

EVALUATION OF MODEL PREDICTIVE CONTROL METHOD FOR  
COLLISION AVOIDANCE OF AUTOMATED VEHICLES

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This work is dedicated to my parents, my siblings,  
my husband and my sweet Arel.

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## SYMBOLS

$m$	mass
$v$	velocity
$\delta_f$	front steering angle
$\delta_r$	rear steering angle
$l_f$	front axle to the center of the mass
$l_r$	rear axle to the center of the mass
$\psi$	yaw angle
$x, y$	local coordinates
$v_x$	longitudinal velocity
$v_y$	lateral velocity
$I_z$	moment of inertia
$\dot{\psi}$	yaw rate
$\ddot{\psi}$	yaw acceleration
$X, Y$	global position
$c_f, c_r$	cornering stiffness of the front and rear tires
$\alpha_f, \alpha_r$	front and rear tires slip angles
$T_s$	sampling time
$t$	time
$\dot{y}$	lateral velocity

## ABBREVIATIONS

MPC	Model Predictive Control
SMC	Sliding Mode Controller
FLC	Fuzzy Logic Controller
CAS	Collision Avoidance System
ADAS	Advanced Driver Assistance System
MMPC	Multi-Constrained Model Predictive Control
CPU	Central Processing Unit
LTV	Linear Time-Varying
DOF	Degree of Freedom
MIMO	Multi-Input Multi-Output
QP	Quadratic Program
SISO	Single-Input Single-Output
ZOH	Zero-Order Hold
MV	Manipulated Variables
OV	Output Variables
TSCC	Lines of Time-Scaled Collision Cone

## ABSTRACT

Ozdemir, Hikmet D. M.S.E.C.E., Purdue University, August 2020. Evaluation of Model Predictive Control Method for Collision Avoidance of Automated Vehicles. Major Professor: Lingxi Li.

Collision avoidance design plays an essential role in autonomous vehicle technology. It's an attractive research area that will need much experimentation in the future. This research area is very important for providing the maximum safety to automated vehicles, which have to be tested several times under different circumstances for safety before use in real life.

This thesis proposes a method for designing and presenting a collision avoidance maneuver by using a model predictive controller with a moving obstacle for automated vehicles. It consists of a plant model, an adaptive MPC controller, and a reference trajectory. The proposed strategy applies a dynamic bicycle model as the plant model, adaptive model predictive controller for the lateral control, and a custom reference trajectory for the scenario design. The model was developed using the Model Predictive Control Toolbox and Automated Driving Toolbox in Matlab. Built-in tools available in Matlab/Simulink were used to verify the modeling approach and analyze the performance of the system.

The major contribution of this thesis work was implementing a novel dynamic obstacle avoidance control method for automated vehicles. The study used validated parameters obtained from previous research. The novelty of this research was performing the studies using a MPC based controller instead of a sliding mode controller, that was primarily used in other studies. The results obtained from the study are compared with the validated models. The comparisons consisted of the lateral overlap, lateral error, and steering angle simulation results between the models. Additionally,

this study also included outcomes for the yaw angle. The comparisons and other outcomes obtained in this study indicated that the developed control model produced reasonably acceptable results and recommendations for future studies.

# 1. INTRODUCTION

## 1.1 Thesis Background

After the fast development of automated vehicle technology in the last decade, many collision avoidance approaches have been proposed to improve driving safety. Collision avoidance system (CAS), was firstly proposed in the 1950's. Cadillac developed a prototype by using a radar that detects the objects in front of the car. They decided not to manufacture due to the associated high cost [1]. In 1977, Toyota introduced the adaptive cruise control system on their cars only in Japan. In 1995, they first tested the collision avoidance system based on radar sensors in a research lab in California, which was never implemented. Nowadays, we have many different types of advanced driver-assistance systems such as maneuver control systems, adaptive cruise control, lane change assistance, and collision avoidance systems.

All these systems are using technology to minimize human error through various type of warnings [2]. The standards established by the Society of Automotive Engineers indicate that the driving automation or the advanced driver-assistance systems (ADAS) can be categorized at six-levels, starting from no to full driving automation. Table 1.1 shows the details of the driving automation criteria. Level 0 encompasses the manual control of all tasks of driving done by a human. Level 1 includes some basic functions of driving assistance, such as cruise control, which supports the vehicle driver to reduce or increase the speed. Level 2 is the partial automation by ADAS. The driver allows the function to act and is responsible for the result' so the vehicle can perform steering and acceleration in this level of automation. In Level 3, the automated system is in charge and performs the primary driving tasks. There have been fatalities of on-road testing of prototypes for vehicles at Level 2 and Level 3 [2]. At Level 4 automation, the vehicle can perform all driving tasks under specific circum-

stances. Full automation is referred to as Level 5, and these vehicles can complete all driving responsibilities under all circumstances without the need for human attention or cooperation. Algorithms for ADAS and autonomous driving for automated driving cars have been further discussed and analyzed in [3].

Table 1.1.: Levels of Driving Automation

	Past	Past	2018	2020	2020-2025	2025-2030
Level	0	1	2	3	4	5
	No Automation	Driver Assistance	Partial Automation	Conditional Automation	High Automation	Full Automation

## 1.2 Literature Review

Automated driving technology consists of sensing, understanding, preparation, and action. Technology has a fast journey, and a large amount of research has been conducted in the field. However, it is challenging to understand the whole spectrum of the technology. Most of the analyses describe problems under several boundary circumstances. Researchers can examine many issues under a specific condition, but it's not feasible to estimate solutions for the whole autonomous driving system all at once.

In the compared research, for the lateral motion, the desired trajectory has been designed as a sinusoidal or exponential trajectory. A sliding mode controller was created to ensure that the vehicle tracks the desired path [4]. Reichardt and Schick [5] provided an electrical field analysis of the obstacles and examined the autonomous vehicle as an electrical charge in this field. Forces managed in that charge determined the vehicle's path. Brandt et al. [6] suggested an automatic collision prevention system based on the theory of elastic band. In this method, the desired trajectory of the autonomous vehicle is called the elastic band, resembling links connected to springs. There is a force applied on the links by the obstacle. Accordingly, a secure

trajectory can be formed. In this approach, a nonlinear equation has to be solved, which can then be treated as a real-time implementation problem.

The obstacle avoidance maneuver is not an individual task to overcome. The vehicle stabilization becomes an essential problem ensuring that the controller does not fail. In the case of a stabilization failure, the vehicle can end up with an accident. When the collision event is prioritized rather than stabilization, path tracking can be successful in terms of results. A new control design has also been tested for collision avoidance [7]. In this study the framework was determined to be essential, and it can disturb the stabilization criteria to avoid the collision. Model predictive and feedback controllers have been used for their experimental data and avoiding the obstacle successfully.

In [8], the authors proposed algorithms for creating the path-control and lateral-control of an autonomous vehicle with vision systems for a collision-free trajectory. They have shown that controlling the lateral motion starts with a reference line, then the obstacle detection and the lateral control algorithm follows the simulated obstacle avoidance path. The expediency relationship of the algorithms used for the camera, radar, and lidar navigation between reference lines and collision avoidance is presented.

For a collision-free path which concentrates on geometric features of the obstacles and kinematic limitations, it is necessary for path planning to be implemented in the collision avoidance systems [9]. In the earlier research conducted in 1980's, trajectory planning for autonomous vehicles for moving between points essentially converged on computing a time optimization and collision-free path [10, 11]. This was improved in the following decades by several different successful calculation and implementation methods. For path-planning and tracking strategies, a 3-D virtual range was created. Researchers have also tested a trigonometric function for the path and the exponential function for the obstacles that can cause the desired trajectory for collision avoidance when a vehicle expected to collide [12].

Path tracking for collision avoidance maneuvers has also been studied [9]. The controller for path-tracking is implemented to be expressed as a multi-constrained model predictive control (MMPC) problem and the calculation of the front steering angle. For stabilization, the vehicle is proposed to be at high speed and the state constraints are defined to be the lateral position, yaw rate, and side slip angle where the input constraint was the steering wheel angle. This approach has been simulated in many different scenarios in both static and dynamic environments, and has consistently yielded accurate results [9]. The MMPC-based path-tracking has been demonstrated to provide a high dynamic tracking performance and manages outstanding maneuverability.

For real-time implementations, path planning has to be both dynamic to deal with any unknown obstacles met on the path and also fast enough to succeed. Its ability to estimate optimal control strategy satisfying both the hard and soft constraints and adaptability is critical. For this reason the model predictive control (MPC) is a preferred candidate to control of autonomous vehicles. The nonlinear model predictive controller (NMPC) has also been used to avoid obstacles for automated vehicles that travel at realistic speeds [13].

Different methods of obstacle avoidance, such as the NMPC were examined in several tracking scenarios for static obstacles on constrained roadways [13]. Meanwhile, to clarify the vehicle dynamics, a bicycle model is preferred for the prediction of future vehicle states in the NMPC framework. The CPU time can be intensive in those cases, so the implementation of NMPC controllers in real-time becomes challenging. Several researchers have investigated methods to estimate these schemes for real-time applications [13]. This research shows that the NMPC controllers achieved satisfying results for online tracking performance in practical scenarios and against real-time constraints at average road speeds. The research also showed that extended horizon predictions for controllers provide better responses than shorter horizons. Comparison studies have identified controllers with significant reductions in deviations while successfully avoiding obstacles.



The model-predictive control method is widely used in systems with linear or non-linear constraints as one of the optimal controller methods. It uses a mathematical dynamic model of the system to predict future states and optimize the control method performance [9, 14]. These researchers also developed MPC linked path planning algorithms based on the bicycle vehicle model. Based on the Borelli concept, Yakub [15] used MPC merging with a feed-forward controller.

There were various experiments of MPC for tracking control by other researchers. For instance, Falcone et al. [9] discussed how MPC managed the predictive control for the steering angle input, path tracking and obstacle avoidance. MPC controllers in non-linear and linear time-varying (LTV) domains were tested using a rapid prototyping module. Nevertheless, the research of this method requires tremendous computational support because it solves optimization problems in real-time. This was demonstrated by Tomatsu et al. [16] who implemented MPC for trajectory tracking on excavators at slow speeds. There are numerous studies conducted on using the MPC controller for vehicles at slow pace [17–19]. For solving optimization problems on autonomous vehicles, Beal [20] practiced predictive control models using C programming.

The main challenge in path tracking control is the controller’s ability to navigate various types of road curvatures under different speeds and road conditions. Model predictive controller designs were examined for path tracking to avoid obstacles with various changes in speed [21]. Through controlling the front steering angle, these controllers predetermined paths for autonomous vehicles to avoid static obstacles. These methods were implemented by solving a single non-linear MPC problem to avoid static obstacles. Typical vehicle models used in these studies were generated based on a 3 DOF non-linear vehicle model. The controller in these studies used the X, Y global axes, and took the yaw rate as an input to output the steering of the dynamic vehicle system. These studies used Stanley controller and MPC as the non-linear controllers both at low and middle velocities and performed comparisons

in several realistic scenarios. As a result of this research, the MPC controller was concluded to have better performance both at mid and high velocity.

Planning structure for autonomous vehicles that can form safe trajectories in complicated driving scenarios, which is usually encountered in high traffic situations, has also been analyzed by researchers [22]. The approach used in these studies followed a levels approach. The initial level of the recommended structure forms a Model Predictive Control strategy using a convex programming method for avoiding collision. The limitations on road curvature were integrated into this optimization system. The next layer is mainly responsible for handling the random responses that are usually from other actors of the traffic. Optimization of the speed and trajectory according to exterior changes were simultaneously handled, and produced onward lines of time-scaled collision cones (TSCC). This research demonstrated successful implementation of the controls optimization while resulting in safe trajectories. The demonstration of the system architecture in these studies presented various driving scenarios under the dynamic and static obstacle environment, such as overtaking, lane changing and jaywalking.

Improvements in the automatic driving algorithm frameworks in the recent years have attracted worldwide attention [23], however current safety controls are still not deemed enough when tested against accidents [24]. Other researchers have shown control strategies to maneuver developments for collision avoidance between the vehicles and the front vehicle [25, 26]. Obstacle avoidance strategy in autonomous vehicles has shown to be based mainly on an understanding of environmental information and then creating control commands to safely operate a vehicle around obstacles [27, 28].

