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ASYMPTOTIC SOLUTIONS OF LOVE WAVE PROPAGATION IN A COVERED HALF-SPACE WITH INHOMOGENEOUS INITIAL STRESSES G_3^1

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Abstract. The dispersive behavior of Love waves in an elastic half-space substrate covered by an elastic layer under the effect of inhomogeneous initial stresses has been investigated. Classical linearized theory of elastic waves in initially stressed bodies for small deformations is used and the well-known WKB high-frequency asymptotic technique is applied for the theoretical derivations. Numerical results on the action of the influence of the initial stresses on the wave propagation velocity for a geophysical example are presented and discussed.

Keywords: Love wave, dispersion, inhomogeneous initial stresses, WKB.

AMS Subject Classification: 74J15.

1. Introduction

Surface waves play an important role in geophysical studies for site characterization, determination of shear wave velocity profiles, damping ratios, fault detection and study of the earthquakes. On the other hand, acoustic surface waves also have enormous applications in material sciences, electronic devises, non-destructive testing and damage detection etc. An important issue in the study of this type of elastodynamics problems is the study of the effect of initial stresses on the wave propagation characteristics. Initial stresses in Earth's crustal layers might occur under the action of geostatic and geodynamic forces, for example as a result of difference of temperature, slow process of creep, differential external forces, gravity variations and in composite materials or structural elements during their manufacturing or assembling processes. It also important to note that the stress magnitudes in the Earth's crust are not homogeneous throughout the crustal layers and show linear increase with depth. These stresses have a profound influence on the propagation of surface waves.

Propagation of seismic waves is a complicated process and analytical solutions of the elastodynamics equations in general types of media cannot be solved exactly and usually numerical methods must be used. High-frequency asymptotic theory is another alternative and powerful technique introduced to solve these kind of equations approximately.

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The high-frequency asymptotic methods are presented in the form of the ray series, for this reason, the ray method is also often called the ray series method or the Asymptotic Ray Theory (ART). In the present study, dispersive behavior of surface Love waves in a half-space substrate covered by a layer, which are assumed to be linear and elastic, under the effect of linearly varying initial stress patterns is investigated. Classical linear elasticity theory with small initial deformations is applied and the Wentzel-Kramers-Brillouin (WKB) high-frequency asymptotic technique is adopted for the theoretical derivations.

Seismic ray theory has been described elaborately in several text books, for example, Babich and Buldyrev [1], Cerveny [4], Chapman [5]. Number of studies which applied the high-frequency asymptotic approximation to wave propagation problems are enormous and here we will present a few of recent studies in the direction of our study purpose. Li et al. [7] studied the propagation behaviors of Love waves in inhomogeneous medium using WKB method and obtained the dispersion relations of Love waves for different gradient variation of material constants. Jin et al. [6] also applied the WKB method to solve the Rayleigh surface wave propagation in a homogeneous isotropic elastic structures with curved surfaces of arbitrary form. Liu et al. [8] on the basis of WKB method derived the dispersion equations for Love wave propagation in layered graded composites structures using the shear spring model for the rigid, imperfect, and slip interface cases. Cao et al. [3] employed the WKB technique for the asymptotic solutions of propagation of Rayleigh surface waves in a transversely isotropic graded piezoelectric half-space when material properties varying continuously along depth direction. Liu et al. [9] obtained the asymptotic solutions of Love waves by applying the WKB method and solved the fourth order differential equation with variable coefficients to investigate the effects of gradient variations of the piezoelectric and dielectric constants. Qian et al. [10], [11] investigated the existence and propagation behavior of transverse surface waves in a layered structure concerning a gradient metal layer by WKB method and obtained the dispersion equation for such structures. Balogun and Achenbach [2] studied the surface waves generated by a time-harmonic line load on an isotropic linearly elastic half-space whose elastic moduli and mass density vary with the depth direction.

