is as follows:

S = {0.08, 0.22, 0.38, 0.47, 0.65, 0.71, 0.78, 1.00, 1.14}

x_{max} = 1.14

x_{min} = 0.08

RV_{FNPV} (project I) = (.78)(.5[(1.14-.08)/(1.14-.08+1.14-.78)]
+ (.5)[1-(1.14-.47)/(1.14-.08+.78-.47]) = 0.49

RV_{FNPV} (project II) = (.65)(.5[(1-.08)/(1.14-.08+1-.65)]
+ (.5)[1-(1.14-.22)/(1.14-.08+.65-.22]) = 0.34

RV_{FNPV} (project III)= (.38)(.5[(.71-.08)/(1.14-.08+.71-.38)]
+ (.5)[1-(1.14-.08)/(1.14-.08+.38-.08)]) = 0.13

Step 14. The ranked values of each measure can be normalised by dividing each ranked value by the total of ranked values. Table (15) shows the normalised ranked values for the three fuzzy measures. The normalised values are then used in developing the efficient plane as shown in figure (9).

It should be noted that no project dominates the other two alternative projects. Project I provides the highest financial return, but it has few non-financial benefits and is very risky. Project III provides many non-financial benefits and has the lowest risk but also the lowest NPV. Project II is in-between the other two projects with respect to the three measures. The selection of a project depends on the decision makers' preferences between NPV, risk and the non-financial criteria, and can be undertaken by considering the crisp values in table (15). If any of the projects had not been on the efficient plane, then they would not be considered in this final step.

The calculations in step 13 could be repeated for different values of the optimism/pessimism index to explore the sensitivity of the decision in step 14.

CONCLUSIONS

In this paper a model for AMT investment decision making has been developed. The model is based on an empirically grounded framework and builds upon several theoretical themes in the AMT investment literature. In particular it draws on models based on the analytic hierarchy process and uses both fuzzy numbers and linguistic variables. The model therefore synthesises several aspects of existing models and provides a comprehensive overview of the problem.

The proposed model does not provide a single measure that combines all factors, rather a distinction was drawn between factors that can be quantified and factors that can only be subjectively assessed. Hence, Accola's suggestion of a three-dimensional framework was followed in which a separate measure for each dimension: financial return, non-financial benefits and risk, was developed. The final decision is subject to decision makers' preferences. However, if all benefits can be reasonably quantified in cash flows terms, then the three-dimensional framework can be collapsed into two-dimensions, financial return and risk.

In developing the model two issues/problems have been identified. The first issue relates to fuzzy linguistic variables where we note that Wilhelm and Parsaei's suggestion seems complex and that the fuzzy operations of concentration and dilation when applied to concepts such as "important" lead to counter-intuitive results in the context of investment appraisal. We have suggested a refinement in the construction of fuzzy linguistic scales - making such scales dependent on decision maker's preferences through the use of pairwise comparisons. The proposed method also avoids the use of concentration and dilation in the construction of the scale.

The second issue relates to the difficulties that can arise when fuzzy variables are to be ranked, as they must be when choosing between competing investment projects. This issue has been addressed through the development of a new ranking method building on existing methods described in the literature.

We have set out a model to demonstrate that the analytic hierarchy technique can be combined with fuzzy set theory in investment decision making. The technical issues can be overcome and the use of triangular fuzzy numbers together with Chiu and Park's simplifying assumptions make the specific calculations quite simple. However, this apparent simplicity and availability of specialist software should not obscure the inherent difficulties in the mathematics of fuzzy sets theory. We drew attention in particular, to the issue of deconvolution in subtraction and division of fuzzy numbers.

REFERENCES

Abdel-Kader, M. G. (1997). 'Investment decisions in advanced manufacturing systems: a review and identification of research areas' In P. Atrill & L. Lindley (eds), *Issues in Accounting and Finance*, pp. 189-216, Aldershot (UK), Ashgate.

Abdel-Kader, M. & Dugdale, D. (1996). 'Investment decisions in advanced manufacturing technology', *Applied Accounting Research Series*, Working Paper No. 5, University of the West of England, Bristol, UK

Abdel-Kader, M. & Dugdale, D. (1998). 'Investment in advanced manufacturing technology: a study of practice in large UK companies', *Management Accounting Research*, 9(3), September, pp. 261-284.

Abdel-Kader, M., Dugdale, D. & Taylor, P. (1998). Investment Decisions in Advanced Manufacturing Technology: A Fuzzy Set Theory Approach, Aldershot, Ashgate.

ACARD (1983). *New Opportunities in Manufacturing: The Management of Technology*, London, Her Majesty's Stationery Office, October.

Accola, W. (1994). 'Assessing risk and uncertainty in new technology investments', *Accounting Horizons*, 8 (3), pp. 19-35.

Accola, W., Agrawal, S. & Holsapple, C. (1995). 'Adaptive decision support systems for evaluating risk and uncertainty in capital investment decisions: opportunities for future research', *Managerial Finance*, 21(3), pp. 1-16.

Angelis, D. & Lee, C. (1996). 'Strategic investment analysis using activity based costing concepts and analytical hierarchy process techniques', *International Journal of Production Research*, 34(5), pp. 1331-1345.

Attaran, M. (1992). 'Flexible manufacturing systems: implementing an automated factory', *Information Systems Management*, 9(2), pp. 44-47.

Azzone, G., Bertele, U. & Masella, C. (1992). 'Selecting techniques for the financial justification of advanced manufacturing technologies: contingent approach' In H. Parsaei, W. Sullivan, & T. Hanley (eds), *Economic and Financial Justification of Advanced Manufacturing Technologies*, pp. 19-35, Amsterdam, Elsevier Science Publishers.