There has also been research about developing integrated controllers for avoiding collision. Obstacle avoidance in these research were based on utilizing a nonlinear model predictive controller. In this approach, it is critical for the path to be suddenly modified when a dynamic obstacle unexpectedly appears. Collision avoidance is achieved entirely by creating a moving function to predict the obstacle position

changes in the predictive horizon [29]. The system uses a risk index that occurs between the vehicle and obstacles position on the predictive horizon.

PID controllers have also been commonly used in lane detection systems, mainly for path tracking and obstacle avoidance strategies. Researchers have used and reported satisfactory results using the estimations of these approaches for control and prediction of vehicles heading to an obstacle at an angle and successfully avoiding it [30].

There have been numerous studies on various types of control approaches to find the most suitable one to handle safety conditions. The fuzzy-based control approach is one of the commonly preferred ones after the MPC approach. In summary, there are three control strategies regularly used for collision avoidance [10]. The first approach is the optimal control strategy that uses adjustable input parameters and settles the optimal cost using the current state of the system to avoid obstacles. This approach functions by using the information obtained based on the coordinate system of a vehicle, the object velocity, and the current lane of the vehicle. The second strategy is based on the fuzzy based control that is achieved by using the driver behavioral intelligence to solve the complicated cases about obstacle avoidance [31]. The studies presented on this strategy indicate that more precise knowledge of input and outputs allow for advantages in an optimal control approach. In one of the studies it is demonstrated that the first layer is used for fuzzy controller for obstacle detection for managing the possibility of collision, and the second layer for obstacle avoidance for the high chance of collision [32]. The third strategy utilizes the nonlinear model predictive controller (NMPC). Using NMPC for collision avoidance, it has been shown that the control framework can be divided into high and low levels [33]. Following the required path to avoid a collision, mainly a high level is used.

In a fuzzy based control strategy for obstacle avoidance in complex traffic scenarios, static and dynamic obstacles have been examined [34]. Ulrich, [35] has also used a standard automated Level 2 type vehicle to acquire data on how drivers avoid fixed obstacles and develop a fuzzy controller network. The same data was also collected

by a robot driver and compared with the initial driver. The success and quality of the control strategy have been demonstrated by examining 300 fuzzy rules, where only two have failed. It was also shown that using a single fuzzy controller was not efficient due to the large number of input and output variables [36]. This has also resulted in the controller not being able to handle the rules separated in detail, hence the amount of fuzzy rules increased rapidly. The number of fuzzy rules is only in linear growth instead of exponential, and hierarchical fuzzy controller is required to meet the real-time control specifications. These issues have led to research to develop an algorithm that avoids obstacles on autonomous vehicles in a shorter time. An electric three-wheeled vehicle was tested for avoiding obstacles by using a fuzzy logic controller [37], where the considerations derived from a popular bicycle model. The study examined many different road conditions and successfully reached the set goals. To provide a brief overview, all the literature review presented in this chapter is summarized in Table 1.1.

Table 1.2.: State of the Art on Different Control Methods  
for Collision Avoidance.

Controller	Research Title	Features	Results
MPC	Collision Avoidance and Stabilization for Autonomous Vehicles in Emergency Scenarios [7]	A single controller for collision avoidance, vehicle stabilization, and path tracking used.	The performance, ability of the controller and the convenience of this prioritization approach for avoiding collision and stabilization was successful.
MPC	Visual Navigation along Reference Lines and Collision Avoidance for Autonomous Vehicles [8]	Visual navigation algorithms including image processing for reference lines and obstacles, and lateral control with reference lines and for obstacle avoidance is proposed.	The feasibility of the algorithms for visual navigation along reference lines and collision avoidance is tested.

Table 1.2.: continued

MPC	Path Planning and Tracking for Vehicle Collision Avoidance Based on Model Predictive Control With Multi-constraints [12]	A framework for path planning built on a 3-D potential field based on the data of road and obstacles.	The results demonstrate that the controller was able to stabilize the vehicle on a low-friction-coefficient road with a moving obstacle and satisfy the tracking performance.
MPC	Obstacle Avoidance in Real Time With Nonlinear Model Predictive Control of Autonomous Vehicles [13]	The NMPC system uses a simplified bicycle model within the controller, in realistic road conditions.	The NMPC method handled dynamic trajectory changes and unanticipated obstacles at normal road speeds.

Table 1.2.: continued

MPC	<p>Model Predictive Controller for Path Tracking and Obstacle Avoidance Manoeuvre on Autonomous Vehicle [21]</p>	<p>This paper proposed a path tracking MPC controller for an 3 DOF non-linear autonomous vehicle at various speed on avoidance obstacle.</p>	<p>It was showed that the small lateral position error at small speed and at large speed large error at yaw rate observed.</p>
MPC	<p>Motion Planning Framework for Autonomous Vehicles: A Time Scaled Collision Cone Interleaved Model Predictive Control Approach [22]</p>	<p>A framework generated a global plan with complex constraints for a realistic planning horizon.</p>	<p>The framework is compared with standalone MPC validated in different scenarios including pedestrian jaywalking, merging and overtaking.</p>

Table 1.2.: continued

MPC	Dynamic Trajectory Planning and Tracking for Autonomous Vehicle With Obstacle Avoidance Based on Model Predictive Control [29]	Simultaneous trajectory for dynamic path planning and tracking are integrated as a single-level NPMC controller, when dynamic obstacle suddenly appears, the trajectory should be adjusted respectively.	Moving a function to predict the position of the obstacle changes in the predictive horizon has designed.
FLC	Fuzzy-based Collision Avoidance System for Autonomous Driving in Complicated Traffic Scenarios [34]	The Fuzzy controller has both the lane change and adaptive cruise control with optimal rules to enable effective collision avoidance when there are both static and dynamic obstacles.	The results shows that the vehicle could change lane, slow down or stop depending on the traffic situation.



Table 1.2.: continued

FLC	Collision Avoidance System for fixed Obstacles - Fuzzy Controller Network for Robot Driving of an Autonomous Vehicle - [35]	Collision avoidance using the fuzzy controller based on real data of human driver for fixed objects are examined.	The simulation results shows that the vehicle had a good performance.
FLC	The study of obstacle avoidance algorithm for vehicles based on hierarchical fuzzy controller [36]	Introduces the theory and structure of fuzzy controller, then uses the theory of obstacle avoidance based on the hierarchical fuzzy controller.	Results show that the hierarchical fuzzy control algorithm is ideal and feasible.
FLC	Obstacle Detection System Fuzzy Controller Applied to an Electric Three-Wheeled Vehicle [37]	Bicycle model by using the fuzzy controller under many different road conditions experimented.	A fuzzy control due to the high number of the input variables has been used to test static object with small dimensions was presented.

Table 1.2.: continued

SMC	Collision Avoidance Maneuver for an Autonomous Vehicle [4]	Avoiding collisions with fix and moving obstacles in the path of the vehicle was proposed.	The method showed an effective result.
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### 1.3 Thesis Organization

Literature review on the topic has indicated that there is not only one right method for controlling and planning the collision avoidance maneuver. All the methods presented in this chapter have their unique benefits and limitations. The major methods, including the (i) model predictive controller, (ii) sliding mode controller and (iii) fuzzy logic controller, are further described in this thesis.

In Chapter 2, kinetic and dynamic bicycle models are discussed and demonstrated. Benefits of the dynamic vehicle model are also summarized.

In Chapter 3, the control methods used in automated vehicles is explained by the guidance of the previous research are explained. The structures of these control methods are explained, with an emphasis on the model predictive control methods.

Chapter 4, proposes the design of the model predictive controllers. The controller has been applied to traffic scenario and results are presented. Comparisons between the model predictive control and the sliding mode control are presented and discussed.

Chapter 5, draws the conclusions. Findings and recommendations are summarized and presented in Chapter 6.

### 1.4 Thesis Contributions

The primary contributions of this thesis are summarized as follows:

- A collision avoidance control system for automated vehicles is developed.

- The properties of the proposed control model are discussed in detail.
- A MPC controller is demonstrated based on built-in Simulink toolboxes to control the lateral position and yaw angle.
- The model predictive controller is tuned to control the lateral maneuver successfully.
- The collision avoidance system results are verified by comparing against a validated study having similar scenarios and parameters but using a different type of controller.

## 2. VEHICLE MODEL

This chapter gives information about the vehicle dynamics models and under which circumstances they are used. Advantages and disadvantages of the models are briefly explained. In vehicle dynamics, there are various types of degrees of freedom. The most manageable dynamic model is the two-degree-of-freedom bicycle model which is used in this thesis. It represents the lateral motion and yaw angle. This model doesn't include the longitudinal direction because it does not have any impact on lateral motion and yaw angle. The steering angle maneuver delivers a vehicle direction on the road. In this research, the vehicle is doing an automated lane-change maneuver. For that reason, the longitudinal velocity is considered constant. In experiments to control the lane-change maneuver, kinematic or dynamic models is preferred.

### 2.1 Kinematic Vehicle Model

The kinematic vehicle model represents the geometry of vehicle motion [38]. Figure 2.1, represents the bicycle vehicle model, front two tires are modelled as one tire, and the rear two tires modelled as one. The control input for the front steering angle is  $\delta_f$ , and that for the rear steering angle is  $\delta_r$ . We assume  $\delta_r = 0$  and that there is a no slip condition generally because many vehicles are front wheel drive. We ignored the force effects to the motion in this model. The advantage of this model is that there are only two parameters to identify. The distance from the front axle to the center of the mass  $L_f$  and the distance from the rear axle to the center of the mass of a vehicle is defined by  $L_r$ . The length of the vehicle can be calculated by  $L_f + L_r = L$ .

X and Y are the global position of the vehicle where  $x$  and  $y$  are used for to represent the local coordinates [39]. The yaw angle of the vehicle is  $\psi$  and velocity is denoted as  $v$ .

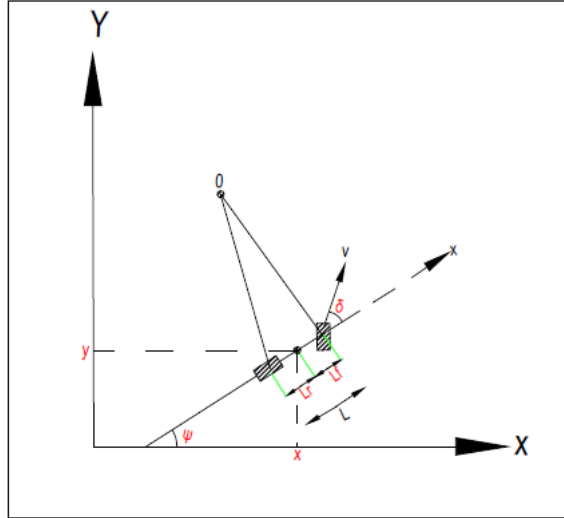


Fig. 2.1.: Kinematic Bicycle Model

The equations for the front and rear tire motion are,

$$\dot{x}_f \sin(\delta_f + \psi) - \dot{y}_f \cos(\delta_f + \psi) = 0 \quad (2.1)$$

$$\dot{x}_r \sin(\delta_r + \psi) - \dot{y}_r \cos(\delta_r + \psi) = 0 \quad (2.2)$$

where

$$\dot{x}_r \sin \psi - \dot{y}_r \cos \psi = 0. \quad (2.3)$$

These equations can be utilized to obtain the front tires position on an  $x$  and  $y$  axis.

$$x_f = x + L \cos \psi \quad (2.4)$$

$$y_f = y + L \sin \psi. \quad (2.5)$$

Substituting (2.3) and (2.4) into (2.1) yields:

$$\frac{d}{dt} (x + L \cos \psi) \sin(\delta + \psi) - \frac{d}{dt} (y + L \sin \psi) \cos(\delta + \psi) = 0 \quad (2.6)$$

$$(\dot{x} - \dot{\psi} L \sin \psi) \sin(\delta + \psi) - (\dot{y} - \dot{\psi} L \cos \psi) \cos(\delta + \psi) = 0 \quad (2.7)$$

In (2.7) the equations  $\sin(\delta + \psi)$  and  $\cos(\delta + \psi)$  multiplied with the equations inside the parentheses which yields:

$$\dot{x} \sin(\delta + \psi) - \dot{y} \cos(\delta + \psi) - \dot{\psi} L \sin \psi \sin(\delta + \psi) - \dot{\psi} L \cos \psi \cos(\delta + \psi) = 0. \quad (2.8)$$