2. Formulation of the problem

Consider an elastic half-space covered by an elastic layer with thickness h as shown in Figure 1. Here we determine the positions of the points by the Cartesian system of coordinates $Oxyz$ with O being any point on the free surface. The layer and the half-space occupy the regions $\{-\infty < x < +\infty, -\infty < y < +\infty, 0 \leq z \leq h\}$ and $\{-\infty < x < +\infty, -\infty < y < +\infty, h \leq z < +\infty\}$, respectively. We assume that Love wave propagate in the positive direction of x axis. Thus, the displacement component v along y direction is non-zero while the displacement components along x and z directions, u and w both are zero, i.e., $u = w = 0$, $v = v(x, z, t)$. Let the system be under initial compressive stress σ_z^0 along z direction and compressive or tensile initial stress σ_x^0 along x direction, respectively. The initial compressive stress σ_z^0 may be due to weight of the material of the layer and the half-space or some other external loads. However, the initial stress σ_x^0 might have been generated through other processes such as creep, temperature difference or some external forces. Note that the following notation will be used through the formulations:

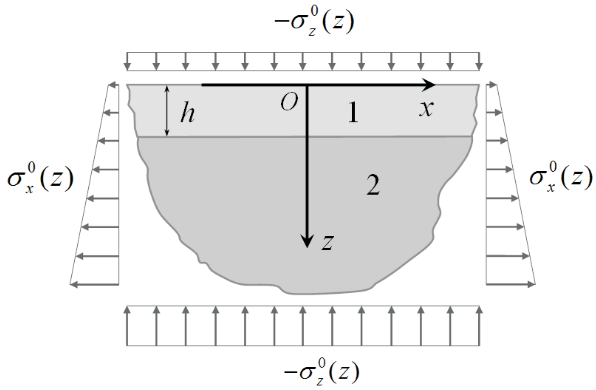

FIGURE 1. The geometry of the problem.

The values related with the covering layer and the half-space are denoted by upper indices 1 and 2, respectively. The values related to the initial stresses, though, are denoted by upper index 0. The dynamical equations of motion for initially stressed incompressible medium assuming small initial deformations are written as:

$$\frac{\partial \sigma_{xy}^{(m)}}{\partial x} + \frac{\partial \sigma_{yz}^{(m)}}{\partial z} + \sigma_x^0 \frac{\partial^2 v^{(m)}}{\partial x^2} + \sigma_z^0 \frac{\partial^2 v^{(m)}}{\partial z^2} = \rho^{(m)} \frac{\partial^2 v^{(m)}}{\partial t^2}, \quad (m = 1, 2), \tag{1}$$

where $\sigma_{xy}^{(m)}$ and $\sigma_{yz}^{(m)}$ are components of the Cauchy stress tensor, $v^{(m)}$ are the components of the displacement vector along y direction, $\rho^{(m)}$ are the mass density of the layer and the half-space, respectively. Note that, constitutive relations for a linear isotropic elastic solid are given by:

$$\sigma_{xy}^{(m)} = \mu^{(m)} \frac{\partial v^{(m)}}{\partial x}, \quad \sigma_{yz}^{(m)} = \mu^{(m)} \frac{\partial v^{(m)}}{\partial z}, \quad (m = 1, 2), \tag{2}$$

where $\mu^{(m)}$ is the shear modulus or Lame's second parameter. The strain components in equation (2) can be calculated through the following formula:

$$\varepsilon_x^{(m)} = \frac{\partial u^{(m)}}{\partial x}, \quad \varepsilon_z^{(m)} = \frac{\partial w^{(m)}}{\partial z}, \quad \varepsilon_{zx}^{(m)} = \frac{1}{2}\left(\frac{\partial u^{(m)}}{\partial z} + \frac{\partial w^{(m)}}{\partial x}\right), \quad (m = 1, 2). \tag{3}$$