Bellman, R. & Zadeh, L. (1970). 'Decision-making in a fuzzy environment', *Management Science*, 17(4), pp. 141-164.

Boucher, T. & MacStravic, E. (1991). 'Multiattribute evaluation within a present value framework and its relation to the analytic hierarchy process', *The Engineering Economist*, 37(1), pp. 1-32.

Buckley, J. (1985). 'Fuzzy hierarchical analysis', *Fuzzy Sets and Systems*, 17, pp. 233-247.

Canada, J. & Sullivan, W. (1990). 'Persistent pitfalls and applicable approaches for justification of advanced manufacturing systems', *Engineering Costs and Production Economics*, 18, pp. 247-253.

Chen, I. & Small, M. (1996). 'Planning for advanced manufacturing technology: a research framework', *International Journal of Operations and Production Management*, 16(5), pp. 4-24.

Chen, S. (1995). 'An Empirical examination of capital budgeting techniques: impact of investment types and firm characteristics', *The Engineering Economist*, 40(2), pp. 145-170.

Chen, Shan-Huo (1985). 'Ranking fuzzy numbers with maximizing set and minimizing set', *Fuzzy Sets and Systems*, **17**(2), pp. 113-129.

Chen, Shu-Chen & Hwang, C., in Collaboration with Hwang, F. (1992). *Fuzzy Multiple Attribute Decision Making: Methods and Applications*, New York, Springer-Verlag.

Cheung, J. (1993). 'Managerial flexibility in capital investment decisions literature', *Journal of Accounting Literature*, 12, pp. 29-66.

Chiu, C. & Park, C. (1994). 'Fuzzy cash flow analysis using present worth criterion', *The Engineering Economist*, 39(2), pp. 113-138.

Clark, J., Hindelang, T. & Pritchard, R. (1984). *Capital Budgeting: Planning and Control of Capital Expenditures*, Englewood Cliffs. Prentice-Hall, Inc.

Currie, W. (1994). *The Strategic Management of Advanced Manufacturing Technology*, London, The Chartered Institute of Management Accountants.

Dubois, D. & Prade, H. (1980). *Fuzzy Sets and Systems: Theory and Applications*, New York, Academic Press.

Dugdale, D. & Abdel-Kader, M. G. (1998). 'A hierarchical framework for evaluating investment decisions in advanced manufacturing technology: an empirical approach', *Journal of Applied Accounting Research*, IV, October.

Dugdale, D. & Jones, C. (1995). 'Financial justification of advanced manufacturing technology' In D. Ashton, T. Hopper, & R. Scapens (eds), *Issues in Management Accounting*, 2nd ed., pp. 191-213, London, Prentice Hall.

Franz, D., Duke, J. & Omer, K. (1995). 'Capital budgeting: development of a decision approach for determining expected value of cash flows' In P. Siegel, A. De Korvin, & K. Omer (eds), *Applications of Fuzzy Sets and the Theory of Evidence to Accounting*, London, Jai Press Inc.

Jain, R. (1976). 'Decision making in the presence of fuzzy variables', *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-**6**(10), pp. 698-703.

Jain, R. (1977). 'A procedure for multiple-aspects decision making using fuzzy sets', *International Journal of Systems Science*, 8(1), pp. 1-7.

Kandel, A. (1986). *Fuzzy Mathematical Techniques with Applications*, Reading (UK), Addison- Wesley Publishing Company.

Kaplan, R. (1986). 'Must CIM be justify by faith alone?', *Harvard Business Review*, March-April, pp. 87-93.

Kaplan, R. & Atkinson, A. (1989). *Advanced Management Accounting*, 2nd ed., New Jersey, Prentice-Hall.

Kaufmann, A. & Gupta, M. (1985). *Introduction to Fuzzy Arithmetic: Theory and Applications*, New York, Van Nostrand Reinhold Company.

Kaufmann, A. & Gupta, M. (1991). *Fuzzy Mathematical Models in Engineering and Management Science*, North-Holland, Elsevier Science Publishing Company Inc.

Kim, K. & Park, K. (1990). ‘Ranking fuzzy numbers with index of optimism’, *Fuzzy Sets and Systems*, 35, pp. 143-150.

Knott, K. & Getto, R. (1982). ‘A model for evaluating alternative robot systems under uncertainty’, *International Journal of Production Research*, 20, pp. 155-165.

Laarhoven, P. & Pedrgcz, W. (1983). ‘A fuzzy extension of Saaty’s priority theory’, *Fuzzy Sets and Systems*, 11(3), pp. 229-241.

Lavelle, J. & Liggett, H. (1992). ‘Economic methods for evaluating investments in advanced manufacturing technologies’ In H. Parsaei, W. Sullivan, & T. Hanley (eds), *Economic and financial Justification of Advanced Manufacturing Technologies*, pp. 119-139, Amsterdam, Elsevier Science Publishers.

Lefley, F. (1994). ‘Capital investment appraisal of advanced manufacturing technology’, *International Journal of Production Research*, 32(12), pp. 2751-2776.

Lefley, F. (1996). ‘Strategic methodologies of investment appraisal of AMT projects: a review and synthesis’, *The Engineering Economist*, 41(4), pp. 345-363.

Liang, G. & Wang, M. (1991). ‘A fuzzy multi-criteria decision-making method for facility site selection’, *International Journal of Production Research*, 29(11), pp. 2313-2330.

Lumby, S. (1995). *Investment Appraisal and Financial Decisions*, 5th ed., London, Chapman & Hall.

Meredith, J. & Suresh, N. (1986). ‘Justification techniques for advanced manufacturing technologies’, *International Journal of Production Research*, 24(5), pp. 1043-1057.

Naik, B. & Chakravarty, A. (1992). ‘Strategic acquisition of new manufacturing technology’, *International Journal of Production Research*, 30(7), pp. 1575-1601.

