DRTC Workshop on Informetrics and Scientometrics
16-19 March 1998, Bangalore

Paper : AB

CLASSICAL LAWS OF INFORMETRICS : An Overview

I.K. Ravichandra Rao Professor and Head, Documentation Research and Training Centre, Indian Statistical Institute, 8th Mile, Mysore Road, Bangalore 560 059

[*Classical laws of bibliometrics – Bradford's law, Lotka's and Zipf's law – are discussed, with emphasis on to law of scattering and inverse square law of scientific productivity. Two different approaches to bibliometric distributions – size and rank frequency approaches, characteristics of bibliometric distributions are discussed]*

1. LAW OF SCATTERING

The topic in bibliometrics that has received a great deal of attention is the problem related to the scattering of articles. The tabulation of the distribution of the number of references on a specific subject area among the journals is the traditional way of summarizing scattering of articles. In most of the bibliographies covering a short period on a particular scientific subject, it may be observed that :

i) most of the journals contribute only one article each; the other articles in the journals obviously are not relevant to the said subject.

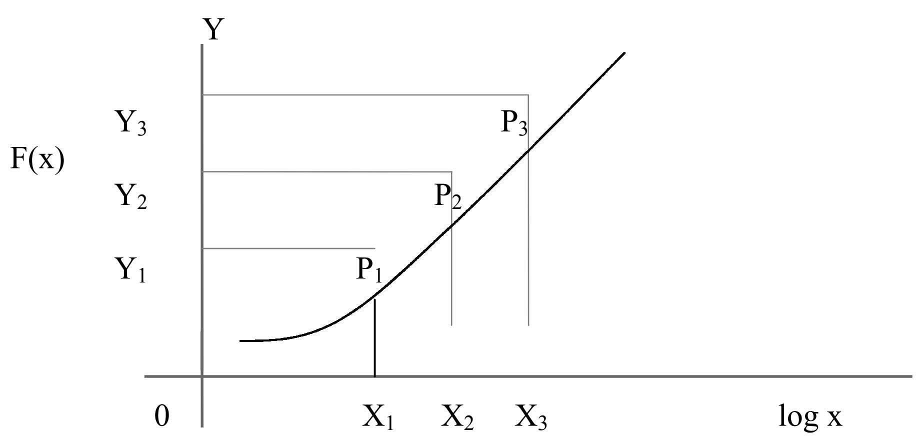
ii) a few journals contribute on an average 5 to 10 articles each;

iii) very few journals, as compared to the first two groups, contribute a large number of articles

This was first observed by Bradford (1934). In his article on “sources of information”, he studied the extent in which literature in a single discipline is scattered over a range of journals. His study was based on the literature in geophysics and lubrication. He plotted the partial sums of references against the natural logarithm of the partial sum of numbers of journals, and he noticed that the resulting graph is a straight line. On the basis of this observation, he suggested following log-linear relation to describe a scattering phenomena.

$$F(x) = a + b * \log x$$

F(x) is the partial sum of references (-- cumulative number of references) contained in the first x most productive journals; a and b are constants. The following figure is a typical log-linear curve, as observed by Bradford.



Baed on this graph, by plotting $Y_1 P_1$, $Y_2 P_2$ and $Y_3 P_3$ as parrrllel to X- axis, $P_1 X_1$, $P_2 X_2$ and $P_3 X_3$ as perpendiculars to X axis and further $0Y_1 = Y_1 Y_2 = Y_2 Y_3$, Bradford stated that "if scientific journals are arranged in order of decreasing productivity of articles on a given subject, they may be divided into a nuclear of periodicals more particularly devoted to subject and several groups or zones containing the same number of articles on the nuclear, when the zone will be $1:n:n^2:\ldots$".

Later in 1948, Bradford, based on his analytical approach, he again argued that the ratio of the zone size will be as $1:n:n^2$. In his analytical approach, he assumed the collection of journals is ranked (or arranged) in decreasing productivity (-- in terms of number of articles it contains, on a given subject). He then divided these journals into k groups/zones, such that

$$m_1 r_1 = m_2 r_2 = m_3 r_3 = .. = m_k r_k \qquad \text{(A)}$$

where m_i is the number of journals and r_i is the average number of articles per journal in the i^{th} zone. Thus, from the relation (A), we have :

$$m_{i-1} r_{i-1} = m_i r_i$$

$$i = 2, 3, \ldots. k$$

and with the supposition that $n_{i-1} = n_i = n$

Where $n_{i-1} = \dfrac{r_{i-1}}{r_i}$

$$i = 2, 3, 4 \ldots.. k$$

he suggested that the ratio of the zone size will be $1: n:n^2: \ldots.$ This n is known as **Bradford multiplier.**

Bradford infact argued that "**we have no reason why n_1 and n_2 should differ and the simple supposition we can make is that they are equal**". He thus assumed $n_1=n_2=n$. How far his assumption is correct? Based on a small sample of 12 databases, Ravichandra Rao (1997) has shown in his study that n_1 and n_2 are not likely to be equal and it is most likely that Bradford multipliers vary from zone to zone.

Since 1948, many have worked in this area and suggested modifications to this law. For instance, Vickery (1948) observed that Bradford's explanation based on the graphical and his verbal statements are slightly different. By defining, S_{kr} as number of journals in the top most k groups, k = 1,2,3,…., Vickery pointed out that

a) based on Bradford's graph, the following statement

$$S_{2r} : S_{3r} : S_{kr} : \ldots = 1:n:n^2:\ldots \text{ is true}$$

b) based on the verbal statements, the following is true :

$$S_{1r} : S_{2r} - S_{1r} : S_{3r} - S_{2r} : \ldots = 1:n:n^2:\ldots \qquad \text{(B)}$$

Thus, there is a difference between graphical and verbal statements. Wilkingon (1972) elaborately discusses this difference and then based on the verbal statements he derives the following equation :

$$R(n) = j \log\left(\frac{n}{t}+1\right) \qquad \textit{for } n > S_{1r}$$

R(n) is the cumulative number of references in n most productive journals; t is a constant.

