The design of an adaptive optics telescope : the case of DAG

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ABSTRACT

In this paper, we describe in detail the optical design of DAG, a new 4 m telescope for Turkey. DAG is an "adaptive optics friendly" telescope, in a sense that each design decision is taken considering the potential impact on the AO performance (vibrations, static aberrations etc.) The objective is to make this telescope fully ready for AO at first light. It is designed as a Ritchey-Chrétien combination, 56 m focal length, with Nasmyth foci only, and active optics. Its total RMS error is expected to be 45 nm up to Zernike mode 78, and 26 nm for the higher, non AO corrected modes. A final design optimization has been done by the telescope manufacturers, demonstrating that our AO-based requirements can be satisfied, without much difficulty.

Keywords: DAG observatory ; Ritchey-Chrétien telescope ; adaptive optics ; active optics

1. INTRODUCTION

Turkish astronomers have initiated a project to build a 4 m telescope in Eastern Anatolia, near the City of Erzurum, on the Karakaya Tepeleri mountain, at 3170 m altitude. The name of the observatory is DAG, for Dogu Anadolu Gozlemevi, i.e Eastern Anatolia Observatory. The objective is to give national astronomers an easy way to conduct observations (large international telescopes are difficult to access), and build national expertise in astronomical instrumentation research and development. DAG will be a versatile telescope, dedicated to observations in the near infrared (NIR), and visible (VIS), over large fields in seeing limited mode, or medium to small field in adaptive optics (AO) mode.

AO is at the center of DAG optical design. We know that static aberrations and vibrations are the main optical error sources that are extremely difficult to track and mitigate in an existing instrument, and that these errors can be potentially very damaging for AO performance. Therefore, at each step of the design, choices were made in order to minimize the risk of occurrence of vibrations and static errors.

In this report, we discuss and justify all our design choices. The design start with a single parameter - the diameter of the primary mirror, which is fixed at 4 m. Then, we derive all the other parameters, using the AO and seeing limited observation modes requirements as constraints. We end up with a 56 m focal length, F/14 Ritchey-Chrétien configuration, with active optics (aO), a 14' scientific field-of-view (FoV) and a maximum 30' FoV with some vignetting allowed. The overall optical combination wavefront error is expected to be significantly lower than the AO wavefront residual.

The telescope optics and aO is done by the company AMOS *, and the alt-azimuthal mount is done by EIE †. AMOS did an optimization of our initial design, and essentially confirmed the feasibility of our requirements.

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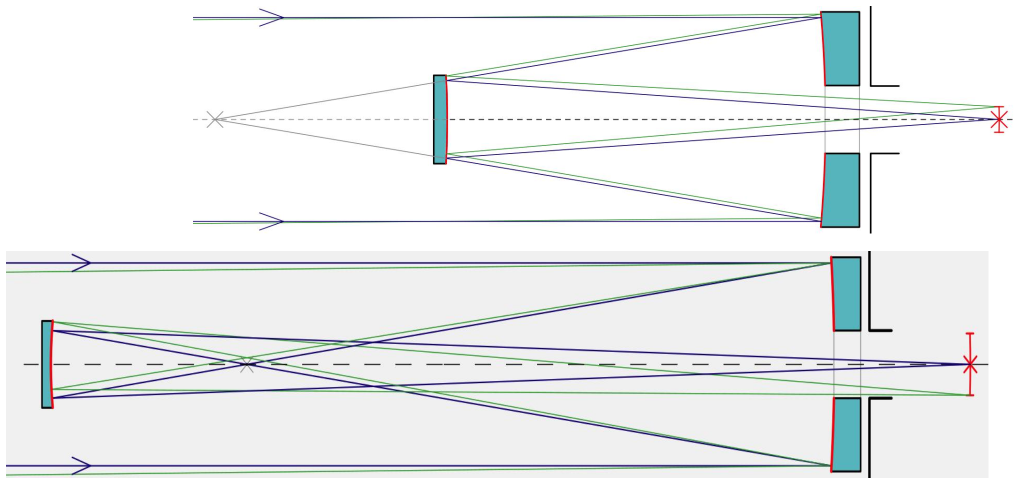


FIGURE 1. Top : Ritchey-Chrétien configuration. Bottom : Gregorian configuration. Both with a Cassegrain focus (not the case for DAG). Note that M1 focal length is the same in these two drawings. Images courtesy `wikipedia`.

2. THE BASIC DESIGN CHOICES

There are two fundamental aspects at the basis of the design : (1) in seeing and AO corrected modes, the limit to the optical performance must be set by the turbulence or the residual turbulence, and not by the telescope optics ; (2) image stability must be maximized.

2.1 Telescope primary mirror diameter and central obscuration

The goal is to build a telescope with a primary mirror 4000 mm in diameter. This choice is defined by the available observatory construction budget. Besides, one of the objective of this project is to gradually build knowledge and expertise in complex astronomical instruments in Turkey, and a medium size observatory is perfect to start.

A central obscuration (defined as the ratio of the secondary to the primary mirrors diameters) of 20 % generates a flux loss of 4 % or 0.04 magnitudes and a central obscuration of 30 % a flux loss of 10 % or 0.1 magnitudes. This is moderate, and most current telescopes have a central obscuration in the range 20–30 %. So we set the range 20–30 % as our target, with 20 % or less as a goal. In any case, a lower light flux input can be compensated with a longer exposure time.

2.2 Optical configuration : a Ritchey-Chrétien

DAG telescope will be a general purpose telescope, dedicated to both seeing limited and AO corrected observations with narrow to moderate field-of-view (FoV). Amongst the different two-mirrors designs developed in the history of telescope design (Schroeder[1]), the Ritchey-Chrétien (RC) configuration (Fig. 1) is by far the most common choice (Hubble Space Telescope, Keck telescopes, VLT at ESO, Gemini, Subaru, ...).