The displacements components of the considered system can be assumed to have the following form:

$$v^{(m)}(x, z, t) = V^{(m)}(z)\, e^{ik(x-ct)}, \quad (m = 1, 2), \tag{4}$$

where k is the wavenumber, c the phase velocity of wave propagation, $i = \sqrt{-1}$, $V^{(1)}(z)$ and $V^{(2)}(z)$ are two undetermined functions with respect to z coordinate only. This way we obtain the following equation for $V^{(1)}(z)$ and $V^{(2)}(z)$ from the equation (1)-(4) as:

$$\frac{\mathrm{d}^2 V^{(1)}(z)}{\mathrm{d}z^2} + k^2 q^{(1)}(z)\, V^{(1)}(z) = 0, \tag{5}$$

$$\frac{\mathrm{d}^2 V^{(2)}(z)}{\mathrm{d}z^2} - k^2 q^{(2)}(z)\, V^{(2)}(z) = 0, \tag{6}$$

where

$$q^{(1)}(z) = \frac{-\mu^{(1)} - \sigma_x^0(z) + \rho^{(1)} c^2}{\mu^{(1)} + \sigma_z^0(z)}, \tag{7}$$

$$q^{(2)}(z) = \frac{\mu^{(2)} + \sigma_x^0(z) - \rho^{(2)} c^2}{\mu^{(2)} + \sigma_z^0(z)}. \tag{8}$$

Equations (5) and (6) are second order differential equation with variable coefficients and in general obtaining the exact solution of the problem is very difficult. However, for high-frequency waves whose wavenumber is very large, i.e., $k >> 1$, the WKB asymptotic approximation method can be applied to obtain approximate solution of the problem. Thus by assuming that k is a large number then, $\varepsilon(= 1/k << 1)$ will be a very small number and the equations (5) and (6) in general form can be recast as:

$$\varepsilon^2 \frac{\mathrm{d}^2 V(z)}{\mathrm{d} z^2} + q(z)\, V(z) = 0. \tag{9}$$

We are looking for the solution of equation (9) in the following form:

$$V(z) \sim e^{i\phi(z)/\varepsilon} \sum_{n=0}^{\infty} \varepsilon^n S_n(z) \tag{10}$$

where ϕ and $S_i, i = 1, 2, 3, ...$ are undetermined functions of z. Substituting equation (10) into (9) and equating the coefficients of each power of ε to zero, we get an infinite number of equations:

$$\phi'^2 S_0 - q S_0 = 0, \tag{11}$$

$$\phi'' S_0 + 2\phi' S_0' = 0, \tag{12}$$

$$S_0'' + i\phi'' S_1 + 2i\phi' S_1' = 0, \tag{13}$$

$$...$$

where superscript $'$ denotes differentiation with respect to the coordinate z. Equation (11) is a first order nonlinear differential equation and is called the *eikonal* equation. Its solutions can be find easily:

$$\phi = \pm \int \sqrt{q(z)}\, dz. \tag{14}$$

The other equations are linear and determine the higher order terms in the expansion. The second equation is called the *transport* equation, then we got the following expression for S_0 from equation (12):

$$S_0 = q^{-1/4}. \tag{15}$$

We have therefore found that a first-term approximation of the general solution of equations (5) and (6) are:

$$v^{(1)}(x, z, t) \sim q^{(1)}(z)^{-1/4} \left(A_1\, e^{-ik \int \sqrt{q^{(1)}(z)} dz} + B_1\, e^{ik \int \sqrt{q^{(1)}(z)} dz} \right) e^{ik(x-ct)}, \tag{16}$$

$$v^{(2)}(x, z, t) \sim q^{(2)}(z)^{-1/4} \left(A_2\, e^{-k \int \sqrt{q^{(2)}(z)} dz} \right) e^{ik(x-ct)}, \tag{17}$$

where A_1, B_1 and A_2 are arbitrary constants. Note that, the solution of the second equation in (17) satisfies the decay condition i.e., $V^{(2)}(z) \to 0$, $z \to \infty$. We assume that the following boundary conditions on the free face plane of the covering layer and on the interface plane between the covering layer and the half-space are satisfied:

$$\left.\sigma_{yz}^{(1)}\right|_{z=0} = 0, \tag{18}$$

$$\left.\sigma_{yz}^{(1)}\right|_{z=h} = \left.\sigma_{yz}^{(2)}\right|_{z=h}, \tag{19}$$