Using trigonometric identities to expand the last two terms on the left hand side, we obtain the equation below,

$$\begin{aligned} \dot{x} \sin(\delta + \psi) - \dot{y} \cos(\delta + \psi) - \dot{\psi} L \sin \psi (\sin \psi \cos \delta + \sin \delta \cos \psi) - \\ \dot{\psi} L \cos \psi (\cos \delta \cos \psi - \sin \delta \sin \psi) = 0. \end{aligned} \quad (2.9)$$

Then expanding and recombining terms we obtain successively,

$$\begin{aligned} \dot{x} \sin(\delta + \psi) - \dot{y} \cos(\delta + \psi) - \dot{\psi} L (\sin^2 \psi \cos \delta + \sin \delta \sin \psi \cos \psi) - \\ \dot{\psi} L (\cos^2 \psi \cos \delta - \cos \psi \sin \delta \sin \psi) = 0, \end{aligned} \quad (2.10)$$

$$\dot{x} \sin(\delta + \psi) - \dot{\psi} L \cos \delta (\sin^2 \psi + \cos^2 \psi) - \dot{y} \cos(\delta + \psi) = 0, \quad (2.11)$$

and

$$\dot{x} \sin(\delta + \psi) - \dot{\psi} L \cos \delta - \dot{y} \cos(\delta + \psi) = 0. \quad (2.12)$$

Then, solving for the yaw rate  $\dot{\psi}$  yields

$$\dot{\psi} = \frac{\dot{x} \sin(\delta + \psi) - \dot{y} \cos(\delta + \psi)}{L \cos \delta}. \quad (2.13)$$

The local coordinates for  $\dot{x}$  and  $\dot{y}$  are given. Plugging that back into (2.13) and simplifying yields:

$$\dot{\psi} = \frac{v_x ((\cos^2 \psi + \sin^2 \psi) \sin \delta)}{L \cos \delta}. \quad (2.14)$$

By using the Pythagorean trigonometric identity theorem  $\cos^2 \psi + \sin^2 \psi = 1$  we obtain the equation below,

$$\dot{\psi} = \frac{v_x \sin \delta}{L \cos \delta}. \quad (2.15)$$

The Equation (2.15) gives the yaw rate of the kinematic vehicle.

$$\dot{\psi} = \frac{v_x \tan \delta}{L}. \quad (2.16)$$

The kinematic vehicle model commonly introduced in smaller velocity analyses due to the loss of the forces on the tire. It is simple to use and can get fast results. It gives similar results on the comparison with the real vehicle model.

## 2.2 Dynamic Vehicle Model

The Dynamic vehicle model can manage more accuracy on predictions than the kinematic vehicle model; also, it brings mathematical complexity. When the vehicle moving at a higher velocity, the forces at the tires increase, that is when we prefer the dynamic model. If the road is slippery than the no-slip condition is not valid. The inertial frame coordinates and the heading angle are defined in the same way as in the kinematic bicycle model [38]. The purpose of modeling is to see the rotation rate of the vehicle moments when the vehicle is moving.

In this thesis, the lateral dynamics of the bicycle model have some assumptions. The forward longitudinal velocity is assumed constant to separate our lateral and longitudinal dynamic models. The model becomes simple, but during acceleration and deceleration out of curves, it can give inaccurate results. It is assumed that there are no front and rear tire forces in the x-axis direction. As shown in Figure 2.2 there are only lateral tire forces of the dynamic vehicle model individually in coordinate

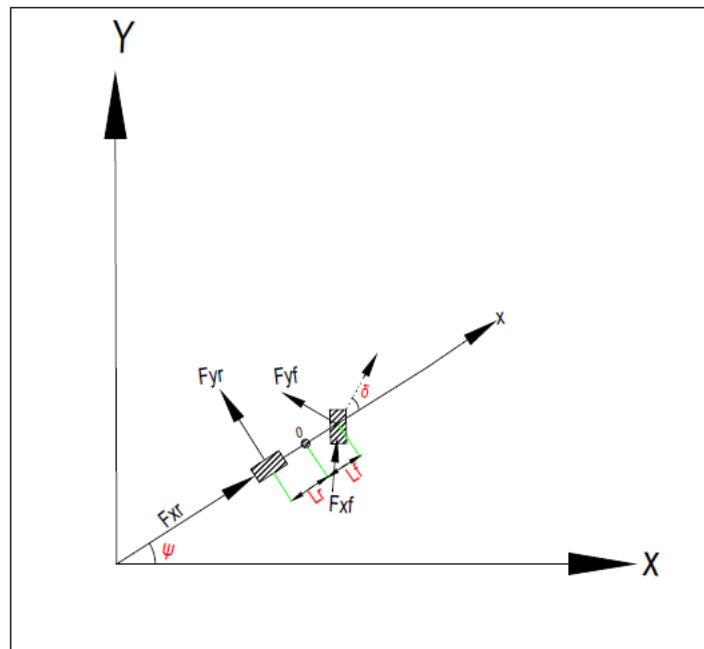


Fig. 2.2.: Dynamic Bicycle Model

frames followed by the wheels. Lastly, the nonlinear effects such as aerodynamic forces, suspension movement, and road inclination are assumed.

The vehicle's center of gravity will be the reference point for calculating Newton's law. The longitudinal velocity is a constant and  $\dot{v}_x = 0$ , as a result of this the force  $F_{xf} = 0$ .

The equations below are relevant to the lateral position and yaw angle, where the yaw angle is calculated with respect to the  $x$ -axis on the global coordinate.

$$m\dot{v}_x = -mv_y\dot{\psi} + F_{xf} \cos \delta + F_{xr} - F_{yf} \sin \delta, \quad (2.17)$$

$$m\dot{v}_y = mv_x\dot{\psi} + F_{xf} \sin \delta + F_{yr} + F_{yf} \cos \delta, \quad (2.18)$$

$$I_z\ddot{\psi} = L_f F_{yf} \cos \delta + L_f F_{xf} \sin \delta - L_r F_{yr}. \quad (2.19)$$

The variables in the equation are,  $v_x$  is the longitudinal velocity and  $v_y$  is the lateral velocity of the vehicle,  $m$  is the vehicle mass, and  $I_z$  is the vehicle's moment of inertia. After substituting the assumptions the differential equations are given by:

$$m\dot{v}_y = mv_x\dot{\psi} + F_{yr} + F_{yf} \cos \delta \quad (2.20)$$

$$I_z\ddot{\psi} = L_f F_{yf} \cos \delta - L_r F_{yr}. \quad (2.21)$$

$\ddot{\psi}$  represents the angular acceleration in the  $z$  direction which combines with yaw inertia to get the equation of the moments as shown at (2.25). Front and rear tires slip angles given by,

$$\alpha_f = \tan^{-1} \left( \frac{v_y + L_f \dot{\psi}}{v_x} \right) - \delta \quad (2.22)$$

$$\alpha_r = \tan^{-1} \left( \frac{v_y - L_r \dot{\psi}}{v_x} \right). \quad (2.23)$$

For small tire slip angles front and rear tire forces are varying linearly. The forces are defined as,

$$F_{yf} = -C_f \alpha_f \quad (2.24)$$

$$F_{yr} = -C_r \alpha_r, \quad (2.25)$$



where  $c_f$  and  $c_r$  are the cornering stiffness of the front and rear tires. Substituting  $F_{yf}$  and  $F_{yr}$  from Equations (2.24) and (2.25) as shown,

$$m\dot{v}_y = mv_x\dot{\psi} - C_f \left( \tan^{-1} \frac{v_y + L_f\dot{\psi}}{v_x} - \delta \right) \cos \delta - C_r \left( \tan^{-1} \frac{v_y + L_r\dot{\psi}}{v_x} \right) \quad (2.26)$$

$$I_z\ddot{\psi} = L_r C_r \left( \tan^{-1} \frac{v_y - L_r\dot{\psi}}{v_x} \right) - L_r C_f \left( \tan^{-1} \frac{v_y + L_f\dot{\psi}}{v_x} - \delta \right) \cos \delta. \quad (2.27)$$

After rearranging the Equations the position of the vehicle is defined as (2.32) and (2.33). These are nonlinear equations.

$$\dot{x} = v_x \cos \psi - v_y \sin \psi \quad (2.28)$$

$$\dot{y} = v_x \sin \psi + v_y \cos \psi \quad (2.29)$$

The linearized equations are shown as,

$$m\dot{v}_y = \frac{-C_f v_y - C_f L_f \dot{\psi}}{v_x} + C_f \delta + \frac{-C_r v_y + C_r L_r \dot{\psi}}{v_x} - mv_x \dot{\psi} \quad (2.30)$$

$$I_z\ddot{\psi} = \frac{L_r C_r v_y - L_r^2 C_r \dot{\psi}}{v_x} + \frac{L_f C_f v_y - L_f^2 C_f \dot{\psi}}{v_x} + L_f C_f \delta \quad (2.31)$$

These can be written as,

$$\dot{v}_y = \frac{-(C_f + C_r)}{mv_x} v_y + \frac{C_f}{mv_x} \delta + \left( \frac{C_r L_r - C_f L_f}{mv_x^2} - 1 \right) \dot{\psi} \quad (2.32)$$

and

$$\ddot{\psi} = \frac{L_r C_r - C_f L_f}{I_z v_x} v_y + \frac{L_f C_f}{I_z} \delta - \frac{L_r^2 C_r + C_f L_f^2}{I_z v_x} \dot{\psi}. \quad (2.33)$$

Mass of the vehicle, cornering stiffness of the front and rear wheels, moment of inertia of the vehicle around center of the mass, the distance between the front axle and the rear axle to the center of the mass values can be seen. The kinematic and dynamic bicycle models are compared in context of MPC controller design in [39].

### 3. CONTROL METHODS FOR AUTOMATED VEHICLES

In recent years, advanced control strategies' importance has rapidly increased in various industries. When we focus on the control methods, we observe various constraints are affecting the control strategy. Traditional control methods address all constraints separately. It is challenging when the system is multi input multi output (MIMO), and the ideal method is to have a single controller that can handle multiple variables. The literature shows that the MPC method is much more beneficial than the other control methods due to its strengths such as the constraints. The parameters have constraints and MPC tries to satisfy it as close as possible.

#### 3.1 Model Predictive Control Theory

Because it can predict future travel based on the current state and can handle multiple constraints MPC became one of the most commonly used controllers. In this thesis, structure and working principles of the model predictive control method are briefly described.

In a model-based control method, the system of the process model is estimated, and control action performed. This control method makes estimates and controls based on time. At every time step, an optimization problem has been solved by the controller, and at sample time tries to find the control signals that give optimal performance. Model predictive control, is an improved control method with feedback. The MPC structure uses control and optimization tools [40]. The purpose of this controller is to get the target control signal for the plant model. Key features are calculation of the plant's input and output measurements at each sampling instant and current state over a finite horizon. Controller optimizes the performance and satisfies the constraints on each control action. In predictive control, parameter settings are compared to other control methods are easier. It can be used to control various type

of dynamic systems, unstable systems to more complex ones. One of the important aspects, it is easily used in the control of multi-variable systems. The result is a linear control rule that can be easy to implement. MPC has fundamental principles that are entirely open for development.

The model predictive control method has many applications today; it provides easy application even in cases with limited system information. The system's easy adjustment of parameters is one of the advantages. It can handle multiple variables easily and have a prediction on upcoming sample time. Plant dynamics can be exploited. It improves steady-state response by a decrease in offset error and transient response by a decrease in rise time, peak time, and settling time. To control a slow-moving process, it is preferred to use time delay situations. On the other hand, the system requires a plant model to control and has a high computational load with a high algorithmic complexity.

Different types of MPC controller use different methods to deal with errors due to operation outside the linear region. The most popular ones are Adaptive, Gain-Scheduled, and Nonlinear MPC strategy. When the system constraints, and the cost function are nonlinear, the nonlinear MPC controller is preferred. It's the most reliable but, at the same time, the most challenging to implement due to the non-convex optimization problem. If the system is nonlinear and it has linear constraints, and a quadratic cost function then one of the linear MPC options can be used, such as Adaptive and Gain-Scheduled. If the optimization problem structure changes, gain scheduled MPC is preferred, if not, Adaptive MPC can handle the system. The adaptive MPC is going to be described briefly during Chapter 4 of the design model.