Indeed, its hyperbolic-hyperbolic two mirrors combination allows to cancel the off-axis coma as well as the spherical aberration. Such configurations are also called *aplanatic Cassegrain* telescopes. The typical un-aberrated FoV (limited by off-axis astigmatism and focal plane curvature) is in the range 10' to 20'.

The secondary mirror (M2) can be located at two positions along the optical axis : before or after the focal plane of the primary mirror (M1) - see Fig. 1. In the first case, M2 is a divergent mirror ; in the second case, M2 is convergent, and the configuration is called a *Gregorian.* The advantage of the Gregorian configuration is that M1 focal plane is available for an instrument or a calibration source, and is a popular design in solar observatories to put filters here. The disadvantage over the RC configuration is that it makes the telescope tube at least 50 % larger, requiring a larger dome, at extra costs. In our case, there are no strong reasons to get access to M1 focus, so we have selected a RC configuration.

2.3 Focal planes location : at the Nasmyth platforms

There are two places where the focal plane can be located on a RC : the Cassegrain focus, behind the primary (Fig. 1), and the two Nasmyth foci, on both sides of the telescope, aligned with the elevation axis (Fig. 2). Nasmyth foci have the enormous advantage of providing a gravity invariant environment. This is not the case for the Cassegrain focus, where gravity vector orientation variation and field derotation requires instrument support structures that minimize both flexures and vibrations. Unfortunately, instrumentation without vibration is almost impossible : there are always cooling pumps or cooling fans in instruments, or any sort of moving mechanisms, transmitting some level of vibrations, whose amplitude can quickly become incompatible with AO corrected, diffraction limited images.

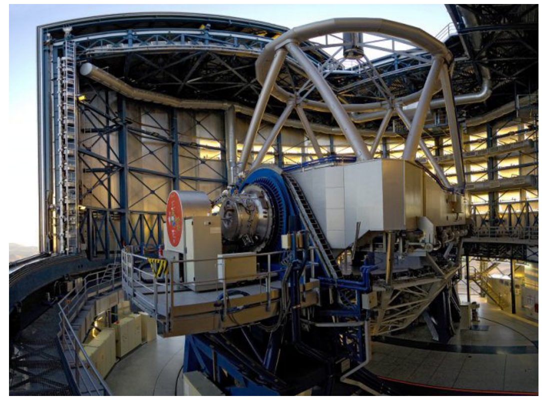


FIGURE 2. Very Large Telescope Nasmyth platform with instrumentation (Image ESO).

Each Nasmyth platform can house at least three instruments, feed with a selection mirror (M4) located at the platform entrance. A platform can be dedicated to large FoV seeing limited instruments, while the other is kept for narrower FoV, high angular resolution instruments (AO corrected). Now the Nasmyth option requires a flat tertiary mirror (M3) at the level of the elevation axis - on a rotation mechanism - to send the beam sideway. Also, Nasmyth platforms require a larger dome than with the Cassegrain configuration, but the later requires a longer telescope fork to make space under M1 (see Gemini telescopes design, for which there is no Nasmyth foci).

To conclude, Nasmyth foci, while requiring a more complex mechanical structure for the telescope (platforms), offer an environment where vibrations management is easier, and instrument design and servicing simpler, than with the Cassegrain focus option. Our choice goes for the Nasmyth option.

2.4 Back focal distance

The back focal distance is defined as the distance between the last reflecting surface of the telescope (here M3) and the Nasmyth focal planes, and is noted $\overline{\mathrm{M3F}}$. It must be equal to M1 radius (2000 mm), plus the telescope fork and elevation axis bearing width (about 1500 mm), plus a space for a derotator (K-mirror design, 500 mm), and a space for an optional field corrector lens system (500 mm). The total is therefore 4500 mm. Now, the derotator can fit into the central space of the elevation axis (a preliminary design has been done - see Baudet et al.,[2] this conference) so 500 mm can be removed from the budget.

Now, a removable calibration source must be located right at the focal plane of the telescope for instruments alignment and other calibration purposes, just after the optional field corrector. After the calibration source, there should be a flat mirror (M4) to send the beam in any of the Nasmyth platform instruments. A space of 200 mm is enough to handle these two pieces of equipment. As a consequence, the back focal distance is set to $\overline{\mathrm{M3F}} = 4200$ mm.

2.5 Tertiary mirror size and location

M3 is an elliptic mirror, located above the primary, inclined at an angle of 45° wrt the telescope optical axis. It is supported by a rotating mechanism, attached to M1, to redirect the beam to either of the two Nasmyth foci. Initial design of the telescope mount indicates that the height of M3 vertex cannot be smaller than 1000 mm. The diameter of M3, projected into M1 plane, must be of course smaller or equal to M2 support structure shadow diameter, but must be large enough to handle off-axis beams.

2.6 Telescope nominal focal length

DAG design must be done with the most demanding observation mode : adaptive optics (AO) corrected diffraction limited imaging, in the near infrared (NIR). Diffraction limited imaging in the visible (V band) is not considered because such a mode is beyond current technological AO capability, and if becomes the case in the future, there is no fundamental impossibility to handle wavefront errors at this level.