$$\left.v^{(1)}\right|_{z=h} = \left.v^{(2)}\right|_{z=h}. \tag{20}$$

Substituting of the equations (16) and (17) and their corresponding stress displacement components into the equations of motion (1) and considering the boundary conditions (18)-(20) yields the system of three homogenous algebraic equations for A_1, B_1 and A_2. For a nontrivial solution the determinant of the coefficients must vanish giving the dispersion equation of Love wave propagation,

$$\det \|\alpha_{ij}\| = 0, \quad i,j = 1,2,3. \tag{21}$$

This completes the formulation of the problem and in the case where $\sigma_x^0 = \sigma_z^0 = 0$ this formulation transforms to the corresponding one made within the scope of the classical linear theory of elastodynamics.

For explicit expressions of dispersion equation (21) and consequently obtaining related dispersion curves we need to determine the variation pattern of the initial stresses in the system. It is known that approximately initial stress magnitudes in the Earths crust show a linear increase with depth (Zang and Stephansson [12]). Accordingly, here we also assume that the initial stresses vary linearly with depth and the variation pattern of the initial stresses in both normal and transverse directions are taken as the following relations:

$$\sigma_x^0(z) = \sigma_x^0 \cdot (1 + mz), \tag{22}$$

$$\sigma_z^0(z) = \sigma_z^0 \cdot (1 + nz), \tag{23}$$

where σ_x^0 and σ_z^0 denote the magnitudes and m, n denote the gradient coefficients of the inhomogeneous initial stresses in the transverse Ox and in the normal Oz directions, respectively. Substituting these equations into the equations (7) and (8), respectively, we got:

$$q^{(1)}(z) = \frac{-\mu^{(1)} - \sigma_x^0 \cdot (1 + mz) + \rho^{(1)} c^2}{\mu^{(1)} + \sigma_z^0 \cdot (1 + nz)}, \tag{24}$$

$$q^{(2)}(z) = \frac{\mu^{(2)} + \sigma_x^0 \cdot (1 + mz) - \rho^{(2)} c^2}{\mu^{(2)} + \sigma_z^0 \cdot (1 + nz)}. \tag{25}$$

To obtain the displacement components in the covering layer and in the half-space as given by equations (16) and (17) we have to integrate $\sqrt{q^{(1)}(z)}$ and $\sqrt{q^{(2)}(z)}$. However, without loss of generality, we assume that $q(z) = \frac{a\,z+b}{c\,z+d}$ to simplify the calculations, and integrate it in the following general form:

$$\int \sqrt{\frac{a\,z+b}{c\,z+d}} = \frac{\sqrt{a^2d^2 - 2abcd + b^2c^2}}{\sqrt{-ac^3}} \cdot \arctan\left(\frac{c^2\sqrt{q(z)}\sqrt{a^2d^2 - 2abcd + b^2c^2}}{\sqrt{-ac^3}\,(ad - bc)}\right)$$
$$+ \sqrt{\frac{c}{a}}\,(ad - bc)\left(\sqrt{a}\,c^{3/2} - \frac{c^{5/2}\,(az + b)}{\sqrt{a}\,(cz + d)}\right)^{-1}\sqrt{q(z)},$$

where the expressions for the parameters a, b, c and d in the case of $q^{(1)}(z)$ and $q^{(2)}(z)$ are given by:

$$q^{(1)}(z):\begin{cases} a=-\sigma_x^0\, m \\ b=-\mu^{(1)}-\sigma_x^0+\rho^{(1)}c^2 \\ c=\sigma_z^0\, n \\ d=\mu^{(1)}+\sigma_z^0 \end{cases},\quad q^{(2)}(z):\begin{cases} a=\sigma_x^0\, m \\ b=\mu^{(2)}+\sigma_x^0-\rho^{(2)}c^2 \\ c=\sigma_z^0\, n \\ d=\mu^{(2)}+\sigma_z^0 \end{cases}.$$

Inserting these results to the displacement components in the covering layer and the half-space, equations (16) and (17), and considering the boundary conditions, equations (18)-(20), yields the system of three homogenous algebraic equations as discussed in the earlier sections. For a nontrivial solution the determinant of the coefficients must vanish giving the dispersion equation of Love wave propagation. Since the expressions for the components of the matrix of the corresponding dispersion determinant are cumbersome we are omitting here these details. The explicit expressions of the α_{ij} in the dispersion equation (21) for two typical cases when ($\sigma_x^0(z)=mz$, $\sigma_z^0=0$) and ($\sigma_x^0=0$, $\sigma_z^0(z)=nz$) as special cases are presented in the Appendix A.