### **3.1.1 Receding Horizon Concept**

In this section, the technology behind the MPC control algorithm is presented and briefly compared with similar and different approaches. The purposes of the MPC controller are limiting violation of the input and output constraints, optimizing the

input variables and output variables to stay in the assigned limits, and also can handle multiple variables when there are not many sensors controlling the variables [41]. The concept behind it, is that, while the driver looks at the road ahead, he judges the present status, and the previous response. He predicts his response up to a horizon ahead, As shown in Figure 3.1 where it is called prediction horizon. Depending on the prediction horizon, the driver proceeds to the direction. MPC presents a structure that can substitute a simple path tracking control law with continuously introduced in the constraints of the parameters which has an optimization attempting to reduce the errors of path tracking. MPC can calculate non-linearity, future predictions, and operating constraints of the control system framework. MPC working principle depends on the prediction states and the present states of the output. The purpose of the MPC control calculations is to determine the prediction of the control movements of the response as close as possible to the set-points.

The control of the single-input single-output plant discussed in this chapter. The current time is represented as time step  $k$  in a discrete-time setting. At that moment, the plant output is  $y(k)$ , also in the Figure 3.1, the previous output trajectory is shown. The output should follow the set-point trajectory  $s(t/k)$ .  $N_p$  is the prediction horizon that shows how far the MPC looks into the future and represents the number of future time steps. MPC controller aims to find the most suitable predictive path closest to the reference. Control signal  $u(k+i)$  is the predicted control action at  $k+i$  given  $u(k)$ , likewise  $y(k+i)$  is the predicted output at  $k+i$  at given  $y(k)$ . Figure 3.2 shows the model-based predictive control approach.

The reference trajectory represents an essential perspective of closed-loop performance and starts at output  $y(k)$ . The reference trajectory could be also a linear function as shown in Figure 3.2. For instance, it determines a trajectory where the plant returns to the set-point trajectory even though there is a disturbance. It is considered that the reference trajectory accesses the set-point and the systems response speed expressed as  $T_{ref}$ .

$$\varepsilon(k) = s(k) - y(k) \quad (3.1)$$

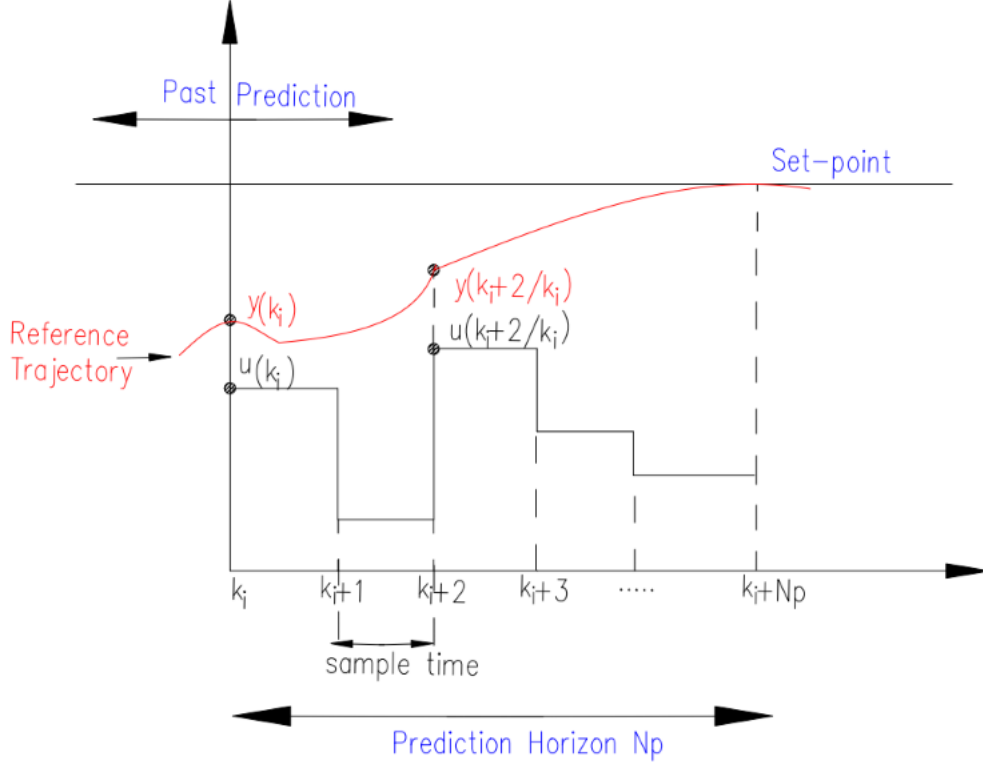


Fig. 3.1.: The MPC Action Structure [40]

$$\varepsilon(k+i) = e^{-iT_s/T_{ref}} \varepsilon(k) \quad (3.2)$$

$$= \lambda^i \varepsilon(k) \quad (3.3)$$

The error is calculated at Equation 3.1 and the reference trajectory is picked  $i$  step later and the output is represented above where the  $T_s$  is denoted as sampling time [42]. The reference trajectory defined as follows,

$$r(k+i|k) = s(k+i) - \varepsilon(k+i) = s(k+i) - e^{-iT_s/T_{ref}} \varepsilon(k) \quad (3.4)$$

Predictive control starts from the current time and has an internal model to predict the behavior of the system throughout the  $N_p$ . The internal model assumed linear for simplicity of the calculation. Predicted system behavior relies on the measured input trajectory  $\hat{u}(k+i)$ . The aim is to choose the input that gives the most suitable predictive behavior. In the representation  $\hat{u}$  has been preferred rather than  $u$ , because

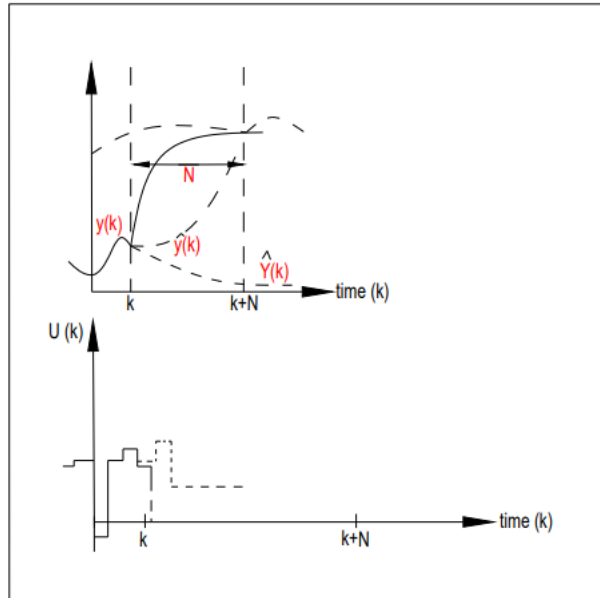


Fig. 3.2.: Model Predictive Control

at a time step  $k$ , it gives a prediction of what the input at time  $k+i$  could be. While deciding the input value  $u(k)$ , it's assumed  $y(k)$  the output value is known. The output value  $y(k)$  depends on the previous inputs  $u(k-1), u(k-2), u(k-3), \dots$  but not  $u(k)$ . The input trajectory could be chosen as the reference output  $(k+N)$  directly on the prediction horizon. As it is shown in Figure 3.3, it differs for the first three steps of the prediction horizon then the remaining is  $\hat{u}(k|k), \hat{u}(k+1|k), \dots, \hat{u}(k+N_p-1|k)$  constant. The aim is to choose the input trajectory with the smallest input energy and that satisfies Equation 3.5.

$$\hat{y}(k+N_p|k) = r(k+N_p|k) \quad (3.5)$$

Furthermore, the actual input signal will be applied as  $u(k) = \hat{u}(k|k)$ . After selecting the future input trajectory, only the first element of the trajectory is applied as the input signal to the plant. After that input signal, the whole cycle is repeated, also for the prediction and input trajectories. A sampling interval moves one step forward, and a new output measured  $y(k+1)$  in the system. So it starts calculating all over again with new reference trajectory  $r(k+i|k+1)$ ,  $i = 2, 3, \dots$ ; predictions done

through the horizon  $k + 1 + i$ ,  $i = 1, 2, 3, \dots$ ; new trajectory chosen  $\hat{u}(k + 1 + i|k + 1)$ ,  $i = 0, 1, 2, \dots, N_p - 1$  and the input signal applied  $u(k + 1) = \hat{u}(k + 1|k + 1)$ . The length of the prediction horizon stays same but it shifts forward one sampling interval at each time step to control the plant where it's called the receding horizon control.

### 3.1.2 The Structure of Model Predictive Control

The general structure of the state feedback strategy of MPC is presented in Figure 3.4. In here  $r(\cdot)$  is reference trajectory, the output signal of the plant is  $y(k_j)$ ,  $u(k_j)$  is the plant input signal,  $x(k_j)$  is the state at time  $k_j$  from the plant which can be measured in some cases. If it cannot be measured then a state estimator has to be used and feedback to the MPC controller. The MPC uses a model of the plant to predict the future behavior of the output and optimizer to make sure the output matches as close as possible the expected reference. The aim of the optimization is to reach the most suitable value for the best result despite the unmeasured disturbance such as wind. Optimizer tries to satisfy constraints while solving an online optimization problem at each time step, and the controller attempts to reduce the error between the predicted path and the reference. Additionally, the controller minimizes the input signal for immediate changes at each time step. By using the control input  $u$ , the optimization algorithm decreases the cost function  $J$ . The cost function is,

$$J = \sum_{i=1}^p w_e e_{k+i}^2 + \sum_{i=0}^{p-1} w_{\Delta u} \Delta u_{k+i}^2, \quad (3.6)$$

where,  $w_e$  and  $w_{\Delta u}$  are the weighting coefficients, the square of  $\Delta u_{k+i}$  is the manipulated variable,  $p$  is the prediction horizon, and the square of  $e_{k+i}$  is the error obtained by subtracting the reference variable from the controlled variable. There are some constraints that MPC tries to stay inside the boundaries for the most desirable outcome, which gives the optimal solution and the automated car moves as close as possible to the reference.

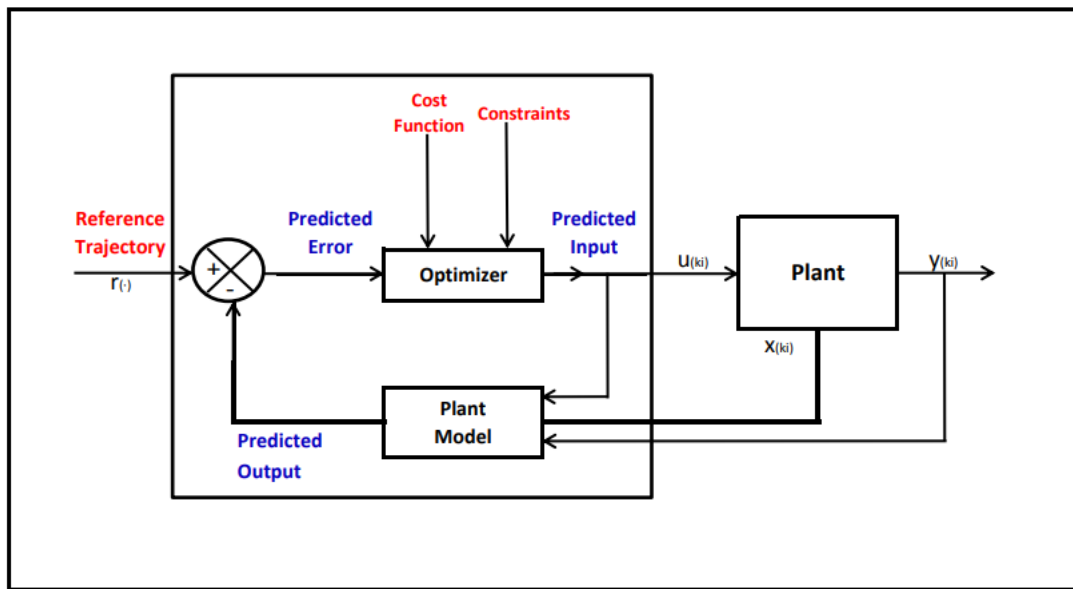


Fig. 3.3.: State Feedback Model of the MPC [40]



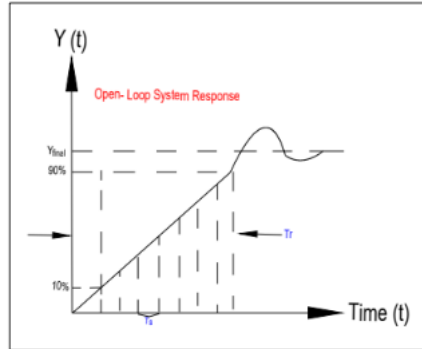


Fig. 3.4.: Sampling Time

To design an MPC controller, there are some parameters that have to be chosen correctly to get higher performance. The first one is the sample time,  $T_s$ , which defines the rate of the controller, applies the control algorithm in the system. As it is shown at Equation 3.7 sampling time is chosen by looking at the step response of the open-loop system and trying to feed with 10 to 20 samples at that rise time to get the most desirable sample time.

$$\frac{T_r}{20} \leq T_s \leq \frac{T_r}{10} \quad (3.7)$$

Rise time,  $T_r$ , is the time it takes to go from 10% to 90% of the steady-state response as shown in Figure 3.5. It depends on the disturbance entering and how fast you want to react to set-point changes. If  $T_s$  is too big, the controller has a hard time responding fast enough to disturbances and set-point changes; if it is too small, it gives a fast response that creates an extreme load on calculation.