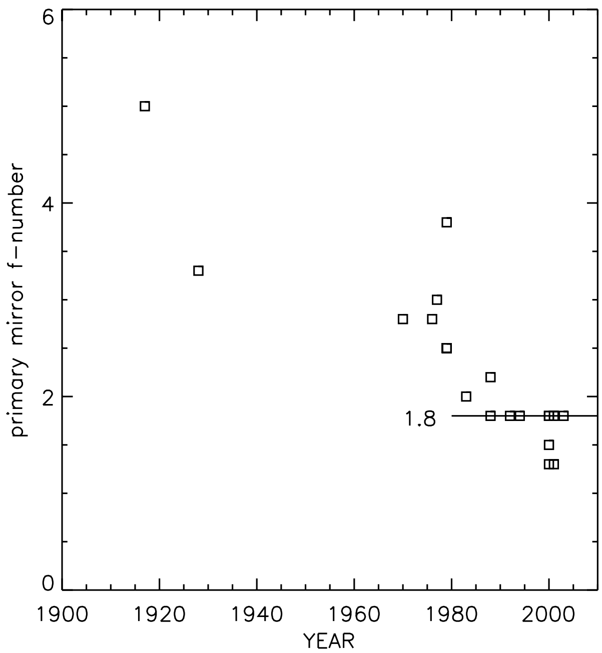


FIGURE 3. M1 f-# trend since 1900.

Current AO systems allow for diffraction limited imaging starting at 0.8 μm - and this is an optimistic value - for bright natural guide stars and excellent seeing (0.2”-0.3”). In the case of DAG, the median seeing is larger than this (about 0.7”), therefore we can expect the diffraction limit to be reached at 1 μm instead. Besides, current NIR detectors have pixel sizes in the range 27 μm (CONICA/VLT) to 7 μm (NIRC-2/KECK) - future detectors will essentially have higher sensitivity and possibly smaller pixel sizes, but not much smaller.

With the telescope diameter and the knowledge of the available NIR detectors pixel scales, we can now compute what must be the telescope focal length to make a diffraction limited PSF Nyquist sampled at 1 μm. With $D_1 = 4$ m, we find a PSF full-width-at-half-maximum (FWHM) of 0.25 μrad, or 51.7 masec, so the pixel size must be half this, i.e. $\Delta x_f = 0.125$ μrad or 25.8 masec. With F_T the telescope focal length, we have

$$F_T \text{ [m]} = \frac{\Delta x_{\text{pixel}} \ [\mu m] \times 206'264.81}{\Delta x_f \ [\text{"}] \times 10^6}$$

and we see that the telescope focal length is proportional to the camera pixel size. There are good reasons to keep the focal length not too long, in particular to keep the back focal length at a value compatible with the 4000 mm specified for the Nasmyth plane location. Therefore, it is reasonable to select for the pixel size the same 7 μm as for NIRC-2 on Keck (we know that this technology is available, and will certainly be available in the near future - or an equivalent one). With this pixel size, we find

$$F_T = \frac{7 \times 206'264.81}{25.8 \times 10^{-3} \times 10^6} = 56 \text{ m}$$

So, the telescope f-number is $F_T/D_T = 14$. This value is typical of 4 m class telescopes.

2.7 Telescope overall length, M1 f-number

Telescope tube length is essentially set in our case by the distance between the primary and the secondary mirrors, and must be limited to minimize the dome size. For 4 m class telescopes designs, tube lengths are in the range 5 to 8 m approximately, which is basically proportional to the focal length F_1. The shorter F_1, the smaller the dome but the more curved M1 surface (for a given diameter), therefore the more expensive the mirror machining.

As a consequence, for a given telescope, there is an optimal value of the primary f-number that minimize the overall cost of the observatory. But this optimal value is difficult to set, there are no models. This being said, since the year 1980 and the venue of modern, actively controlled mirrors shape (active optics, NTT at ESO being the precursor), no primary mirror has been build with a f-number larger than 2 (Fig. 3), and 1.8 is clearly the norm (van Belle[3]), even for 4 m class telescopes - and for a budget in accord with DAG’s allocated construction budget. It is therefore reasonable to chose a primary f-number of 1.8 as well. M1 focal length is then 7.2 m.

2.8 Imaging modes, geometrical field-of-view and detector size

Let us examine the required geometrical field-of-view (FoV) for the different imaging modes foreseen for DAG, ignoring off-axis aberrations for now. Later, we will compute the off-axis aberrations, and check if the required FoV is achievable or not, and for what level of image quality.

2.8.1 Single guide star diffraction limited AO mode

In single guide star AO (SGAO), the isoplanatic patch angular diameter is tiny and depends on the altitude of the dominant turbulent layers and the seeing angle (Roddier[4]),

$$\mathrm{FoV}_{\mathrm{SGAO}}\ ["] = \frac{6546}{\langle h \rangle\, \omega_0}$$

where ω_0 is the seeing angle in asec, and $\langle h \rangle$ the average altitude of the turbulent layers in m. In most locations except Antarctica, the isoplanatic patch is in the range 1" to 10". This corresponds, for a focal length of 56 m, to a linear size of the FoV of 0.27 mm to 2.7 mm, or about 40 to 400 pixels if the pixel size is 7 μm. Larger detectors size are easily achievable with current NIR technology.

2.8.2 Improved seeing mode - ground layer AO

The width of the field over which ground layer AO (GLAO, see Rigaut[5]) correction is efficient is on the order of several arc-minutes to several tens of arc-minutes. Once again, it strongly depends on the turbulent layers' altitude, for instance on the Mauna Kea summit (Hawai'i Island) the turbulence is very close to the ground and significant GLAO correction (PSF FWHM of 0.2") is achievable over 0.5° to 1° FoV (Chun et al.[6]).

In DAG case, as we do not know yet the structure of the turbulent profile, it is wise to assume a conservative value (working for most observatories) : a corrected field of 5' (10' at most) with a corrected GLAO PSF FWHM of about 0.2".