3. Numerical results

Now we perform numerical calculations to study the quantitative and qualitative influence of initial stresses on dispersion of Love wave propagation. In the following numerical example, we assume that $\rho^{(1)}=2800$ kg/m^3, $\beta^{(1)}=3000$ m/s and $\rho^{(2)}=3200$ kg/m^3, $\beta^{(2)}=5000$ m/s and $h=10$ km; where $\rho^{(1)}$, $\rho^{(2)}$ are the mass density and $\beta^{(1)}$, $\beta^{(2)}$ are the shear wave velocities in the layer and the half-space, respectively, and h is the thickness of the crustal layer. As discussed above, in the Earth's crust the lithostatic stress magnitudes increases linearly with depth, therefore we assume that the gradient coefficients of the inhomogeneous initial stresses in the transverse and in the normal directions are $m=9000$ MPa/km and $n=27000$ MPa/km, respectively. We considered different possible combinations of the initial stresses in the system as the following cases:

Case 1. $\sigma_{\mathrm{x}}^0>0$, $\sigma_{\mathrm{z}}=0$;
Case 2. $\sigma_{\mathrm{x}}^0<0$, $\sigma_{\mathrm{z}}=0$;
Case 3. $\sigma_{\mathrm{x}}^0=0$, $\sigma_{\mathrm{z}}<0$;
Case 4. $\sigma_{\mathrm{x}}^0>0$, $\sigma_{\mathrm{z}}<0$;
Case 5. $\sigma_{\mathrm{x}}^0<0$, $\sigma_{\mathrm{z}}<0$.

Here we will present only the graphs obtained for the case 1 and case 4. Figure 2 and Figure 3 show the dispersion curves for the first four modes of Love wave propagation in this example for the initial stress cases 1 and 4, respectively. Note that each curve in the graphs is obtained for different values of the initial stresses σ_x^0 and σ_z^0 as indicated in the figures. Dispersion curves related to the free stress cases and constant initial stress cases are also given in these figures for comparison. Figure 2 shows that the initial stretching stresses in Ox direction in the covering layer and in the half-space causes to increase the wave propagation velocity for the first four mode of the propagation. According to the graphs which are not given here as a result of the initial compression in Oz direction (case 3) the wave propagation velocity for the first and second modes of the wave propagation