The other essential parameters are the prediction and control horizon. Previous research has determined that choosing the prediction horizon to be 20 to 30 samples so that it covers open loop the transient response. If the stability properties need to improve, then the length of the prediction horizon has to be increased. The control horizon, should be limited to at most 2 to 4 time steps, in order to limit the complexity of the quadratic program (QP) optimization problem solved at each control interval.

As seen in Equation 3.8 the control horizon  $m$ , has to be set to 10% to 20% of the prediction horizon. If the control horizon is smaller than the calculation load, and only the first few moves has a important effect on the predicted output, the rest is constant and doesn't have any insignificant effect. So if the control horizon is chosen as same as the prediction horizon, it causes a complexity on the optimization problem. Thus we must have,

$$\frac{P}{10} \leq m \leq \frac{2P}{10} \quad (3.8)$$

Plant manipulated variables, MV; constraints are the limits you have in actuators. Plant output variables, OV, are the constraints that limit the outputs controlled for the physical system. The most important part of the constraints is when more constraints are added; then, the optimization problem is more complex to solve. They are classified as hard and soft constraints depending on whether they can be violated or not. For the optimization problem to be solvable, it is not wise to give hard constraints to both input and output; they can conflict with each other. The most common use is to set the output constrains soft. The ratio between the output and input weights are the last essential parameters. If the ratio is high, the controller is more competitive but weak. If the controller needs to be robust, then the input weights have to increase. For balanced performance is to weight the ratio of input and output correlated with each other.

### 3.1.3 Models for Model Predictive Control

In this section, there will be a brief description of the models used for the MPC algorithms. There are different types of discrete models used to calculate the predicted outputs. The impulse and step response are the most common models used in MPC algorithms.

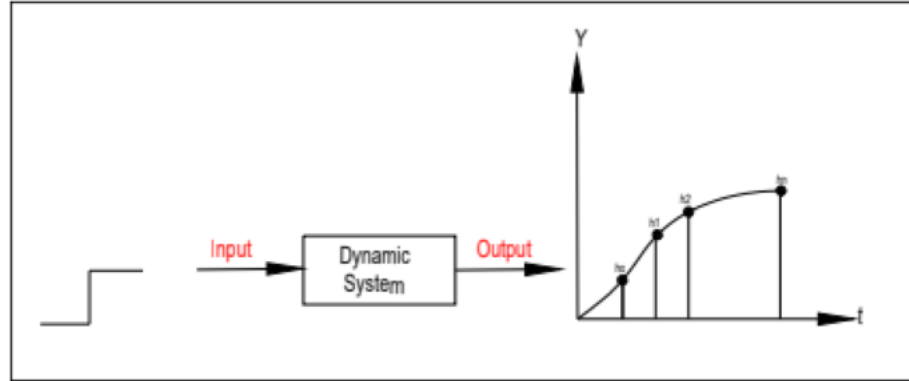


Fig. 3.5.: Step Response

### 3.1.3.1 Finite Step Response Model

It is the most popular model that has the open-loop step response shown in Figure 3.6. Implementation is simple, when the step response is applied to the system, the model parameters can be found by measuring the system output.

$$S = [s_1 s_2 s_3 s_4 s_5 \dots s_N] \quad (3.9)$$

It is achieved by assigning a unit step input change to a system running at a steady state. The coefficients are the output values at the single time step. The step response model is a vector of step response coefficients. The step response coefficient,  $s_i$ , is the unit step input change after the  $i^{th}$  sample time.  $N$  is the model length. Also can be described as the sum of the impulse response coefficient to that point is the step response coefficient.

### 3.1.3.2 Finite Impulse Response Model

This model is similar to step response. The finite impulse response model is similar to the step response. In this model, a unit pulse is applied to the input, and the coefficients are the  $i^{th}$  impulse response coefficient of the values of the output.

There is a close connection between the step response  $S_i$  and impulse response  $H_i$  shown in Equation 3.10.

$$H_i = S_i - S_{i-1} \quad (3.10)$$

$$S_i = \sum_{j=1}^n h_j \quad (3.11)$$

. in which  $\{h_j\}_{j=1}^n$  are the impulse response coefficients (Markov parameters) are differences at the step response coefficient at each time step. Overall there are two significant constraints for both of these response models.

### 3.1.3.3 Transfer Function Model

The transfer function model requires fewer parameters. The input signal is  $u(t)$ , and the output signal is  $y(t)$ . The model can be represented as (3.14) where,

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_{na} \cdot z^{-na}, \quad (3.12)$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{nb} \cdot z^{-nb}, \quad (3.13)$$

$$A(z^{-1})y(t) = B(z^{-1})u(t - 1). \quad (3.14)$$

The predicted output using this model is,

$$y\left(t + \frac{k}{t}\right) = \frac{Bz^{-1}}{Az^{-1}}u\left(t + \frac{k}{t}\right). \quad (3.15)$$

### 3.1.3.4 State Space Model

The state-space equation is mostly preferred for multivariate systems. It is often used in predictive functional control algorithms. The state space equation is,

$$x(k + 1) = Ax(k) + Bu(k), \quad (3.16)$$

The output equation is,

$$y(k) = C_y x(k) \quad (3.17)$$

and the estimator state is,

$$z(k) = C_z x(k). \quad (3.18)$$

Here  $x$  is the  $n$ -dimensional state vector,  $u$  is the one-dimensional input vector,  $y$  is dimensional measured outputs,  $z$  is the outputs to be controlled at  $m_z$  dimensional vector. The variables at  $y$  and  $z$  often overlapped with a broad scope, and generally, they are the same. So all controlled outputs are continuously measured. In the equation  $k$  represents the time step, often  $y$  is accepted as  $z$ ,  $C$  applied as  $C_y$  and  $C_z$ .

This model can be generalized to include the measured and not-measured errors and the measurement noise. Measurement values are taken  $y(k)$ . The required input values calculated  $u(k)$  applied to the system. There is always a delay between the measured  $y(k)$  to  $u(k)$ , so the measured output doesn't have direct feed between  $u(k)$  to  $y(k)$ . The controlled output  $z(k)$  can be related to  $u(k)$ ,

$$z(k) = z(k) + D_z u(k). \quad (3.19)$$

### 3.1.4 Single-Input and Single-Output Systems

A single output controlled by a single manipulated variable is called single input single output systems [43]. Heating control and air conditioning control systems are examples of SISO systems.

The controlled system is a set of elements put together for a specific function as represented in Figure 3.7. The control of the set of elements calculates the model; therefore, there is a single input and single output at the model.

### 3.1.5 Multi-Input and Multi-Output Systems

If the systems have more than one cycle it is called Multi-input multi-output systems [43]. There are some applications such as manufacturing that can not be controlled by single input single output. It is challenging to tune many controller gains. The MPC can also handle constraints. An example is given for multivariate

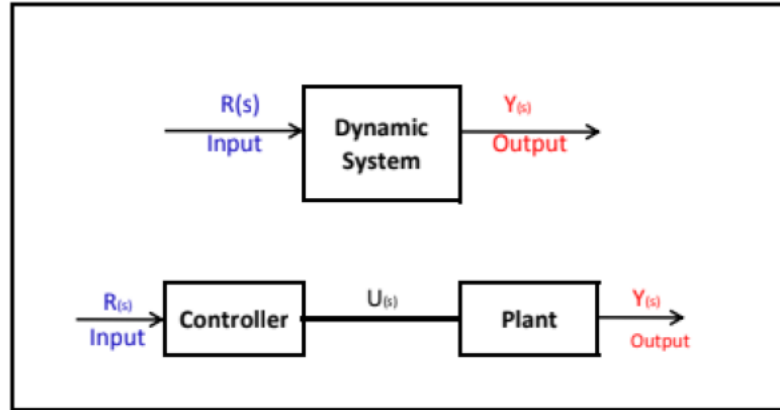


Fig. 3.6.: Single-Input and Single-Output Systems

system analysis for two input, and two output in Figure 3.8. Any input at MIMO systems effected by the other output and the outputs are effected by the other input.

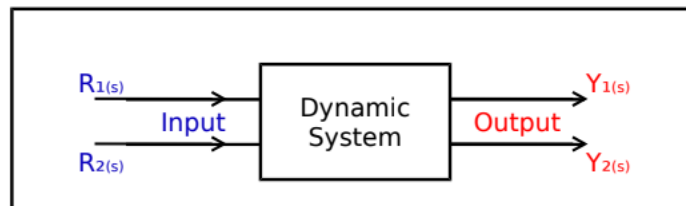


Fig. 3.7.: Multi-Input and Multi-Output Systems

The system variables can be represented with the equations by using the transfer functions relation.

$$Y_i(s) = G_{ii}(s)R_i(s) + G_{ij}(s)R_j(s) \quad (3.20)$$

$$Y_1(s) = G_{11}(s)R_1(s) + G_{12}(s)R_2(s) \quad (3.21)$$

$$Y_2(s) = G_{21}(s)R_1(s) + G_{22}(s)R_2(s) \quad (3.22)$$

In these equations, the  $G_{ij}$  transfer function is the relation between the  $i$ , output variable with  $j$ , input variable. The block diagram of the equation set shown in Figure 3.9.

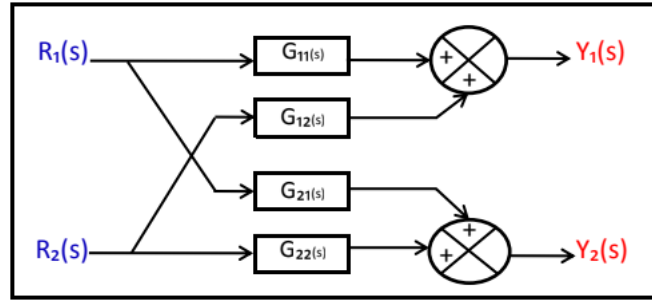


Fig. 3.8.: An Example of a Multi-Input and Multi-Output Systems.