From the relation as stated in (B), Vickery derived :

$$S_{kr} = s\,(n^k - 1)$$

where $s = S_{1r} / (n-1)$ and n is the Bradford multiplier. Further he argued that

$$\log S_{kr} - \log S_{(k-1)\,r} \simeq \log n$$

It implies that the curve approaches a straight line as the logarithmic interval tends to log n. With the assumption that (A) is true, Leimkuhler (1967) derived an equivalence of Bradford's law to a log form. By defining F(x) as relative cumulative number of references in topmost x proportion of journals, Leimkuhler suggested that

$$F(X) = \frac{\log(1+\beta x)}{\log(1+\beta)}$$

$$0 \le x \le 1$$
$$\beta > 0$$

The derivation of the above distribution function is given here. Let us consider a collection of journals arranged in decreasing order of productivity and divided into m groups of relative sizes $d_1, d_2, d_3, d_4, \ldots. d_m$; d_i is the proportion of journals in i^{th} group; $\sum_{i=1}^{m} d_i = 1$. If the groups are chosen such that each group yields the same number of references, then according to Bradford :

$$d_i = b_m\, d_{i-1} = b_m^{i-1}\, d$$

$$i = 1,2,3,\ldots m$$
$$m = 2,3,4,\ldots$$
$$0 \le d_i \le 1 \le b_m$$

b_m is known as Bradford multiplier, in this multiplier, m indicates that there are m divisions. Since $\sum d_i = 1$, we have

$$\sum_{i=1}^{m} d_i = 1 = d_1 \frac{\left(b_m^m - 1\right)}{\left(b_m - 1\right)}$$

d_1 is therefore :

$$d_1 = \frac{b_m - 1}{b_m^m - 1}$$

Thus,

$$d_i = b_m^{i-1} d_1 = \frac{b_m^{i-1}\left(b_m - 1\right)}{(b_m^m - 1)}$$

The cumulative proportion of journals ($D_{j,m}$) in the first j of m group and containing the fraction j/m of all of references is defined by :

$$D_{j,m} = \sum_{i=1}^{j} d_i = \frac{b_m^j - 1}{b_m^m - 1}$$

j = 1, 2, 3,…m

Define a parameter b, such that $b_m=b^{1/m}$, b is only a function of the total number of references and is independent of m, the number of zones into which the collection has been divided. Thus, for each j, we have

$$D_{j,m} = \frac{b^{j/m} - 1}{b - 1}$$

Further, for each j, if we can associate a value of x such that $x=D_{j,\,m}$ and F(x)=j/m, we then have:

$$x = \frac{b^{F(x)} - 1}{b-1}$$

Thus, for β=b-1, we have :

$$F(x) = \frac{\log(1+\beta x)}{\log(1+\beta)} \;, \quad 0 \le x \le 1$$

$$\beta > 0$$

F(x) is called the Bradford distribution. Its density function is given by :

$$f(x) = \frac{\beta}{(1+\beta x)\,\log(1+\beta x)}$$

The mean of the distribution is

$$\mu = \frac{1}{\log(1+\beta)} - \frac{1}{\beta}$$

In Wilkinson's equation, if we define F(n) = R(n)/R(N) and substituting for R(n) and R(N) we have.

$$F(n) = \frac{j \log\left(n/t+1\right)}{j\,\log\,(N/t+1)}$$

Since x = n/N, Leimkuhler's equation can be obtained from the above equation as follows:

$$F(x) = \frac{\log\,(1+\beta x)}{\log(1+\beta)} \quad \text{for } \beta = \frac{N}{t}.$$

Brookes (1969a, 1969b) on the basis of the graphical formulation argued that

$$R(n) = k \log \frac{n}{s}$$

where k and s are constants. The j and t in Willkinson's equation and k and s in Brooke's equation are not equal (k≠j and s≠t). This again indicates that, the verbal and graphical equations are not equal. In fact, they do not even converge to the same limit for a large n.

1.1 Bradford-Zipf Distribution

Brookes (1968), after reviewing the literature of journal productivity, restates the law of scattering in the form of a hypothesis where the number of pertinent articles yielded by the *n*th ranking periodical exceeds that of the (n+1)th by $b \log \frac{n}{n-1}$ where b is the ratio between successive equity yielding zone of periodicals. Brookes then agrees with Vickery that "Bradford's law requires

$$R(n) = R(n^2) - R(n) = R(n^3) - R(n^2) = \ldots$$

for all integral values of n greater than unity". R(n) is the cumulative number of references found in the n most productive journals. According to this relation $R(n^m)=mR(n)$, where m is a positive integer. Brookes therefore claims that "the only function which perfectly satisfies the condition is:

$$R(n) = k \log n$$

where k is a constant. We then have $R(n^m)=k \log n^m = m' \log n$ as required. The graph of $R(n)=k \log n$ is obtained by plotting R(n) on a linear scale along the y-axis and n on a logarithmic scale along the x-axis. This graph is likely to be a straight line with slope k. Thus, Brookes has developed the graphical formulation of the law. However, in most cases, a straight line is attained only in part. The cumulative total of relevant papers found in the first n journals, when all the journals are ranked in order of decreasing productivity is unlikely to be zero for n=1; but in the equation as suggested by Brookes, R(1)=0. This form of the Bradford's distribution is therefore a crude approximation.

The density function r(n) which expresses the number of reference corresponding to the nth journal is therefore given by:

$$r(n) = \frac{d}{dn} R(n) = \frac{k}{n}$$

which is similar to Zipf's function. In fact,

$$r(n) = R(n) - R(n-1)$$

$$= k \log n - k \log (n-1) = k \log (1 - 1/n)$$

$$= k \left[\frac{1}{n} + \frac{1}{2n^2} + \frac{1}{3n^3} + \ldots \right] \simeq \frac{k}{n}$$

as n increases. He calls this the complete Bradford-Zipf curve. He also points out that the following distribution are likely to fit into the Bradford-Zipf curve:

1. Distribution of number of books published by each publisher;
2. Distribution of number of references to each periodical; and
3. Distribution of the number of the times a particular books is issued

Brookes (1969a, 1969b) in another paper further attempts to get a better fits by expressing the curve in two parts as show below :

$$R(n) = \alpha\, n^{\beta} \qquad \begin{Bmatrix} 1 \le n \le c \\ \alpha, \beta > 0 \end{Bmatrix}$$

$$= N \log \frac{n}{s} \qquad \begin{Bmatrix} c < n \le N \\ s > 0 \end{Bmatrix}$$

In another paper Brookes (1977), also argued that a mixed Poisson model describes the Bradford law and he claims that it elucidates the uncertainties surrounding the law and its applications. Since Poisson probabilities converge to zero for a large n, much repaidly it is unlikely that it can describe a tail of the journal productivity curve.