Nelson, C. (1986). ‘A scoring model for flexible manufacturing systems project selection’, *European Journal of Operational Research*, 24, pp. 346-359

Nixon, B. (1995). ‘Technology investment and management accounting practice’, *British Journal of Management*, 6, pp. 271-288.

O'Brien, C. & Smith, S. (1993). ‘Design of the decision process for strategic investment in advanced manufacturing systems’, *International Journal of Production Economics*, 30-31, pp. 309-322.

Park, C. & Son, Y. (1988). ‘An economic evaluation model for advanced manufacturing systems’, *The Engineering Economist*, 34(1), pp. 1-26.

Parsaei, H. & Wilhelm, M. (1989). ‘A justification methodology for automated manufacturing technologies’, *Computers and Industrial Engineering*, 16(3), pp. 363-373.

Pike, R., Sharp, J. & Price, D. (1989). ‘AMT investment in the larger UK firm’, *International* Journal of Operations and Production Management, 9(2), pp. 13-26.

Pohjola, V. & Turunen, I. (1990). ‘Estimating the internal rate of return from fuzzy data’, *Engineering Costs and Production Economics*, 18, pp. 215-221.

Putrus, R. (1990). ‘Accounting for intangible in integrated manufacturing’, *Information Strategy: The Executives Journal*, 6(4), pp. 25-30.

Rayburn, L. (1989). *Principles of Cost Accounting Using a Cost Management Approach*, 4th. ed., Boston, IRWIN.

Rangone, A. (1997). ‘Linking organizationl effectiveness, key success factors and performance measures: an analytical framework’, *Management Accounting Research*, 8, pp. 207-219.

Ronen, J. & Sorter, G. (1972). ‘Relevant accounting’, *Journal of Business*, 45, pp. 258-282.

Saaty, T., 1980, *The Analytic Hierarchy Process*, New York, McGraw Hill.

Slagmulder, R. (1997). ‘Using management control systems to achieve alignment between strategic investment decisions and strategy’, *Management Accounting Research*, 8, pp. 103-139.

Slagmulder, R. & Bruggeman, W. (1992). ‘Justification of strategic investment in flexible manufacturing technology’, *Integrated Manufacturing Systems*, 3, pp. 4-14.

Slagmulder, R., Bruggeman, W. & Wassenhove, L. (1995). ‘An empirical study of capital budgeting practices for strategic investments in CIM technologies’, *International Journal of Production Economics*, 40, pp. 121-152.

Srinivasan, V. & Millen, R. (1986). ‘Evaluating flexible manufacturing systems as a strategic investment’ In K. Stecke, & R. Suri (eds), *Proceedings of the Second ORSA/TIMS Conference on Flexible Manufacturing Systems: Operations Research Models and Applications*, , pp. 83- 93, Amsterdam, Elsevier Science Publishers.

Ward, T. (1985). ‘Discounted fuzzy cash flow analysis’, *Proceedings of Institute of Industrial Engineers*, December, pp. 476-481.

Ward, T. (1989). ‘*Fuzzy discounted cash flow analysis*. *Applications of Fuzzy Set Methodologies in Industrial Engineering’* In G. Evans, W. Karwowski, & M. Wilhelm (eds), *Applications of Fuzzy Set Methodologies in Industrial Engineering*, pp. 91-102, Amsterdam, Elsevier.

Wilhelm, M. & Parsaei, H. (1988). ‘A Model for selecting and evaluating flexible automation systems’, *Proceedings of the Institute of Industrial Engineers Integrated Systems Conference*, pp. 341-346.

Wilhelm, M. & Parsaei, H. (1991). ‘A fuzzy linguistic approach to implementing a strategy for computer integrated manufacturing’, *Fuzzy Sets and Systems*, 42, pp. 191-204.

Zadeh, L. (1965). Fuzzy Sets, *Information and Control*, **8**, 338-353.

Zadeh, L. (1972). ‘A fuzzy set theoretic interpretation of linguistic hedges’, *Journal of Cybernetics*, 2(3), pp. 4-34.

Zadeh, L. (1973). ‘Outline of a new approach to the analysis of complex systems and decisions processes’, *IEEE Transactions of Systems, Man, and Cybernetics*, SMC-3(1), pp. 28-44.

Zadeh, L. (1975a). ‘The concept of a linguistic variable and its application to approximate reasoning - I’, *Information Science*, 8, pp. 199-249.

Zadeh, L. (1975b). ‘The concept of a linguistic variable and its application to approximate reasoning - II’, *Information Science*, 8, pp. 301-357.

Zadeh, L. (1975c). ‘The concept of a linguistic variable and its application to approximate reasoning - III’, *Information Science*, 9, pp. 43-80.

Zebda, A. (1984). ‘The investigation of cost variances: a fuzzy set theory approach’, *Decision Sciences*, 15(3), pp. 359-388.

Zebda, A. (1989). ‘Fuzzy set theory and accounting’, *Journal of Accounting Literature*, 8, pp. 76-105.