A PSF four times larger than SGAO means that we can have an angular pixel size Δx_f four times larger, therefore a focal length F_T four times shorter for the same linear pixel size (7 μm), i.e. 14 m. This focal length reduction can be achieved with optics added before the SGAO detector. An other alternative is to have a GLAO dedicated camera with a larger pixel size, (28 μm) on the same Nasmyth platform. In the first case a 5' (10') FoV would correspond to a linear width of 21 mm (42 mm), and in the second case to a linear width of 84 mm (168 mm). In both cases the detector would be about 3'000 (6'000) pixels wide. Such detectors are potentially available for a 5' FoV, but for 10' mosaics of detectors would be required.

2.8.3 Seeing limited imaging mode

This is technically the simplest case, with potentially the largest FoV and focal plane extension. Many nearby galactic clusters and close galaxies are in the range 1' to 10' in diameter, and for larger objects (Andromeda galaxy for instance), it is still possible to do a mosaic. Assuming a seeing angle of 0.5" in the visible (500 nm) (because the design has to be defined by the best cases and not the worst), and a Nyquist sampled PSF, a FoV of 10' would require a detector $10 \times 60"/0.25" = 2400$ pixels wide, and 7200 pixels for a 30' FoV. This number is independent of the telescope focal length as we can see : a shorter F_T would only make the linear extension of the FoV smaller, but the number of required pixels would remain the same. Note also that because the seeing angle is a slow function of the wavelength (Roddier[4]) these values would essentially be identical in the NIR domain.

Panoramic detectors can be designed for 3600 pixels using mosaics of sub-array (see for instance HAWK-I/VLT) but 7200 px arrays are more challenging. The seeing limited mode is therefore the mode that defines the maximum FoV. We will set the geometric FoV specification to 10' un-vignetted, and 30' un-vignetted as a goal.

2.9 Normalized parameters of the two mirrors telescope

Two mirror telescopes optical design is made simpler by the use of the four so-called *normalized parameters* (see Schroeder[1]), defined below,

$$\begin{array}{r|l} k & \text{ratio } D_2/D_1 \textbf{ when the telescope FoV is } \mathbf{0^\circ} \\ m_2 & \text{ratio } F_T/F_1\text{, M2 lateral magnification} \\ \rho & \text{ratio of the radius of curvature of the mirrors } R_2/R_1 \\ \beta & \text{back focal distance (M1 vertex to focal plane) in units of } F_1. \end{array}$$

and we have the following relations,

$$k = \frac{1+\beta}{1+m_2}, \quad m_2 = \frac{F_T}{F_1}, \quad \rho = \frac{m_2\, k}{m_2 - 1}, \quad \beta = \frac{1}{F_1}\left(\overline{\mathrm{M3F}} + \overline{\mathrm{M1M3}}\right)$$

2.10 Secondary mirror optical diameter and FoV

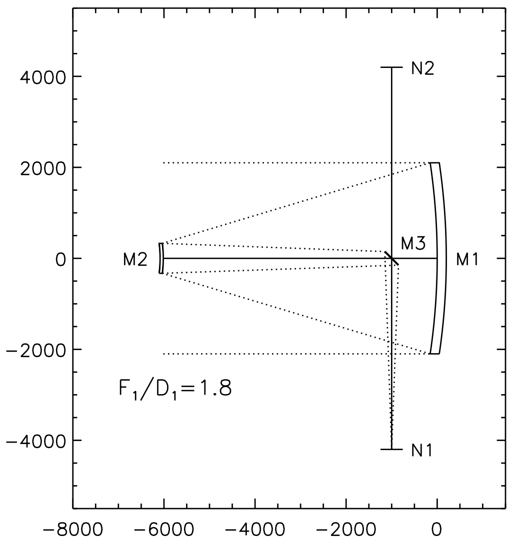


FIGURE 4. DAG nominal optical layout. All dimensions to scale.

Because marginal rays for an off-axis source should be reflected by M2, M2 diameter must be larger than the simple diameter of the M1-reflected beam at the level of M2, and must be equal, for a FoV of angular **diameter** $\theta_{\rm FoV}$, to

$$D_2 = k\, D_1 + \theta_{\rm FoV}\, F_1\, (1-k)$$

where D_1 and F_1 are the primary mirror diameter and focal length, and k is one of the normalized parameters.

2.11 Primary and secondary mirrors conic constants

In an aplanatic telescope, M1 and M2 conic constants [‡] are chosen in order to cancel both the spherical aberration and the 3rd order off-axis coma. Expressed in terms of the normalized parameters, the aplanatic conic constants are given by

$$K_1 = -\frac{2\,(1+\beta)}{m_2^2\,(m_2-\beta)} - 1$$

$$K_2 = -\left(\frac{m_2+1}{m_2-1}\right)^2 - \frac{2\,m_2\,(m_2+1)}{(m_2-\beta)\,(m_2-1)^3}$$

For a RC the conic constants are < -1 therefore both mirrors are hyperbolic.

3. TELESCOPE NOMINAL CONFIGURATION - SUMMARY

In table 1, we summarize our choices and the resulting optical prescriptions computed using the formulas above. The telescope optical layout is shown in Fig. 4.

TABLE 1. Telescope optics specifications, **lengths units are mm**, z-axis positive from sky to the ground.