1.2 Size-Frequency Model

Kendall (1960) in his analysis of the bibliography upto 1958, pertaining to Operation Research claims that the scattering of articles in journals is similar to that of income distribution. He equates **journals to persons** and **number of articles to the size of income**. Thus, he obtains a pattern in which many journals (people) have a few articles (low income) in a subject, comparing to a few journals (few millionaires) with a large number of articles in a subject. Kendall substantatites this fact in the following model:

$$J_p = \frac{1}{p^{(\rho+1)}}$$

where J_p is the relative number of journals having p reference each; p=1, 2, 3, ….. He also shows that this distribution is structurally similar to the Zipf distribution. In fact he is the first to publish the Bradord-Zipf equivalence. Naranan (1970, 1971) shows with certain assumptions that a frequency distribution of the number of journals with p articles is given by :

$$J(p) \propto p^{-}\theta; \quad \theta > 0, \quad p = 1, 2, 3, \ldots.$$

He fits this function to the data collected by Goffman and by many others. His assumptions are:

1. The number of journals in a subject grows exponentially in time;
2. Each journal augments in the number of papers in the subject exponentially in time;
3. The rate of growth of articles in each individual journal is the same.

But, Hubert (1976) argues that Narnan's interpretation of the original form of Bradford's law does not follow a stochastic argument based on his assumptions. Infact Hubert gets an equation similar to the "Naranan variable". Naranan considers "number of papers" as a variable; Hubert by considering the number of journals with p papers as a variable (say, Y_p- number of journals with p papers) shows:

$$E\,(Y_p) = kp^{-\theta} \qquad p = 1, 2, 3, \ldots. \quad \theta > 0$$

where k and θ are constants; E is the expected value in a statistical sense.

1.3 Rank Frequency Model

By defining the rank of a journal as the "number of distinct journals, including itself, that have occurred at least as frequently as it has" (Hubert (1977), p 465) and also defining f(r) as the number of occurrences of a journal of rank r, Hubert suggested the following law for journal productivity:

$$f(r) = \alpha r^{\beta}$$

where α and β are positive constants. His approach is an inverse of the size-frequency approach. In the size-frequency formulation, N_j is the number of journals which have j references each. For an observed sample of K references in N distinct journals, it will be

$$\sum_{j=1}^{n} N_j = N \qquad \sum_{j=1}^{n} j N_j = K$$

where n is the number of frequency classes. From empirical evidences, Hubert assumes that the expected value of N_j for each j is:

$$E(N_j) = K\, C_n(b)\, j^{-(b+1)}$$

where

$$C_n(b) = \left[\sum_{j=1}^{n} j^{-(b+1)}\right]^{-1}$$

It may be noted that $C_n(b)$ is a Zeta function and the function $E(N_j)$ is similar to that of Zipf's function. For a large n, $E(N_j)$ implies that the frequency of references (x) is a function of the rank of journal of the following form :

$$x = f(r) = (r/c)^{-1/b}$$

or $$x = f(r) = (r/c)^{-1/b}$$

$$f(r) = \alpha r^{-\beta},\ \beta = 1/b \text{ and } \alpha = c^{1/b}$$

One estimates b with the size-frequency approach and β is estimated with the rank-frequency approach. Hubert also observes that for a few sets of data, β is more suitable than b.

1.4 Other Studies

Chung (1977) suggests a model to portray the scattering of articles on a subject among journals. Chung class this model a non-Bradfordian model. It is given below :

$$p(x) = \frac{1}{x^{\alpha}}$$

$$x = 1, 2, 3, \ldots..$$

This model explains the relation between the number of references and the proportion of periodicals containing x references. This model appears to be similar to Naranan's model or a generalized version of Lotka's law. **Also, for x=1, p(x)=1 which is very unlikely**.

Since 1948, many have worked in this area and suggested different models to explain law of scattering. Simon (1955) proposed a Beta model under the following two assumptions.

i) There is a constant probability α that the k^{th} paper be published in a new journal that has not published in the first (k-1) papers.
ii) The probability the k^{th} paper is published in a journal that has published i paper is prepositional to i*f (i, k-1); i.e. to the total number of papers of all journals that have published exactly i papers. The β model is given by

$$j(r) = \frac{N}{r^{(1+\rho)}}$$

N is the total number of periodicals containing at least one paper n the subject, ρ is the distance between origin and the point at which straight line meets at x-axis, and j(r) is the distribution of the number of journals j with exactly r papers.

Cole (1958) suggested a semi-log model, as mentioned by Bradford to explain law of scattering. Groos(1967) observed a S-shape curve (with a droop, at the end of the curve) to explain law of scattering. Fairthorne (1969) and also Asai (1981) suggested a log model. Karmeshu and others (1982) presented two models to explain the mechanism that could produce Bradford distributions. These models are called as subdivisions model and multiple factor model. Burrell (1988) suggested Waring process to explain general features of Bradford's law. Basu (1992, 1995) suggested a model to explain distribution of articles in journals based on probabilistic considerations.

Some of these models are based on size-frequency approach with probabilistic considerations and most of them are based on rank-frequency approach. Some of models are theoretical in nature and they are not tested with real-life data. To identify a suitable model to explain the law of scattering, Ravichandra Rao (1997) fitted about 24 different models to the 12 different sets of data. He has observed that log-normal model fits much better than the many other models, including the log-liner model.

Law of scattering is an area where much work has been done. However, till now, no one has come out with a single model which fits fairly well to most of the data sets. Various models, suggested by the different authors are summarized in Table 1.

Table 1 : Different Models to Explain law of Scattering

Sl.	Author	Model	Remark
1.	Bradford	$F(x) = a + b * \log(x)$	F(x) : no. of articles contained in the x top most periodicals; periodicals are arranged in decreasing productivity.
2.	Vickery	$S_{kr} = s(n^k-1)$	S_{kr} is the no. of journals in the most productive k groups; n is a constant $s = S_{1r} / (n-1)$ This is derived under the assumption that Bradford's verbal statement is correct.

3.	Wilkinson	$R(n) = j \log (\frac{n}{t} + 1)$	$n > S_{1r}$; R(n) is the cumulative no. of references in n most productive journals; t is a constant.
4.	Cole	F(x) = 1 + b. log (x)	As in the case of (1)
5.	Brookes	$R(n) = k.\log \frac{n}{s}$ $R(n) = \alpha n^{\beta}$	R (n) is as in case of (3) For $n \le c$ for > c and R(n) is as in case of (3)
6.	Leimkuhler	$F(x) = \frac{\log(1+\beta x)}{\log(1+\beta)}$	F(x) is the relative cumulative frequency of references contained in top most x proposition of journals; $\beta > 0$.
7.	Hasper	$R(n) = j \log \left(\frac{r}{a} + 1 \right) + R(0)$	R(n) is as defined in (3) a is constant.
8.	Asai	Y = A log (x +c) + B	A and B are constants; x is the rank of journals.
9.	Brookes (1984)	$R(n) = j_1 \log (1 + n / a_1)$ $R(n) = j_2 \log (1 + n / a_2)$	X<C, R(n) is as defined in (3) $X \ge C$, R(n) is as defined in (3)
10.	Egghe	Y = A log [B + CX + D log (1+CX)]	
11.	Basu(1992)	Y = x-x log x $Y = (x - bx \log x)^a$	

2. INVERSE SQUARE LAW OF SCIENTIFIC PRODUCTIVITY

In an "Informetrics Production Process" (Egghe(1990)), often known as generalized source-item relationship, distributions of publications over different authors are well recognized. Authors are considered as sources and papers as items. Since 1926, such a distribution is explained by an "inverse square law of scientific productivity" known as Lotka's law. It is discussed in detail in this section.