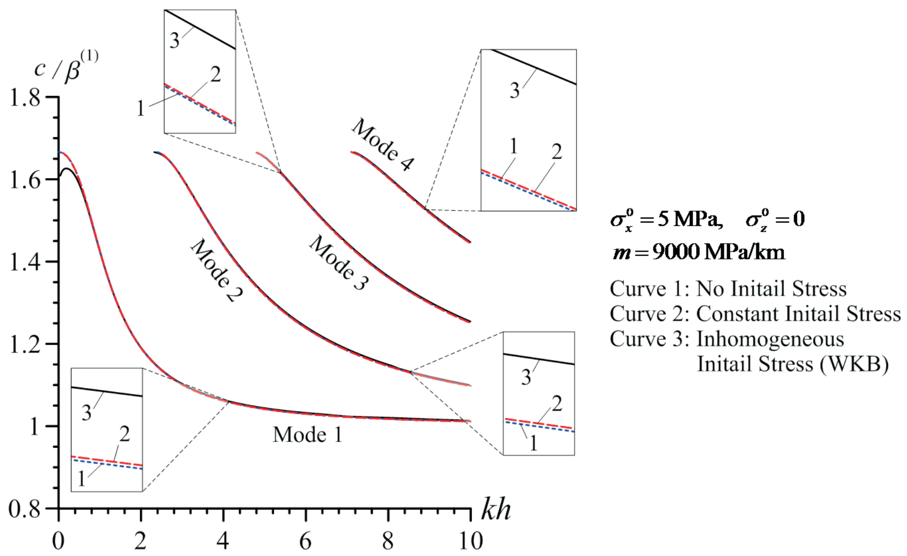

Figure 2. The influence of the inhomogeneous initial stress on the dispersion curves for the first four mode of Love wave propagation for Case 1.

also increases. However, the results indicate that existence of initial stretching in Ox direction (case 2 and case 5) causes to decrease the wave propagation velocity for the first and the second modes of the propagation. Figure 3 shows that as a result of initial stress pattern case 4, the wave propagation velocity also increases for the first and the second modes of the propagation. It is also important to note that the effect of inhomogeneous initial stresses on the dispersion of Love waves are almost similar to the constant initial stresses qualitatively for the first two modes of the the propagation, but with more intensive magnitudes.

4. Conclusion

In this study we exploited WKB high-frequency analysis to solve seismic Love wave dispersion problem under the effect of different inhomogeneous initial stress patterns. Theoretical derivations are obtained in the framework of classical linearized theory of elastic waves in initially stressed bodies for small deformations and numerical examples are given and discussed. The results indicate that depending on the initial stress pattern the wave propagation velocity might be increased or decreased, however, the effect of inhomogeneous initial stresses are more significant in comparison to the constant ones.

Appendix A.
The expressions of the components α_{ij}

1. For the case when $\sigma_x^0(z) = mz$, $\sigma_z^0 = 0$:

$$\alpha_{11} = -k\exp\left(\frac{k\mu_1\,\xi_1^{\frac{3}{2}}\,2i}{3m\sigma_x^0}\right)\xi_1^{\frac{1}{4}}\,i + \frac{m\exp\left(\frac{k\mu_1\,\xi_1^{\frac{3}{2}}\,2i}{3m\sigma_x^0}\right)\sigma_x^0}{4\mu_1\,\xi_1^{\frac{5}{4}}},$$

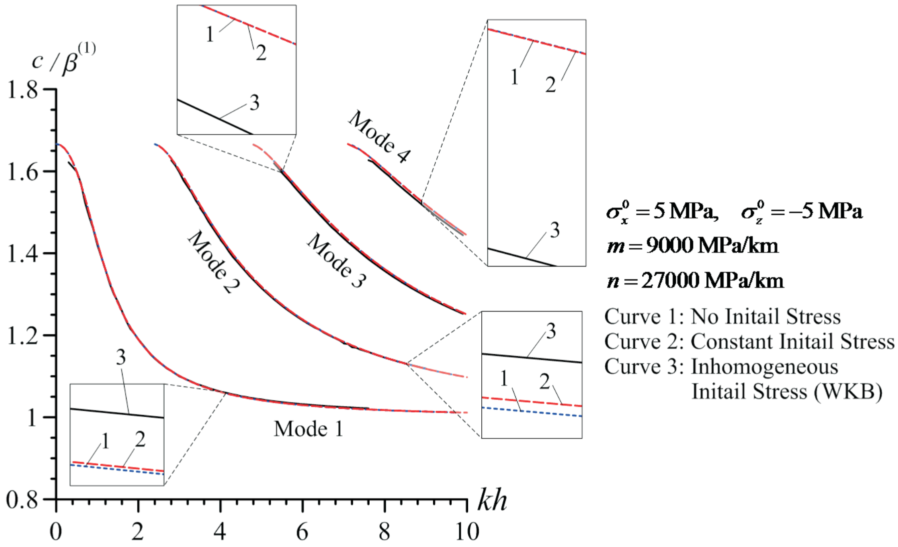

FIGURE 3. The influence of the inhomogeneous initial stress on the dispersion curves for the first four mode of Love wave propagation for Case 4.