TABLE 1

Panel A: *The geometric mean for the linguistic values*

Q_i	e_i		f_i		g_i	
Q_1 *(VI)*	$(1\times2\times4\times6\times8)^{1/5}$	= 3.29	$(1\times3\times5\times7\times9)^{1/5}$	= 3.94	$(1\times4\times6\times7\times9)^{1/5}$	= 4.32
Q_2 *(I)*	$(1/4\times1\times2\times2\times3)^{1/5}$	= 1.24	$(1/3\times1\times2\times3\times4)^{1/5}$	= 1.51	$(1/2\times1\times3\times4\times5)^{1/5}$	= 1.97
Q_3 *(MI)*	$(1/6\times1/3\times1\times1\times3/2)^{1/5}$	= 0.61	$(1/5\times1/2\times1\times3/2\times2)^{1/5}$	= 0.79	$(1/4\times1\times1\times2\times3)^{1/5}$	= 1.08
Q_4 *(MU)*	$(1/8\times1/4\times1/2\times1\times1)^{1/5}$	= 0.43	$(1/7\times1/3\times2/3\times1\times5/4)^{1/5}$	= 0.52	$(1/6\times1/2\times1\times1\times3/2)^{1/5}$	= 0.66
Q_5 *(U)*	$(1/9\times1/5\times1/3\times2/3\times1)^{1/5}$	= 0.34	$(1/9\times1/4\times1/2\times4/5\times1)^{1/5}$	= 0.41	$(1/8\times1/3\times2/3\times4/5\times1)^{1/5}$	= 0.47
Total		5.91		7.17		8.50

Panel B: *Weights of the linguistic values*

W_i	$k_i = e_i$ / 8.50	$l_i = f_i$ / 7.17	$m_i = g_i$ / 5.91
W_1 *(VI)*	0.39	0.55	0.73
W_2 *(I)*	0.14	0.21	0.33
W_3 *(MI)*	0.07	0.11	0.18
W_4 *(MU)*	0.05	0.07	0.11
W_5 *(U)*	0.04	0.06	0.08

TABLE 2

Possible interpretations of linguistic terms

	Interpretation when used to express:	
Fuzzy numbers	**Importance of non-financial Criteria/ sub-criteria**	**Alternative's ability to achieve sub-criterion requirements**
(.04, .06, .08)	Unimportant	Very low
(.09, .13, .19)	More-or-less unimportant	Low
(.16, .24, .37)	More-or-less important	Medium
(.30, .45, .70)	Important	High
(.69, 1.00, 1.00)	Very important	Very high

TABLE 3

The fuzzy linguistic scale for risk factors

Fuzzy numbers	**VD**	**LM**	**PT**	**RS**	**EC**
(0.00, 0.06, 0.08)	Non	Infinite	Very high	Very high	Excellent
(0.09, 0.11, 0.19)	Very low	Very long	High	High	Very good
(0.16, 0.24, 0.37)	Low	Long	Average	Average	Good
(0.30, 0.45, 0.70)	Average	Average	Low	Low	Average
(o.60, 0.70, 0.85)	High	short	Very low	Very low	Poor
(0.80, 1.00, 1.00)	Very high	Very short	Non	Non	Very poor

VD: Volatility of demand, LM: Life of market, PT: Proven technology, RS: Reliability of supplier, EC: Experience in the company.

TABLE 4

Stepwise description of AMT investment decisions algorithm

1. Form a committee of decision makers, then decide the appropriate financial measure, non-financial criteria and risk measures and identify the available alternatives for AMT investment projects.

2. Check each alternative for consistency with overall company strategy. Consider only those alternatives which have the desired consistency.

3. Develop fuzzy estimates of project cash flows including those items which can be sensibly quantified in cash flow terms and calculate the appropriate fuzzy financial measure(s).

4. Perform sensitivity analysis in order to identify any key variables which might be especially sensitive ('risky').

5. Identify those intangible benefits which have not be quantified in the financial analysis. These benefits will probably fall under three headings: product, process and market. A hierarchical structure may be useful in summarising the attributes of each alternative.

6. Develop a fuzzy linguistic scale using pair-wise comparisons or a ready-made one to evaluate the fuzzy importance of the non-financial investment criteria and sub-criteria and to assign fuzzy ratings to the AMT investment projects.

7. Tabulate the importance assigned to the non-financial criteria and sub-criteria by the decision maker(s).

8. Tabulate the ratings assigned to each alternative with respect to each sub-criterion by the decision maker(s).

9. Calculate the fuzzy non-financial criteria measure for each alternative using equation (29).

10. Consider any action which might reduce the risk inherent in sensitive variables. Then identify the risk possibilities associated with each alternative. A hierarchical representation may be useful in summarising the inherent risks. These risks will probably fall under two headings: market and technology.

11. Weigh each risk factor using a fuzzy linguistic scale (or some quantitative proxies) and evaluate each alternative with respect to each risk factor.

12. Calculate the fuzzy risk measure for each alternative ($FRMA_k$) using equation (33)

13. Calculate the ranked value for each alternative under each fuzzy measure (financial, non-financial and risk) using equations (35) - (39).

14. Develop an efficient plane using ranked values calculated in the previous step and choose an alternative located on this plane which satisfies the decision maker's preferences regarding financial return, non-financial benefits and risk.

TABLE 5

Estimated fuzzy initial investments (I_0) and fuzzy net cash flows after tax (x_t) (£000,000s)

	Project I	Project II	Project III
I_o	(0.8, 1.0, 1.1)	(1.2, 1.4, 1.6)	(1.6, 1.8, 1.9)
x_1	(.33, .35, .37)	(.38, .40, .41)	(.41, .42, .43)
x_2	(.33, .35, .37)	(.38, .40, .41)	(.41, .42, .43)
x_3	(.33, .35, .37)	(.38, .40, .41)	(.41, .42, .43)
x_4	(.33, .35, .37)	(.38, .40, .41)	(.41, .42, .43)
x_5	(.33, .35, .37)	(.38, .40, .41)	(.41, .42, .43)
x_6	(.31, .33, .35)	(.37, .39, .41)	(.41, .42, .43)
x_7	(.28, .30, .31)	(.34, .36, .40)	(.38, .40, .42)

TABLE 6

Sensitivity of FNPV to changes in the annual net cash flows after tax (£000,000s)