In recent past, distribution of articles/papers is approximated by a number of related distributions; the following are some of the important models or distributions :

- Law of Inverse Square (Lotka (1926))
- generalized bibliometric distributions (Bookstein (1976))
- negative binominal and as a special case sometimes geometric distribution (Ravichandra Rao (1980))
- lognormal distribution, etc. (Ravichandra Rao (1995), Shocklay (1957)

In most of these studies, the number of publications are considered as a measure of scientific productivity. As pointed out by Egghe (1993) and Lindsey (1980), there are three methods of counting of the number of publications. They are :

- method of total counting / normal counting : assigning every author a weight one for each of his/her publications during a time period, irrespective of whether he/she is a first author or a second author, etc.
- method of straight counting : assigning only the first author a weight one for each of his/her publications during a time period; for other authors a weight zero; In deriving inverse square law of scientific productivity, Lotka adopted this method while collecting data from Author Index of the Chemical Abstracts and Auerbach's Geschichtstafeln der physik.

- method of fractional counting: assigning every author a weight 1/n in a n-authored paper published during a time period.

In distribution of papers, baed on the first two methods, the domain of the variable (-- the number of papers) is positive integers; i.e. $i = 1,2,3, \dots m$ and thus the variable is discrete in nature. Lotka's law and similar other models or distributions are likely to fit such data. In distributions of papers, based on the third method, the domain of the variable is the positive real numbers; i.e. $0 < i \leq m$. In the above said three methods, the first two methods give rise to an incorrect size of population of authors/papers. Only the third method gives the correct size of population of authors as well as papers.

2.1 Lotka's Law

Lotka (1926), in his classic paper published in 1926 in the *Journal of the Washington Academy of Sciences* on the frequency distribution of scientific productivity presented an analysis of the number of publications listed in *Chemical Abstracts* from 1907 to 1916 with the frequency of publications by particular authors. He excluded the names of corporate authors, but only considered the names of authors whose names begin with A and B as listed in the index. He neither adopted a sampling design nor computed a sample size by using scientific methods. He also made a similar study in the field of physics. He applied the same process to the name index of *Auerbach's Geschichtstafeln der physic* which covers the entire range of history upto the year 1900. For these two sets of data, he computed the theoretical frequencies of publications of authors using the least square method. For large values of X (when X is the number of papers published by an author), the Fluctuation is too high; perhaps, owing to the limited number of persons in the sample. He, therefore, considered only the first 17 points of the data in physics and first 30 points in chemistry. Based on his data, he proposed the following inverse square law of scientific productivity.

$$y_x = \frac{6}{\pi^2 x^\alpha} \qquad x = 1, 2, 3, \dots\dots \qquad \alpha > 0$$

y_x is the relative frequency of authors publishing x number of papers. The value of α is found to be 2 for physicists and 1.89 for chemists. This difference in the value of α is possibly due to the sampling error, especially in the data pertaining to chemists. Lotka's law is based on a meager set of data and it has not been statistically tested. It is indeed reasonable to say that it **is more a conjecture than a "law**". If N is the total number of authors, Ny_1, in Lotka's equation, gives the number of authors who have published single paper each. Thus, Lotka's equation is determined in its general form by three parameters :

1. The number of scientists with minimal productivity (authors with single paper each - Ny_1);
2. The maximal productivity of a scientist (x_{max});
3. The characteristic exponent α.

Lotka's equation can also written in the following form :

$$y_x = \frac{k}{x^\alpha}$$

$x = 1, 2, 3, \ldots$

$k = 6/\pi^2$ for $\alpha = 2$

Thus, Lotka's equation suggests that the proportion of single author papers (k) is a function of α. That is,

$$k(\alpha) = \left[\sum_{x=1}^{x_{\max}} \frac{1}{x^\alpha} \right]^{-1}$$

This suggests that increase of α is accompanied by the increase of low productivity scientists. This implies that for a given N and for a large value of α, the proportion of highly productive scientists will decrease. Yoblonsky (1980), therefore, argues that the larger the parameter α, the greater is the gap between the productivity of individual groups of scientists. In this sense, the α is considered as a measure of inequality in the distribution of scientific papers.

The other models which are used to describe scientific productivity and which are related to Lotka's law are discussed in the following sections.

2.2 Scientific Productivity

'Scientific productivity' is frequently measured in terms of published output. This is because, the data on the number of publications by the authors can easily be collected and are also quite reliable. Many have used the number of publications of an individual scientist as a measure of his/her scientific productivity. Dennis (1954) studies the relationship between quantity of publication by outstanding scientists and scientific recognition. His study was based on 71 members of the *National Academic of Sciences*. He observed that all these scientists substantially contribute to literature, with the range of publication s between 27 and 768 and the median is 145. He also asserts that the members of the National Academy of Sciences contribute a large number of publications. He shows that almost without exception., those who have published many papers have also achieved eminence by being listed in the Encyclopedia Britannica. He obtains a similar result in a study on the pattern of publications by European scientists. He observes a close correlation between the quantity of scientific publication and the achievement of eminence as a contributor. We can therefore take the number of publications as a measure of scientific productivity. Since the number of articles published by an author can usually take only discrete values, it is reasonable to assume a discrete probability distribution function to describe the phenomenon of scientific productivity. In recent years, there have been many analytical approaches different from Lotka's Law for scientific productivity. Narin (1976) reviewed the early studies of scientific productivity and he concluded that "scientific talent is highly concentrated in a limited number of individuals". He, therefore, points out that the science policy should be designed to encourage the most productive scientists. He further observes that there are very few studies in his area.

Price (1971) conjectures that the number of the elite in science is small compared to the total number of scientists. Taking this clue and further assuming that exchange among scientists in general is a rare event, Griffith et al (1971) suggest that informal contacts in science might follow a Poisson distribution. Using the data gathered by Crane (1971) they attempt to estimate the value of the parameter of the Poisson distribution that would fit into the empirical data on scientists in the areas of rural sociology and finite mathematics. They suggest the following two groups :

1. A majority having a low average rate of contact which follow a Poisson distribution;
2. Elites whose frequency of contact deviates considerably from what the Poisson distribution would predict which is in fact about 8 times higher than the majority.