$$\alpha_{12} = \frac{k\xi_1^{\frac{1}{4}}\, i}{\exp\left(\frac{k\mu_1\,\xi_1^{\frac{3}{2}}\,2i}{3m\sigma_x^0}\right)} + \frac{m\,\sigma_x^0}{4\mu_1\,\xi_1^{\frac{5}{4}}\exp\left(\frac{k\mu_1\,\xi_1^{\frac{3}{2}}\,2i}{3m\sigma_x^0}\right)}, \qquad \alpha_{13} = 0,$$

$$\alpha_{21} = -\mu_1 k \exp\left(\frac{k\,\xi_2^{\frac{3}{2}}\,\mu_1\,2i}{3m\sigma_x^0}\right)\xi_2^{\frac{1}{4}}\, i + \frac{m\exp\left(\frac{k\,\xi_2^{\frac{3}{2}}\,\mu_1\,2i}{3m\sigma_x^0}\right)\sigma_x^0}{4\xi_2^{\frac{5}{4}}},$$

$$\alpha_{22} = \frac{k\mu_1\,\xi_2^{\frac{1}{4}}\, i}{\exp\left(\frac{k\,\xi_2^{\frac{3}{2}}\,\mu_1\,2i}{3m\sigma_x^0}\right)} + \frac{m\sigma_x^0}{4\xi_2^{\frac{5}{4}}\exp\left(\frac{k\,\xi_2^{\frac{3}{2}}\,\mu_1\,2i}{3m\sigma_x^0}\right)}, \qquad \alpha_{23} = \frac{k\mu_2\,\xi_3^{\frac{1}{4}}}{\exp\left(\frac{2k\,\xi_3^{\frac{3}{2}}\,\mu_2}{3m\sigma_x^0}\right)} + \frac{m\sigma_x^0\,\xi_3^{-\frac{5}{4}}}{4\exp\left(\frac{2k\,\xi_3^{\frac{3}{2}}\,\mu_2}{3m\sigma_x^0}\right)},$$

$$\alpha_{31} = \frac{\exp\left(\frac{k\,\xi_2^{\frac{3}{2}}\,\mu_1\,2i}{3m\sigma_x^0}\right)}{\xi_2^{\frac{1}{4}}}, \qquad \alpha_{32} = \frac{\left(\exp\left(\frac{k\,\xi_2^{\frac{3}{2}}\,\mu_1\,2i}{3m\sigma_x^0}\right)\right)^{-1}}{\xi_2^{\frac{1}{4}}}, \qquad \alpha_{33} = -\frac{\left(\exp\left(\frac{2k\,\xi_3^{\frac{3}{2}}\,\mu_2}{3m\sigma_x^0}\right)\right)^{-1}}{\xi_3^{\frac{1}{4}}},$$

where $\xi_1 = \frac{\rho^{(1)}c^2-\mu^{(1)}}{\mu^{(1)}}$, $\xi_2 = \frac{\rho^{(1)}c^2-\mu^{(1)}-hm\sigma_x^0}{\mu^{(1)}}$, $\xi_3 = \frac{\mu^{(2)}-\rho^{(2)}c^2+hm\sigma_x^0}{\mu^{(2)}}$.

2. For the case when $\sigma_x^0 = 0$, $\sigma_z^0(z) = nz$:

$$\alpha_{11} = -\frac{\left(\exp\left(\frac{k\mu_1\sqrt{\zeta_1}\,2i}{n\sigma_z^0}\right)\right)^{-1}\left(k\sqrt{\zeta_1}2i-\frac{k\zeta_1 i}{\sqrt{\zeta_1}}\right)}{\zeta_1^{\frac{1}{4}}} + \frac{n\left(\exp\left(\frac{k\mu_1\sqrt{\zeta_1}\,2i}{n\sigma_z^0}\right)\right)^{-1}\zeta_1\sigma_z^0}{4\mu_1\zeta_1^{\frac{5}{4}}},$$