	Project I	Project II	Project III
Original estimates	(0.47, 0.78, 1.14)	(0.22, 0.65, 1.00)	(0.08, 0.38, 0.71)
+ 5%	(0.55, 0.87, 1.24)	(0.32, 0.76, 1.11)	(0.18, 0.48, 0.83)
+ 10%	(0.63, 0.96, 1.34)	(0.41, 0.86, 1.22)	(0.28, 0.59, 0.94)
+ 15%	(0.71, 1.05, 1.43)	(0.50, 0.96, 1.33)	(0.38, 0.70, 1.06)
- 5%	(0.39, 0.69, 1.05)	(0.13, 0.55, 0.89)	(-0.02, 0.27, 0.60)
- 10%	(0.31, 0.60, 0.95)	(0.04, 0.45, 0.78)	(-0.12, 0.16, 0.48)
- 15%	(0.23, 0.51, 0.85)	(-0.05, 0.34, 0.67)	(-0.22, 0.05, 0.36)

TABLE 7

Sensitivity of FNPV to changes in the discount rate (£000,000s)

	Project I	Project II	Project III
Original estimates	(0.47, 0.78, 1.14)	(0.22, 0.65, 1.00)	(0.08, 0.38, 0.71)
+ 5%	(0.44, 0.76, 1.12)	(0.19, 0.63, 0.98)	(0.05, 0.34, 0.68)
+ 10%	(0.42, 0.73, 1.10)	(0.19, 0.60, .95)	(0.02, 0.32, 0.65)
+ 15%	(0.40, 0.71, 1.07)	(0.14, 0.57, 0.92)	(-0.01, 0.29, 0.63)
- 5%	(0.50, 0.80, 1.17)	(0.25, 0.68, 1.03)	(0.11, 0.40, 0.74)
- 10%	(0.52, 0.83, 1.19)	(0.28, 0.71, 1.06)	(0.15, 0.43, 0.77)
- 15%	(0.55, 0.85, 1.21)	(0.32, 0.74, 1.09)	(0.18, 0.47, 0.80)

TABLE 8

Sensitivity of FNPV to changes in the project life (£000,000s)

	Project I	Project II	Project III
7 years	(0.47, 0.78, 1.14)	(0.22, 0.65, 1.00)	(0.08, 0.38, 0.71)
6 years	(0.33, 0.61, 0.95)	(0.05, 0.44, 0.75)	(-0.11, 0.14, 0.45)
5 years	(0.15, 0.40, 0.72)	(-0.16, 0.20, 0.48)	(-0.34, -0.12, 0.16)
4 years	(-0.05, 0.16, 0.45)	(-0.40, -0.08, 0.19)	(-0.60, -0.41, 0.14)

TABLE 9

The linguistic importance of the non-financial criteria

Criterion	Importance (Ci)	Importance (C_i)
Quality	Very important	(0.69, 1.00, 1.00)
Flexibility		
Variety	Very important	(0.69, 1.00, 1.00)
Volume	Very important	(0.69, 1.00, 1.00)
Customer requirements		
New feature	Very important	(0.69, 1.00, 1.00)
Responsiveness	Important	(0.30, 0.45, 0.70)
Delivery	More-or-less important	(0.16, 0.24, 0.37)

TABLE 10

The scale ratings assigned to the alternatives

Criterion	Project I	Project II	Project III
Quality	Very low	Very high	Very high
	(0.00, 0.10, 0.19)	(0.70, 1.00, 1.00)	(0.70, 1.00, 1.00)
Flexibility			
Variety	Low	Medium	Very high
	(0.15, 0.25, 0.40)	(0.30, 0.50, 0.70)	(0.70, 1.00, 1.00)
Volume	Low	Medium	Very high
	(0.15, 0.25, 0.40)	(0.30, 0.50, 0.70)	(0.70, 1.00, 1.00)
Customer requirements			
New feature	Low	Very high	Very high
	(0.15, 0.25, 0.40)	(0.70, 1.00, 1.00)	(0.70, 1.00, 1.00)
Responsiveness	Low	Medium	Very high
	(0.15, 0.25, 0.40)	(0.30, 0.50, 0.70)	(0.70, 1.00, 1.00)
Delivery	Low	Medium	Very high
	(0.15, 0.25, 0.40)	(0.30, 0.50, 0.70)	(0.70, 1.00, 1.00)

TABLE 11

The importance weight of the risk factors

Factor	Weight	Weight
Volatility of demand	More-or-less important	(0.16, 0.24, 0.37)
Life of market	Important	(0.30, 0.45, 0.70)
Proven technology	Very important	(0.69, 1.00, 1.00)
Reliability of supplier	Very important	(0.69, 1.00, 1.00)
Experience in the company	More-or-less unimportant	(0.09, 0.11, 0.19)

TABLE 12

The evaluation of the alternatives with respect to risk factors

Factors	Project I	Project II	Project III
Volatility of demand	Average	Very low	Non
	(.30, .45, .70)	(.09, .11, .19)	(.00, .06, .08)
Life of market	Long	Infinite	Infinite
	(.16, .24, .37)	(.00, .06, .08)	(.00, .06, .08)
Proven technology	Low	High	High
	(.30, .45, .70)	(.09, .11, .19)	(.09, .11, .19)
Reliability of supplier	High	Average	Very high
	(.09, .11, .19)	(.16, .24, .37)	(.00, .06, .08)
Experience in the company	Excellent	Good	Very poor
	(.00, .06, .08)	(.16, .24, .37)	(.80, 1.0, 1.0)

TABLE 13

Summary of the three fuzzy measures

Fuzzy measures	Project I	Project II	Project III
FNPV	(0.47, 0.78, 1.14)	(0.22, 0.65, 1.00)	(0.08, 0.38, 0.71)
FNFM	(0.06, 0.17, 0.30)	(0.25, 0.56, 0.69)	(0.38, 0.78, 0.85)
FRM	(0.07, 0.16, 0.28)	(0.04, 0.09, 0.15)	(0.03, 0.06, 0.11)