Telescope	F_T	D_T	F_T/D_T			
	56’000	4’000	14			
Primary mirror	F_1	D_1	F_1/D_1	R_1	K_1	
	7’200	4’000	1.8	14’400	-1.006512	
Secondary mirror	F_2	D_2	F_2/D_2	R_2	K_2	D_2/D_1
(30’ FoV)	-1359.618	710.720	1.9	-2719.236	-1.737038	0.178
Mirrors separations	$\overline{\rm M1M2}$	$\overline{\rm M2M3}$	$\overline{\rm M1M3}$	$\overline{\rm M3F}$		
	-6015.190	5015.190	-1’000	4’200		
Normalized parameters	k	m_2	ρ	β		
	0.164557	7.777778	0.188836	0.444444		

‡ defined as $K = -e^2$ where e is the surface eccentricity

4. OFF-AXIS ABERRATIONS AND PRACTICAL FIELD-OF-VIEW

Off-axis aberrations and vignetting are the reasons why the FoV cannot be arbitrarily large, even if large detectors were available. Aplanatic RC telescopes have off-axis astigmatism, and focal plane field curvature, generating defocus on the field edge if the detector is flat.

4.1 Astigmatism

For a RC telescope, the angular radius of the geometric astigmatism spot, projected on the sky, is given by (Schroeder[1])

$$\text{AAS} \;\; [\text{rad}] = \left[\frac{m_2\,(2\,m_2+1)\,\beta}{2\,m_2\,(1+\beta)}\right]\frac{\theta^2}{2f_T}$$

where m_2 and β are the normalized parameters, θ the off-axis angle, and f_T the telescope f-number. Astigmatism increases with the square of the off-axis angle, so the FoV limitation can be very strong depending on the design and the aberration tolerancing. Astigmatism is therefore the first cause of FoV limitation on a RC telescope.

4.2 Focal plane curvature

The focal plane of a two mirror telescope cannot be flat (Schroeder[1]) and in first approximation the surface is a sphere. For an aplanatic configuration, the radius of curvature of the focal plane is given by

$$\mathcal{R}_c = \frac{R_1\,m_2^2\,(1+\beta)}{2\,(m_2+1)\,[m_2^2-\beta\,(m_2-1)]}$$

so the shorter R_1, the stronger the field curvature. Today, electronic detectors are flat, and it is not possible to bend them to fit the focal plane surface as it was done with photographic plates. As a consequence the beam on the field edge is defocused, and objects become blurry. Field curvature is the other case of FoV limitation for a RC telescope.

4.3 The practical FoV without field curvature correction

AMOS (Advanced Mechanical and Optical Systems) is the company in charge of the telescope manufacturing. Based on our prescriptions, they have optimized our telescope design considering a FoV of 10', which is our basis. With this design, and assuming a **flat focal plane**, the image performance degradation on the edge of the FoV has been computed using a ZEMAX model (without turbulence), at 1 μm, and is shown in table 2.

TABLE 2. Off-axis degradation of performance without field lens corrector (AMOS).

	center FoV	1' FoV	5' FoV	10' FoV
WFE (nm RMS)	0	4.5	113.4	453.7
spot radius μm	0	0.6838	17.1275	68.5168
Strehl ratio	1	0.9992	0.6017	0.0003

It is clear from this result that a field lens is absolutely required to compensate the focal plane curvature, unless the instruments can handle a curved detector, or the FoV is limited to a SGAO size. AMOS has demonstrated that with a 3-lenses field corrector (500 mm in diameter and 500 mm in length), image quality becomes essentially diffraction limited from 350 nm to 3 μm.

5. AMOS OPTIMIZED TELESCOPE DESIGN

TABLE 3. AMOS telescope optics final design, units are mm. M1 mechanical diameter is 4'000 mm.

Telescope	F_T	D_T (opt.)	F_T/D_T			
	56'000	3'940	14.2			
Primary mirror	F_1	D_1 (opt.)	F_1/D_1	R_1	K_1	
	7'210	3'940	1.83	14'420	-1.006574	
Secondary mirror	F_2	D_2 (opt.)	F_2/D_2	R_2	K_2	D_2 (mech.)
(10' FoV)	-1362.880	704	1.936	-2725.760	-1.738819	764
Mirrors separations	$\overline{\mathrm{M1M2}}$	$\overline{\mathrm{M2M3}}$	$\overline{\mathrm{M1M3}}$	$\overline{\mathrm{M3F}}$		
	-6022.591	5022.591	-1'000	4'200		
Normalized parameters	k	m_2	ρ	β		
	0.164689	7.766990	0.189026	0.443828		

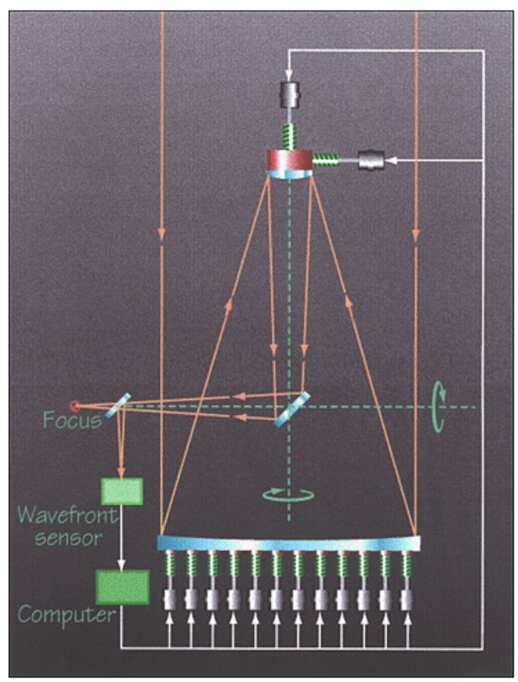
Besides, M3 elliptical diameters are 830-by-588 mm optical, 890-by-648 mechanical, and the focal plane radius of curvature is 1254.661 mm. The reason for not having an optical diameter of 4 m is that mirror polishing quality cannot be certified up to the mirror edge and a margin of 20 mm is considered. Overall, AMOS's design is very close to our specifications, differences are only marginal.