### 3.1.6 Continuous-Time to Discrete-Time Models

In the MPC strategy, linearization is important. A method is presented to transform linear continuous-time models to linear discrete-time in this section. The discrete models perform calculations to produce control commands. A sample and zero-order hold (ZOH) operation is used to convert a continuous signal to a discrete signal where  $h$  is the sampling period shown in Figure [40].

$$f(kh), kh \leq t < (k+1)h, \quad (3.23)$$

## 3.2 Simple Sliding Mode Controller

In this section, a single input single output sliding mode controller overview given. It is a nonlinear control strategy that has easy tuning and gives accurate results. The structure of a sliding mode controller has two phases. The first phase is to obtain

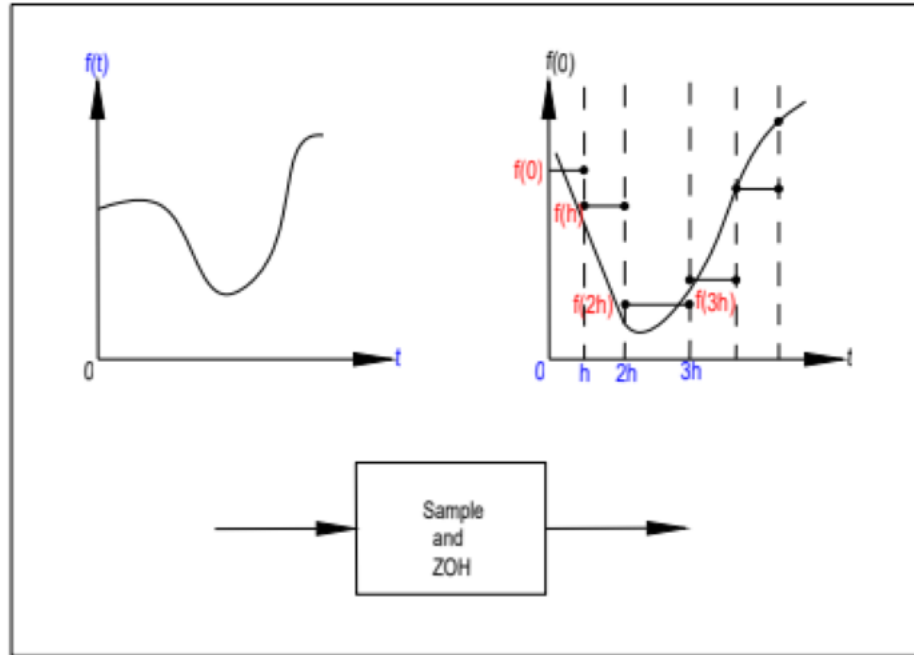


Fig. 3.9.: Sample and ZOH Conversion [40]

the desired system behavior with a custom-made surface design. The second phase is to make sure the closed-loop system is stable to the sliding surface; the feedback gains of the controller have to be selected [40]. The sliding surface is a derivative of state space. On the sliding surface, the controller keeps the states on the close neighborhood of the sliding surface. The advantages and disadvantages are described at the most common controller comparison Table 3.1 at the end of the chapter.

### 3.3 Fuzzy Logic Controller

In this section, a closed-loop system with a fuzzy controller presented. The fuzzy logic controller has common use in the industry as much as a model predictive controller due to efficiency, reliability, and it's success at the control application. Many products have a fuzzy logic controller, such as control on traffic lights, washing machines, and room temperatures. The fuzzy logic control has rules which is used in controller design. The closed-loop fuzzy logic architecture utilizes the control error



$e$  and the rate of change  $\dot{e}$  [40]. The controller has two main components, first one is the inference engine, and the other one is the defuzzifier. Inference engine is the core prepares the inputs using the knowledge base. The fuzzy database collects the information the input-output data and describes to the plant the input variables to the fuzzy rule base and also the output variables. The output of the inference engine is converted by the defuzzifier into crisp values. The fuzzifier does the conversion from the crisp input values to fuzzy values.

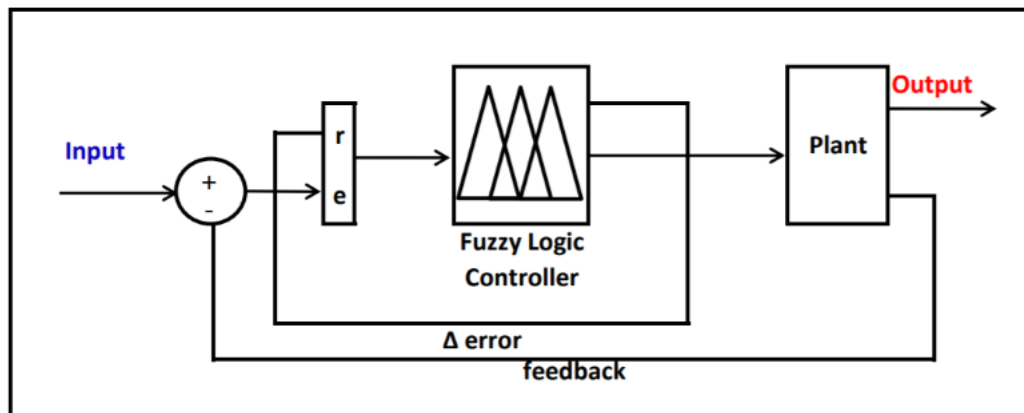


Fig. 3.10.: Closed-loop System with a Fuzzy Logic Controller

While designing the fuzzy logic controller, some assumptions have to be made. The plant has to be assumed observable and controllable. It has to be assumed there is a solution, and it should be more than the optimum solution. The controller design should have an acceptable range of exactness. In the design of the controller, stability and optimizing are not addressed explicitly.

Designing a fuzzy logic controller has six steps, the controller's input and output ranges should be identified for the input, and fuzzy output sets have to be created. The input and output should be converted into if-then rules, and the rules matrix is developed. The inference engine and the defuzzifier is chosen. Finally, the controller is ready for implementation and testing.

Table 3.1.: Advantages and Disadvantages of Control Methods

Controller Type	Advantages	Disadvantages
<b>Model Predictive Control</b>	<ul style="list-style-type: none"> <li>-Control of multiple variables</li> <li>-Cost effective</li> <li>-Model based optimization</li> <li>-Constraint satisfaction</li> <li>-Incorporates predictions</li> <li>-Steady state response improvement</li> <li>-Priorities</li> </ul>	<ul style="list-style-type: none"> <li>-Effort for designing accurate modeling</li> <li>-Computational complexity</li> </ul>
<b>Fuzzy Logic Control</b>	<ul style="list-style-type: none"> <li>-Simple and flexible</li> <li>-Precision</li> <li>-The development is cheaper than MPC</li> <li>-Customizable</li> <li>-Human like thinking</li> <li>-Reliable</li> </ul>	<ul style="list-style-type: none"> <li>-Requires a lot of data to be applied</li> <li>-Not useful for smaller or larger data</li> <li>-Needs high human expertise</li> <li>-The rules must be updated with time</li> </ul>

Table 3.1.: continued

<b>Sliding Mode Control</b>	<ul style="list-style-type: none"> <li>-Simple structure</li> <li>-System stability guaranteed by using Lyapunov function</li> <li>-Easy to realize the controllers with switching elements</li> <li>-The output can be restricted to a certain region of operation</li> <li>-While system sliding, it's not affected by matched uncertainties and behavior is governed by a reduced set of differential equations</li> </ul>	<ul style="list-style-type: none"> <li>-Chattering in order to eliminate matched external disturbances and elimination of the existing system also creating a new one according to desired features</li> </ul>
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## 4. MODELING

Several years ago, autonomous vehicles attracted intense awareness from the automotive industry due to their potential for improving comfort and safety in driving. Automated collision avoidance control can reduce the number of accidents, fatalities, and increase in traffic density moved safety in a critical position in intelligent transportation systems. Due to the complex work environment, researchers applied modern control theory and sensing technologies to control smart vehicles. Fully autonomous driving is a remaining complicated task, hence generating a collision-free trajectory is the key for the next level intelligent vehicles.

Autonomous vehicles are driven by artificial intelligence, which has a place in everyday life. The vehicle applies the sensors and collects data of the environment from them. The data collected is transmitted to the central computer system to enable the vehicle to perform its maneuvers, such as steering control, acceleration, and deceleration accurately. The computer gives the best outcome for safe drive. There are many industries in which autonomous vehicles will widely be used over the next decades, such as professional driving, parking garages, military, and delivery services.

There are many challenges to succeed. In some states, it is legal to test autonomous vehicles on public roadways. To control the reliability in terms of fatalities and accidents, the control strategy has to examine for several cases. After the fatality of a pedestrian in Arizona in 2018, artificial intelligence was questioned. Following that accident, there were nine confirmed Level 2 accidents in which the autopilot was involved in the Florida area. It is mandatory to investigate, examine, and research widely.

## 4.1 Collision Avoidance Design by Adaptive MPC

The collision avoidance design is presented at Figure 4.2. The strategy is designed using Matlab/Simulink. For the controller design Simulink/ Model Predictive Control Toolbox was used and the reference trajectory was designed using the Automated Driving Toolbox. In the following, sections each block will be described in detail.

### 4.1.1 Plant Model

An MPC controller contains a dynamic plant model. It is briefly discussed in Chapter 2, along with the kinematic and dynamic vehicle model parameters. Our goal is to control the lateral position and yaw angle for passing the obstacle. In this section, the parameters and the dynamic plant model will be explained.

The global position of the car is represented depending on the distance to the X and Y axes.  $V_y$  is the lateral velocity;  $V_x$  is the longitudinal velocity. The MPC controller needs a reference trajectory for the car to control. The reference values for our goal are measured with respect to  $V_x$ . The car model is linearized in this thesis, and the longitudinal velocity is constant during the vehicle travels. The steering angle is  $\delta$ , yaw angle is  $\psi$ , and reference yaw angle is  $\psi_{ref}$ . It is created as state-space model which shown in Equation 4.1 and 4.2. System dynamic matrices are A and B which are time-invariant when longitudinal speed is constant. There are four states used for calculation first lateral velocity  $V_y$ , second yaw angle represented as  $\psi$ , third yaw rate  $\dot{\psi}$  and last the lateral position  $\dot{y}$ .

$$\dot{x} = Ax + Bu \quad (4.1)$$

$$y = Cx + Du \quad (4.2)$$

The model takes longitudinal velocity and steering angle as input and the lateral position with yaw angle and states of the model as output. Matrices are shown in the continuous-time model which will be linearized and converted into discrete-time

model inside the controller because the MPC uses discrete-time. Only  $\dot{x}$  is considered to calculate,  $y$  is not calculated.

$$A = \begin{pmatrix} -(2C_f+2C_r)/m/V_x & 0 & -V_x-(2C_fI_f-2C_rI_r)/m/V_x & 0 \\ 0 & 0 & 1 & 0 \\ -(2C_fI_f-2C_rI_r)/I_z/V_x & 0 & -(2C_fI_f^2+2C_rI_r^2)/I_z/V_x & 0 \\ 1 & V_x & 0 & 0 \end{pmatrix} \quad (4.3)$$

$$B = \begin{pmatrix} 2C_f/m & 0 \\ 2C_fI_f/I_z & 0 \end{pmatrix} \quad (4.4)$$

$$C = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad (4.5)$$

$$D = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (4.6)$$

In Chapter 2, the parameter information of the matrices variables is given. The parameter values for calculating the dynamic model of the plant model in this research are presented in Table 4.1. By using these values the model can be examined with different controllers in simulations and give stable results.

#### 4.1.2 Adaptive Model Predictive Controller

In this section, design of an adaptive MPC controller that autonomously steers a car in a lane change maneuver for collision avoidance is presented. The controller block is inside the Model Predictive Control Toolbox in Simulink. This block also performs optimization to get the optimal value for giving to the plant.

The measured output of the lateral position with the yaw angle from the plant enters the MPC block as an input. The next input is the reference for the trajectory with yaw angle and lateral position for vehicle changing lanes for a collision-free path, which is explained in the following reference trajectory section. The adaptive MPC's

Table 4.1.: Values of Dynamic Bicycle Model Parameters

<b>Parameter</b>	<b>Value</b>
$m$	976 <i>kg</i>
$C_r$	33000 <i>N/deg</i>
$C_f$	19000 <i>N/deg</i>
$I_z$	5400 <i>kgm<sup>2</sup></i>
$L_r$	1.6 <i>m</i>
$I_f$	1.2 <i>m</i>

main difference from the regular MPC is that at each time step it presents a new linear plant model for present operating conditions. In this research, it assumed that there is no measured disturbance. The last is connecting controller output, which is steering angle to the plant input. All these connections can be seen in Figure 4.2.

The MPC block has an optimizer and a plant model inside, which is shown in Figure 3.3. The controller has constraints, weights, sample time, prediction, and control horizons, which have to be specified in the controller and tuned for the most reliable result the performance can be measured. Inside the MPC structure, there is an MPC designer where we set the number of inputs and outputs. We set the controller sample time to 0.1 sec and then linearize. For the predictions, MPC uses an internal plant model and an optimizer to make sure it is the optimal control action. While linearizing, it takes the plant model data and uses it as an internal plant. It shows the input and output responses, so the next step will be finding the best control action by changing the parameters.

The signal information, units, and also when there isn't much magnitude difference between the signals has been adjusted. The scale factor is used as one. The change is made inside the I/O attributes. Then the parameters are saved inside the edit scenario. In Figure 4.1 it is displayed.