TABLE 14

The ranked values of the three fuzzy measures

	Project I	Project II	Project III
RV_{FNPV}	0.49	0.34	0.13
RV_{FNFM}	0.03	0.32	0.59
RV_{FRM}	0.08	0.03	0.01

Table 15

The normalised ranked values of the three fuzzy measures

	Project I	Project II	Project III
RV_{FNPV}	0.51	0.35	0.14
RV_{FNFM}	0.03	0.34	0.63
RV_{FRM}	0.67	0.25	0.08

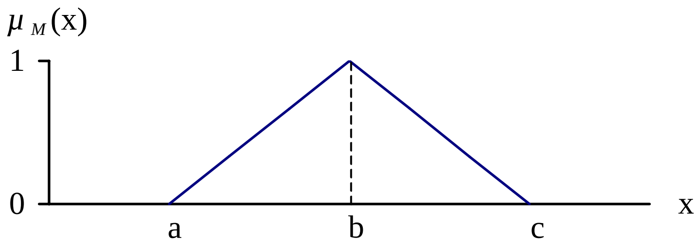

Figure 1. A triangular fuzzy number ***M = (a, b, c)***

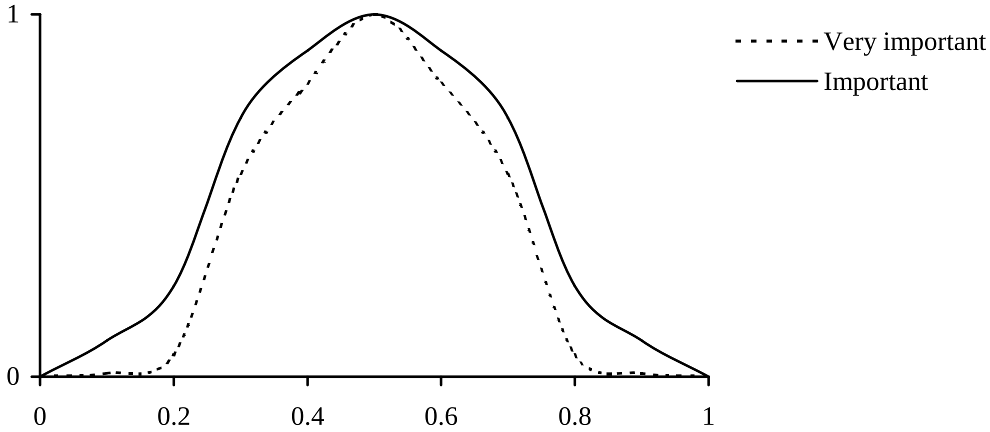