$$\alpha_{12} = \frac{\exp\left(\frac{k\mu_1\sqrt{\zeta_1}\,2i}{n\sigma_z^0}\right)\left(k\sqrt{\zeta_1}2i-\frac{k\zeta_1 i}{\sqrt{\zeta_1}}\right)}{\zeta_1^{\frac{1}{4}}} + \frac{n\exp\left(\frac{k\mu_1\sqrt{\zeta_1}\,2i}{n\sigma_z^0}\right)\zeta_1\sigma_z^0}{4\mu_1\zeta_1^{\frac{5}{4}}}, \qquad \alpha_{13} = 0,$$

$$\alpha_{21} = -\frac{\left(k\sqrt{\zeta_2}2i-\frac{k\zeta_2 i}{\sqrt{\zeta_2}}\right)\mu_1}{\exp\left(\frac{k\left(\mu^{(1)}+hn\sigma_z^0\right)\sqrt{\zeta_2}\,2i}{n\sigma_z^0}\right)\zeta_2^{\frac{1}{4}}} + \frac{n\sigma_z^0\zeta_2\mu_1\left(\mu^{(1)}+hn\sigma_z^0\right)^{-1}}{4\exp\left(\frac{k\left(\mu^{(1)}+hn\sigma_z^0\right)\sqrt{\zeta_2}\,2i}{n\sigma_z^0}\right)\zeta_2^{\frac{5}{4}}},$$

$$\alpha_{22} = \left(\frac{\left(k\sqrt{\zeta_2}2i-\frac{k\zeta_2 i}{\sqrt{\zeta_2}}\right)\mu_1}{\zeta_2^{\frac{1}{4}}} - \frac{n\sigma_z^0\zeta_2\mu_1}{4\left(\mu^{(1)}+hn\sigma_z^0\right)\zeta_2^{\frac{5}{4}}}\right)\exp\left(\frac{k\left(\mu^{(1)}+hn\sigma_z^0\right)\sqrt{\zeta_2}\,2i}{n\sigma_z^0}\right),$$

$$\alpha_{23}=\frac{\left(2k\sqrt{\zeta_3}-\frac{k\zeta_3}{\sqrt{\zeta_3}}\right)\mu^{(2)}}{\exp\left(\frac{2\,k\left(\mu^{(2)}+h\,n\,\sigma_z^0\right)\sqrt{\zeta_3}}{n\,\sigma_z^0}\right)\zeta_3{}^{\frac{1}{4}}}-\frac{n\mu^{(2)}\zeta_3\sigma_z^0}{4\exp\left(\frac{2\,k\left(\mu^{(2)}+h\,n\,\sigma_z^0\right)\sqrt{\zeta_3}}{n\,\sigma_z^0}\right)\left(\mu^{(2)}+hn\sigma_z^0\right)\zeta_3{}^{\frac{5}{4}}},$$

$$\alpha_{31}=\left(\exp\left(\frac{k\left(\mu^{(1)}+hn\sigma_z^0\right)\sqrt{\zeta_2}\,2\,i}{n\sigma_z^0}\right)\zeta_2{}^{\frac{1}{4}}\right)^{-1},\qquad \alpha_{32}=\frac{\exp\left(\frac{k\left(\mu^{(1)}+h\,n\,\sigma_z^0\right)\sqrt{\zeta_2}\,2\,i}{n\,\sigma_z^0}\right)}{\zeta_2{}^{\frac{1}{4}}},$$

$$\alpha_{33}=-\left(\exp\left(\frac{2\,k\left(\mu^{(2)}+hn\sigma_z^0\right)\sqrt{\zeta_3}}{n\sigma_z^0}\right)\zeta_3{}^{\frac{1}{4}}\right)^{-1},$$

where $\zeta_1=\frac{\rho^{(1)}c^2-\mu^{(1)}}{\mu^{(1)}}$, $\zeta_2=\frac{\rho^{(1)}c^2-\mu^{(1)}}{\mu^{(1)}+hn\sigma_z^0}$, $\zeta_3=\frac{\mu^{(2)}-\rho^{(2)}c^2}{\mu^{(2)}+hn\sigma_z^0}$.

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