6. ACTIVE OPTICS, ADAPTIVE OPTICS AND THE WAVEFRONT ERROR BUDGET PRESCRIPTION

DAG telescope optics is designed for adaptive optics and seeing limited modes of observation. The primary mirror will be a thin meniscus, controlled in shape by an active optics system. The secondary and tertiary mirrors will be passive mirrors, controlled in position and angle. AO implies specific requirements for the mirrors manufacturing errors, studied and specified below.

6.1 Active optics

Since 1980, thanks to the introduction of the so-called active optics (aO) technology [§] in telescope design (see Noethe & Wilson[7]), primary mirrors thicknesses (and mass) have dramatically decreased. By controlling the mirror shape against gravity and thermal-induced deformation (see Fig. 5), aO has made it possible to significantly reduce mirrors thicknesses, down to values of a few tens of centimeters (VLT's primary mirror is 20 cm thick) [¶]. Before 1980, indeed, mirror stiffness was simply obtained by massive thickness - with the rule of thumb that the thickness needed to be at least 1/8th of its diameter. That would make 50 cm, and a mass of approximately 40 t for DAG's primary! VLT primary mirror is supported and controlled by 150 axial force actuators (in a so-called keynote configuration). The



(Drawing by Ed Janssen, ESO)

FIGURE 5. Active optics system (courtesy ESO).

[§] New Technology Telescope (NTT), ESO

[¶] http ://www.eso.org/paranal/telescopes/ut/actopt.html average inter-actuator separation is 58 cm. 16 Zernike modes can be corrected - and this is sufficient to cover all the gravity/thermal induced wavefront aberrations.

Now of course the thinner the mirror, the lighter it is, but at the cost of an increased flexibility which would require a larger number of modes to be compensated. In fact a quick geometrical calculation shows that the maximum number of Zernike modes that can be adjusted with N_{act} actuators over a **infinitely** flexible mirror is given by

$$j_{\max} = 2\,\frac{N_{\mathrm{act}}}{\pi} + 5\sqrt{\frac{N_{\mathrm{act}}}{\pi}} + 3 \tag{1}$$

i.e. 133 Zernike modes for the VLT. Now, if it is possible to imagine adaptive primaries in the not so distant future[‖], with the clear advantage of simplifying focal plane AO systems design, this cannot be a objective for DAG telescope : cost and technical risk would be unacceptably high.

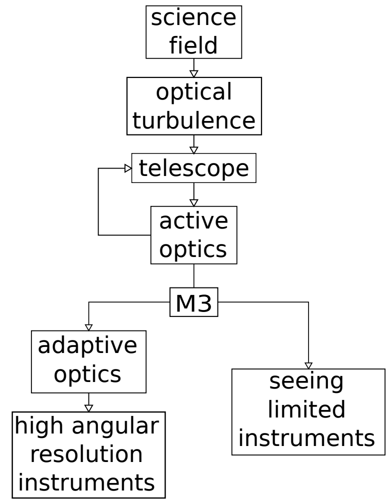


FIGURE 6. System's components from sky to instruments.

On the other hand, a stiff 50 cm and inactive primary mirror would be also expensive, extremely heavy, and manufacturing would take time (long cool off period), and is clearly not a solution anymore. In between the infinitely flexible (impossible) and the thick primary options, the best choice is a relatively thin mirror (a meniscus) hold and controlled by an aO system, working in closed loop on a guide star, the loop rate being on the order of a few seconds or minutes (loop frequency 0.01 Hz to 0.1 Hz) - enough to compensate the slow deformations induced by gravity and thermal changes. Our optical error budget will therefore be done assuming that an aO system is implemented.

6.2 Reference conditions for mirror manufacturing wavefront errors budgeting

There are several key aspects and facts at the root of the mirrors wavefront errors (WFE) budgeting :

- M1 will be a thin mirror actively controlled in shape ;
- M2 will be controlled in decentering, piston, and tip-tilt ; stiffness will be ensured by thickness ;
- M3 will be a flat, elliptical mirror, inclined at 45° with respect to the telescope optical axis ; stiffness will be ensured by thickness ;
- a ground layer adaptive optics system (GLAO) will be installed at one of the Nasmyth platform to feed science instruments with an improved seeing (0.2") over a moderate field of view (5') ;
- the GLAO system could be turned into a single natural guide star high angular resolution AO system, using the same deformable mirror and wavefront sensors ;
- seeing limited large FoV instruments (imagers, spectrographs) will be located at the other non-AO Nasmyth focus (port). aO will be active during seeing limited observation as well ;
- optical turbulence (OT) aberrations cannot be compensated by the telescope aO because turbulence time scale - on the order of 1 to 10 ms - is much faster than the aO system loop rate. In other words, OT aberrations will pass through the telescope without being affected at all.

As seen from the instruments focal plane, aberrations will be a combination of (1) OT aberrations (residual if AO is on), (2) aO-corrected telescope slow varying static aberrations and (3) instruments internal optics aberrations.

The AO system will be dimensioned to compensate for OT aberrations, but as residual telescope aberrations will be seen by the AO system (Fig. 6), these will naturally be compensated, up to the AO system's cutoff frequency. Now, because the AO system deformable mirror stroke is dedicated to correct OT, the telescope low order static aberration should be kept under a reasonable level, discussed below. Besides, telescope high order aberrations above the AO system's cutoff frequency will not be corrected at all. These high order static aberrations therefore have to be kept under a certain level, discussed below as well.