Figure 2. Membership functions of 'very important' and 'important' values

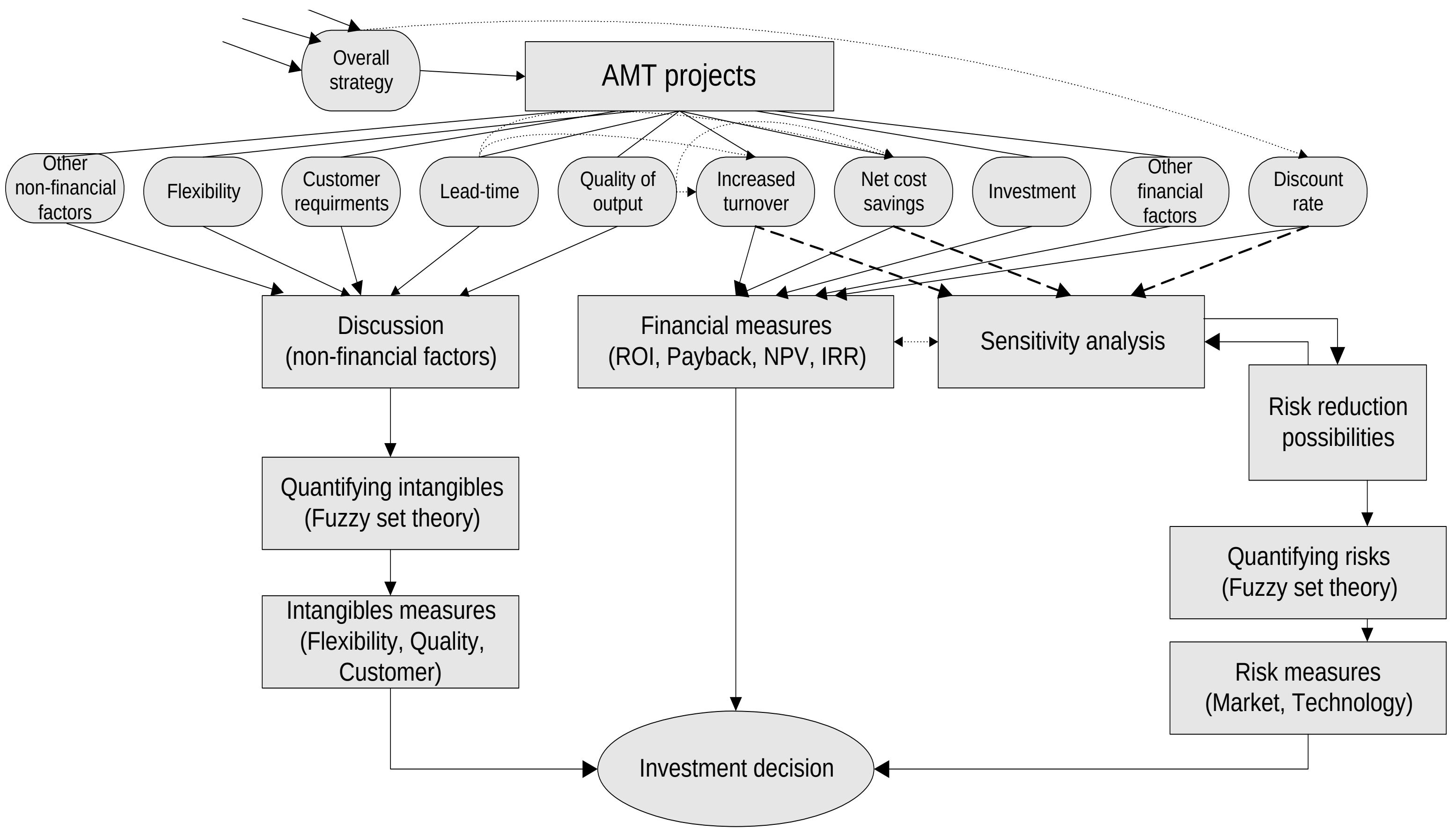

Figure 3. A model for AMT investment decision making

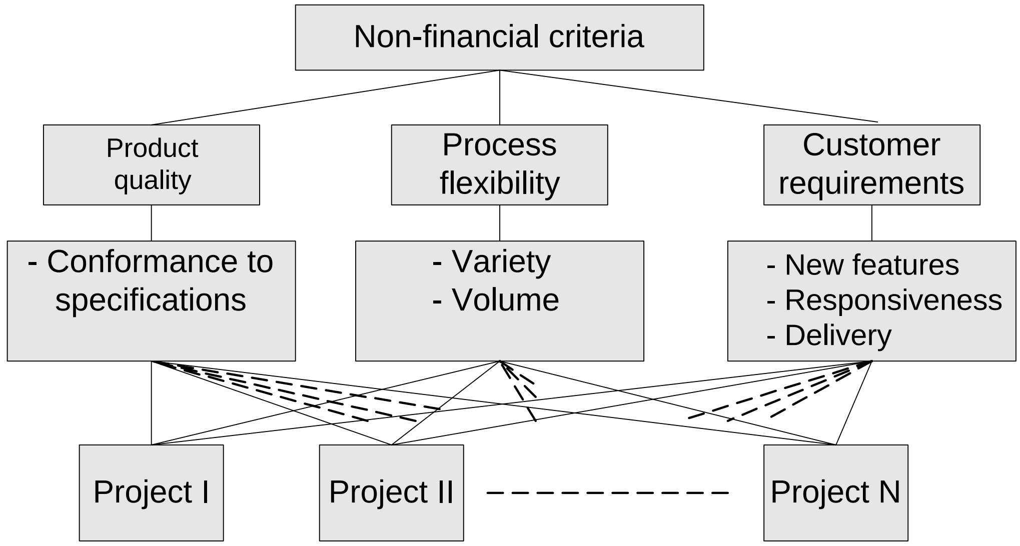

Figure 4. The non-financial criteria hierarchy

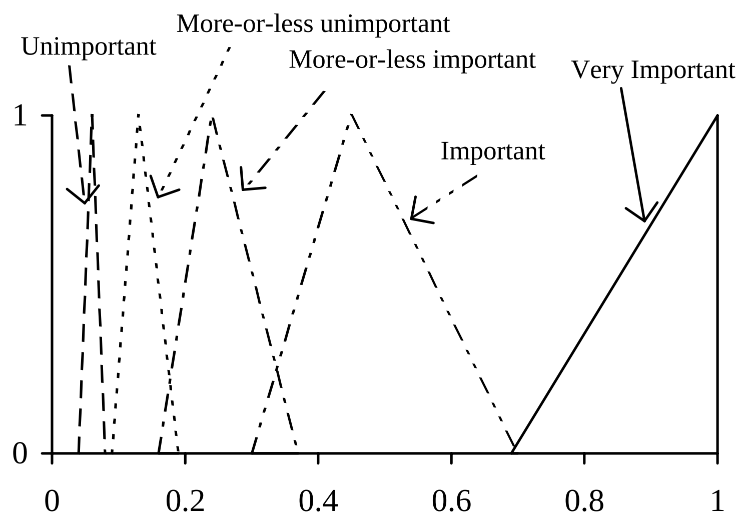

Figure 5. Membership functions of linguistic scale

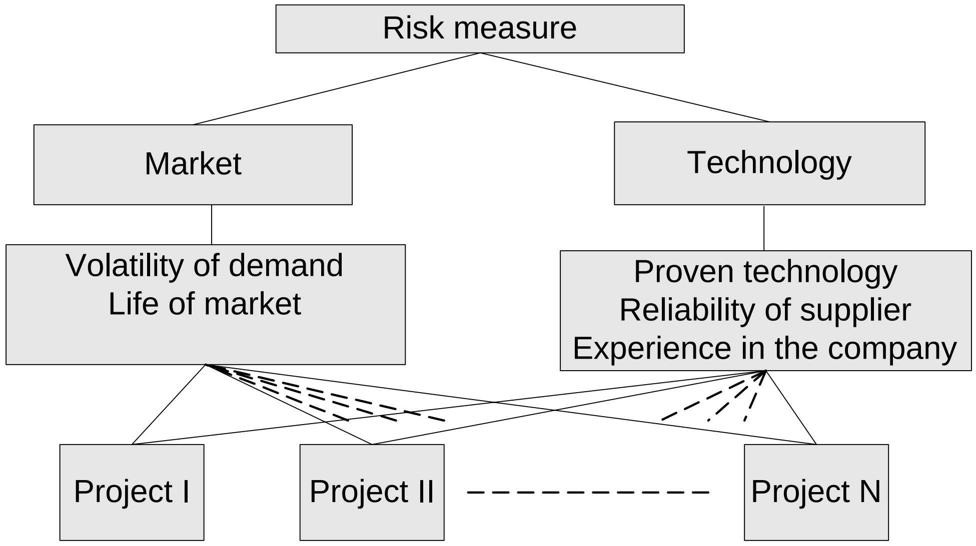

Figure 6. The risk measure hierarchy

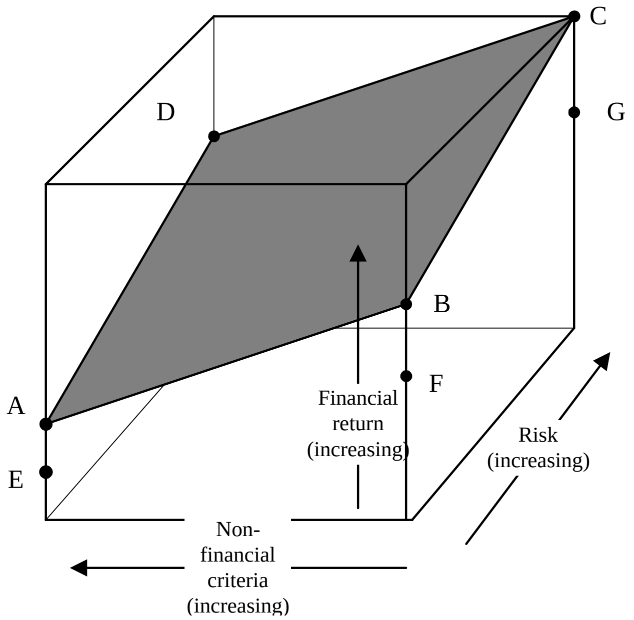