Their study suggests that Price's elite concept must be valid as far as the size of the elite group is concerned. Price also claims that "any population of size N contains an effective elite of size √ N" (Price (1971), p.74). be tested by statistical techniques. Neelameghan and others (1970a, 1970b) in heir papers on the pattern of duplication of discovery, use regression analysis, analysis of variance and a modified Poisson distribution to analyses the data and study the trend of duplication, the distribution of duplication, and the relationship between duplication and discovery. Their observations are: (1) the total number of discoveries, new discoveries, and duplications are predictable by assuming a linear relationship between the respective pairs of variables; (2) the patten of duplication is not due to chance alone; there is a regularity in its behaviours in the statistical sense; (3) the reputed discoveries and duplications both increase with time; (4) the behaviour of the functional relationship between the number of reported discoveries of antibiotics and the number of duplications in relation to time are different; (5) from 1937 to 1966, the rate of duplication has been nearly halved.

2.3 Probability Distribution of Scientific Papers

There are many analytical approaches apart from Lotka's law describing scientific productivity. Williams (1944) examined the geometric model and Fisher's logarithmic series in a study of publication patterns of biologists. Simon (1955) proposed a Beta function which he calls "Yule distribution". Shockeley (1957) proposed a lognormal model. Price (1976) proposed the Cumulative Advantage Distribution, similar to a Beta function. Coile (1977) proposed a continuous distribution (Weibull) for the data collected by Hersh (1942) and compared his results with those obtained by the use of other models. On the other hand, Bookestein (1977) argued the Lotka's Law is invariant under the impact of society on patterns of scientific productivity.

2.4 Social Change and Scientific Productivity

The environment of scientific research has undergone enormous changes both qualitatively and quantitatively. Under these circumstances, it is reasonable to presume that the patterns of scientific productivity have also charged. Bookstein (1977) shows that social change can affect overall levels of production, but not the pattern of individual productivity under this assumption that the scientific productivity follows Lotka's law. Bookstein also states that Lotka's law is invariant under the following two impacts of society on patterns of scientific productivity.

1. Society's ability to increase the calibre of scientists; and
2. Society's ability to encourage scientists by the way of rewards or threats to get the maximum research output.

Ravichandra Rao (1980) in his analytical study shows that the negative binominal distribution describes a pattern of scientific productivity under the "success-breeds-success" condition in a wide variety of social circumstances.

2.5 A Stochastic Model of Concentration

Ravichandra Rao (1995) suggested a model concerning the concentration of papers over authors, based on the data on author productivity in Mathematics. This model is mostly used in econometrics to analyse the market concentration over time and size distribution of firm (Hay and Moris (1991)). Like in the market concentration, a familiar common feature of distributions of papers is that they are generated by a stochastic process in which the variable (no. of papers) is subjected to cumulative random shocks over time and thus the distribution of papers at a given point in time is the product of a series of random growth patterns. In this section, a general non-linear model has been proposed to explain the distribution of the levels of scientific productivity by scientists, in the context of stochastic process. The general model suggested is :

$$\frac{y_{t+1,s}}{y_{t,s}} = \alpha \; y_{t,s}^{(\beta_s - 1)}$$

where $y_{t,s}$ refers to the number of authors who have published t papers during a sub-period s. In this section, an empirical study is explained only for one year (i.e. for s=1). Hence the stochastic model suggested above, would simply be, for a given period :

$$\frac{y_{t+1}}{y_t} = \alpha \; y_t^{(\beta - 1)} \quad \text{, where}$$

y_t is the number of authors who have published t papers each; y_t is the number of authors in group t. y_t thus refers to those authors who have published one paper less as compared to y_{t+1}.

The term “growth” is used below in a sense that at what proportion the number of authors are moving from one group to the other group. Say, for example, y_{t+1}/y_t gives the proportion at which authors capable of moving to the next higher productive group.

As in the case of size distribution of firms, it is conjectured here that the productivity of scientists may also be analyzed under two different situations. The first is of the type where the ratio of the number of authors in group t+1 to group t is constant; the constant ratio is common to all scientists. i.e., we have

$$\frac{y_{t+1}}{y_t} = \alpha$$

This ratio can be treated as "productivity-growth" of scientists; rate at which they are moving from low productive group to high productive group. The second situation is of the type of where the number of authors with (t+1) papers is related to the initial size of authors, who are less productive

$$R_t = \frac{y_{t+1}}{y_t} = \alpha\, y_t^{(\beta-1)} \qquad (2)$$

Further,

$$\frac{dR_t}{dy_t} = \alpha\,(\beta-1) y_t^{(\beta-2)} \qquad (3)$$

Equation (3) is either negative or equal to zero or positive depending on the value of β and the effect of initial size of growth is determined by the value of β, since $\alpha>0$; if $\beta = 1$, the exponent of y_t is zero in equation (2). If $\beta>1$ scientists of "high productivity" grow much faster than scientists of "less productivity" and vice versa for $\beta<1$. The case where β is less than 1 implies that the elite group consisting of very high productive group is quite small in the total population i.e., the tendency for a variate to return to the "mean of populations". Such situation is also called as regression. For the data in mathematics, Ravichandra Rao observed that $\beta = 0.918068$. It implies that mathematicians of " less productivity" publish papers much faster (with a higher rate of publications!) than mathematicians of "higher productivity".

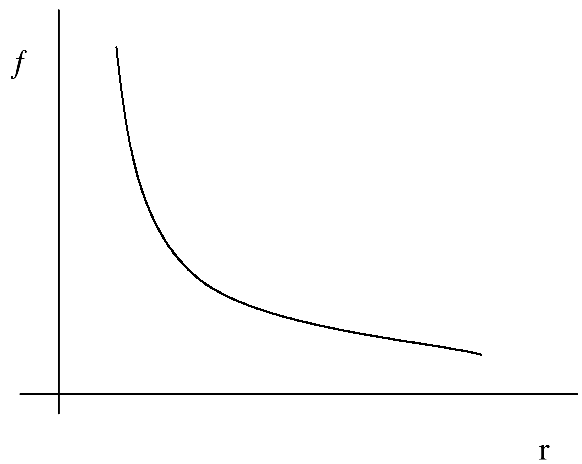
3. ZIPF's LAW

This is a third well known bibliometric law/distribution. It is argued by many that Zipf's law has a number of good applications in Information Retrieval environment; particularly, to select the terms for indexing in automated text retrieval systems. Further, it is closely related to Bradford's distribution.

Zipf (1949) developed and extended an empirical law, as observed by Estoup (1916), governing a relation between the rank of a word and the frequency of its appearance in a long text. If r is a rank of a word and f is its frequency, then Zipf's law is stated as follow:

$$r f = c$$

where c is a constant. Zipf drved his law from a general "principle of least efforts". Words whose cost of usage is small or whose transmit demands the least effort are frequently used in a large text. A hypothetical, but a typical example of a Zipfian curve is shown in the figure below. The other models which are used to describe word frequency distributions and which are related to Zipf's law are discussed in the following sections.