[‖]see recent progress in adaptive secondary mirrors at the Large Binocular Telescope (LBT)

6.3 Telescope wavefront error budget

An AO system is able to correct wavefront aberrations, into the entrance pupil, up to the AO cutoff spatial frequency given by

$$f_{\mathrm{AO}} = \frac{1}{2\,\Lambda_{\mathrm{AO}}} \quad [\mathrm{m}^{-1}] \tag{2}$$

where Λ_{AO} is the average distance between the actuators of the AO system's deformable mirror (DM), as seen from the entrance pupil plane. This spatial cutoff frequency separates the WFE into two components, the low frequency and high frequency aberrations. A preliminary study of DAG AO modes made us conclude that the best DM actuator pitch is, as seen from the entrance pupil plane, equal to 40 cm. Therefore, phase aberrations at scales larger than $2\Lambda_{\mathrm{AO}} = 80$ cm are reduced by the AO system, while structures at smaller scales cannot be corrected.

6.3.1 Low order wavefront error mirrors specification

Telescope residual static error will be seen by the AO system wavefront sensor (WFS), therefore the AO system will automatically try to compensate for these static aberrations by applying an offset to the DM commands. The DM mirror has a limited stroke, though, so an offset is acceptable only if it is significantly smaller than the DM stroke dedicated to the correction of OT.

In the case of DAG's telescope AO system, a pitch Λ_{AO} of 40 cm allows for about 80 actuators regularly spaced inside the 4 m entrance pupil. Assuming that the DM is flexible enough, it will be possible to compensate for all the Zernike polynomials up to index $j_{\max}(80) = 79$ or radial order $n = 11$ (Eq. 1) that we limit to 78 (last polynomial of the nearest radial order, $n = 11$).

Pre-dimensioning of the future AO system DM demonstrate that we require a stroke equal to 6 times the WFE RMS of the tilt corrected turbulent low order wavefront (including the Zernike polynomials Z_4 to Z_{78}), which is 5265 nm (both ways, so the the maximum required P2V is 10'530 nm (about 10 μm). This value ensures that the DM will be saturated less than 0.1 % of the time for a seeing of 1.5".

TABLE 4. Telescope Zernike modes specification for radial orders 2 to 11. The values indicate the total RMS per radial order (quadratic addition), for instance the 25 nm of radial order 2 are to be distributed amongst the two astigmatisms Z_5 and Z_6.

j, n-indexes	$\lvert a_j \rvert$ nm spec.	goal
5-6, 2	25	17
9-10, 3	19	13
11-15, 4	23	9
16-21, 5	19	7
22-28, 6	13	5
quadratic sum a_{29} to a_{78}		
29-78, 7-11	18	8
total error nm	49	26

Now if we attribute to the correction of the static aberrations a P2V equal to the RMS of the tilt corrected wavefront for a seeing of 1.5" (877 nm), the available stroke for OT correction still will make a saturation only 0.4 % of the time. This is still acceptable. Now an average P2V of 877 nm corresponds to a RMS of 124 nm. This means that in AO mode, we can safely accept a low order aberration of 124 nm RMS and P2V 877 nm, knowing that this aberration could be corrected by the AO system.

In seeing limited mode, V-band (500 nm), such a WFE would correspond to a Strehl drop of a factor $\exp -\sigma^2 = \exp -(124 \times 2 \times \pi/500)^2 = 0.08$! This is clearly not acceptable. **So, contrarily to what we would expect, the specifications for the low order aberrations are not set by the AO mode, but by the seeing limited mode**.

So, how much RMS error can we accept in this mode? Using the Maréchal relation again, and accepting a Strehl drop by a factor 0.9 (or 0.8), we get a maximum static RMS of 26 nm (or 38 nm). We therefore proposed to set the later value as the specification, and the former as the goal, and we also assumed a distribution of this error budget onto the Zernike coefficients similar to what is found in the turbulent phase, following Noll's[8] distribution, in order to be close to what the turbulence generates and not increasing the impact of a given aberration with respect to the other.

After discussion with AMOS, though, we found that using Noll's distribution was generating a requirement on the spherical aberration that was extremely hard to satisfy, and some margin was also required for the other higher order modes. We therefore relaxed our total low frequency budget to follow what the manufacturing was realistically able to do, and ended up with the specifications given in table 4. The Strehl drop would then be 0.7 at 500 nm, which is still acceptable in seeing limited mode.

This WFE budget is to be applied to the wavefront error inside the telescope entrance pupil, defined by the primary mirror. Indeed, the AO system DM will be conjugated to the telescope pupil, so it is critical to meet these specifications at this surface. Now, the way this budget is distributed amongst the three mirrors of the combination is left to the manufacturer, as it might be for instance easier to manufacture M3 to a tighter tolerance than M2. Also, these specifications have to be met regardless of the telescope elevation, from zenith down to 20° above horizon.

6.3.2 High order wavefront error mirrors specification

Let us now discuss the high order aberrations, i.e. the ones at spatial scales smaller than $2\Lambda_{\mathrm{AO}}$. These aberrations, generated by a polishing errors residuals and mirror supports print-through, cannot be corrected at all by the AO system. They are on the same spatial scale domain as what is called *fitting error* in the AO jargon.

Fitting error comes from uncorrected OT aberrations at scales under $2\Lambda_{\mathrm{AO}}$. It is well described using the so-called structure function (SF) tool, which is a measure of the mean square difference of the phase between two points in the pupil, separated by a vector $\mathbf{\Delta u}$

$$D_{\varphi}(\mathbf{\Delta u}) \equiv \left\langle \left[\varphi(\mathbf{\Delta u}+\mathbf{r}) - \varphi(\mathbf{r})\right]^2 \right\rangle_{\mathbf{r}} \tag{3}$$

where the average is taken over each point inside the entrance pupil. Zernike polynomials are indeed not practical to describe the high order aberrations, as an infinite number of coefficients would be required : the SF approach is preferred here. Also, at these spatial scales, it is particularly critical to avoid high amplitude of the mirror support structure print through errors, because as it would not be corrected, structures would appears in the PSF wings as satellite speckles, and worse, these errors could even interfer with the AO WFS aliasing error, potentially damaging the AO performance very significantly.