Figure 7. A three dimensional analysis for selecting AMT investment projects

Adapted from Accola (1994, p. 24)

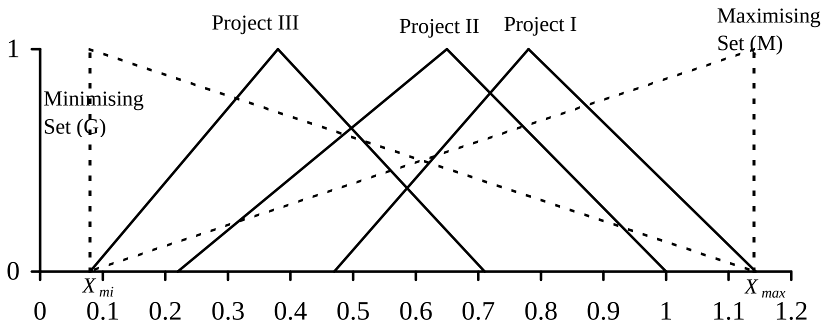

Figure 8a. The *FNPV* for the three projects

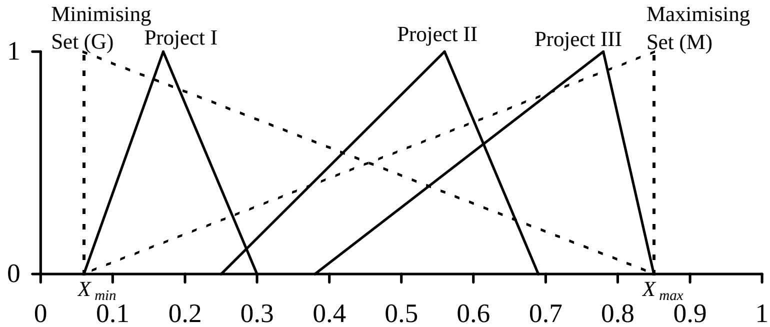

Figure 8b. The *FNFM* for the three projects

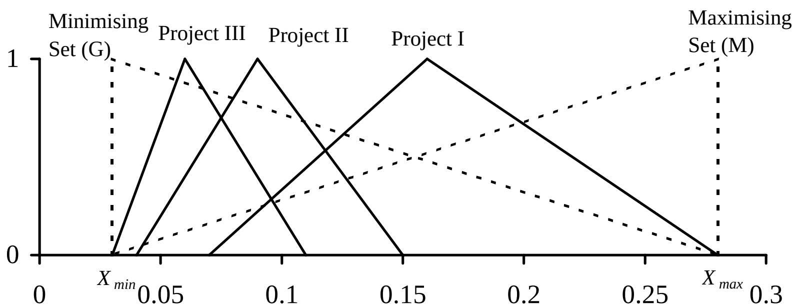

Figure 8c. The *FRM* for the three projects

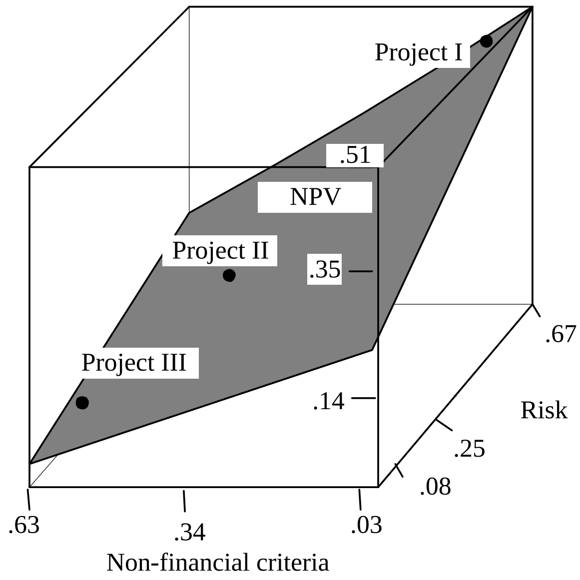

Figure 9. A graphical presentation of the example