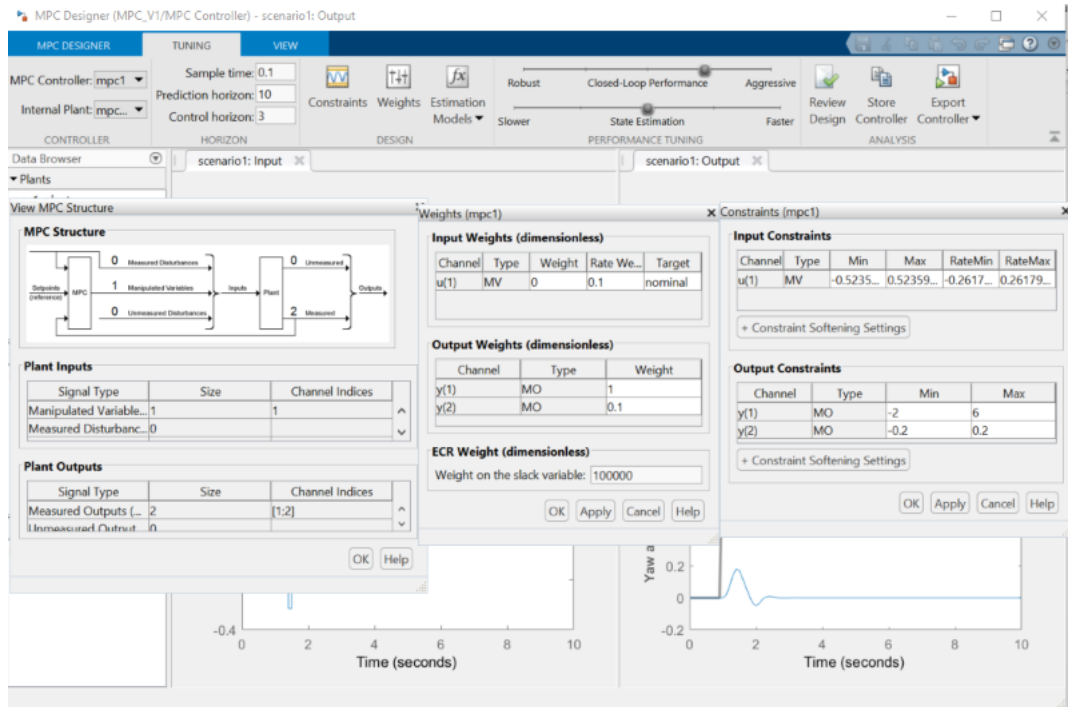


Fig. 4.1.: Structure Inside the MPC Designer

The next part is the tuning, where we decide the prediction and control horizon. For the prediction horizon, it is preferred to be 10 when it is increased the response goes slow. For the control horizon, it is raised and then decreased for the more desirable result; it is preferred to be three. There are weights to be set to an amount other than zero for the input and outputs to have a destination. The lateral position is the most important, so it is set to one. There is a slider on top of the MPC designer user interface. For aggressive control, it is slid to the right side.

The last part is assigning the constraint parameters. It mentioned in the previous chapters that there are soft and hard constraints. The input constraints are hard in this research, and the output constraints are soft. Earlier it is described that when we are assuming constraints for the better result if the input is soft, the output should be hard or vice versa. In this research, it was thought that the steering angle should be a maximum of 30 degrees and the rate of change of steering angle 16 deg/sec to get more solid results. The data is entered in radians then it will be converted into



degrees in the Simulink model. After all these adjusted parameters, the controller is ready to compile and update the MPC block in our model.

The adaptive MPC uses an update plant model in which the output is the last input to be connected to the controller. It updates the plant model at each time step for current condition. Inside the block, it has the Matlab function of the state space matrices of the state-space model. It has a conversion to the discrete-time model from the continuous-time model and updates the nominal conditions. The inputs to the blocks are longitudinal velocity, steering angle, and states.

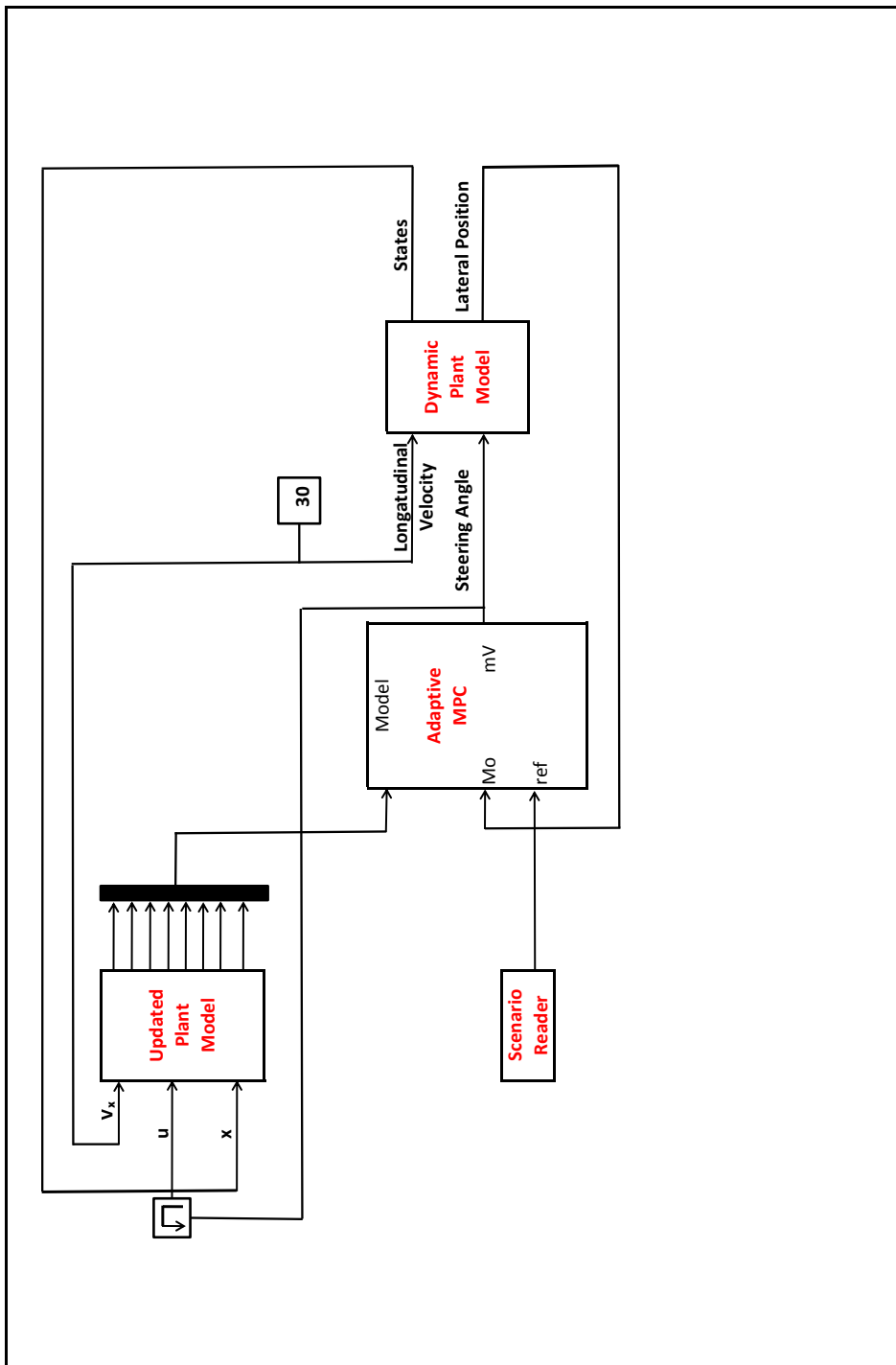


Fig. 4.2.: Collision Avoidance Simulink Design Using Adaptive MPC

### 4.1.3 Reference Trajectory Design

In this section, designing a reference trajectory for the MPC controller will be described. One of the main inputs to the MPC controller is the reference trajectory, and the aim is to force the plant output as close as possible to the reference trajectory by applying the MPC controller, as shown in Figure 4.3.

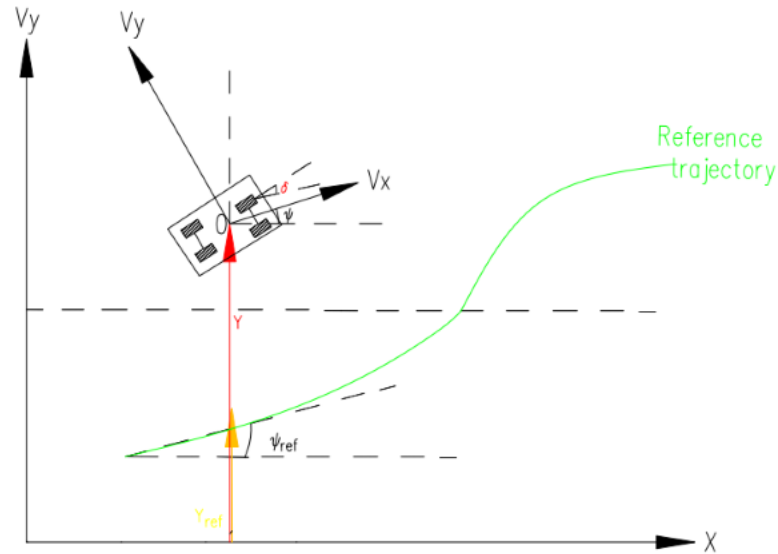


Fig. 4.3.: Lateral Vehicle Dynamics

In this thesis for designing the reference trajectory, the Driving Scenario Designer inside the Automated Driving Toolbox used. For the scenario examined in this thesis, a road with two lanes and 4 meters wide is created. On to the road, an ego car with 30 m/sec and another vehicle as a dynamic obstacle with 5m/sec were used. We Added way-points and generated the collision avoidance maneuver successfully. The scenario was exported to Matlab as a block that outputs the lateral position and yaw angle values to be used as input inside the controller.

## 4.2 Simulations and Results

This research is about implementing a collision avoidance model controlling the lateral maneuver and yaw angle for autonomous vehicles. The results are compared with the scenario proposed in [4], which used a sliding mode controller for a collision-free path. In the scenario, a vehicle moving at 5m/s is considered as the dynamic obstacle. The following vehicle is at a speed of 30 m/s. The lateral overlap, lateral error, tire steering angle comparisons are presented. Additionally, the yaw angle is also displayed in the Simulink results.

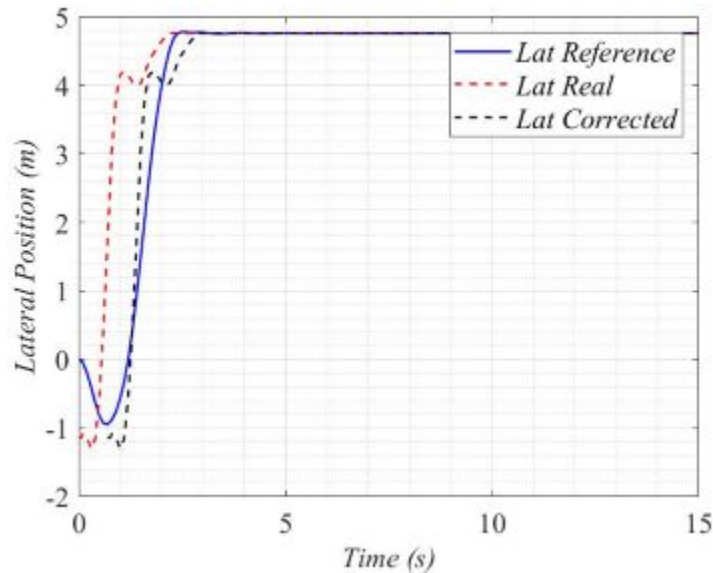


Fig. 4.4.: Lateral Position

In the parameters considered for the study were the mass ( $m$ ), sampling time and moment of inertia of the vehicle ( $I_z$ ) which were obtained from the research paper [4]. The simulation results for the lateral overlap scenario are shown in Figure 4.4, where the controller followed the expected trajectory with a similar response having a time discrepancy of less than a second. The discrepancy between the results is credited to the difference in the input reference trajectory. In Figure 4.4 it is shown that the Lat-Real proceeds slower than the reference. There is a time delay of 0.7 seconds. When

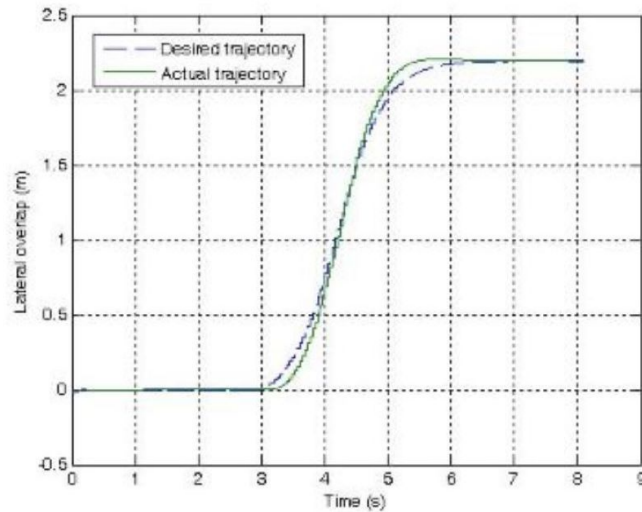


Fig. 4.5.: Reported Lateral Position [4]

the signal is shifted, it is clear that the controller presented a good performance. The lateral position presented in [4] can be seen in Figure 4.5. The lateral error obtained from this study is shown in Figure 4.6 as a result of difference at the lateral overlap. The corrected lateral error presents how smaller is the difference if there is 0.7 sec. shift.