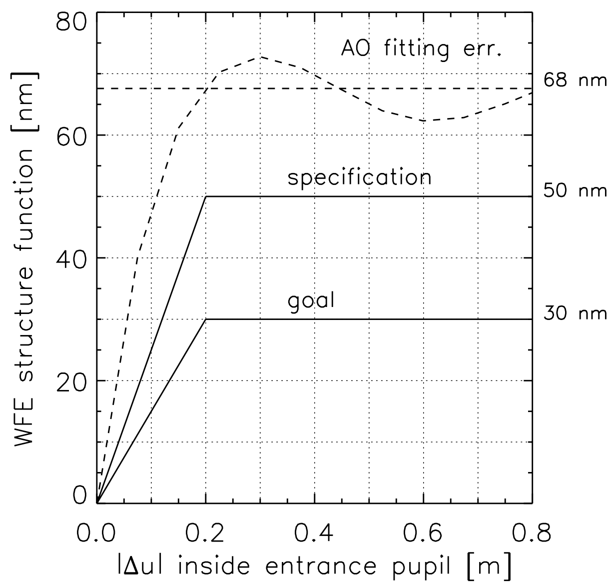


FIGURE 7. Specification and goal for the structure function wavefront error of the three mirrors combination, at high spatial frequencies, in the range 0 to 0.8 m. Dash line shows the AO fitting error SF, for comparison. Note that what is actually shown here is the square root of half of the phase structure function, expressed in wavefront error.

When the AO system is working in favorable conditions, i.e. when the guide star is bright and the wind is weak, the performance of the system is essentially limited by the fitting error (WFS aliasing is another error but lower than the fitting error, and in this context can be ignored). As a consequence, it is important that the high order mirror error stays at a lower level than the residual uncorrected OT aberrations. If we consider the favorable case of a 0.5" seeing, and a DM pitch of 0.4 m, we get a structure function fitting error with an RMS of 68 nm (computed from a method described in Jolissaint[9]) - see Fig. 7.

What we require is that the Strehl decrease due to the high order telescope error should be no more than 0.85 (goal 0.95) at 800 nm, which is the shortest AO wavelength we are probably going to use for a long time. These Strehl specifications translate, using Maréchal's law, into 50 nm and 30 nm (goal) WFE. To summarize :

1. the AO corrected Strehl ratio in good observing conditions, when including high order telescope errors, shall not be lower than 0.85 (goal 0.95) times the fitting error Strehl at 800 nm ; this translates according to Maréchal's law into 50 nm (goal 30 nm) for the high order telescope errors ;
2. the telescope high order errors **statistical average** (RMS) distribution for each spatial scales under $2\Lambda_{\mathrm{AO}}$ shall be limited to the high order mirror error structure function described in Fig. 7 - this is order to avoid deleterious effects of the mirrors PT errors (which are unavoidable) ;

3. for each spatial scale under $2\Lambda_{\mathrm{AO}}$, the peak-to-valley optical path difference (P2V OPD) shall be not more than **four times** the specified WFE structure function value given in Fig. 7 ;
4. high order telescope errors shall not create systematic structures (on top of the usual diffraction rings) that would prevent an efficient reduction of photometric and astrometry data : in other words, there should be no satellites dots in the PSF core that could be interpreted as faint companions.

The SF adds in quadrature from a surface to another. The mirror manufacturer can distribute the SF specification between the three mirrors of the combination according to what is deemed convenient.

Because the high order WFE of the mirrors is in principle isotropic and homogeneous over the entrance pupil, the specifications/goal given in Fig. 7 have to be understood as independent of the orientation of the separation vector $\Delta\mathbf{u}$: in other words the manufacturer has to make sure that the SF spec/goal is isotropically and homogeneously satisfied, i.e. identical for all possible orientations of $\Delta\mathbf{u}$ and for all positions $\mathbf{r}$ in the entrance pupil.

6.4 AMOS final design wavefront error budget

AMOS has a long experience in the design of active optics systems. After a detailed analysis, they delivered the following estimation of what could be achieved, on a real implementation of an aO system, considering manufacturing and support residual errors after aO correction :

6.4.1 Low order wavefront error

Table 5. Low order telescope WFE budget, units nm.

	Z_{5-6}	Z_{7-10}	Z_{11-15}	Z_{16-21}	Z_{22-28}	Z_{29-78}	total
expected WFE	19	17	23	19	12	18	45
requirement	25	19	23	19	13	18	49
margin	16	7	3	2	4	3	19

6.4.2 High order wavefront error

Concerning the high order error, the overall WFE structure function is well inside the allocated envelope, with a maximum of 28 nm at a separation of 250 mm, and a plateau around 26 nm beyond 300 mm separation. Essentially, our goal specification is reached. The margin in high order error is therefore comfortable, at about 43 nm (quadratically).

7. CONCLUSIONS

We have given in this paper all the steps for the design of an adaptive optics based astronomical telescope, considering the basic constraints set by high angular resolution observations. This design method has been applied on DAG, the future Turkish 4 m telescope. Manufacturer's final analysis has validated our approach in a sense that the telescope wavefront error specifications have been demonstrated to be realistic. The conclusion is that with current technology, including active optics control, the conception of a medium size adaptive optics friendly telescope is not a challenge.

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