A Zipfian Curve

Based on observations of many empirical distributions of word frequencies, Simon (1955) characterizes word frequency distribution by a Yule distribution rather than by a commonly used contagious distribution such as the negative binomial or its limiting form called the Fisher's logarithmic series. The Yule distribution is

$$f(r) = \rho\, \beta\, (r, \rho + 1), \qquad r = 1, 2, 3, 4, \ldots.. \quad \rho > 0$$

$f(r)$ is the probability mass function; it is the theoretical relative frequency of words whose rank is r; β (.) is a Beta function. He also observes that the Yule distribution fits the tail much better than the top of the frequency curve. He further points out that f(r) is proportional to $r^{-(\rho-1)}$ for a sufficiently large r. The ratio of relative frequencies of 1's and 2's in the Yule distribution for ρ=2, as in the case of Lotka's law, is 4:1. But Simon, on the other hand, observes in actual practice that the values of this ratio are close to 3:1 and sometimes even less. He, therefore, argues that the Yule distribution with ρ approximately equal to 1 fits the empirical data in most cases, especially towards the top of the curve.

Booth (1967) suggests a method for estimating the Zipf constant c (in the equation rf = c) for a given author. He argues that

1. His estimate provides a measure of the author's richness in vocabulary, and
2. The general form of the law of occurrence for law frequency words is independent of the detailed validity of Zipf's law for the distribution as a whole.

Booth further observes that a rank distribution of rare words can be expressed as r(1+f) = c where r is the rank of a rare word occurring f times. Donohue (1973), in his book draws attention to Goffman's suggestion to derive a truncation point between high and low frequency words. The formula is as follows:

$$T = \frac{-1\sqrt{1+8\,I_1}}{2}$$

where T is the truncation point, I_1 is the number of distinct words of which occurs only once in a text. This formula is unlikely to be most useful in a highly positively skewed distribution (Ravichandra Rao, 1978, 1981) Mandelbrot (1952) shows that "under quite general conditions that word frequencies should follow a law of the type:

$$p(r) = (B-1)\,V^{B-1}\,(r+V)^{-B}$$

where B and V are constants". In this formula, r is the rank of a word and p(r) is the relative frequency of words. This equation is equivalent to the following form :

$p(r) = c(r^{-}-a)^{-s}$

where c, a and s are contents.

4. CHARACTERISTICS OF BIBLIOMETRIC DISTRIBUTIONS

4.1 Success-bareeds- Success Phenomenon

Price (1976) and many others argue that the success-needs-success phenomenon characterizes bibliometric distributions as they do in the other social processes. Price gives a few examples where such a phenomenon occurs in bibliometrics. They are :

1. A journal which has been frequently used is more likely to be used again than an infrequently used journal ;
2. An article in a journal which has been cited many times is more likely to be cited again than the one which has been rarely cited;
3. An author of many papers is more likely to publish again than the one who is less prolific.

Similarly, Ravichandra Rao (1981) in his thesis argued that :

1. Those documents which have been borrowed frequently are more likely to be borrowed again than those borrowed infrequently in an academic library in an academic year;
2. Those users who borrow documents frequently are more likely to borrow documents again than those who seldom borrowed documents in an academic year.

In statistics, such a phenomenon is generally described by a hyperbolic distribution function. Price characterized such a distribution by a Cumulative Advantage Distribution. In statistics, it is known as Yule distribution. The probability mass function of the Cumulative Advantage Distribution is :

$$p(x)=(m+1)B(x, m+2)$$

$x=1, 2, 3, \ldots.$

$m>0$

Price also points that the success-breeds-success phenomenon can be described by a negative binomial distribution. Tague (1981) and Ravichandra Rao (1980) show analytically that a negative binomial distribution describes the success-breeds-success phenomenon. Ravichandra Rao empirically shows that the negative binomial distribution describes the success breeds-success phenomenon much better than the Cumulative Advantage Distribution.

4.2 Other Characteristics

Bird (1977) in a study of the characteristics of bibliometric distributions, argues that increasing sample size leads to an increasing number of represented classes and an increase in both the mean and variance of distribution. He also shows for several bibliometric data that Yule characteristic :

$$k = \frac{1 + (sample\ \mathrm{var}iance - mean) / mean^2}{number\ of\ classes}$$

changes with the sample size.

Bookstein (1976) briefly discussed different bibliometric distributions that allow one to understand them as being different versions of a single theoretical distribution. He suggests the following to describe the bibliometric processes :

$$f(x) = \frac{k}{x^{\alpha}}$$

$x=1, 2, 3, \ldots.$

$K, \alpha>0$

This function can be used to describe Zipf's Lotka's and Bradford's laws as follows :

1. f(x), the number of words occurring x times, is proportional to $1/x^{\alpha}$;
2. f(x), the number of authors who have published x papers, is proportional to $1/x^{\alpha}$;
3. f(x), the number of journals which contain x articles in a given subject, is proportional to $1/x^{\alpha.}$

This is also the view of Mandelbrot (1952). He suggested the following formula :

$$f(x) = \frac{k}{(1+\alpha x)^{\beta}}$$

f(x) is the number of words with rank x; k is a normalizing factor; $\alpha$and β are constants. Thus, one can argue that the bibliometric distributions have the following characteristics in common :

1. Distributions are reverse-J shaped;
2. Distributions are highly skewed;
3. Generally, the distributions have long tails;
4. The most general form of the distribution is :

   $f(x)=c(x+a)^{k}$

   c, a and k are constants;
5. The bibliometric distributions are usually due to a success-breeds-success phenomenon; it can be described by a negative binomial described by a negative binomial distributions

4.3 80-20 Rule

Many have observed in library and information field that :

i. Most of the documents are hardly circulated/used/cited and very few books are frequently circulated/used/cited

ii. Most of the authors publish very few articles and very few authors publish most frequently.

This phenomenon is explained fairly by a 80-20 rule. i.e. 80% of the documents contribute to 20% of the total circulations/uses/citations received; 20% of the documents contribute 80% of the total circulations/uses/citations received; the 20% of the such documents are usually called "core documents/core collection". Similarly many have observed that 20% of the authors (at any given time and ; called most productive authors) contribute 80% of he total publications and 80% of the authors contribute hardly 20% of the publications.

In general, if we define

x = x (n) : the fraction of sources having n or more items

θ = θ (n) : the fraction of items yielded by the sources having n or more items.