The tuning of the controller was challenging for satisfying performance. Each input and output, parameters need to be tuned. The controller parameters such as constraints, weights, sample time, prediction horizon and control horizon were changed several times to obtain the most desirable result presented in this thesis. When the prediction horizon was tuned and increased up to 30, the result was late to respond. If we do less 10 than it cannot handle to control. The sample of time is 0.1. When the control horizon was increased to more than 4, it started giving lazy results and didn't present any fast change. The steering angle was set to -0.5 rad. to 0.5 rad. and the steering rate was set to -15 deg. to 15 deg. in the controller at constraints for a better ride as a hard constraint. It is assumed from the controller to execute aggressive lateral control. At simulation, the tire steering angle result

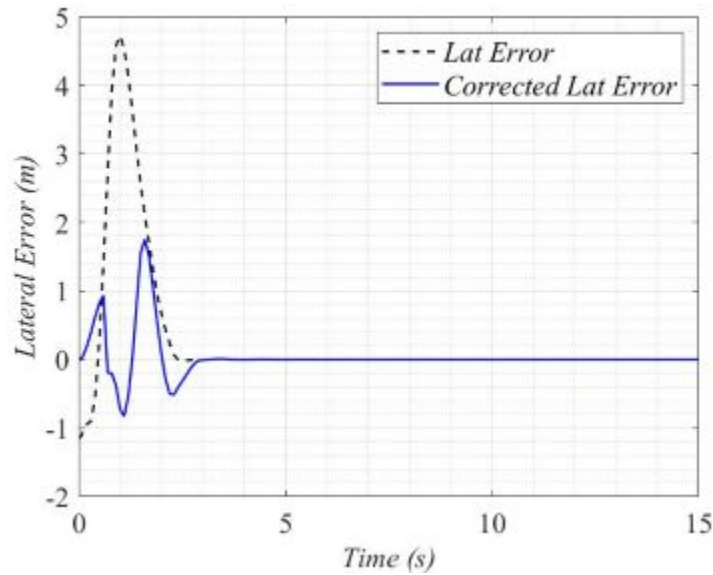


Fig. 4.6.: Lateral Error

converted into degrees from radians in the Figure 4.7. shows it stayed in the limits as we set the controller constraints. When we compared the steering angle results with the reported, the hard constraints can be much smaller than what we set. At the reported lateral position to avoid collision the maneuver started after 3 sec. an exponential trajectory was used where they have a rapid change in the steering angle Figure 4.8. At the simulation results the controller is tuned to get the most desirable performance.

The output lateral position and the yaw angle had a soft constraint. For the lateral position, it stayed between -2 to 6 and yaw angle between the -0.2 to 0.4 as shown in Figure 4.10.

It both demonstrates that it followed the trajectory as desired, and the controller did an acceptable performance.

In this study, a model is experimented to demonstrate that the adaptive MPC could handle higher longitudinal velocity in a lateral position control. For smaller longitudinal velocity, it has a more significant delay so it is recommended to use regular MPC under that condition. When the model is compared with [4] on the

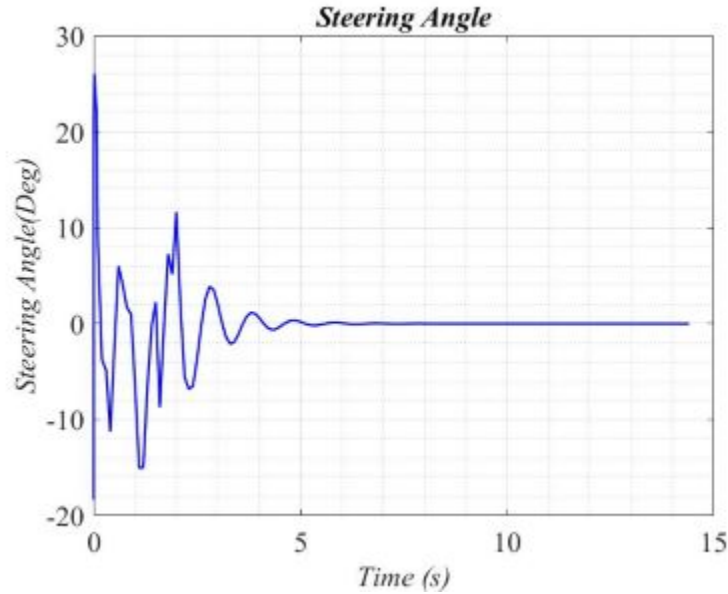


Fig. 4.7.: Steering Angle

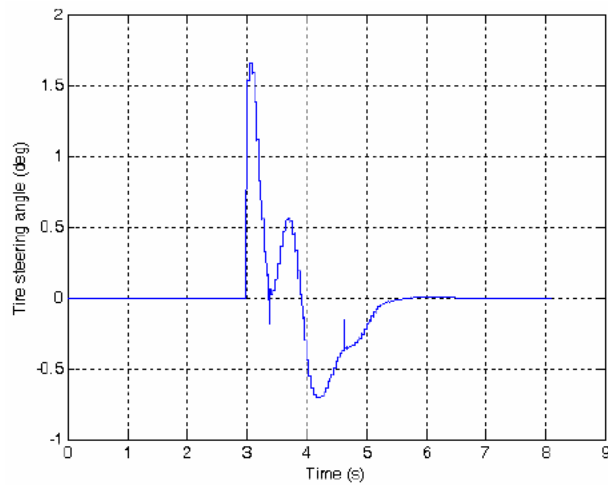


Fig. 4.8.: Steering Angle Reported [4]

lateral overlap basis, the actual trajectory followed the desired path closer by using the sliding mode controller. The yaw angle control performed an acceptable result by tracking the reference closely as shown in Figure 4.9. Relatively, the overlap error was less. The tire steering angle input is smaller than the model presented in this thesis, which can also be a reason for the difference in the results. All the

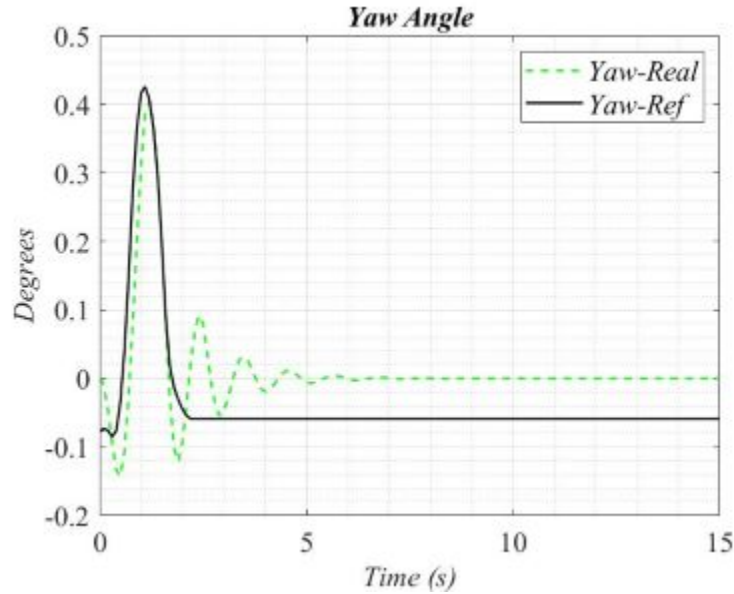


Fig. 4.9.: Yaw Angle

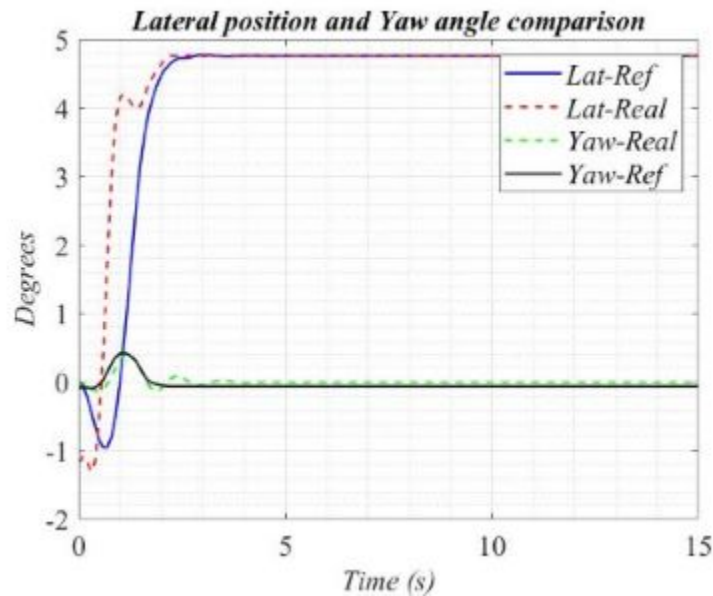


Fig. 4.10.: Lateral Position and Yaw Angle Comparison

computational results for lateral position and yaw angle are compared in Figure 4.10. The comparisons indicate that the MPC controller successfully completed the lateral control for collision avoidance in the specific dynamic environment.



## 5. CONCLUSION

The Model Predictive Controller (MPC) is robust, reliable, and highly scalable for handling obstacle avoidance. In this thesis, the MPC controller was implemented to develop a control method to avoid a collision with a dynamic obstacle in the path. The parameters included in the study were the lateral overlap, lateral error, and tire steering angle. The responses were compared with the results presented in [4], where they analyzed a similar system using the sliding mode controller for the same scenario. In this thesis, the yaw angle is also presented in the studies.

The reason for the differences in the simulation results can be contrast of the controller structure which is explained briefly for both of the controllers in Chapter 3. The sliding mode controller used an exponential trajectory; whereas in this thesis, the reference trajectory was obtained from the driving scenario designer app.

Based on the comparisons, the sliding mode controller (SMC) followed the desired trajectory closer than the MPC. The controllers were originally generated for different objectives, where the main difference is that one is model-based and the other is model-free. MPC is an optimal controller suitable for the constrained method; on the other hand, SMC mainly used for an uncertain plant. SMC has also exhibited to be strong in precise tracking, robustness against disturbances and unpredicted inaccuracies. MPC has a computational complexity and requires significant effort to obtain the final result.

The simulation results in this thesis have shown that with further improvement, the MPC can provide a more reliable performance for obstacle avoidance for the given parameters considered in this study.

## 6. FUTURE WORK

The results of the simulations introduced the idea that there are still challenges in this model to explore and testing it with another popular method called fuzzy logic controller. For this future research, a combination of radar, lidar and camera can be implemented for testing real-time performance.

The lateral motion model was tested using Matlab/Simulink. In this research, the longitudinal velocity was constant and can be tested with changing variables in the future study. The most critical improvement to the controller tuning should be to minimize the time delay obtained in the response by further tuning the controller parameters.

In the future, more detailed scenarios with different constraints need to be considered. This can be done using an extended approach for dealing with disturbances from the measurement where the disturbance can be measured before it enters the system or could be rejected before entering the system.

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