Burrell (1985) has shown (under the assumption that f(n) : the number of sources with n items, follows negative bibominal distribution) that

$$\theta\ (x, \mu) = x\left(\frac{1}{\mu\ \log\left(\dfrac{\mu - 1}{\mu}\right)} \bullet \log x + 1\right)$$

where μ is the mean number of circulations. For θ = 0.8, x = 0.57 if $\mu \simeq 2.0$ and x = 0.48 if μ = 5. i.e., the greater the μ, the smaller is the x. i.e., libraries with lower average borrowing

(μ) tend to require larger proportions (x) of their collections to account for say $\theta = 0.8$ (80%) of the borrowings.

Egghe (1986) in his study on the other hand argued that for $\alpha = 2$, in Lotka's law,

$$\theta\ (x, \mu) = \frac{\pi^2 \mu - 6E - 6 \log\left(6/x\pi^2\right)}{\pi^2\ \mu}$$

where E = 0.5772 is Euler's constant. Thus, Egghe also observe that x increase if and only if μ decreases.

For much of the library applications, we may be satisfied with 80-20 rule!, especially to identify the core collection. Thus, most of the bibliometric data may be analysed using Lorenz Curve.

5. RANK AND SIZE FREQUENCY DISTRIBUTIONS

Explanations of Bradford's law, Zipf's law and many other similar laws or rank-size relations are to be usually sought in the statistical theory of events that might show these relations instead of the nature of object to which the relations apply. Let us consider a frequency density function y= f(x) in which the independent random variable X represents non-negative integer (size), and f(x) represents the relative frequency of objects in a population of N objects. Therefore,

$$\sum f(x)\ dx = 1$$

Let F(x) be the number of objects having a size greater than or equal to x. That is :

$$F(x) = \int_x^{\infty} N f(t)dt$$

This is exactly the rank of the objects whose size x, under the assumption that the ranks of objects of equal size are assigned arbitrarily among them. The rank-frequency curve is therefore essentially the cumulative curve of the size-frequency distribution. For example, Zipf's law, f(x)=k/x, holds approximately if the size frequency is of the form k/x^2, since

$$\int_x^{\infty} \frac{k}{t^2} dt = \frac{k}{x}$$

Hubert (1978) similarly argues that even for Bradford's law, the rank-frequency and size frequency distributions are similar.

It is worthwhile to mention here that Zipf;s law and Bradford's law are due to a rank-frequency analysis of the data. Lotka's law is due to a size frequency analysis of the data.

Although the rank-frequency and size-frequency distribution are closely related to each other, based on Shannon's theory of information, Brookes (1980) argues that the rank-frequency analysis is richer than conventional size-frequency analysis. But the fact is that not only a simple rule of assigning a rank to an entity can lead us to ties, the rank-frequency approach is also tedious in nature since it involves ranking of each individual. Brookes' argument is given below.

If N entities are uniquely ranked, then there are N! possible different orders. The information yielded by any of these rank orders is than (from Shannon's information theory) given by :

I (R) = log N!

On the other hand, if there are f_1 entities with the score of one, f_2 entities with the score of two, and so no, we have a conventional frequency distribution wherein $\sum_r f_r = N$. The number of all possible permutations of N entities which give exactly the same frequency distribution is :

$$\frac{N!}{f_1!\,f_2!\,f_3\,!.....f_r!}$$

The corresponding information measure is :

$$I(F) = \log\left[\frac{N!}{f_1!f_2!f_3!....f_r!}\right]$$

Thus we have

$$I(R) - I(F) = \sum_r \log(f_r!)$$

This shows that the different between I(R) and I (F) is zero only if the frequency distribution is such that f_r=1 for every r, in which case the size frequency distribution is indistinguishable from the rank-frequency distribution. But if $f_r > 1$, for any r,

I(R)>I(F)

Hence, Brookes argues that the rank-frequency approach is richer than size-frequency approach.

6. CONCLUDING REMARKS

Most of the theoretical works in informetrics / bibliometrics are centred around Bradford's law and Lotka's law; the Bradford's law is derived based on the rank-frequency approach and the Lotka's law is based on the size frequency approach. They are closely related. However, often 80-20 rule confirming to many of library phenomena and they are very much useful in taking library related decisions.

7. BIBLIOGRAPHICAL REFERENCES

1. ASAI (I A) (1981). General formulation of Bradford's distribution : The Graph-oriented approach (Journal of ASIS, 32 (2), 113-9).

2. ASLIB proceedings. 1, 1949, p. 102.

3. BASU, (Aparna) (1992). Hierarchical distributions and predgerion. (Journal of Human Society for --- science, 43(7), 494-500).

4. BASU, (Aparna) (1995). Models of Bradford law as solution of differential comparative proceedings of the 5h Biennial conference of the International Society of Scientometrics and Informetrics, Illinois, USA; p 53-62

5. BIRD (P R) (1977). Some sampling characteristics of bibliometric distributions. (Journal of Information. 1, 2; 64-80).

6. BOOKSTEIN (Abraham) (1976). Bibliometric distributions (Library Quarterly. 46, 4;16-23)

7. BOOKSTEIN (Abraham) (1977). Patterns of scientific productivily and social change: A discussion of Lotka's law and bibliometric symmetry. (Journal of American Society for Information Science. 28, 4; 206- 10).

8. BOOTH (Andrew D) (1967). A "law" of occurrence for words of low frequency. Information and Control. 10, 386-93)

9. BRADFORD (S C) (1934). Sources of information on specific subjects. (Engineering. Jan 26; 85-6).

10. BRADFORD (S C) (1948). Documentation Crosby Lockwood. London. p156.

11. BROOKES (B C) (1968). The derivation and application of the Bradford-Zipf distribution. (Journal of Documentation. 24, 4; 247-65).

12. BROOKES (B C) (1969a). The complete Bradford-Zipf bibliography. (Journal of Documentation, 25, 1; 58.60).

13. BROOKES (B C) (1969b). Bradford's law and the bibliography of science. (Nature 224; 953-6).

14. BROOKES (B C) (1977). Theory of the Bradford laws. (Journal of documentation. 33; 180-209).

15. BROOKES (B C) (1980). The foundation of information science, Part 11: Quantitative aspects: Classes of things and the challenge of human individuality. (Journal of Information Science, 2, 209; 21).

16. BURRELL (Q L). (1985). The 80-20 rule : Library lore or statistical law? (Journal of Documentation, 41 (1985) no.1, 24).

17. BURRELL, (Q.L) (1988). Modelling of the Bradford phenomena . (Journal of Doc. 44(1), 1-8).

18. CHUNG (Young M) (1977). A non-Bradfordian model for scattering of articles in a subject among periodicals. Ph. D dissertation. Case-Western Reserve University. U.S.A.

19. COLE (Russel C) (1977). Lotka's frequency distribution of scientific productivity. (Journal of the American Society for Information Science. 28, 6; 366-70).

20. CRANE (Diana) (1971). Transational networks in basic sciences. (International organizations, 25, 3; 585-601)

21. DENNIS (Wayne) (1954). Bibliography of eminent scientists. (Science. 79; 180-3).

22. DONOHUE (Joseph C) (1973). Understanding scientific literature: a bibliographic approach. The MIT Press, Cambridge.

23. EGGHE (L). (1993). "Consequence of Lotka's law in the case of fractional counting of authorship and of list author counts" Math. Comput. Modelling V18(9) P63-77, 1993.

24. EGGHE (L) Rousseau. (1990). Introduction to Informetrics: Quantitative methods in library Doc & Inf. Science. Elsevier Science Publisher, NY, 1990.

25. EGGHE (L) (1986). The 80/20 rule. (Scientometrics Vol.10, Nos.1-2. 55-68).

26. EGGHE (L). (1985). Consequences of Lotka's law for the law of Bradford. Journal of Documentation 41(3) : 173-189. 1985.

27. ESTOUP (J B)(1916). Les Grammer stenographignes. Paris. Institute stenographigue.

28. FAIRTHORNE (Robert A)(1969). Empirical hyperbolic distributions (Brandford-Zipf-Mandelbrot) for bibliometric description and prediction. (Journal of Documentation. 25, 4; 319-43).

29. GRIFFITH (Belver C) and others (1971). Informal contacts in science: A Probabilistic model for communication processes. (Science, 173, 164-5).

30. GROSS (O V) (1988). Bradford law and the Keenan-Atherton data. (American Documentation, 18;46)

31. HAY (Donald A) and MORRIS (Derek J) (1991). Industrial Economics and Organization: Theory and Evidence" Oxford university press.p537-8

32. HERSH (A H) (1942). Drosophila and the course of research. (Ohio Journal of Science. 42; 5; 198-200).

33. HUBERT (J 1) 1977). bibliometric models for journal productivity. (Social Indieators Research; 4; 441-73).

34. HUBERT (J J) (1978). Relationship between the forms of Bradford's law. (Journal of the American Society for Information Science. 29, 3; 159-61).

35. HUBERT (John J) (1976). On the Naranan interprelalion of Bradford's law. (Journal of the Ameriean Society for Information Scienee. 27; 339-41).

36. KARMESH (N.C). and others (1984). Rationales for Bradford's law (Scientometrics, 6(4) 233-41).

37. KENDALL (M G) 1960). Bibliography ol operational research. (Operation Research Quarterly, 11, ½, 31-6).

38. LEIMKUHLER (Ferdianand F) (1967). Bradford distribution. (American Journal of Documentation, 23, 3; 197-207).

39. LINDSEY (D) (1980). "Production and citation measures in the society of science: The problem of multiple authorship". (Social studies of science, 10;145-162)

40. LOTKA (A J) (1926) Frequency distribution of scientific productivity (Journal of Washington Academy of Science 16; 317-23)

41. MANDELBROT (Benoit) (1952) Information theory of the statistical structure of language. (Proceeding of the Symposium on Application of Communication Theory, London, Sept. 1452. Butterworth. P.486-500)

42. NARANAN (S) (1970) Bradford's law of bibliography of science: an interpretation. (Nature. 227; 631-2).

43. NARANAN (S) (I971). Power law relations In science bibliography: a self-interpretation, (Journal of Documentation. 27; 83-97).

44. NARIN (Frencis) (1976). Evaluative bibliometrics: the use of publication and citation analysis In the evaluation of scientific activity. Computer Horizons, Inc. Project No.704R.

45. NEELAMEGHAN (A), BUCHE (V V) and GUPTA (B S S) (1970a). Discowery and duplication of antibiotics: A Correlation. (Current Science, 34;511-2)

46. NEELAMEGHAN (A), BUCHE (V V) and GUPTA (B S S) (1970b). Distribution pattern of the duplication of discoveries. (Nature, 227;1059-1060).

47. PRICE (Derek de Solla) (1971). Some remarks on elitism in information and the invincible college phenomenon in science. (Journal of the American Society for information Science. 22; 74-5).

48. PICE (Derek de Solla) (1976). General theory of bibliometric and other cumulative advantage processes. (Journal of the American Society for Information Science. 27 (2); 292-307).

49. RAVICHANDA RAO (I K) (1997). An analysis of Bradford multiplies and a model to explain the law of Scattering Proceedings of the 6th International conference on Informetrics and Scientometrics, Jeruslem. 341-349.

50. RAVICHANDRA RAO (I K) (1978). Distributions of transactions over users and over documents in Canadian academic libraries: a dissertation proposal. School of Library and Information Science: the University to Western Ontario, London, Ontario (Unpublished).

51. RAVICHANDRA RAO (I K) (1980). Distribution of scientific productivity and social change. (Journal of American Society for Information Science. 31, 2: 111-21)

52. RAVICHANDRA RAO (I K) (1981). Documents and user distribution in transaction records Or Canadian university libraries. Ph. D Thesis. Faculty of Graduate Studies, School of Library and Information Science: the University of Western Ontario, London, Ontario, Canada.

53. RAVICHANDRA RAO (I K) (1995). A Stochastic approach to analysis of distribution of -- in mathemation : Lotka's revised. Proceeding of the 5th Bichmial Conf. Of the International Society for Scientometrics and Informetrics, Illinous, USA, p455-464.

54. SHOCKLEY (William) (1957) . On the statistics of individual variations of productivity in research laboratories. (proceedings of IPE. 279-90)

55. SIMON (Herbet A) (1955). On a class of skew distribution functions. (Biometrika, 42; 425-40).

56. TAGUE (J M) (1981). Success-breeds-success phenomenon and bibliometric processes. (J. Amer. Soc. For Inf. Sc. 32, 4; 280-6).

57. VICKERY (B C) (1948). Bradford's law of scattering. (Journal of Documentation. 4, 3; 198-203).

58. WILKINSON (Elizabeth) (1972). Ambiguity of Bradford's law. (Journal of Documentation. 28, 2; 122-30).

59. WILLIAMS (C B) (1944). Numbers of publication written by biologists. (Annals of Human Genetics. 12; 143-6).

60. WITTIG (Glenn R) (1978). Statistical bibliography-a historical footnote. (Journal of Documentation. 34, 3; 240-1).

61. YABLONSKY (A 1) (1980). On fundamental regularities of the distribution of scientific productivity. (Scientometrics. 2, 1; 3-34).

62. ZIPF (Ceorge) (1949). Human behaviour and the principle of least effort. New York. Addison-Wesley Press.